A Text Book of Physics

C. R. DASGUPTA

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For H. S. courses of West Bengal, Assam, Tripura and +2 Courses of Central and Delhi Board & other states and for Joint Entrance and I. I. T. admission test.

A Text Book of
PHYSICS

PART I

[Mechanics, General Properties of Matter, Heat and Sound]

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&
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'A Hand Book of Practical Physics (Pass & Hons)' etc.

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Preface to the Eighth Edition

The present Edition of the book has been thoroughly revised. I am grateful to many teachers and students for favourable response given to the book. The fact that the seventh Edition is exhausted within a short time encouraged me very much to revise the entire text of the book.

The contents of the book, including the worked out numerical problems, have been brought uptodate in the light of recent H. S., J.E.E. and I.I.T examinations. An appendix giving the methods of calculating moment of inertia of different bodies is added in accordance to the I.I.T syllabus. Harder numerical problems will enable students to prepare properly for different competitive examinations.

I hope that the present enlarged and revised edition will receive appreciation from the teachers and the taught. Suggestions for further improvement of the book will be thankfully received.

Department of Physics
City College, Calcutta
7th June, 1988

C. R. Dasgupta

Preface to the Fifth Edition

West Bengal Higher Secondary Council is going to introduce some alterations and changes in the syllabi of some subjects including Physics from 1985. Students who will be admitted in class XI in 1985 will appear in the final higher secondary examination in 1987 under this revised syllabus. In some subjects the revision of syllabus is reported to be extensive but it is not so in Physics. General Physics and Heat in paper I may be said to have undergone very little change while in the section 'Vibrations and Waves' the subject-matter has been kept almost the same, the only significant change being in the order of chapters. The present edition has been adapted according to the new syllabus which is printed at the beginning of the book.

The present edition retains other features of the previous editions which incidentally have given the book a bias towards the I.I.T-J.E.E. and other competitive examinations in addition to a comprehensive discussion of the H.S. course.

I shall feel my labours amply rewarded if the present edition receives the same patronage as before by the teachers and the students.

Department of Physics
City College, Calcutta
July, 1985

C. R. Dasgupta

Preface to the First Edition

A drastic change has again been brought about in the educational pattern of the state. Scrapping the old XI-class H. S. course, a 10-year secondary course followed by a 2-year H. S. course has been introduced. Students, who will pass the 'Madhyamik Examination' in 1976, will be admitted in class XI of the new 'plus two' H. S. course. A separate authority named 'Higher Secondary Council' has been set up for conducting the 'plus two course' in schools and colleges. The council, has in the meantime, prescribed syllabi for different subjects to be taught in the H. S. classes. The present volume has been written according to the syllabus of Physics meant for 'general stream'. The whole book is split up into two volumes—volume I for first paper and volume II for second paper.

The most important feature of this new scheme is the introduction of vocational stream. The Council has, most judiciously, made provision for those candidates who want to opt for the general stream after completing the vocational stream. For this purpose, the council has announced a 'bridge course'. The present volumes fully cover the syllabi of vocational as well as bridge course for Physics.

The author's book in Physics for the old H. S. course was widely introduced in different schools of West Bengal. It is hoped that the present volume will also be able to satisfy the expectations of the students as well as the teachers.

I take this opportunity to express my gratitude to Sri Bibhas Mitra, Head Master, Mitra Institution (Main) who took great pains in going over the entire manuscript.

As the book has been rushed through the press, some errors due to 'Printer's devil' may creep in and the author will be grateful to the readers if such errors are brought to his notice.

Department of Physics
City College, Calcutta
September, 1976

C. R. Dasgupta

REVISED SYLLABUS FOR PAPER I

1. MECHANICS

Particle dynamics : Rest and motion, reference frame, displacement, velocity and acceleration, momentum, kinematical equations (in one dimension), elementary problems, Scalars and Vectors. Composition and resolution of vectors. Representation of vector by co-ordinates. Addition and subtraction of vectors by geometrical and analytical methods. Relative velocity and acceleration. Newton's laws of motion, inertia, units of force, impulse and impulsive forces, conservation of linear momentum—elastic collisions of particles moving in the same line, jets and rockets. Friction, static and kinetic friction, coefficient of friction.

Statics : Centre of mass, Centre of gravity, conditions of equilibrium of a system of forces.

Dynamics of circular motion : Rotational motion of a particle, angular velocity, angular acceleration, relation between angular velocity, linear velocity, angular momentum, moment of a force about a point and about an axis, couple, torque, relation between angular acceleration and torque (statement only), centripetal force, centrifugal force (as a pseudo-force).

Work, energy and power : Definition of work, relevant units, work done by and against a force, Mechanical energy—kinetic and potential form. Conservation of energy —the case of a freely falling body. Power—definition, units.

2. GENERAL PROPERTIES OF MATTER

Gravitation : Newton's law of universal gravitation (statement and mathematical relation). Constant of Gravitation, Definition & value with units, (No experimental determination), Examples of gravitational attraction between heavenly bodies (e.g. the sun & the earth, the earth & the moon). Gravitational attraction of the earth, relation between G and g, Variation of g. Motion of planets & Satellites (essential points regarding the nature of the orbits should be mentioned avoiding Kepler's law), Weightlessness in orbiting satellites.

Elastic properties of matter : Definitions. Statement & explanation of the terms Stress, Strain, elastic limit, Hooke's law, elastic modulii, Young's modulus, bulk modulus, rigidity modulus, Poission's ratio. Principle of measurement of Y (No experimental details).

Hydrostatics : Density, Specific gravity (methods of determination of Sp. gr. not required), Archimedes' Principle (statement & demonstration), Pressure in fluids, Pressure inside a liquid, unit of pressure, transmission of fluid pressure, Pascal's law and Principle of multiplication of force, Hydraulic Press. Air pressure and its measurement, Torricelli's and Fortin's Barometer. Siphon, Principles of lift pump, compression pump, vacuum pump (No mathematical deduction).

3. HEAT

Thermal expansions of solids and liquids, Simple demonstrations, Coefficients of expansion of solids ; relation between them. Applications of expansions of solids, Real and apparent expansions for liquids, relation between expansion coefficients. Anomalous expansion of water. Effect on marine life.

Thermal expansion of gases. Boyle's law, Charles' law, Equation of state of an ideal gas ; Volume and pressure coefficient, Absolute scale of temperature.

Calorimetry : Preliminary definitions, principle of calorimetry (no questions on measurement to be set), Calorimetric problems.

Change of state : Latent heat (brief discussions of determination), Evaporation and boiling. Effects of pressure on melting point and boiling point. Vapour pressure. Relative humidity. Dew, fog and cloud.

Mechanical equivalent of heat : Heat as a form of energy. Relation between the calorie and the erg. Determination of mechanical equivalent of heat. First law of thermodynamics. Isothermal and adiabatic expansion of gases (brief discussion.) Specific heats of gases (elementary idea).

Kinetic theory of gases : Evidence of molecular structure of matter and of random molecular motion. Brownian movement (qualitative description). Basic assumptions of kinetic theory of ideal gases. Pressure of an ideal gas (mention of the formula ; derivation not required), concept of temperature from kinetic theory.

Transmission of heat : Conduction of heat, thermal conductivity. Convection of heat, convection current, Radiation ; radiation as a form of energy.

4. VIBRATIONS AND WAVES

Vibrations : Periodic motion ; oscillations and its characteristics, periodic time, frequency, amplitude, simple harmonic motion, relation between S. H. M., and uniform circular motion ; displacement, velocity, and acceleration of S. H. M., Energy in S. H. M. ; characteristics of S. H. M. Time-period, frequency, amplitude and phase. Graphical and mathematical representation of S.H.M. Simple pendulum, time-period of simple pendulum. Superposition of two S.H.M. of same frequency in the same direction (a) in phase (b) in opposite phase—graphical treatment. Free vibrations—longitudinal and transverse. Forced vibration, resonance. Damped vibration (qualitative discussion with example).

Waves : Elastic waves—longitudinal and transverse ; characteristics of propagating waves, nature of the medium, wavelength, amplitude of wave, time period, frequency, velocity of wave and their relation. Properties of waves ; Laws of reflection and refraction of wave with reference to sound waves ; formation of echo ; determination of depth of sea with the help of echo.

Sound Waves : Longitudinal elastic wave, velocity of sound wave ; statement and explanation of Newton's formula and Laplace's correction.

Superposition of waves : Graphical representation only (i) two S. H. waves of nearly equal frequency and amplitude and moving in the same direction (beat) ; determination of unknown frequency from beat frequency ; (ii) two S.H. waves of equal amplitude and frequency travelling in opposite direction (standing or stationary waves) ; characteristics of standing waves and comparison with progressive wave.

Transverse standing waves on stretched wire ; laws of transverse vibration of strings, fundamental, harmonic. Longitudinal standing waves in air column.

(i) Vibration of air column in a tube closed at one end. Determination of velocity of sound or the frequency of a tuning fork by resonance with vibration of air column in a tube closed at one end.

(ii) Vibration of air column in a tube open at both ends.

Effect of temperature and humidity on the frequency of air column. Qualitative discussion. Musical sound and noise—characteristics of musical sound—loudness and intensity, pitch and frequency, quality (qualitative discussion only).

Light as wave phenomenon ; finite velocity of light. Mention characteristics of wave phenomena associated with light wave (without going through details). Transverse characteristics of light wave.

CONTENTS

Introduction : (i—iv)

Physics ; The aim of study of Physics ; Subdivisions of Physics ; General characteristics of matter ; Energy and its different forms ; Conservation of energy.

Units and Measurement : (v—xxii)

1. Physical quantity 2. Units 3. Different systems of units 4. Range of variation of length and time 5. S.I. or International system of units 6. Light year 7. Dimensions of physical quantities 8. Measurement of length 9. Measurement of mass 10. Measurement of time. *Exercises.*

MECHANICS

1. Kinematics and Kinematical equations : (1—24)

Introduction : **1.1** Rest and motion **1.2** Reference frame **1.3** Translation and Rotation **1.4** Definitions of some terms in connection with translational motion **1.5** Kinematical equations of rectilinear motion **1.6** Equations of motion according to calculus. *Exercises.*

2. Scalar and Vector : (25—49)

2.1 Scalar and Vector **2.2** Representation of a scalar and a vector **2.3** A vector in geometry **2.4** Some facts about vectors **2.5** Addition of scalars **2.6** Addition of vectors **2.7** Vector addition follows commutative and associative laws **2.8** Illustration of parallelogram law **2.9** Sum of several vectors : Polygon rule **2.10** Subtraction of vectors **2.11** Multiplication of a vector by a number **2.12** Product of two vectors **2.13** Resolution of vectors **2.14** Composition of a number of vectors by resolution **2.15** Representation of a vector by co-ordinates : Position vector **2.16** Relative velocity **2.17** Calculation of relative velocity **2.18** Relative acceleration. *Exercises.*

3. Newton's laws of motion : (50—89)

3.1 Kinetics **3.2** Newton's laws of motion **3.3** Discussion of the first law **3.4** Mass of a body is a measure of its inertia **3.5** Force **3.6** Discussion of the second law **3.7** Physical independence of force **3.8** Inertial mass **3.9** Different effects of force on a body **3.10** Impulse of a force **3.11** Composition of forces **3.12** Resolution of forces **3.13** Discussion of the third law of motion **3.14** Different kinds of action and reaction **3.15** Reaction in a lift **3.16** A problem in connection with the application of the laws of motion **3.17** Law of conservation of linear momentum **3.18** Newton's third law of motion from the principle of conservation of momentum **3.19** Experimental demonstration **3.20** Inelastic and elastic collisions **3.21** Elastic collision between two particles moving in the same straight line **3.22** Co-efficient of restitution : Resilience **3.23** Inelastic collision between two spheres **3.24** Direct collision between a sphere and a fixed plane **3.25** Rocket **3.26** Jet. *Exercises.*

4. Friction : (90—106)

4.1 What is friction ? **4.2** Static friction and limiting friction **4.3** Co-efficient of friction **4.4** Laws of static friction **4.5** Angle of friction and cone of friction **4.6** Angle of repose **4.7** Acceleration down an inclined plane. **4.8** Equilibrium of a body on a rough inclined plane **4.9** Sliding or kinetic friction **4.10** Rolling friction **4.11** Fluid friction **4.12** Effect of friction and its removal. *Exercises.*

5. **Statics :** (107—131)

5.1 Introduction 5.2 Composition of any number of coplanar forces acting at a point 5.3 Rotation of a body and moment of a force 5.4 Moment of any number of coplanar forces about a point 5.5 Couple 5.6 General conditions of equilibrium of any number of coplanar forces 5.7 Equilibrium under two and three forces 5.8 Like and unlike parallel forces 5.9 Centre of mass 5.10 Position of C.M. of a number of particles 5.11 Centre of gravity 5.12 Position of C.G. of a number of coplanar particles 5.13 Difference between centre of gravity and centre of mass 5.14 Equilibrium of bodies. *Exercises.*

6. **Dynamics of rotational motion :** (132—166)

6.1 Rotational motion of a particle 6.2 Angular velocity 6.3 Relation between angular velocity and linear speed 6.4 Angular acceleration 6.5 Relation between angular and linear acceleration 6.6 Kinematical equations of linear and rotational motion 6.7 Rotation of a rigid body : Moment of inertia 6.8 Relation between the torque and the angular acceleration 6.9 Angular momentum or moment of momentum and its relation with torque 6.10 Vector representation of angular velocity, angular acceleration and angular momentum 6.11 Principle of conservation of angular momentum 6.12 Motion in a circular path ; Radial or Normal acceleration 6.13 Centripetal force 6.14 Centrifugal force 6.15 Some illustrations of centripetal force 6.16 Some effects and applications of centrifugal force 6.17 Comparison between linear motion and rotational motion. 6.18 Motion of a body in a vertical circle. *Exercises.*

7. **Work, Power and Energy :** (167—192)

7.1 Work 7.2 Different units of work 7.3 Relation between foot-poundal and erg. 7.4 Work done by a variable force 7.5 Work done by a couple 7.6 Power 7.7 Different units of power 7.8 Relation between horsepower and watt 7.9 Energy 7.10 Kinetic energy 7.11 Potential energy 7.12 Measure of potential energy of a body raised to a certain vertical height 7.13 Gravitational potential energy may be negative 7.14 Elastic potential energy of an extended spring 7.15 Distinction between energy and power 7.16 Transformation and conservation of energy 7.17 Principle of conservation of mechanical energy 7.18 Dissipation of energy 7.19 Work done against friction ; Dissipation of energy 7.20 Solar energy is the ultimate source of energy. 7.21 Mass and energy 7.22 Conservative and non-conservative forces. *Exercises.*

GENERAL PROPERTIES OF MATTER

1. **Gravitation :** (195—236)

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1.21 Weightless condition **1.22** Some terms in connection with a simple pendulum **1.23** Laws of pendulum **1.24** Experimental verification of the laws of pendulum **1.25** An interesting example of variation of time-period of a simple pendulum **1.26** Second's pendulum and Pendulum clock **1.27** Determination of the value of *g* by simple pendulum. **1.28** Drawbacks of simple pendulum **1.29** Determination of the height of a hill by a simple pendulum. *Exercises.*

2. Elasticity : (237—256)

2.1 Elasticity **2.2** Some important definitions in connection with elasticity **2.3** Force constant of a spring **2.4** Elastic behaviour of a solid : load-extension graph **2.5** Hooke's law **2.6** Different kinds of strain and modulii of elasticity **2.7** Relations among the elastic constants **2.8** Comparison between three modulii of elasticity **2.9** Work done in stretching a wire **2.10** Determination of Young's modulus in a laboratory. *Exercises.*

3. Hydrostatics : (257—273)

3.1 Density of a substance **3.2** Specific gravity **3.3** Difference between specific gravity and density **3.4** Pressure **3.5** Pressure of a liquid at a point and thrust **3.6** Calculation of pressure at a point in a liquid **3.7** Liquid, at rest, exerts pressure at a point within it in all directions with equal magnitude **3.8** Thrust exerted by a liquid on the base of a vessel depends on the area of the base and the height of the liquid **3.9** Equilibrium of two liquids in a U-tube **3.10** Total lateral thrust on a surface immersed vertically in a liquid **3.11** Pascal's law for the transmission of liquid pressure **3.12** Principle of multiplication of thrust from Pascal's law **3.13** Hydraulic press **3.14** Mechanical advantage, velocity ratio and the efficiency of a hydraulic press **3.15** Conservation of energy in the hydraulic press. *Exercises.*

4. Floating bodies and Archimedes' Principle : (274—296)

4.1 Calculation of resultant thrust on a body immersed in a liquid **4.2** Apparent loss of weight of a body immersed in a liquid **4.3** Experiment to demonstrate the apparent loss of weight of a body **4.4** Archimedes' principle **4.5** Experimental verification of Archimedes' principle **4.6** Applications of Archimedes' principle **4.7** Floatation and immersion of a body **4.8** Condition of equilibrium of floating bodies **4.9** Stability of a floating body **4.10** A floating body displaces liquid whose weight is same as the weight of the body **4.11** Two important facts in connection with a floating body **4.12** Some cases of floatation **4.13** Buoyancy of air and application of Archimedes' principle to gases **4.14** Buoyancy correction of weighing in a common balance. *Exercises.*

5. Atmospheric pressure and Air pressure pumps : (297—320)

5.1 Atmospheric pressure **5.2** Experiments to demonstrate the existence of atmospheric pressure **5.3** Barometer **5.4** Magnitude of atmospheric pressure **5.5** Normal or Standard atmospheric pressure **5.6** Height of liquid column in different barometers **5.7** Advantages and disadvantages of mercury, water and glycerine as barometric liquids **5.8** Height of the homogeneous atmosphere **5.9** Weather forecasting ; effect of water-vapour on atomspheric pressure **5.10** Determination of height by a barometer **5.11** Measurement of gas pressure by a manometer **5.12** Suction or Common pump **5.13** Lift pump **5.14** The siphon **5.15** The exhaust pump or the air pump **5.16** Rotary pump **5.17** Air condensing or compression pump **5.18** Applications of air compression pump. *Exercises.*

6. **Surface tension and Viscosity :** (321—334)

6.1 Surface tension 6.2 Surface energy 6.3 Factors influencing surface tension of a liquid 6.4 Some phenomena in connection with surface tension 6.5 Pressure difference in a curved liquid surface 6.6 Angle of contact 6.7 Capillarity or Capillary action 6.8 Measurement of surface tension from capillary rise of liquid 6.9 Simple molecular explanation of surface tension 6.10 Motion of a liquid 6.11 Streamlining 6.12 Viscosity 6.13 Coefficient of viscosity 6.14 Critical velocity and Reynold's number 6.15 Flow of a liquid through a narrow tube : Poiseuille's equation 6.16 Motion of a body through viscous medium ; Stokes' law. *Exercises.*

HEAT

1. **Heat and Temperature :** (337—346)

1.1 Heat 1.2 Nature of heat 1.3 Temperature 1.4 Difference between heat and temperature 1.5 Thermal equilibrium ; the Zeroth law of thermodynamics 1.6 Thermometers 1.7 Mercury thermometers 1.8 Thermometric scales 1.9 Alcohol thermometer 1.10 Comparison between mercury and alcohol as thermometric liquid 1.11 Range of temperature. *Exercises.*

2. **Calorimetry :** (347—365)

2.1 Calorimetry 2.2 Units of measurement of heat 2.3 Relation between calorie and Btu. 2.4 Specific heat 2.5 Definition of specific heat 2.6 Amount of heat either absorbed or given out by a body for a rise or a fall of temperature 2.7 Thermal capacity of a body 2.8 Water equivalent of a body 2.9 Difference between thermal capacity and water equivalent 2.10 Principle of calorimetry 2.11 Determination of water equivalent of a calorimeter 2.12 Determination of specific heat of a solid by the method of mixtures 2.13 Determination of specific heat of a liquid by the method of mixtures 2.14 Dulong and Petit's law: Atomic heat 2.15 Application of calorimetry in the measurement of high temperature 2.16 Effects of high specific heat of water . 2.17 Calorific values or heating values of fuels. *Exercises.*

3. **Expansion of solids :** (366—390)

3.1 Expansion of solid 3.2 Different materials expand differently 3.3 Enormous force is developed due to expansion or contraction 3.4 Coefficient of linear expansion of solids 3.5 Coefficient of surface or superficial expansion 3.6 Coefficient of volume or cubical expansion 3.7 Relation between the three coefficients of expansion 3.8 Determination of coefficient of linear expansion of a solid 3.9 Practical applications of the expansion of the solids 3.10 Differential expansion 3.11 Compensated pendulum 3.12 Compensated balance wheel of a watch 3.13 Graham's mercurial pendulum 3.14 Thermal stress. 3.15 Volume expansion of hollow vessel 3.16 Change of density of a solid due to change of temperature. *Exercises.*

4. **Expansion of liquids :** (391—409)

4.1 Expansion of liquids 4.2 Coefficient of apparent expansion of a liquid 4.3 Coefficient of real expansion of a liquid 4.4 Relation between the coefficients of apparent and real expansion 4.5 Determination of coefficient of apparent expansion of a liquid 4.6 Relation between density and coefficient of real expansion of liquid 4.7 Apparent weight of a body immersed in a liquid at different temperatures 4.8 Determination of coefficient of real expansion of liquid by Dulong and Petit's method 4.9 Temperature correction of barometer reading

4.10 Anomalous expansion of water 4.11 Experimental study of anomalous behaviour of water 4.12 Hope's experiment to demonstrate the maximum density of water at 4°C 4.13 Consequence of anomalous expansion of water 4.14 A problem in connection with the volume of water. *Exercises.*

5. **Expansion of gases :** **(410—432)**

5.1 Introduction **5.2** Effects of pressure and temperature on the expansion of gas ; Gas laws **5.3** Experimental verification of Boyle's law **5.4** Absolute scale of temperature **5.5** Alternative form of Charles' law and pressure law **5.6** Combination of Charles' and Boyle's laws **5.7** Dalton's law of partial pressure **5.8** The ideal gas equation **5.9** Magnitude of universal gas constant **5.10** Magnitude of ordinary gas constant **5.11** Relation between the pressure, temperature and density of a gas **5.12** Coefficient of expansion of gases **5.13** Reason for taking initial volume at 0°C in calculating the volume coefficient of a gas **5.14** Relation between the coefficients of expansion of a perfect gas **5.15** Experimental determination of the two coefficients of expansion of gas. *Exercises.*

6. **Change of state :** **(433—477)**

6.1 Introduction **6.2** Melting and solidification **6.3** Melting point and freezing point of a substance **6.4** Supercooling **6.5** Latent heat **6.6** Latent heat of fusion **6.7** Determination of latent heat of fusion of ice **6.8** Change of volume during melting and solidification **6.9** Effects of pressure on melting point **6.10** Regelation **6.11** Freezing point of a solution **6.12** Laws of fusion **6.13** Vapour and vaporisation **6.14** Different ways of vaporisation **6.15** Difference between evaporation and boiling **6.16** Factors influencing rate of evaporation **6.17** Cold caused by evaporation **6.18** Boiling of liquid **6.19** Definition of boiling point of a liquid **6.20** At the boiling point, the vapour pressure of a liquid is equal to the superincumbent pressure **6.21** Similarity between fusion and boiling **6.22** Approximate determination of height by observation of boiling points (Hypsometry) **6.23** Latent heat of vaporisation **6.24** Method for determining the latent heat of steam **6.25** Graphical representation of the steps by which ice is heated through fusion and vaporisation **6.26** Effects of pressure on boiling point **6.27** Factors influencing the boiling point of a liquid **6.28** Laws of ebullition **6.29** Presence of water vapour in the atmosphere **6.30** Saturated and unsaturated vapour **6.31** Difference between saturated and unsaturated vapours **6.32** Triple point **6.33** Dew point **6.34** Humidity and relative humidity **6.35** Effect of relative humidity on daily life **6.36** Hygrometers **6.37** Mass of wet air **6.38** Condensation of water-vapour present in atmosphere. *Exercises.*

7. **Mechanical equivalent of Heat :** **(478—499)**

7.1 Heat is a form of energy **7.2** Internal energy of a body **7.3** Work done by a gas **7.4** Internal energy and latent heat **7.5** First law of thermodynamics and mechanical equivalent of heat **7.6** Definition and units of J **7.7** The value of J **7.8** Specific heat of gases **7.9** Cp is greater than Cv **7.10** Importance of the ratio of two specific heats of a gas **7.11** Isothermal process **7.12** Adiabatic process **7.13** Adiabatic relations for a perfect gas **7.14** Slopes of isothermal and adiabatic curves. *Exercises.*

8. **Kinetic theory of Gases :** **(500—512)**

8.1 Molecular structure of matter **8.2** Brownian motion **8.3** Basic assumptions of kinetic theory of gases **8.4** Pressure of a gas according to kinetic theory **8.5** Root mean square velocity or R.M.S velocity **8.6** Concept of temperature according to kinetic theory **8.7** Variation of r.m.s. speed **8.8** Some important deductions from the kinetic theory of gases **8.9** Degrees of freedom and the law of equipartition of energy **8.10** Ratio of two specific heats of a gas **8.11** Limitation of the ideal gas equation. *Exercises.*

9. **Transmission of Heat :** (513—550)

9.1 Different ways of transmission of heat. CONDUCTION—**9.2** Thermal conductivity and coefficient of thermal conductivity **9.3** Temperature variation with distance in a lagged and unlagged bar **9.4** Thermal resistance and thermal resistivity **9.5** Determination of thermal conductivity of metals by Searle's apparatus **9.6** Comparison of conductivities of different substances **9.7** Conduction of heat through a composite wall **9.8** Conduction of heat through a slab of varying thickness ; Freezing of lake **9.9** Variable state, steady state and thermal diffusion **9.10** Different substances have different diffusivity **9.11** Good and bad conductors and their uses **9.12** Experiment to show low conductivity of water and comparatively high conductivity of mercury **9.13** Some practical illustrations of conduction of heat CONVECTION—**9·14** Some experiments of convection of heat **9.15** Explanation of convection current **9.16** Practical applications of convection of heat for different purposes and in nature **9.17** Convective equilibrium of the atmosphere RADIATION—**9.18** Properties of radiant heat **9.19** Nature of radiant heat **9.20** Instruments for detection and measurement of radiant heat **9.21** Some important illustrations in connection with radiation and absorption **9.22** Emission and absorption of radiation **9.23** Good radiators are good absorbers ; Ritchie's experiment **9.24** Thermosflask **9.25** Black body **9.26** Kirchoff's law **9.27** Prevost's theory of exchange **9.28** Stefan's law **9.29** Newton's law of cooling from Stefan-Boltzmann law **9.30** Application of the law. *Exercises.*

SOUND

1. **Simple Harmonic Motion :** (553—578)

1.1 Periodic motion : Definition **1.2** Oscillation or vibration **1.3** Simple harmonic motion : Definition **1.4** Relation between S.H.M. and circular motion **1.5** Some important terms in connection with S.H.M. **1.6** Displacement, velocity, acceleration etc. of a particle executing S.H.M. **1.7** Time-period and frequency of S.H.M. **1.8** Illustrations of simple harmonic motion **1.9** Graphical representation of S.H.M. **1.10** Mathematical equation of S.H.M. **1.11** Energy in a simple harmonic motion **1.12** Composition of two collinear S.H.M.'s by graphical method *Exercises.*

2. **Vibrations :** (579—586)

2.1 Nature of vibrations **2.2** Free and Forced vibrations **2.3** Resonance **2.4** Damped oscillation **2.5** Sharpness of resonance. *Exercises.*

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"

Nature,
and Nature's laws,
lay hid in night ;
God said,
Let Newton be !
and all was light.

"

ALEXANDER POPE

INTRODUCTION

Physics :

This earth is full of matters. When we look around us, we see a great variety of substances. A *body* or a *substance* is that which may be perceived by one or more of our senses and has some weight. Chair, table, paper, pen, tree and such other things are illustrations of different bodies or substances. Some of these substances are solid, some liquid and some gaseous. Some, again, have good odour, some are odourless and some have bad odour. Many of them have colours, some may be colourless. How are these varieties of bodies formed ? How do they behave ? What are their structures ? These and such other numerous questions have troubled the mind of man from the very dawn of civilisation. The inquisitive mind of man has since then, been trying to find answers to those questions. The information gathered by them has given rise to different branches of science.

The material of which a body is made is called *matter*. For example, wood is a kind of material but a chair made of wood is a body.

Besides matter, attention of man was also drawn to another important thing—viz, *Energy*. *Capacity for doing work is called energy*. The universe exists because energy exists. Without energy, the universe will come to a standstill. Our familiarity with different forms of energy grows through its association with material bodies. For example, heat is a form of energy. But unlike material bodies heat has no shape, colour or odour. To realise the existence of heat, we are to observe the physical change which heat brings about in a body. Absorbing heat, a body changes its temperature, increases in length or in volume. In the same way, electricity is another form of energy. To realise its existence, we are to observe the effects when electricity flows through a body. When we touch a current-carrying wire or look at it, we do not feel the existence of electricity. But when electric current goes to a fan, the fan starts whirling and then we realise the existence of it. So material bodies are essential for realising the existence of energy.

Definition : *Anything that is perceptible to our senses and has weight as well as free existence, is called a body. The material of which a body is made is called matter. Capacity of a body for doing work is called its energy.*

The aim of study of Physics :

The simple aim of study of physics is to unfold secrets of nature and to explain the natural phenomenon that gave rise to many legends in the past. Poor *Rahu* to-day has nothing to devour to produce a solar eclipse. We are satisfied now that a solar eclipse is nothing but the formation of shadow with the sun as the source of light and the moon as the obstacle with its position in between the sun and the earth. *Indradeva* no longer hurls thunder at the demon *Britra* to produce lightning and thunder. Physics has made us believe that this phenomenon is nothing but a discharge of electricity between neighbouring charged clouds etc.

Thus we see how the study of physics aims at explaining natural phenomena.

Subdivisions of Physics : Long ago, all natural sciences like chemistry, astronomy, zoology etc. were included in Physics. Since the middle of nineteenth century, nature seems to unfold her secrets with vengeance and the knowledge and information about each branch of natural science became so vast, that separation of the information into different independent branches was urgently felt. The science of Physics now mainly deals with matter and energy. For convenience of study the whole science of Physics is divided into the following six subdivisions :

(1) *General Physics* (2) *Sound* (3) *Heat* (4) *Light* (5) *Magnetism* and (6) *Electricity*.

General characteristics of matter :

Matter, in whatever state it may exist, possesses some general characteristics, such as,—

(1) **Gravitational or universal attraction** : This means that there exists an attraction between any two particles in the universe. There exists an attraction between the sun and the earth which makes the earth move round the sun. The falling of a fruit on the ground when released from its stalk shows that the earth attracts the fruit. High and ebb tides are due to the attraction of the sun and the moon on the sea-water. The weight of a body is due to this attraction of the earth on the body.

(2) **Extension** : It means that every material body occupies some space. The space occupied by a body is called its *volume*. Every body—solid, liquid or gas has its own volume.

(3) **Impenetrability** : It means that two pieces of matter cannot simultaneously occupy the same space. When a nail is driven into a wall, apparently it looks like the penetration of nail into the materials of the wall. As a matter of fact, when the nail enters, the mortars of the wall move aside yielding space for the nail to enter.

(4) **Divisibility** : A piece of matter can be subdivided into small parts. This is known as divisibility. For example, if we write with a piece of chalk on a black-board, the piece will be broken into smaller parts.

We can carry out the division and the subdivision of a piece of matter to a limit after which no further subdivision retaining the characteristics of the original substance is possible. At this limit, the subdivided particles are called *molecules*. Thus, molecules are the finest constituent particles of a substance that can exist by themselves exhibiting the characteristics of the substance.

When molecules are subdivided, we arrive at what we commonly call, the ultimate point of division. These ultimate subdivided particles are called *atoms*. According to Dalton, atoms are considered to be the ultimate point of division of elements.

Modern physics however, states that an atom is a much complicated composite particle which admits of further subdivisions into elementary particles like electrons, protons and neutrons.

(5) **Cohesion and Adhesion :** The attraction between the molecules of the same substance is known as *cohesion.* Due to cohesion a solid substance retains its shape and size. In the case of liquid, the cohesion is much less, for which liquid has no shape of its own. The cohesion is extremely small in gases.

The attraction between the molecules of different substances is called *adhesion.* Soldering is a very good example of adhesion. The molecules of the soldering material keep the soldered parts rigidly connected due to the existence of a strong force of adhesion. Water particles stick to a glass vessel due to the force of adhesion.

(6) **Porosity :** Material bodies possess pores both sensible and insensible. A piece of blotting paper can absorb ink because, ink enters into the pores of the blotting paper. In the same way, a piece of brick, chamois leather, charcoal etc. possess sensible pores. In some cases, the pores are too small to be sensible. Such insensible pores are known as inter-molecular pores. When sugar is dissolved in water, sugar particles enter into the small spaces existing between the molecules of water.

(7) **Inertia :** A material body has the innate tendency to remain in a state in which it is originally placed. This is known as *inertia* of a material body. *The mass of a body is a measure of its inertia.*

(8) **Elasticity :** If a piece of rubber be pressed between two fingers, the shape of the piece changes but it regains the shape as soon as the pressure is withdrawn. This property of rubber is known as its *elasticity.* It is present in different degrees in all material bodies.

Energy and its different forms :

It has already been mentioned that the capacity of a body for doing work is known as its energy. Energy is generally divided into seven different forms.

(1) *Mechanical energy* (2) *Heat energy* (3) *Light energy* (4) *Sound energy* (5) *Magnetic energy* (6) *Electric energy* and (7) *Chemical energy.*

Transformation of energy : The different forms of energy, as stated above, are inter-related *i.e.* one form can be transformed into another. As a matter of fact, almost all natural phenomena are the examples of such transformation of energy. Below are given some illustrations of it.

Water flows from higher level to lower level. At the higher level, water possesses potential energy which is gradually transformed into kinetic energy as it flows down to the lower level. This kinetic energy of water may again be transformed into electrical energy.

When electric current flows through the filament of an electric bulb, we get light. Here, electric energy is transformed into light energy.

In steam engine, superheated steam is used to run the locomotives. Here, heat energy of the steam is transformed into mechanical energy.

In this way, numerous examples can be cited illustrating the transformation of energy.

Conservation of energy : When energy, in one form, is transformed into another, no energy is lost. The amount of energy lost by a body during transformation is always equal to the amount of energy gained by the other. Indeed, *we can not create any new energy nor can we destroy any.* Scientists are of opinion that the total amount of energy that existed when this universe was created, is equal to the amount that exists now. This is known as the *principle of conservation of energy.*

UNITS AND MEASUREMENT

1. **Physical quantity :** That which is measurable is generally termed as a quantity. In Physics, we come across innumerable quantities of different kinds, such as, mass, length, velocity, acceleration, force, potential etc. These are called physical quantities.

2. **Units :** For expressing the magnitude of a quantity we require some standard reference which is called the '*unit*' of measurement for that quantity.

Thus, when we say that the room is 20 ft. long, the length is a physical quantity and 'foot' is taken as a unit of measurement of length. Similarly, if we say that Bombay mail has taken 48 hours to reach Calcutta, hour is taken as a unit for the measurement of 'time' which is a physical quantity. In this way, it is found that each quantity requires a unit for its measurement. But there are thousands of quantities in Physics. Do they have thousands of units ? Fortunately, it is found that although the quantities are numerous in number, the units are not. If we fix up units of only three quantities, the units of almost all other quantities may be formed from them. These three quantities are (i) **Length** (ii) **Mass** and (iii) **Time.** The units of these three quantities do not depend on one another. They are known as **Fundamental units.** Units of other quantities which can be formed from one or more of the fundamental units are called **Derived units.** Units of area, volume, force, acceleration etc. are all derived units.

Definition : The units of three quantities, which are independent of one another and from which the units of other quantities may be derived are called fundamental units.

Those units which are derivable from one or more of the fundamental units, are known as derived units.

3. **Different systems of units :**

There are, in general, three systems to express the above-mentioned three fundamental units : (1) *C.G.S.* or *French* or *Metric system* (2) *F.P.S.* or *British system* and (3) *M.K.S. system.*

(i) **C.G.S. System :** In this system, the three fundamental quantities—length, mass and time—are measured in the units of *centimetre* (C), *gramme* (G) and *second* (S) respectively.

Centimetre : The distance between two fine lines on a platinum iridium bar (called the *standard metre*) kept at 0°C at the international office of weights and measures in France is taken as a *Metre.* Because the standard metre was not very accessible, accurate master copies of it were made and sent to standardising laboratories throughout the world. *Centimetre* is $\frac{1}{100}$th part of a metre. Originally, the metre was intended to be one ten-millionth (10^{-7}) of the distance along the meridian from the equator through Paris, to the North pole. One of the advantages of the C.G.S. system is that the units of small and large lengths are related to the metre by either multiples or sub-multiples of 10 as will be evident from the table below :

10 Millimetres (mm)=1 Centimetre (cm)
10 Centimetres =1 Decimetre
10 Decimetres =1 Metre (m)
10 Metres =1 Decametre
10 Decametres =1 Hectometre
10 Hectometres =1 Kilometres (Km)

Gramme : The mass of a certain cylindrical peice of platinum-iridium alloy kept in Paris is called a standard *Kilogram* which is known as the International Prototype Kilogram. Gram is $\frac{1}{1000}$th part of a kilogram. Usually, the mass of 1 cubic centimetre (c.c.) of water at 4°C is taken as 1 gramme.

Second : Due to rotation of the earth about its own axis, the sun appears to travel across the sky from east to west. The interval between two successive crossings of the meridian at a place by the sun is called a *solar day*. For some reasons, the length of the solar day does not remain constant throughout the year but changes slowly. The average taken over a year is known as *mean solar day*. $\frac{1}{24}$th part of a mean solar day is an hour. $\frac{1}{60}$th part of an hour is a minute, while $\frac{1}{60}$th part of a minute is called a second. In other words.

$$1 \text{ Second} = \frac{1}{60\times60\times24} = \frac{1}{86400}\text{th part of a mean solar day.}$$

Sidereal day : It is the time interval between two successive passages of a fixed distant star across the meridian at a place. It is also equal to the time taken by the earth to make one complete rotation about its own axis. Sidereal day is, however, shorter than the solar day by about 4 minutes. It is generally used in Astronomy.

(ii) **F.P.S. system :** In this system, the three fundamental units are (a) *foot* (b) *pound* and (c) *second.*

Foot : It is the unit of length in the F.P.S. system. It is one third of the length of what we call a *standard yard.* A standard yard is defined as the distance between the centres of two lines on two gold plugs at 62°F, kept at the office of the British Exchequer.

For the measurement of smaller lengths, subdivisions of foot are not much in practical use except inches and *sutas.*

1 yard=3 ft.; 1 ft.=12 inches ; 1 inch=8 *sutas.*

For the measurement of longer distance, two practical units are in common use :—

1 mile=1760 yd.=1760×3=5280 ft.
1 Furlong=$\frac{1}{8}$th of a mile=220 yds.

The students are advised to keep in memory the following conservation table :

1 inch =2·54 cm.	1 cm=0·394 inch
1 foot =30·48 cm.	1 metre=39·37 inch
1 mile =1·609 km.	1 kilometre=0·621 mile

Pound : It is the unit of mass in F.P.S. system. It is the mass of a piece of platinum kept at the standards office at Westminster, London. This is marked as 'P.S. 1844', 1 lb and is called the Imperial Avoirdupois pound. The other units of mass, in use, in the F.P.S. system are as follows :

16 drums=1 ounce (oz)
16 ounces =1 pound (lb)
28 pounds=1 quarter
4 quarters=1 hundred-weight (cwt)
20 hundred-weight=1 ton.

Remember the following conversion table :

1 lb=453·6 gm. 1 kilogram=2·205 lb.

Second : 'Second' is also the unit of time in the F.P.S. system.

(iii) **M.K.S. System** : In this system, the three fundamental units are (a) *Metre* (b) *Kilogram* and (c) *Second.*

4. Range of variation of length and time :

The range over which the magnitudes of length and time can vary, is indeed enormous. In some cases like the atomic and sub-atomic dimensions, the length to be measured is very very minute, while in some other cases like the distances between heavenly bodies, it is so very big that human mind finds it difficult to comprehend such distances. In the same way, we are sometimes faced with the problem of measuring an extremely small interval of time like the period of revolution of an electron round the nucleus of an atom while, on the other extreme case, we are sometimes called upon to measure enormous lengths of time like the age of the earth or the half-life of uranium etc. In the following table, the orders of magnitude of length and time over which measurements can be done have been shown. These values will give an idea as to the enormous range over which these two quantities can vary.

Lengths	Magnitude in metres	Time intervals	Magnitude in sec.
Distance to the great nebula in Andromeda (nearest galaxy)	10^{22}	Age of the earth	10^{17}
Diameter of our galaxy	10^{20}	Time elapsed since the man appeared on the earth	10^{13}
Distance to the nearest star Alpha centauri	10^{17}	Time of earth's rotation round the sun	10^{7}
Radius of the sun	10^{9}	Time taken by light to reach the earth from the sun	10^{3}
Mean distance between the earth and the moon	10^{8}	Half life of a muon	10^{-3}
Radius of the earth	10^{7}	Time for electron to move round the proton in H_2- atom	10^{-15}
Thickness of a sheet of paper	10^{-4}		
Diameter of hydrogen atom	10^{-10}		
Diameter of a proton	10^{-15}		

5. SI or International system of units :

Different systems of units are found to be prevalent in different countries. For example, countries of erstwhile British empire use F.P.S. system, France and some other countries use C.G.S. system, some again use M.K.S. system. To remove the anomalies in measurement existing in different countries, the question of acceptance of an international system of units was the subject of discussion among the scientists for some time past. International scientific societies of different countries recommended in 1960, an international system of units of measurement. This system has been prepared by adding some standard units with those of M.K.S. system. It is known as *SI* (System international d' unites) system. Six basic units of *SI* system with their symbols are given below :

Quantities	Units	Symbols	Quantities	Units	Symbols
1. Length	Metre	m	4. Electric current	Ampere	A
2. Mass	Kilogram	Kg	5. Temperature	Degree Kelvin	°K
3. Time	Second	S	6. Illuminating Power	Candela	cd

In the table below are given *SI* units of some other derived units along with their symbols :

Quantities	Units	Symbols	Formula in fundamental units
1. Energy	Joule	J	Kgm^2S^{-2}
2. Force	Newton	N	$KgmS^{-2}$
3. Power	Watt	W	Kgm^2s^{-3}
4. Electric charge	Coulomb	C	As
5. ,, potential	Volt	V	$Kgm^2s^{-3}A^{-1}$
6. ,, resistance	Ohm	Ω	$Kgm^2s^{-3}A^{-2}$
7. Frequency	Hertz	Hz	s^{-1}

6. **Light year :** It is a special unit of length used to denote the enormous distances between various astronomical bodies. These astronomical distances are so huge that if expressed in miles, they would yield gigantic figures. They are, therefore, expressed in terms of light-years.

Light-year means the distance that light travels in one year. It has been ascertained that light can travel a distance of about 186000 miles in 1 sec. or 3×10^5 kilometres in 1 sec.

Hence, 1 light-year$=3 \times 10^5 \times 365 \times 24 \times 60 \times 60 = 9{\cdot}45 \times 10^{12}$ km.

Also, 1 light-year$=186000 \times 365 \times 24 \times 60 \times 60 = 5{\cdot}86 \times 10^{12}$ miles

7. **Dimensions of physical quantities :**

We have already learnt that derived units can be formed from one or more than one fundamental unit. So the value of the derived unit depends on the values of the fundamental unit or units from which it is derived. The units of length, mass and time are represented by [L], [M], and [T] respectively. They represent the nature of the unit and not the magnitude.

Now, the unit of *area* which is the product of two unit lengths, is represented by [L]×[L] or [L²]. Here, area is said to be of *two dimensions* in length. As area does not depend on the units of mass and time, its dimension is *zero* in each of them and so its unit is represented in respect of dimension by $M^0L^2T^0$. This indicates that the magnitude of the unit of area is independent of the units of mass and time and is proportional to the *square* of the unit of length used. Similarly, the *volume*, being a product of three units of lengths and independent of mass and time, is represented dimensionally by $M^0L^3T^0$.

Definition : *By dimension of a physical quantity, we mean the powers to which the fundamental unit or units need be raised to obtain the derived unit of the quantity.*

Dimensional formula of some important quantities :

The dimensional formula of a physical quantity means an expression which shows which of the fundamental units have been used and in what way to form the derived unit of that physical quantity. We shall discuss below dimensional formulae of some important quantities.

(i) $$\text{Velocity}=\frac{\text{Distance}}{\text{time}}=\frac{[L]}{[T]}=[LT^{-1}]$$

(ii) $$\text{Acceleration}=\frac{\text{Velocity}}{\text{Time}}=\frac{[LT^{-1}]}{[T]}=[LT^{-2}]$$

(iii) $$\text{Force}=\text{mass}\times\text{acceleration}=[M]\times[LT^{-2}]=[MLT^{-2}]$$

(iv) $$\text{Pressure}=\frac{\text{Force}}{\text{Area}}=\frac{[MLT^{-2}]}{[L^2]}=[ML^{-1}T^{-2}]$$

(v) $$\text{Work}=\text{Force}\times\text{Distance}=[MLT^{-2}]\times[L]=[ML^2T^{-2}]$$

(vi) $$\text{Density}=\frac{\text{Mass}}{\text{Volume}}=\frac{[M]}{[L^3]}=[ML^{-3}]$$

(vii) $$\text{Longitudinal strain}=\frac{\text{Change of length}}{\text{Original length}}=\frac{[L]}{[L]}=\text{a pure number.}$$

(viii) $$\text{Young's modulus}=\frac{\text{Stress}}{\text{Strain}}=\frac{\text{Force}}{\text{Area}}=\frac{[MLT^{-2}]}{[L^2]}=[ML^{-1}T^{-2}]$$

[Strain being a ratio has no dimension]

(ix) $$\text{Energy}=\text{Work done}=[ML^2T^{-2}]$$

Dimensional equations and their uses :

When the dimension of a quantity is found and expressed in the form of an equation, the equation is called the *dimensional equation*. Thus the dimension of force is $[MLT^{-2}]$ and $[F]=[MLT^{-2}]$ is the dimensional equation of force. The following are the chief uses of dimensional equation :

(a) To convert one system of units to another.
(b) To check equations established from theoretical consideration.
(c) To derive correct equation connecting some physical quantities.

MEASUREMENT OF LENGTH, MASS AND TIME

8. **Measurement of length** : For ordinary measurement of length we use a scale. It is a properly seasoned thin rectangular bar of box-wood of length

one metre. On one edge of the surface there is a scale graduated in centimetres

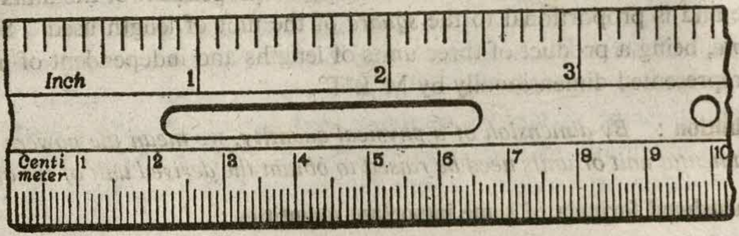


Fig. 1

subdivided to millimetres and on the other edge the scale is graduated in inches subdivided to tenths of an inch as shown in fig. 1.

Let us suppose that we are to measure the length of a straight line *AB*. The scale should be held along the line *AB* with graduated surface vertical. The end *A* should be on a complete division of the scale. In the fig. 2, the end *A* coincides

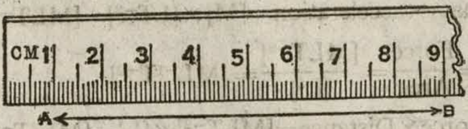


Fig. 2

with 1 cm. mark. The other end *B*, is between 8·9 and 9 cm. The excess part, here, according to eye-estimation, may be taken as 0·5 mm. So, the reading for the *B*-end is 8·95 cm. Then,

$$\text{the length } AB = B\text{-end reading} - A\text{-end reading}$$
$$= 8{\cdot}95 - 1 = 7{\cdot}95 \text{ cm.}$$

Now, we should not rest content with one set of reading. It is better to get an average value by making use of different parts of the scale for similar observations.

The eye-estimation as applied in the case of an ordinary scale reading involves error of the order of fraction of a millimetre. Hence, for an accurate measurement of length, an ordinary metre-scale is not a convenient appliance.

Vernier : This appliance was originally invented by a French mathematician named P. Vernier. It enables us to measure length with greater accuracy than eye-estimation. Fig. 3 shows a model to explain the underlying principle.

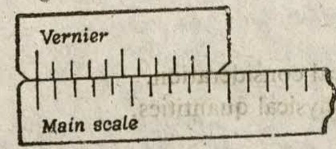


Fig. 3

The vernier scale which is a small auxiliary scale capable of sliding along the edge of the main or principal scale, is ordinarily divided into 10 equal divisions, the total length of which is equal to 9 divisions of the main scale. Each division of the main scale, being 1 mm., 10 divisions of vernier scale=9 mm. or 1 div. of the vernier scale$=\frac{9}{10}$ mm.

Vernier constant : It is the difference between one division of the main scale and one division of the vernier scale. In the present case, the vernier constant is,

$$\left(1-\frac{9}{10}\right)\text{ mm}=\frac{1}{10}\text{ mm}=0{\cdot}1\text{ mm}=0{\cdot}01\text{ cm}.$$

This is the smallest measurable distance with the help of above vernier. In other words, the above vernier is capable of giving correct readings upto $\frac{1}{100}$th of a centimetre.

General formula :

Suppose, m vernier divisions $=(m-1)$ smallest divisions of the main scale

or, 1 ,, division $=\dfrac{(m-1)}{m}$,, ,, ,, ,,

$$\therefore\ V.C.=\left(1-\frac{m-1}{m}\right)\times\text{value of the smallest div. of the main scale.}$$

$$=\frac{1}{m}\times\text{value of the smallest div. of the main scale.}$$

Use of the Vernier scale : Suppose, the length of a thin wire AB is to be measured by a vernier scale. Place the scale in such a way that the 0-mark of the main scale coincides with the left-end A of the wire. Slide the vernier so that its first line, known as the *pointer* may just touch the other end B of the wire. [Fig. 4].

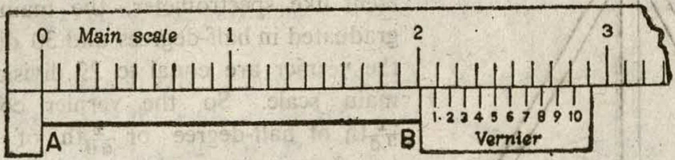


Fig. 4

In the figure, the pointer has just crossed the 2 cm. mark. In this case, the main scale reading is 2 cm. Now, see that the mark 5 of the vernier is in one line with a mark of the main scale graduation. Hence, the reading for the fractional part in the difference between 5 divisions of the main scale and 5 divisions of the vernier scale *i.e.* $0{\cdot}1\times5=0{\cdot}5$ mm $=0{\cdot}05$ cm.

Thus, the length of the wire $=2+0{\cdot}05=2{\cdot}05$ cm.

Vernier Calipers or Slide Calipers : It consists of a steel plate or bar, sometimes nickel-plated, having a scale engraved on it. In metric calipers, the graduations are in centimetres and millimetres and in more precision instrument, in half millimetres. This is called the main-scale of the calipers. There is another piece which carries a vernier. It can slide along the main scale. In the common type of instrument, 10 divisions of the vernier scale are equal to 9 divisions of the main scale, each division of the main scale being equal to 1 mm. Thus, the *V.C.* of the instrument is 0·1 mm. or 0·01 cm.

The article to be measured is placed between the two jaws of the calipers. The sliding piece containing the vernier is slowly moved along the bar by turning the slow-motion screw till it just touches one end of the article, the other end being in contact with the fixed jaw *A* as shown in fig. 5.

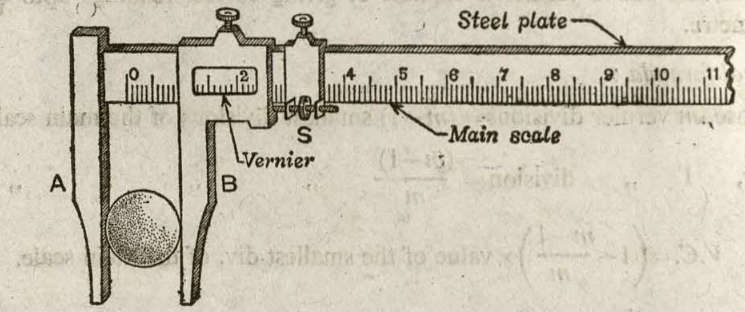


Fig. 5

The length of the article is measured from the position of the pointer (*i.e.* the first line of the vernier) on the main scale and noting the particular mark of the vernier scale coinciding with a certain mark of the main scale. The calculation is done in the usual way as described earlier.

Angular Vernier : This type of vernier is generally found in optical instruments like spectrometers, theodolite, sextants etc. In common type of instrument like spectrometer, the main scale is graduated in half-degrees and 30 divisions of the vernier are equal to 29 divisions of the main scale. So the vernier constant is $\frac{1}{30}$th of half-degree or $\frac{1}{60}$th of a degree *i.e.* 1 minute. This angular vernier is, therefore, capable of measuring an angle upto the smallest minute. The method of measurement is same as that of a straight vernier.

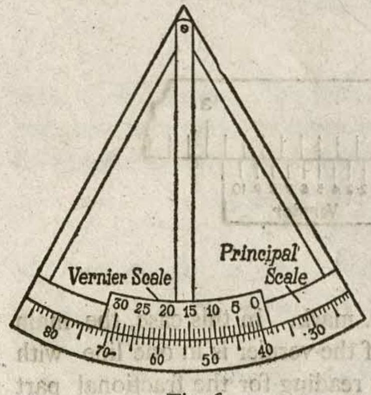


Fig. 6

The screw gauge or micrometer screw :

When greater accuracy in the measurement of small lengths like the diameter of a thin wire, the thickness of a thin sheet etc., is required, screw gauge is used.

The instrument (Fig. 7) consists of the screw *AB* passing through a barrel on which is engraved a scale divided ordinarily in millimetres. The screw *AB* is cut in such a way that one complete rotation of it produces a linear

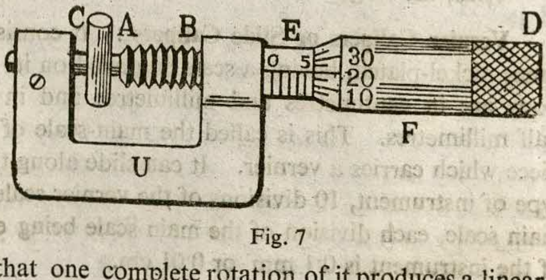


Fig. 7

motion of the screw equal to 1 mm. of the scale. This is called the *pitch* of the instrument. Here, the pitch is 1 mm. The scale graduated round the thimble of the screw ordinarily contains 100 equal divisions. This scale is called the circular scale.

The *least count* (L.C.) of the instrument or the *micrometer constant*, as it is often called, is obtained by dividing the pitch by the number of divisions on the circular scale. Here, the least count is $\frac{1}{100}$th of a millimetre or 0·01 mm. Hence, the instrument can read with accuracy upto $\frac{1}{100}$th of a millimeter.

[In some cases, however, two complete revolutions of the screw produce a linear motion through 1 mm., giving pitch equal to 0·5 mm. In this case, the circular scale on the thimble contains 50 divisions instead of 100. So L.C. of the instrument remains the same.]

Suppose that the diameter of a wire is to be measured and also suppose that the instrument has no error ; that is, when the two jaws *C* and *A* are in contact, both the linear and circular scales read zero. Then the wire is placed in the gap between the jaws *A* and *C* and the milled head *D* is turned until the two ends lightly press the wire from the two opposite ends. The complete number of millimeters is read off from the scale on the barrel and the fractional part from the circular scale. Suppose, that mark 20 of the circular scale coincides with the reference line of the main scale. Then the reading of the fractional part is (20×0·01)mm. or 0·2 mm.

Spherometer : It belongs to the general group of instruments known as micrometer screws with the additional advantage that with its help, the radius of curvature of a spherical surface either convex or concave may be conveniently measured. It is due to this particular use that the instrument is named *Spherometer*.

This instrument (fig. 8) primarily consists of a tripod, the legs of which are of equal height and the ends A, B and C of the legs constitute the vertices of an equilateral triangle. A fine screw D which works through the centre of the tripod, often called the central leg, terminates to a milled head T with a circular disc S, graduated usually in 100 equal divisions on its rim. This is called the circular scale. A vertical scale M, usually divided in millimetres, is attached to one arm of the tripod with its divisions close to the edge of the disc. This is known as the linear scale.

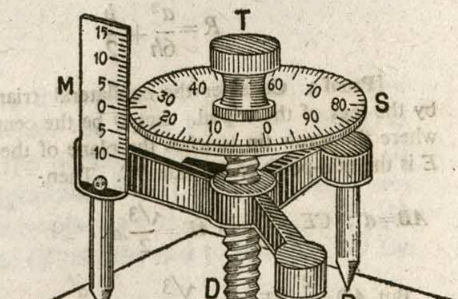


Fig. 8

Pitch of the instrument is determined by turning the circular disc S through one complete rotation and noting the linear displacement of the central leg over the linear scale M. If the central leg moves through 1 mm, the pitch of the instrument is 1 mm. The least count of the instrument is determined by the usual method of dividing the pitch by the total number of divisions on the circular scale.

Measurement of radius of curvature of a spherical surface :

The instrument is first placed on a sheet of glass plate, generally known as the *base plate* and the screw is rotated downwards by turning the milled head T until the tip of the central leg just touches the surface. The main scale and the circular scale readings are taken from which the total reading of the position of the screw tip is obtained. This is known as the initial reading. Next the central leg is sufficiently raised and the convex surface is placed over the base plate. The spherometer is then placed over the spherical surface and the central leg is turned downwards till it just touches the spherical surface. In this position, the reading of the spherometer is taken. The difference between this reading and the initial reading gives the value *h* [fig. 9(a)]. In the case of a concave surface, the spherometer is placed on the spherical surface and the central leg is lowered just to touch the bottom of the concave surface [fig 9(b)]. If *a* be the length of each side of the equilateral triangle ABC formed by the fixed legs of the tripod [fig. 10], then the radius of curvature R of the spherical surface is given by

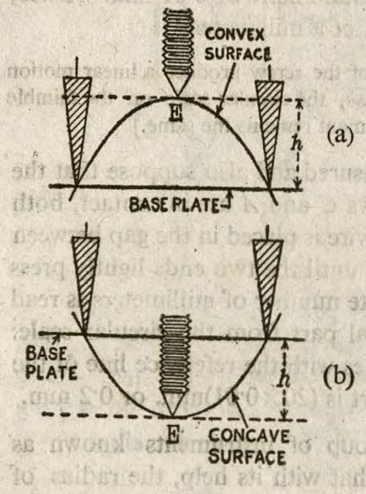


Fig. 9

$$R=\frac{a^2}{6h}+\frac{h}{2}$$

[**Proof :** Consider the equilateral triangle *ABC* formed by the legs of the tripod. Let *D* be the centre of the triangle where the screw tip touches the plane of the triangle [Fig. 10]. *E* is the mid-point of the side *BC*. Then,

$$AB=a\ ;\ CE=\frac{a}{2}\text{ and }AE=\frac{\sqrt{3}}{2}.a$$

$$\text{But } AD=\frac{2}{3}.AE=\frac{2}{3}.\ \frac{\sqrt{3}}{2}.\ a=\frac{a}{\sqrt{3}}.$$

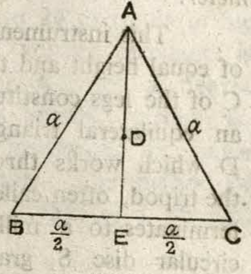


Fig. 10

Now consider Fig. 10 (a) which shows the section of the sphere of which the spherical surface in question is a part. Had the surface been plane, the screw tip would have touched the point *D*. So, *ED*=*h*. Now, from the geometry of the circle, we have, $(AD)^2=ED\times DM$

$$\text{or, }\left(\frac{a}{\sqrt{3}}\right)^2=h\times(2R-h)\qquad\text{or, }\frac{a^2}{3}=2Rh-h^2$$

$$\therefore R=\frac{a^2}{6h}+\frac{h}{2}\Big]$$

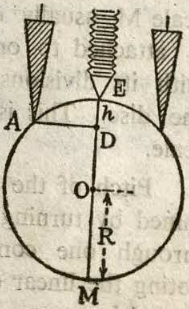


Fig. 10 (a)

9. Measurement of mass :

Mass of a body means the matter contained in it. Although we generally do not make any difference between mass and weight of a body, yet they are basically different quantities. Ordinarily, a common balance is used for the measurement of mass of a body. By a common balance we cannot determine the weight of a body.

The **Common balance** primarily consists of the following parts [fig. 11] : A vertical pillar H is fixed on a base provided with levelling screws L, L. Through the vertical pillar passes a rod which can be rasied or lowered by means of a key K at the base. At the end of the vertical rod, there is a small piece of agate plate. On it rests an agate or hard-steel knife edge C, called the *fulcrum*, rigidly attached at the middle of the horizontal beam AB of the balance. The beam is a light but rigid frame-work of thin metal rods. At the end of the beam there are two adjustable screw-weights (a, a) and two knife-edges. The edges of all the three knife-edges are in the same horizontal plane. From the two knife-edges at the two ends of the beam, are suspended two stirrups which support two scale-pans (S, S) of equal weight. There is a pointer P fitted at the centre of beam at right angles to it so that when the beam oscillates about the fulcrum, the pointer moves over an ivory white scale fitted at the base of the pillar. When the scale pans are empty or weights on the scale-pans are counterpoised the pointer stands at the middle of the scale. The beam is graduated lengthwise (graduations not shown in the figure) in order to make the final counterpoise possible by placing a '*rider*' of known weight on a suitable mark on the beam. A plumb line (V) is provided to test whether the base is horizontal and the pillar is vertical.

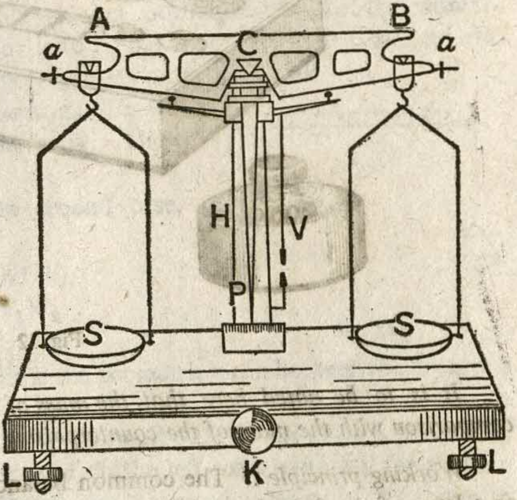


Fig. 11

Although a weight box is not a part of the balance proper, yet it is always necessary for the measurement of mass of a substance by a common balance. Fig. 12 shows a weight box. In different pockets of the box, standard weights are arranged systematically.

Method of ordinary measurement of mass: Assuming the balance to be accurate in all respects, for ordinary measurement of mass of a body, the body should be placed on the left-hand side scale pan and suitable counterposing weights from the weight box should be placed, by trial, on the right hand side pan till the balance beam is horizontal as indicated by the pointer resting on the middle line of the scale. The 'weights' that are placed on the right-hand scale pan balance the weight of the substance (See working principle). The total mass of the counterpoising weights gives the mass of the substance.

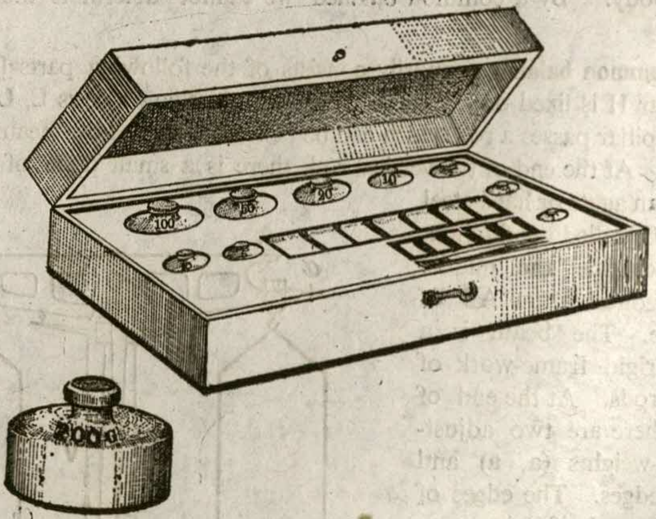


Fig. 12

It is to be noted here that the mass of the substance has been determined in comparison with the mass of the counterpoising weights.

Working principle : The common balance, as a machine, may be considered as a lever of the first class with fulcrum at the centre of the balance beam. Here the effort is the counterpoising weights put on the right scale pan and load is the substance to be weighed placed on the left scale pan. Suppose on the left scale pan is placed a body of unknown mass M and on the right scale pan standard weights of mass m are placed when the balance beam is horizontal [Fig 13]. Here, the weights of the unknown substance will tend to turn the beam about the fulcrum on the left while the standard weights will do the same on the right *i.e.* the moment of the weight of the mass M about C is equal to that of the standard weights of mass m about the same point. So,

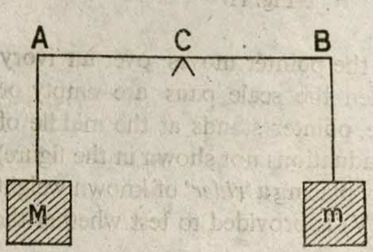


Fig. 13

$$M.g \times AC = m.g \times BC \quad [g = \text{acc}^n \text{ due to gravity}]$$
$$\text{or } M = m.$$

So, the mass of the unknown body is equal to the total mass of the counterpoising weights placed on the right scale-pan.

Defects of a common balance :

A common balance may be called defective if (i) the arms of the balance are of unequal length (ii) the scale pans are of unequal weight (iii) the centre of gravity is not exactly below the fulcrum of the balance when the beam is horizontal.

Weighing a substance correctly with a defective balance :

If the balance has only the first defect *i.e.* the arms are of unequal length, then *Gauss method* may be adopted to find the true weight of a substance. However if any other or all of the defects are present, *Borda's substitution method* gives the true weight of a substance.

(*a*) *Gauss method* (method of double weighing): In this method, the unknown body is to be placed first on the left scale pan and counterpoised. Let the counterpoising weights be W_1. Then the body is to be placed on the right scale pan and again counterpoised by putting weights (W_2, say,) on the left scale pan.

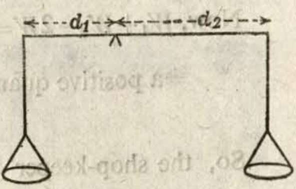


Fig. 14

Let d_1 and d_2 be the lengths of the arms [Fig 14] Let W be the true weight of the body. Then in the first case,

$W.d_1=W_1.d_2$....(i) and in the second case,

$W.\ d_2=W_2.d_1$...(ii)

Multiplying the equations (i) and (ii),

$W^2\ d_1d_2=W_1W_2\ d_1d_2$ or $W^2=W_1W_2$

or $W=\sqrt{W_1 \cdot W_2}$ .. (iii)

That is, the true weight of the body is the geometric mean between the observed weights.

(*b*) *Borda's substitution method* : In this method, the substance is placed on the right scale pan and some shots or sand on the left scale pan until the pointer indicates that the shots and the substance are balanced. Then the substance is removed and in the same pan suitable weights are placed till equilibrium is restored.

The weights put on the right hand scale pan measures the true weight of the substance whatever may be the defects of the balance because the weights used in the second case and the weight of the substance act under the same circumstances in counterpoising the shots.

Examples : (1) *A body is weighed first in the left and then in the right-hand scale pan of a balance the respective weights being* 9·842 *gm. and* 9·833 *gm. Find the true weight of the body and the ratio of the lengths of the arms of the balance.*

Ans. If W be the true weight of the body, then according to the equ (iii) of Gauss method, we have $W=\sqrt{W_1W_2}=\sqrt{9{\cdot}842\times 9{\cdot}833}$ gm.$=9{\cdot}837$ gm.

From equ (i) we get, $W.d_1=W_1.d_2$

$$\text{or, } \frac{d_1}{d_2}=\frac{W_1}{W}=\frac{9{\cdot}842}{9{\cdot}837}=\frac{1{\cdot}0005}{1}$$

(2) *A shop-keeper uses a false balance having unequal arms whose lengths are* d_1 *and* d_2. *He weighs out to two customers W pounds of tea as indicated*

by his balance ; but in serving one of the customers he puts his weights an one scale pan while in serving the other, he puts them in the other. Does he gain or lose and by how much ?

Ans. Since the arms are of unequal length, W lbs of tea as indicated by his balance is not the true weight of the substance. Let the true weight be W_1. When the substance is placed on the left-hand scale pan [Fig 14], we have, $W_1 . d_1 = W . d_2$

$$\text{or } W_1 = W . \frac{d_2}{d_1}$$

For the second time also when the substance is put on the right-hand scale-pan, the weight W as indicated by the defective balance is not the true weight. Let the true weight in this case be W_2. Then, $W_2 . d_2 = W . d_1$ or $W_2 = W . \frac{d_1}{d_2}$

Now, the customers actually take away a quantity of tea whose weight is $(W_1 + W_2)$ lb but the shop-keeper is paid for a quantity of weight $2W$ lb.

$$\text{Now, } W_1 + W_2 - 2W = W . \frac{d_2}{d_1} + W . \frac{d_1}{d_2} - 2W = W \left[\frac{d_1^2 + d_2^2 - 2d_1 d_2}{d_1 d_2} \right] = W . \frac{(d_1 - d_2)^2}{d_1 d_2}$$

= a positive quantity $[(d_1 - d_2)^2$ being a perfect square is always positive]

$$\therefore \quad W_1 + W_2 > 2W$$

So, the shop-keeper loses by an amount $= W \frac{(d_1 - d_2)^2}{d_1 d_2}$

(3) *The only fault in a balance being the inequality in weights of the scale-pans, what is the true weight of a body which balances 10 lb. when placed in one scale pan and 12 lb. when placed on the other ?*

Ans. Let the true weight of the body be W and the difference in the weights of the scale-pans be x, the right-hand scale pan being less in weight than the left-hand. When the substance is kept on the left-hand scale pan (Fig. 15), let the counterposing weights be W_1. From the figure, $W . a = (W_1 + x) . a$ or $W = W_1 + x \ldots$(i)

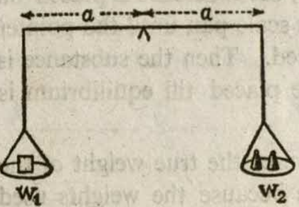


Fig. 15

When the substance is transferred to the right scale-pan, let the counter-poising weight be W_2. Then, $W_2 . a = (W + x) . a$ or $W_2 = W + x \ldots$(ii)

Adding (i) and (ii), we get, $2W + x = W_1 + W_2 + x$ or $W = \frac{W_1 + W_2}{2}$

In the problem, $W_1 = 12$ lb and $W_2 = 10$ lb. So, $W = \frac{12 + 10}{2} = 11$ lb.

Requisites of a good balance :

A good balance ought to have the following requisites :

(1) The balance ought to be **true.** That is, the balance beam should come to horizontal position either when the scale-pans are empty or when same counterpoising weights balance a substance if placed on any of the two scale-pans.

The conditions for a balance to be true are as follows : (a) the arms of the balance must be of equal length (b) the scale-pans must be of equal weight (c) the C.G. of the beam must be vertically below the fulcrum of the balance when the beam is horizontal.

(2) The balance ought to be **stable**. That is, when the balance is slightly disturbed, the beam should readily come to its horizontal position after a few oscillations.

The condition for a balance to be stable is as follows : (a) the fulcrum of the balance beam should be a little above the C.G. of the balance beam.

(3) The balance ought to be **sensitive.** That is, the beam should be sufficiently tilted if there is a slight difference in the weights of the substance and the counterpoising weights placed on the scale pan. A good chemical balance is so sensitive that it can indicate a difference of weight of the order of $\frac{1}{10}$th of a milligram.

The conditions for a balance to be sensitive are as follows : (a) The balance beam should be light (b) the balance beam should be sufficiently long, (c) The C.G. of the beam should be very near the fulcrum.

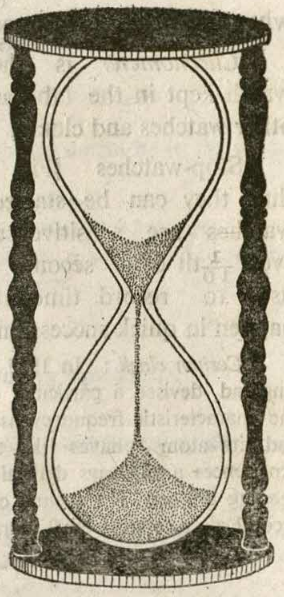


Fig. 16

10. Measurement of time :

Any event which repeats itself after a definite interval of time, may be used for the measurement of time ; for example the oscillations of a pendulum or a coiled spring. The measurement consists of counting the repetitions.

The first device used to measure time was the *sun-dial* which was built some time about 800 B.C. The shadow cast by sun-rays of an opaque obstacle held vertically on a horizontal graduated disc was utilised in recording time. *Sand-clocks* (Fig. 16), were used in churches early in the sixteenth century for measuring time. When sand runs out of the upper compartment of a sand-clock into the lower, a certain time is required. When the sand-clock is inverted, the running out of the sand from the upper chamber into the lower takes place under the same conditions and hence in the same time. Another ancient device for measuring time was *water-clock*. It was probably devised in Egypt, at least 3000 years ago. In the improved type of water-clock as shown in Fig. 17, water was allowed drip from a reservoir into a second vessel in which a wooden figure was kept floating. As the level of water rose, the wooden figure floated

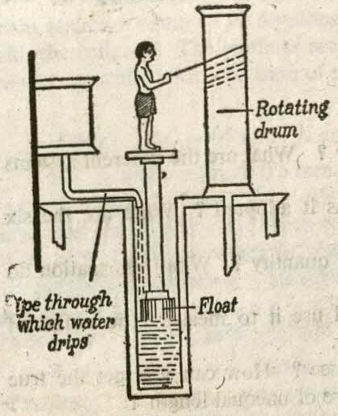


Fig. 17

higher up and the intervals of time were recorded on a revolving drum by the outstretched arm of the doll.

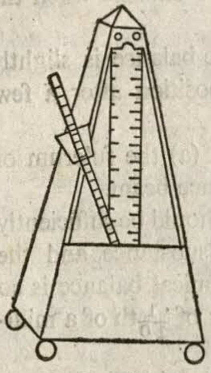


Fig. 18

The principle of clock making was revolutionised by the discovery of pendulum in 1581 by the famous Italian inventor Galileo. At present, we use pendulum clocks, watches etc. for the purpose of measuring time. Besides the conventional clocks and watches, there are metronome, chronometers, stop-watches etc. in use for special purposes.

Metronome, in fact, is an inverted percussion type pendulum (Fig. 18). Its time of swing can be altered within wide limits by moving a small sliding weight up and down on the upper part of the pendulum rod. If this weight be pushed up, the pendulum goes slower and when the weight is lowered, it goes faster.

Chronometer is the most precision type of watch kept in the laboratory for correcting times of other watches and clocks.

Stop-watches [Fig. 19] have the advantage that they can be started or stopped at will. Stop watches are sensitive enough to record $\frac{1}{5}$th or even $\frac{1}{10}$th of a second. These watches are generally used to record time-interval between events which happen in quick succession.

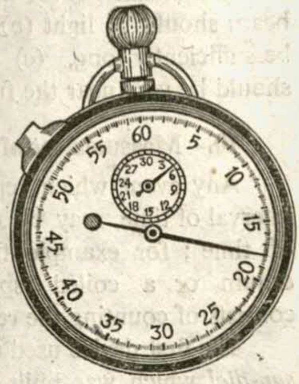


Fig. 19

Cesium clock : In 1955, National Physical Laboratory of England devised a particular type of *atomic clock,* based on the characteristic frequency associated with the cesium atom. Cesium is a radioactive element and its atom behaves like a tiny magnet. In a non-uniform magnetic field, the atom experiences a sideways deflection, the frequency of oscillation being used to control a time-keeping device. The cesium clock is so accurate that two such clocks will differ by only one second after running 5000 years !

Exercises

Essay type :

1. What do you mean by 'units' of a physical quantity ? What are the different systems of units ?

2. What is International system of units ? Why was it adopted ? What are the six basic units in this system ?

3. What do you mean by the dimension of a physical quantity ? What information do they provide us ?

4. Describe a spherometer. Explain how you would use it to measure the radius of curvature of a convex mirror. Establish the underlying theory.

5. What is the working principle of a common balance ? How can you get the true weight of a substance with the help of a balance whose arms are of unequal length ?

6. What are the requisites of a good balance ? What conditions are necessary for a balance to acquire the requisites ?

Short answer type :

7. In defining the standard of length, we have to specify the temperature at which the measurement should be made. Is it justified in calling length a fundamental quantity if another physical quantity *viz.* temperature, has to be specified in choosing a standard ? . Can you suggest why temperature is mentioned in the above case ?

8. Mention some repetitive phenomenon occuring in nature which could serve as time standard ? Why is a second taken as $\frac{1}{86400}$th part of a mean solar day instead of a solar day ?

9. What is the difference between mean solar day and sidereal day ?

10. You are provided with two spherometers of same pitch. One of them has 50 divisions on the circular scale while the other has 100 divisions on its circular scale. Which is more sensitive ?

11. What is a light-year ? Is it a fundamental unit ? What is its area of application ?

12. Which quantity do we get from a common balance—mass of a body or the weight of a body ?

Objective type :

13. Select the best answer :

(a) The latest system of units adopted by International agreement is (i) C.G.S. (ii) F.P.S. (iii) S.I. (iv) M.K.S. system.

(b) The unit of mass in M.K.S. system is (i) Gramme (ii) Pound (iii) Kilogramme.

(c) The dimension of force is (i) MLT^{-2} (ii) $ML^{-2}T$ (iii) $ML^{-2}T^{-2}$.

(d) To make a common balance sensitive, the C.G. of the beam should be (i) far away (ii) very near (iii) vertically below the fulcrum.

(e) Newton is the unit of force in (i) F.P.S. (ii) S.I. (iii) M.K.S. system.

(f) The dimensional formula $ML^{-1}T^{-2}$ belongs to (i) Force (ii) Work (iii) Pressure.

(g) The time-interval between identical phases of the moon is called a lunar month which is (i) longer than (ii) shorter than (iii) equal to a sidereal month.

Numerical problems :

14. Check the dimensional consistency in the case of the following equations :

(i) $S=ut+\frac{1}{2}ft^2$ (ii) $v=u+ft$ (iii) $P.t=m.v$.

15. Find out the dimension of (i) Gravitational constant (ii) Modulus of rigidity and (iii) Frequency.

16. Each division of the main scale of a vernier calipers is 0·5 mm. 9 divisions of the main scale are equal to 10 divisions of the vernier scale. The length of a cylinder is measured with the calipers. The readings are : 78 divisions on the main scale and the 6th division of the vernier coincides with a division of the main scale. What is the length of the cylinder ? [Ans. 3·93 cm.]

[*Hints* : $V.C.=\cdot 05$ mm. ; Length$=(78\times 0{\cdot}5+6\times\cdot 05)$ mm.]

17. A screw of pitch 0·5 mm. is mounted such that its tip can be moved against the end of rod of length 100 cm. The other end of the rod is fixed. A circular scale attached to the screw has 100 divisions and when rotated fully moves against one small division of a linear pitch scale, each division of length 0·5 mm. At 20°C, the pitch scale reading is a little below zero and the circular scale reading is 92 divisions. When the temperature of the rod is raised to 100°C, the reading on the pitch scale is a little above 4 divisions and the reading on the circular scale is 72 divisions. Calculate the present length of the rod. [Ans. 100·24 cm.]

18. A spherometer has a screw of 40 threads to the inch and a linear scale showing fortieths of an inch. How many divisions should there be on the circular scale if the instrument is to read $\frac{1}{1000}$th of an inch ? What will be the readings on the two scales if the length to be measured is 0·394 inch, assuming that both initially read zero ? [Ans. 15 and 19].

19. The tripod of a spherometer constitutes an equilateral triangle, each side of the triangle being 4 cm. long. When the instrument was placed on a base plate, the reading obtained was 4 mm. and when placed upon a concave surface, the reading was 2 mm. Calculate the radius of curvature of the concave surface. [Ans. 13·43 cm.]

[*Hints* : Apply the formula : $R=\frac{a^2}{6h}+\frac{h}{2}$]

20. The fixed legs of a spherometre are at the corners of an equilateral triangle of 4cm side. When placed on a certain spherical surface the instrument reads 5·637 mm. Find the radius of curvature of the surface. [Ans. 5·013 cm]

21. The fixed legs of a spherometer lie at the corners of an equilateral triangle of side 4·8cm. When placed on a convex and concave surfaces of a watch glass, the spherometer reads 1·15 mm and 1·16mm respectively. Assuming that there is no error in the instrument, find the radius of curvature of each surface [Ans 33·45 cm ; 33·16 cm]

22. A common balance has equal arms, 20 cm. in length. A weight 20 gm. rests on one pan, while an unknown weight rests on the other. A one gm. weight is placed on the beam and moved from pivot towards the unknown weight. When the one gm. wt. is 15 cm. from the pivot equilibrium is restored. Find the unknown weight. [Ans. 19·25 gm.]

[*Hints* : $20\times 20=W\times 20+1\times 15$;]

23. The arms of a balance are of unequal length. When a substance is weighed in the balance, keeping it successively on the two scale pans, it is founds to weigh 25 gm. and 9 gm. respectively. What is the true weigh of the substance ? [Ans. 15 gm.]

24. A ration dealer sells equal quantitites of rice, weighing them successively on the two scale pans of a balance whose arms are not of equal length. If the ratio of the arms of the balance is 1·5, find the percentage of loss or gain of the dealer. [Ans. 8·3% loss]

MECHANICS

1

DYNAMICS OF A PARTICLE

KINEMATICS AND KINEMATICAL EQUATIONS

Introduction : Mechanics is said to be the grammer of science. Preliminary ideas in mechanics are essential for a fruitful study of science—specially physics.

Originally mechanics was known as the science of making machines. Now, it is known as the science which deals with motion and effects of force on a body. Mechanics has been divided into two parts :—(i) **Dynamics** and (ii) **Statics.**

Dynamics deals with moving bodies and Statics with bodies at rest under the action of a system of forces.

Dynamics, again, has been divided into two parts, viz. (i) **Kinematics** and (ii) **Kinetics.**

Kinematics deals with motion itself without having any regard to the body that moves or the cause of motion. Kinetics, on the other hand, deals with the mass of the body and the force that causes the body to move.

Body and Particle : Anything having a definite shape, size and volume may be called a body. Most of the things that we see around us have definite shape, size and volume and hence they may be called bodies.

A material particle or simply a particle means a body of negligible size. It is represented by a geometrical point. In mechanics, we are concerned with a particle. We have to consider its position and mass only. Thus, in calculating the acceleration of a train or the force acting on it, it is enough to know the mass and the positions of the train at different times without bothering about the size and volume of the train.

1.1. Rest and Motion : In our every-day life we see various bodies in motion as well as in rest. Buildings, trees, roads, towers, statues etc. are bodies at rest while running trains, horses, cars etc. are bodies in motion. *A moving body means one which changes its position with respect to time while a body at rest, means one which does not change its position with respect to time.*

But we know that the earth rotates round its own axis once in 24 hours and round the sun once in 365 days. How can buildings, trees etc. situated on the moving earth, remain at rest ? When astronauts move far away from the earth in spacecraft, they see all the buildings, trees etc. moving speedily along with the earth. As a matter of fact, nothing in this universe, is at rest. *Absolute rest* is something unknown to us.

If this be so, why do we say that the buildings, trees, towers etc. are at rest ? What sort of rest is it ? Ordinarily, if a body does not change its position in relation to its surrounding objects, we say that the body is at rest. On the other hand, if a body changes its position in relation to its surrounding objects, we say that the body is in motion. These are, therefore, *relative rest* and *motion*. So when we say that a body is at rest or in motion, we always mean relative rest or relative motion with respect to the surrounding objects, assuming that the earth is at rest. In this sense, trees, buildings etc. are at rest because they never change their positions in relation to their surrounding objects.

1.2. Reference frame : To describe the motion of a body, we are to state how the position of the body changes with time. Reference frame is the frame in respect of which such change of position takes place. In the preceding article, it has been mentioned that to determine whether a body is in motion or at rest, we generally consider the earth to be at rest and see whether the body is changing its position with respect to the surrounding objects. The earth, in this case, acts as a reference frame.

If the change of position takes place in a *plane*, the reference frame used to denote such change, is usually two intersecting but mutually perpendicular lines. Suppose OX and OY are two mutually perpendicular lines (Fig. 1.1). A particle, situated at P is at a distance y_1 from the line OX and x_1 from the line OY. Here, the position of the particle is determined with the help of X-Y reference frame. If, now, the particle changes is position to Q whose distance from the lines OX and OY are respectively y_2 and x_2, we can easily find the change of position (PQ) of the particle with the help of the reference frame.

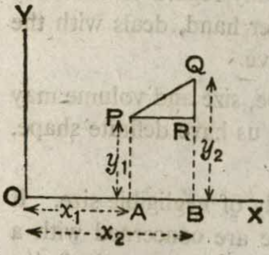


Fig. 1.1

$$PQ=\sqrt{(PR)^2+(RQ)^2}=\sqrt{(AB)^2+(RQ)^2}$$
$$=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Knowing the values of x_1, x_2, y_1 and y_2 from the reference frame we can easily find the value of PQ. The above reference frame was first devised by Descartes' and hence it is known as the *cartesian reference frame*.

To denote the position of a particle in *space*, we may similarly draw three mutually perpendicular lines OX, OY and OZ through any point O and form a right-handed cartesian reference frame [Fig 1.2(a)]. We, then, draw perpendiculars from the position of the particle P on the axes and the intercepts on the axes measured from the origin O will give the position of the particle. Thus, the distance of the point P from the plane XOZ is PQ, from the plane YOX, the distance is RQ and from the plane YOZ, the distance is OR.

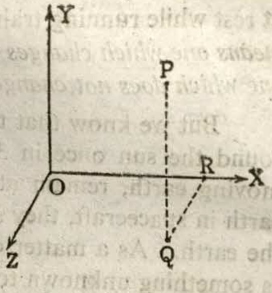


Fig. 1.2(a)

Another useful representation of a point P is by *polar coordinates* (r,θ), where r is the distance of the point P from the origin O [Fig 1.2(b)] called the *radius vector* (i.e. $OP=r$) and θ is the angle between the radius vector and the positive direction of the axis OX.

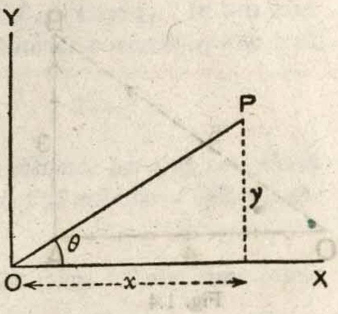


Fig. 1.2(b)

Clearly, $x=r\cos\theta$ and $y=r\sin\theta$, so that $r=(x^2+y^2)^{\frac{1}{2}}$ and $\tan\theta=y/x$.

From the above relations, we can transfer the polar co-ordinates into cartesian co-ordinates and vice versa.

[*Note* : It is to be noted that for complete identification of an *event* in a reference frame *i.e.* for exact location as well as timing of occurrence of the event, we must, in addition to the usual three space co-ordinates x, y and z, have yet another co-ordinate viz, the co-ordinate of time *t*, perpendicular to the former three. A reference frame, with four such co-ordinates, *x*, *y*, *z* and *t* is referred to as *space-time reference frame*—the four axes defining a four dimensional continuum, called *space-time continuum*.

1.3. Translation and Rotation :

Motion may be of two kinds :—(i) *translational* and (ii) *rotational.*

A body is said to have a translatory motion, if all the particles of the body move with same velocity and their paths are all parallel.

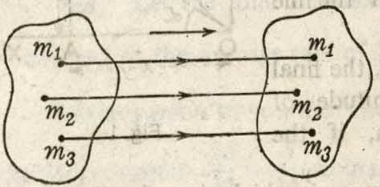


Fig. 1.3

Consider three particles m_1, m_2 and m_3 of a body which is occupying the left-hand position at any instant. After some time, the body goes to the right-hand position moving in the direction of arrow as shown in fig 1.3. The particles m_1, m_2 and m_3 move along parallel paths with the same velocity in the same direction. The motion of the body, in this case, will be called translatory. The motion of a falling stone for example, is translational.

When a particle moves along a circular path about a point or about an axis, its motion is called rotational motion. The motion of the blades of a whirling electric fan, motion of a flywheel etc. are the examples of rotational motion.

There may be complex motion arising out of the combination of the above two types of motion. The motion of a bi-cycle wheel is a combination of rotational and translational motion. The motion of a spinning top is very complex. The earth moves round the sun and at the same time spins about its own axis.

1.4. Definitions of some terms in connection with translational motion :

(*a*) **Displacement :** *Displacement is defined as the distance moved in a specified direction and is measured by the linear distance between the initial and the final positions of the body.*

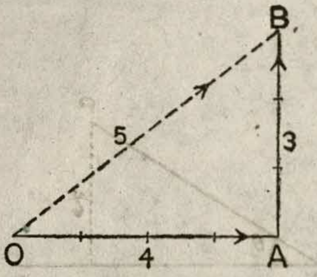


Fig. 1.4

Suppose, a particle is at *O* at any instant (Fig. 1.4). It then moves 4 ft. due east and arrives at *A*. After this, the particle changes direction and moves 3 ft. due north arriving finally at *B*. Here, the displacement of the particle is *OB*, although it went along the path *OAB*, because *O* is the initial and *B* is the final position of the particle. The magnitude of the displacement $OB=\sqrt{(4)^2+(3)^2}=5$ ft.

The displacement is thus, independent of the path taken up by the particle in arriving at *B* from *A*. Suppose a man goes round the college square tank and returns to the starting point. He may have travelled a mile but his displacement is nil because the starting point and the termination point coincide.

The magnitude and the direction of the displacement may best be obtained with the help of the reference frame.

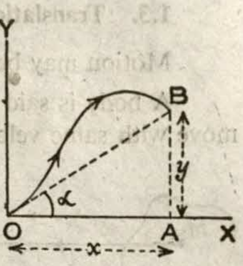


Fig. 1.5

Suppose, the two mutually perpendicular lines *OX* and *OY* denote a reference frame (Fig. 1.5). in which a particle, starting from *O* and travelling in a curved path, arrives at *B*. In this case, the displacement is measured by the straight line *OB*. Now, if the point *B* is at a distance y from the line *OX* and x from the line *OY*, then $OB=\sqrt{x^2+y^2}$.

So, if the co-ordinates (x, y) of the point *B*, the final position of the particle be known, the magnitude of the displacement can be found out. Again, if the line *OB* makes an angle α with the line *OX* then $\tan\alpha=\frac{y}{x}$ which gives the direction of the displacement.

Displacement, therefore, is a quantity which carries a sense of direction as well as of magnitude. Such quantities, which have direction as well as magnitude are called *vector quantities.*

(*b*) **Speed :** *Speed is defined as the rate of change of position with time i.e. the distance travelled in 1 second. So, to find speed, we require the total distance travelled and the total time taken for it, so that,*

$$\text{Speed}=\frac{\text{Distance travelled}}{\text{Time taken}}$$

Speed has only magnitude and no direction. Quantities, which have only magnitude and no direction, are called *scalar quantities.* Speed is, therefore, a scalar quantity.

If a body travels equal distances at equal intervals of time, its speed is said to be uniform. If it does not do so, the speed is variable or non-uniform.

A particle has a variable speed, when in equal intervals of time, its displacements are unequal. In such cases, it is customary to speak of an **average speed.**

Suppose a particle, travelling with variable speed, covers a distance S_1 in time t_1 and then a distance S_2 in time t_2 and finally a distance S_3 in time t_3. In this case, the average speed is obtained by dividing the total distance covered by the total time taken. Thus the average speed $=\dfrac{S_1+S_2+S_3}{t_1+t_2+t_3}$

Examples : (1) *A motor car travels half the distance between two places with a speed of 40 miles/hr and the next half with a speed of 60 miles/hr. What is the average speed of the car?*

Ans. Let the distance between the places be $2x$ miles. Total time taken

$$=\frac{x}{40}+\frac{x}{60}=\frac{x}{24} \text{ hours.}$$

$$\text{Hence, the average speed}=\frac{\text{Total distance travelled}}{\text{Total time}}=\frac{2x}{x/24}$$

$$=48 \text{ miles/hr.}$$

(2) *A person travels half the distance of his destination at the average speed of 24 miles/hr. With what average rate per hour should he travel the rest of his journey so that he may travel the whole path at the average rate of 32 miles/hr?*

[*Jt. Entrance 1985*]

Ans. Let the total distance be $2S$ miles. If the first half *i.e.* the path S is covered at the average rate of 24 miles/hr, the time required $=\dfrac{S}{24}$ hour.

If the person travels the whole path at the average rate of 32 miles/hr, then the time required $=\dfrac{2S}{32}=\dfrac{S}{16}$ hours.

So, the time necessary for the person to travel the second half

$$=\frac{S}{16}-\frac{S}{24}=\frac{S}{48} \text{ hr.}$$

$\therefore$ the reqd. average speed of the person $=\dfrac{S}{S/48}$ miles/hr $=48$ miles/hr.

(*c*) **Velocity :** *Velocity is defined as the rate of change of position in a specified direction i.e. the distance travelled in a specified direction in 1 second.* Velocity is a vector quantity.

$$\text{Velocity}=\frac{\text{Distance travelled in a specified direction}}{\text{time taken}}=\frac{S}{t}.$$

Velocity may be *uniform* or *variable*. If a body travels equal distances at equal intervals of time in a specified direction, its velocity is uniform. If it does not do so, the velocity is variable or non-uniform.

[According to differential notation, if ds be a small displacement in a small interval of time dt, then velocity $v=\dfrac{ds}{dt}$]

Time-displacement graph and velocity :

If the distance S travelled by a body moving in a given direction from some fixed point is plotted against time t, we get a time-displacement graph OBC of the motion [Fig. 1.6]. The velocity of the body at any instant is given by the distance travelled in 1 sec. at that instant.

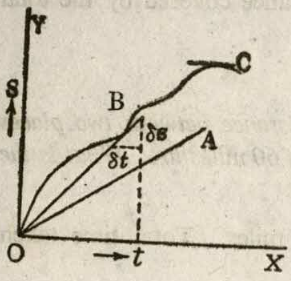


Fig. 1.6

At B in Fig. 1.6, for example if the change of distance is δs made in a time δt, the velocity at $B=\frac{\delta s}{\delta t}$. In the limit when $\delta t \rightarrow 0$, the velocity at B becomes $v=\frac{ds}{dt}$. But when $\delta t \rightarrow 0$, $\frac{\delta s}{\delta t}$ is the gradient of the tangent BO to the curve at B. So, *the gradient of the tangent at B gives the instantaneous velocity of the body at B.*

Now, the tangent at the point C to the curve is parallel to the time-axis. Hence its gradient is nil. This means that the velocity of the body at C is instantaneously zero. If the body moves with such changing velocity represented by a curved line in the time-displacement graph, the velocity is called *variable.*

On the other hand, if the time-displacement graph is a straight line OA, the gradient is constant at all points. This means that the body is moving with *uniform velocity.*

Units of velocity :

C. G. S. System : In the C. G. S. system, the unit of velocity is centimetre per second. If a particle travels a distance of 1 cm. in 1 sec. in a specified direction, its velocity is taken as unit velocity in the C. G. S. system.

F. P. S. system : In the F. P. S. system, the unit of velocity is 'foot per sec'. If a particle travels a distance of 1 foot in 1 sec. in a specified direction, its velocity is taken as unit in the F. P. S. system.

Example : *A person travels half of a circular path of radius 6 metre in 20 seconds. Find the speed and velocity of the man.*

Ans. Starting from A, the person travles along the semi-circular path ACB and arrives at B after 20 seconds. Here, the total path travelled$=\pi r=3{\cdot}14\times 6$ metre.

$$\text{So, his speed}=\frac{\text{Total path travelled}}{\text{time required}}=\frac{3{\cdot}14\times 6}{20}$$

$$=0{\cdot}942 \text{ metre/s}=94{\cdot}2 \text{ cm/s}.$$

Displacement of the person$=AB=2\times 6$ metre.

$$\therefore \text{ His velocity}=\frac{\text{Displacement}}{\text{time}}=\frac{2\times 6}{20}$$

$$=0{\cdot}6 \text{ metre/s}=60 \text{ cm/s along } \overrightarrow{AB}.$$

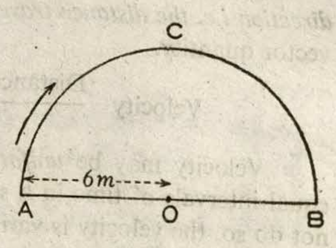


Fig. 1.6(a)

Difference between speed and velocity :

(i) Both are measured in same units.

(ii) Speed is a scalar quantity *i.e.* it has magnitude but no direction. Velocity is a vector quantity *i.e.* it has magnitude and direction.

(iii) Speed may vary only in magnitude. Velocity may vary either (a) in magnitude or (b) in direction or (c) in both.

For example, if a car travels at a speed of 20 miles per hour along a perfectly straight road, it would be correct to say that it has a velocity of 20 miles/hour due south, or whatever the direction of the road may be. On the other hand, if the car travels round a bend with constant speed, its direction of motion will be continuously changing. Hence, it will be incorrect to say, in this case, that the velocity of the car, is 20 miles/hour.

Alternatively, suppose a particle is moving along the circumference of a circle in such a manner that it describes equal lengths of arc in equal intervals of time . In this case, the speed of the particle is uniform but the velocity is not because the direction of velocity is changing at every moment. At points say *A*, *B*, *C*, the magnitude of the velocity is same but the directions are different. Tangents *AP*, *BQ*, *CR* drawn at *A*, *B* and *C* respectively give the directions of the velocities at those points (Fig. 1.7).

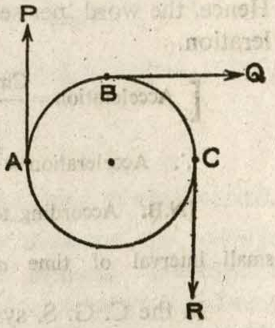


Fig. 1.7

(*d*) **Curvilinear motion and instantaneous velocity :**

Motion along a curved line is called curvilinear motion. The speed in a curvilinear motion may be uniform or variable [Fig. 1.8]. Here, the word 'speed' is used instead of velocity because the motion takes place along a curved path. Now, if the particle moves equal distances along the curved path *ABC* at equal intervals of time, its speed is uniform. If it fails to do so, its speed is variable.

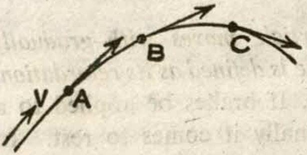


Fig. 1.8

Now, in connection with curvilinear motion, it sometimes becomes necessary to express instantaneous velocity. To determine the instantaneous velocity at any point on the curved line, a tangent is to be drawn to the curved line at that point as shown in the figure. The direction of the tangent will give the direction of the instantaneous velocity at that point and the speed of the particle while crossing the point gives the magnitude of the instantaneous velocity. In the figure, the tangents drawn at *A*, *B*, *C*, etc. give the directions of the instantaneous velocities at those points. (See time-displacement graph on page 6).

(*e*) **Acceleration** : When a train leaves a station, it moves with gradually increasing velocity and finally it attains a constant velocity. At the initial stage, when the velocity of the train was changing, the train may be said to have an acceleration. When a particle moves with gradually increasing velocity, *its rate of change of velocity with time is defined as its acceleration.*

$$\text{Acceleration}=\frac{\text{Change of velocity}}{\text{time}}$$

Suppose, at any instant, a particle has a velocity of 32 ft/sec. It moves with accelerating motion and after 10 seconds, the velocity changes to 52 ft/sec. and after another 10 seconds, the velocity again changes to 72 ft/sec. The particle is moving in this manner with gradually increasing velocity. Here, it is seen that after each 10-second interval, the particle is changing its velocity by 20/ft sec. Hence, the rate of change of its velocity—in other words, its acceleration $=\frac{20}{10}=2$ ft/sec./sec.

Note that the word 'per sec' is appearing twice in the unit of acceleration—first to denote the velocity and the second to denote the rate of change of velocity. Hence, the word 'per sec per sec' or 'per sec^2' is used to express the unit of acceleration.

$$\left[\ \text{Acceleration}=\frac{\text{Change of velocity}}{\text{Time}};\ \text{Change of velocity}=\frac{\text{Distance travelled}}{\text{Time}}\right.$$

$$\left.\therefore\ \text{Acceleration}=\frac{\text{Distance travelled}}{\text{Time}}\times\frac{1}{\text{Time}}=\frac{\text{Distance travelled}}{(\text{Sec})^2}\ \right]$$

[N.B. According to differential notation, if d_v be the small change of velocity in a small interval of time dt, then acceleration $f=\frac{d_v}{dt}$]

In the C. G. S. system, the unit acceleration is cm/sec/sec and in the F.P.S. system, ft/sec/sec.

It is to be remembered that like velocity, acceleration may be uniform or variable.

(*f*) **Retardation** or **Deceleration :** *When a particle moves with gradually decreasing velocity, its rate of change of velocity with time is defined as its retardation.* Retardation may also be called negative acceleration. If brakes be applied to a moving train, its velocity gradually decreases and finally it comes to rest. In this condition, the train may be said to have a retardation. As before, let us take an illustration.

Suppose, at any instant, a particle has a velocity of 32 ft/sec. After 2 seconds its velocity is found to be 28 ft/sec.. and a further period of 2 sec. after, its velocity decreases to 24 ft/sec. The particle is moving in this manner with gradually decreasing velocity. Here, it is seen that after each 2-second interval, the particle is changing its velocity by 4ft/sec. Hence the rate of change of its velocity, in other words, its retardation $=\frac{4}{2}=2$ ft/sec./sec.

The units of acceleration and retardation are identical.

(*g*) **Average acceleration and instantaneous accelaration :**

A particle may have *average acceleration*, like average speed or average velocity, if it moves with variable acceleration. Suppose, a particle is moving with a velocity v_1 during an interval t_1 but with a velocity v_2 during a subsequent interval t_2. In this case, the particle has an acceleration which is variable. The average acceleration of the particle $= \dfrac{v_2 - v_1}{t_2 - t_1} = \dfrac{\delta v}{\delta t} = \dfrac{\text{change of velocity}}{\text{,, ,, interval}}$

When δt is very small, the average acceleration is called the **instantaneous acceleration.**

Velocity-time graph: Let PQR be the velocity-time graph of a particle (Fig. 1.9). The curve shows that the velocity is not uniform. It acquires maximum value at Q, then slows down and comes to rest at the time corresponding to R. In this case, the acceleration and retardation are both non-uniform. A and B represent two different velocities of the particle at two different instants. Drawing perpendiculars AC and BC, we can say, the average acceleration of the particle$= \dfrac{\delta v}{\delta t}$. Now, from the right angled ΔABC,

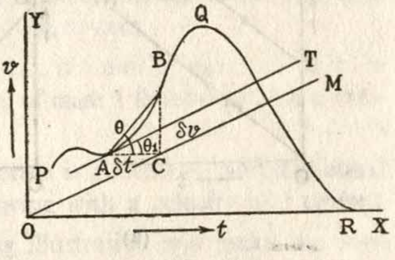


Fig.1.9

$$\frac{\delta v}{\delta t} = \tan\theta$$

So, the average acceleration of the particle$=\tan\theta=$slope of the line AB.

Since the slope at different points of the curve is not the same, the average acceleration of the particle is also not the same at different instants. If a particle moves in such a way that its average acceleration, measured for a number of different intervals does *not* come out to be constant, the particle is said to have a variable acceleration. The acceleration may vary in magnitude or in direction or in both. In such cases, we can find the acceleration of the particle at any given instant, called the *instantaneous acceleration.*

Instantaneous acceleration :

Now, suppose, the point B is approaching the point A. As the interval δt becomes smaller and smaller, the average acceleration between A and B becomes very nearly equal to the instantaneous acceleration at A. Again, with the approach of B towards A, the chord AB turns and finally becomes the tangent AT at A and θ becomes very close to θ_1. In the limit when δt tends to zero, $\dfrac{\delta v}{\delta t} = \dfrac{dv}{dt} =$ the instantaneous acceleration at A. But when $\delta t \to 0$, $\theta = \theta_1 =$the slope of the tangent AT to the curve at A.

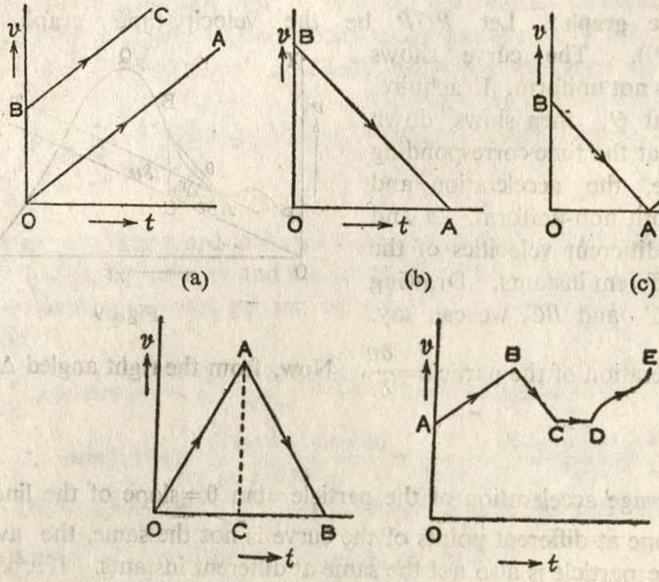
It goes without saying that if the particle accelerates uniformly from rest, the velocity-time graph is a straight line OM whose gradient at every point is constant. Hence we can say that *the instantaneous acceleration at any point in the velocity-time graph equals the slope of the tangent to the graph at that point.*

It may be pointed out here that in mechanics, the average acceleration is more important than the instantaneous acceleration.

Different velocity-time graphs for accelerated and decelerated motions :

Velocity-time graph of the motion of a particle clearly indicates whether the particle has accelerated, decelerated or uniform motion. The following cases may be taken as typical illustrations :

Case (*i*) : The curve BC shows that the particle has an initial velocity $u = OB$ and is moving with uniform acceleration [Fig 1.10(a)]. The curve OA, on the other hand, indicates uniformly accelerated motion from rest because the initial velocity $u=0$ (the straight line passing through the origin).



(a) (b) (c)

(d) Fig. 1.10 (e)

Case (*ii*) : The velocity-time graph BA [Fig 1.10 (b)] indicates that the particle has an initial velocity $u=OB$ and moves with retardation (uniform) until at $t=OA$, its velocity reduces to zero.

Case (*iii*) : The velocity-time graph BA [Fig 1.10(c)] indicates, as before, that the particle starts with an initial velocity $u=OB$ but moving with uniform retardation, the velocity reduces to zero at $t=OA$ and then again it moves with uniform acceleration in the opposite direction attaining the same velocity as the initial one ($OB=DC$) at $t=OD$. In nature we see such motion when a projectile (a stone, say) is thrown vertically upwards with some velocity.

Case (*iv*) : The graph OAB [Fig 1.10(d)] represents that the particle, starting from rest, moves with uniform acceleration, assumes maximum velocity at $t=OC$ and then moves with uniform retardation, finally coming to rest at $t=OB$. In nature, we see such motion when a ping-pong ball is dropped from a certain height. Just before touching the ground, the ball has maximum velocity and then it rebounds upwards and comes to rest momentarily after attaining the greatest height.

Case (*v*) : The graph [Fig 1.10(e)] represents a motion which is a mixture of uniformly accelerated, uniformly retarded and constant velocity motion. *AB* represents the accelerated motion, *BC*, the retarded motion and *CD*, the portion parallel to the time-axis, a motion with uniform velocity.

(*h*) **Momentum :** *It is defined as the quantity of motion produced in a body and is measured by the product of the mass and the velocity of the body.* If '*m*' be the mass of a body and '*v*' its velocity, its momentum $p=m\times v$. Momentum is evidently a vector quantity.

If a number of bodies constitutes a system, the momentum of the system is the vector sum of momenta of different bodies of the system.

Units of momentum : In the *F. P. S.* system, the unit of momentum is lb-ft/sec. and it is equal to the momentum of a body of mass 1 lb moving with a velocity of 1ft/sec.

In the *C. G. S.* system, the unit of momentum is gm.cm/sec. and it is equal to the momentum of a body of mass 1 gm. moving with a velocity of 1 cm/sec.

Conception of momentum : The following illustration will make the conception of momentum clear.

Suppose a motor car is running at a speed of 20 miles/hr. Some effort is necessary to stop the car. Greater effort is needed to stop a loaded lorry running at the same speed because the mass of the lorry is greater than that of the car. If the car moves with double the speed than before, double effort will, evidently be required to stop it. Same is the case with the lorry. If equal forces act upon a light and a heavy body for equal interval of time, the velocity of the lighter body will certainly be greater but their momenta will be equal. The momentum or the quantity of motion, therefore, depends upon the mass as well as the velocity of the body.

Example : *A body of mass 5 kg. is moving along a straight line with a velocity 2·4 metres/sec. What is its momentum?*

Ans. We know, momentum$=m\times v$; Here $m=5\times 1000=5000$ gm ; $v=2{\cdot}4$ metres/sec$=240$ cm/sec. So, the momentum$=5000\times 240=12\times 10^5$ gm. cm/sec.

1.5. Kinematical equations of rectilinear motion :

(*a*) *Determination of the distance traversed by a body moving with uniform velocity V for t sec :*

From the definition of velocity, we know,

In 1 sec, the particle travels a distance V
„ 2 „ „ „ „ „ „ $V\times 2$
∴ „ t „ „ „ „ „ „ $V\times t$

If the distance travelled be S, then $S=V\times t$

i.e. Distance = Uniform velocity × Time

Graphical representation : Let *OX* and *OY* be the axes along which time (t) and the uniform velocity (V) are plotted respectively. Suppose, the length *OC* represents the magnitude of the velocity V. Since, the velocity is uniform, there will be no change of velocity at any time and the velocity-time graph will be a straight line parallel to the axis OX. In this case, the straight line CSN is the velocity-time graph (Fig 1.11).

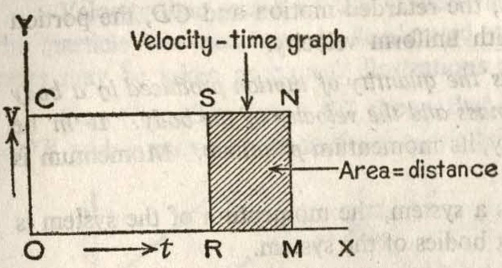


Fig. 1.11

If the length RM represents a small interval of time t_1 then the distance travelled by the particle in that interval=uniform velocity× time$=V\times t_1=OC\times RM=SR\times RM$ =the area of the rectangle $RMNS$.

If, now, OM represents the whole time t, then dividing it into small elements like RM, and summing up all such small rectangular areas, we will get the total distance travelled by the particle in time t.

So the area of the rectangle $OCNM$ = total distance travelled=S.

Hence, it can be said that the area bounded by the time-axis and the velocity-time graph represents the distance travelled by the particle.

Example : *A train is running with a uniform velocity 500 cm/sec. How far will it go in 10 minutes?*

Ans. Here, $t=10$ mnt.$=10\times 60$ sec.; $V=500$ cm/sec. We know, $S=V.\,t$

So, $S=500\times 10\times 60$ cm.$=\dfrac{500\times 10\times 60}{100}$ metres$=3000$ metres$=3$ kilometres.

(*b*) *Determination of the final velocity of a body travelling with uniform acceleration 'f' after 't' seconds* :

Suppose the initial velocity of the particle is u. Since, the acceleration of the particle is 'f', the change of velocity of the particle in 1 sec.$=f$. [See the definition of acceleration]

∴ After 't' seconds, the change of velocity$=f\times t$

Hence, after 't' seconds, the final velocity of the particle v=initial velocity + change of velocity$=u+f.\,t$.

If the particle starts from rest (*i.e.* its initial velocity $u=0$) the final velocity $v=ft$. If, on the other hand, the particle has a retardation f, the final velocity $v=u-ft$.

Graphical representation : OX and OY are two rectangular axes along which time 't' and velocity 'v' are plotted respectively (Fig. 1.12). Suppose, the length OB represents the initial velocity u. If a straight line BA is drawn through B parallel to OX, then it will give the uniform velocity-time graph. But as the particle possesses acceleration, its velocity is not uniform.

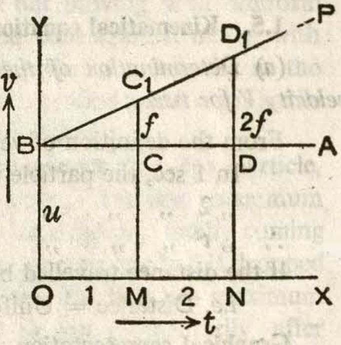


Fig. 1.12

If we suppose $CC_1=f$; $DD_1=2f$ and $OM=1$ second ; $ON=2$ seconds etc., then we can easily prove in the following way that B, C_1, D_1 etc. will be on a straight line BC_1D_1P which will represent the change of velocity with time.

CC_1=increase of velocity in 1 sec$=f\times 1=f\times OM=f\times BC \quad \therefore \quad \frac{CC_1}{BC}=f$

Again, DD_1=increase of velocity in 2 sec.$=f\times 2=f\times ON=f\times BD$

$$\therefore \quad \frac{DD_1}{BD}=f$$

Hence, $\frac{CC_1}{BC}=\frac{DD_1}{BD}$; So, B, C_1, D_1 etc. lie on the straight line BP which represents the equation $v=u+f.t.$

[**N.B.** If the particle starts from rest (*i.e.*, $u=0$), then B will coincide with O and the st. line BP passes through the point O. It then represents the equation $v=ft$ graphically.]

Examples : (1) *A particle starts with a velocity 15 cm/sec. and travels for 10 sec. with uniform acceleration 5 cm/sec.*^{2}. *What is its final velocity?*

Ans. Here, $u=15$ cm./sec ; $t=10$ sec ; $f=5$ cm./sec^2.

We know, $v=u+f.t$, So, $v=15+5\times 10=65$ cm./sec.

(2) *A motor car running with a velocity 30 km./hr. applied brakes and became stationary in 5 seconds. What retardation was produced?*

Ans. Speed$=30$ km./hr.$=\frac{30\times 1000}{60\times 60}$ metre/sec. $=\frac{50}{6}$ metre/sec.

Here, $u=\frac{50}{6}$ metre/sec. ; $t=5$ sec ; $v=0$ (because the car became stationary after 5 sec.). So, from, $v=u-ft$ (negative, because f is retardation).

we have, $0=\frac{50}{6}-f\times 5 \quad \therefore \quad f=\frac{50}{6\times 5}=1{\cdot}66$ metre/sec.^{2}

(3) *A jet plane starting from rest at one end of a runway acquires its take off speed of 160 metres/sec in 25 seconds. What is its acceleartion?*

Ans. *Acceleration* $f=\frac{160}{25}=6{\cdot}4$ *metre/sec*^{2}

(*c*) *Determination of the distance traversed by a particle in 't' seconds moving with an initial velocity 'u' and a uniform acceleration 'f' :*

Because of acceleration, the velocity of the particle is continually changing. Under such condition, if we want to find out the distance traversed by the particle, we are to first ascertain the average velocity of the particle and then we assume that the particle is moving throughout with this average velocity. In this case, the distance travelled by the particle in time 't' with changing velocity will be equal to that travelled in the same time with the average velocity.

Now, the velocity 1 sec after the commencement of the interval 't' $=u+1\times f$
and ,, ,, ,, ,, before ,, termination of the interval 't' $=v-1\times f$
[v=final velocity after the interval 't]

$$\therefore \text{ the average velocity}=\frac{(u+f)+(v-f)}{2}=\frac{u+v}{2}$$

In this way, if the average velocity is calculated by taking velocity at some time after the commencement and the velocity at equal time before the termination of the specified interval, it will be seen that the average velocity in all cases is $\frac{u+v}{2}$. So, the distance travelled by the particle,

$$S = \text{average velocity} \times \text{time} = \frac{u+v}{2} \times t$$

But, we know, $v = u + ft$. $\therefore\ S = \frac{u+u+ft.}{2} \times t = ut + \frac{1}{2}ft^2$

If the particle starts from rest (*i.e.*, $u=0$), $S=\frac{1}{2}ft^2$

If the particle moves with a retardation 'f', then $S = ut - \frac{1}{2}ft^2$.

Graphical proof : (i) **Proof of $S=\frac{1}{2}ft^2$.**

Time 't' is plotted along OX axis and the velocity v along OY axis (Fig. 1.13). Since the particle starts from rest and moves with uniform acceleration, its equation of motion ($v=ft$), is represented by a straight line OP which passes through the origin O. The length OQ expresses a certain interval of time 't'.

Take two very close points A and B on the st. line OP and drop perpendiculars AD and BC on the axis OX. Here, DC may be taken to represent a short interval θ out of the total interval 't'. Draw perpendiculars AE and BF on BC and DA (produced) respectively. The distance travelled by the particle in the short interval θ is, therefore, represented by an area that lies in between the areas of the rectangles $AECD$ and $FBCD$ (according to the graph of $S=Vt$). If the total time be divided into a number of such short intervals like θ, the above fact will hold good for each of the intervals. When the small interval 'θ' approaches zero, the sum of the areas of the outer rectangles like $FBCD$ and the sum of the areas of the inner rectangles like $AECD$ approach each other and each sum approaches the area of the triangle OPQ. Therefore the area of the triangle OPQ represents the space described by the particle in time 't'.

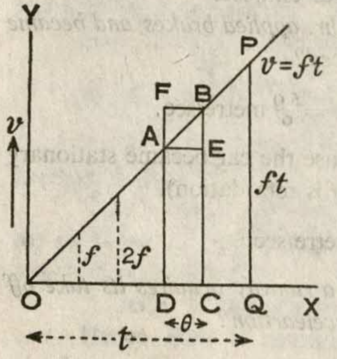


Fig. 1.13

Now, area of $\Delta POQ = \frac{1}{2}\ OQ \times PQ = \frac{1}{2}t \times ft = \frac{1}{2}ft^2\ \therefore\ S = \frac{1}{2}ft^2$.

(ii) **Proof of $S=ut+\frac{1}{2}ft^2$.**

As before, time 't' is plotted along OX axis and the velocity v along the perpendicular OY axis. The length OB represents the initial velocity u and the straight line BP the equation $v=u+ft$ (Fig. 1.14). The straight line BA parallel to OX evidently represents the velocity-time graph when there is no acceleration.

Take any point C on the line BP and draw a perpendicular CMN on OX axis. Suppose, ON represents any time 't'.

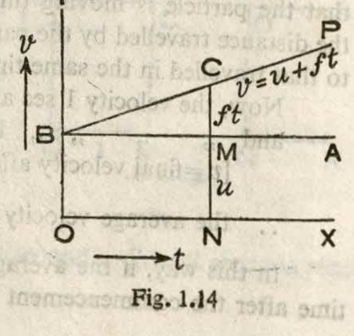


Fig. 1.14

Now, the area of the rectangle *BMNO* measures the distance travelled by the particle in time 't' with initial velocity u but without any acceleration. Again, the area of ΔBCM measures the distance travelled in the same time by the particle with uniform acceleration f but without any initial velocity (ref : proof of $S=\frac{1}{2}ft^2$). Therefore, the total area measures the total space travelled by the particle in time 't' with initial velocity u and uniform acceleration f. In other words, S=area of the rectangle $BMNO$+area of the $\Delta BCM=ON\times OB+\frac{1}{2}BM\times MC=t\times u+\frac{1}{2}t\times ft=ut+\frac{1}{2}ft^2$.

Examples : (1) *A particle, travelling with a velocity 60 cm/sec. covers 500 cm in 5 seconds. What is its acceleration?*

Ans. Here $u=60$ cm/sec ; $t=5$ sec ; $S=500$ cm.

We know, $S=ut+\frac{1}{2}ft^2$ So, $500=60\times5+\frac{1}{2}f\times(5)^2=300+\dfrac{25f}{2}$

or, $200=\dfrac{25f}{2}$ $\therefore$ $f=\dfrac{200\times2}{25}=16$ cm/sec^2

(2) *Starting from rest, a train moves with uniform acceleration and acquires a velocity of 30 miles/hr. in 2 minutes. How far will it go in 5 minutes?*

Ans. At first the uniform acceleration of the train will have to be ascertained. We have in this case, $u=0$; $v=30$ miles/hr$=44$ ft/sec ; $t=2$ mnt$=2\times60$ sec.

We know, $v=u+ft$. or, $44=0+f\times2\times60$ $\therefore$ $f=\dfrac{44}{2\times60}$ ft/sec^2

Now, $f=\dfrac{44}{2\times60}$ ft/sec^2 ; $t=5\times60$ sec ; $u=0$; $S=?$

Again, we know $S=ut+\frac{1}{2}ft^2=0\times5\times60+\frac{1}{2}\times\dfrac{44}{2\times60}\times(5\times60)^2$

$=16500$ ft$=3\frac{1}{8}$ miles.

(3) *Two motor cars start side by side from rest. The first car gets speed with an acceleration of 3 ft/sec² for 15 seconds and then goes on with uniform velocity. The second car moves with a constant acceleration of 25 ft/sec². What will be the distance between the two cars 25 sec, after the start?*

Ans. If S_1 be the distance travelled by the first car in first 15 seconds with acceleration 3 ft/sec^2, then $S_1=\frac{1}{2}f.\ t^2=\frac{1}{2}\times3\times15\times15=337{\cdot}5$ ft.

If V be the velocity produced after 15 seconds, then $V=f.t=3\times15=45$ft/sec.

Now, the distance S_2 travelled by the first car in the remaining 10 seconds with uniform velocity is given by $S_2=V.t=45\times10=450$ ft.

So, the total distance travelled by the first car$=337{\cdot}5+450=787{\cdot}5$ ft.

Again, the distance travelled by the second car with constant acceleration is given by, $S=\frac{1}{2}f.t^2=\frac{1}{2}\times25\times(25)^2=7812{\cdot}5$ ft.

Hence, the distance between the cars 25 sec. after the start$=(7812{\cdot}5-787{\cdot}5)=$ 7025 ft.

(*d*) *Determination of velocity acquired by a particle after travelling a distance 'S' with uniform acceleration 'f' :*

Let u be the initial velocity and S the distance travelled by the particle in time 't'. We know, $v=u+f.t$ and $S=ut+\frac{1}{2}ft^2$

Squaring the 1st equation, $v^2=(u+ft)^2$

$$=u^2+2u.ft+f^2t^2$$
$$=u^2+2f(ut+\tfrac{1}{2}ft^2)$$
$$=u^2+2.S \quad \text{[from 2nd equation]}$$

From this equation, the final velocity v can be found out.

If the particle starts from rest (*i.e.*, $u=0$), $v^2=2f.S$.

If the particle moves with a retardation f, then $v^2=u^2-2.f.S$

Examples : (1) *In describing a distance of 50 ft. the velocity of a particle changes from 15 ft/sec to 30 ft/sec. Calculate the acceleration of the particle.*

Ans. Here $u=15$ ft/sec ; $v=30$ft/sec ; $S=50$ ft.

We know, $v^2=u^2+2.f.S$,

Here, $(30)^2=(15)^2+2.f.\ 50$ or, $(30)^2-(15)^2=2f\times 50$

or, $45\times 15=2.f\times 50 \quad \therefore \ f=\dfrac{45\times 15}{2\times 50}=6{\cdot}7$ ft/sec².

(2) *A body is projected upwards with a certain velocity and it attains a height of 625 ft. What is the velocity of projection?*

Ans. Since the body moves upwards, its direction of motion is opposite to the gravity and hence it will have a ratardation of 32 ft/sec². As a result, its motion will gradually decrease and finally it comes to rest for a moment when it attains the height of 625 ft.

So, $v=0, f=-32$ ft/sec² ; $S=625$ ft ; $u=?$

We know, $v^2=u^2+2.f.S$. or, $0=u^2-2\times 32\times 625$.

or, $u^2=2\times 32\times 625 \ \therefore \ u=\sqrt{2\times 32\times 625}=200$ ft/sec.

(*e*) *Determination of the space traversed by a particle during a particular second of its motion :*

Space described in t th. sec =Space described in t seconds − space described in $(t-1)$ sec.

i.e., $S_t=(ut+\tfrac{1}{2}ft^2)-\{u(t-1)+\tfrac{1}{2}f(t-1)^2\}$

$$=ut+\tfrac{1}{2}ft^2-(ut-u+\tfrac{1}{2}.ft^2-ft+\tfrac{1}{2}f)$$
$$=u+f.t-\tfrac{1}{2}f=u+\tfrac{1}{2}f(2t-1)$$

Examples : (1) *A particle describes 25 cm. in 5th second and 33 cm. in 7th second of its motion. Calculate the initial velocity and the acceleration of the particle.*

Ans. $S_5=u+\tfrac{1}{2}f(2\times 5-1)$ and $S_7=u+\tfrac{1}{2}f(2\times 7-1)$

But $S_5=25$ cm. and $S_7=33$ cm. (given)

So, $25=u+\tfrac{1}{2}f\times 9$ and $33=u+\tfrac{1}{2}f\times 13$

Solving these two equations, we get $u=7$ cm/sec. and $f=4$ cm/sec².

(2) *A particle moves in a straight line with uniform acceleration and covers 40 cm and 50 cm in two successive seconds. Find the distance travelled in the next second.*

Ans. For the distance travelled in a particular second,

we have $S_t=u+\tfrac{1}{2}f(2t-1)$

For the 1st second, $40=u+\tfrac{1}{2}f(2-1)=u+\tfrac{1}{2}f$.

" " 2nd " $50=u+\tfrac{1}{2}f(4-1)=u+\tfrac{3}{2}f$.

Solving, we get $f=10$ cm/s^2 and $u=35$ cm/s.
For the 3rd second, $S_4=35+\frac{1}{2}\times 10(2\times 3-1)=35+25=60$ cm.

(3) *A motor car, travelling with uniform acceleration described one quarter of the whole distance in its last second of motion. Assuming the car to have started from rest, how long was it in motion and through what distance did it move, if it travelled 8 metres in the first second?*

Ans. Considering the motion in the first second, $u=0$, $S=8$ metres ; $t=1$ sec. and $f=?$ We have $S=\frac{1}{2}ft^2=\frac{1}{2}f\times 1$ $\therefore$ $\frac{1}{2}f=8$ or, $f=16$ m/sec^2.

Suppose the whole distance $=S$ and the time for describing the whole distance $=t$ sec. $\therefore$ $S=\frac{1}{2}ft^2=\frac{1}{2}\times 16\times t^2=8t^2\ldots$(i)

Also, the car moves $\frac{1}{4}$ S in the t-th second. So, from $S_t=u+\frac{1}{2}f(2t-1)$ we have, $\frac{1}{4}$ $S=0+\frac{1}{2}\times 16(2t-1)=16t-8$ or, $S=64t-32\ldots$(ii)

From equations (i) and (ii) we get $8t^2=64t-32$ or, $t^2=8t-4$ or. $t^2-8t+4=0$

Solving the equation, $t=4\pm 2\sqrt{3}=7{\cdot}46$ sec or $0{\cdot}54$ sec.

The second value of t being inadmissible, the total time of motion of the car $=7{\cdot}46$ sec.

The total distance travelled [from eqn. (ii)] $S=64\times 7{\cdot}46-32=477{\cdot}44$ metres.

1.6. Equations of motion according to calculus :

We can derive the kinematical equations of motion very conveniently by the application of calculus. Let a particle be in motion along a straight line with constant acceleration f. Then, $f=\frac{dv}{dt}$. or, $dv=f.\,dt$.

Integrating both sides, $\int dv=\int f.dt$ or, $v=ft+C_1$ . .(i) where C_1 is a constant, known as the constant of integration.

If the particle starts from rest, then at $t=0$, $v=0$. Substituting this value in eqn. (i) we get $C_1=0$ and hence $v=ft$

If u and v be the initial and final velocities of the particle at the beginning and at the end of a time interval t, we have $v=u$ at $t=0$, so that $C_1=u$. Substituting this value of C_1 in eqn. (i) we get $v=u+ft$.

If ds be the small distance travelled by the particle in an infinitesimally short interval of time dt during which the velocity v can be supposed to be constant, then, $ds=v.dt=(u+ft)\,dt$.

Integrating, $\int ds=\int (u+ft)dt$. or, $S=ut+\frac{1}{2}ft^2+C_2\ldots$.(ii) where C_2 is another constant of integration.

Since at $t=0$, $S=0$, hence $C_2=0$. $\therefore$ $S=ut+\frac{1}{2}ft^2$

Again since $f=\frac{dv}{dt}$, we can write $f=\frac{dv}{ds}\cdot\frac{ds}{dt}=v\frac{dv}{ds}$ or, $v.dv=f.ds$

Integrating, $\int v.\,dv=\int f.\,ds$ or $\frac{v^2}{2}=f.S+C_3\ldots$(iii) where C_3 is another constant of integration.

Now, at $t=0$, $v=u$ and $S=0$; So, $\frac{u^2}{2}=C_3$ substituting this value in eqn. (iii) we get, $\frac{v^2}{2}=\frac{u^2}{2}+f.S$ or, $v^2=u^2+2f.S.$

Examples : (1) *The displacement x of a particle moving in one dimension is related to the time t by the equation $t=\sqrt{x}+3$; where x is in metres and t in seconds. Find (i) the displacement of the particle when its velocity is zero (ii) the displacement and acceleration after 6 seconds.* [*I.I.T. 1979*]

Ans. $t=\sqrt{x}+3$ or, $\sqrt{x}=t-3$.

Differentiating, $\frac{1}{2\sqrt{x}}\cdot\frac{dx}{dt}=1 \quad \therefore \quad \frac{dx}{dt}=v=2\sqrt{x}$

So, when velocity $v=0$, $2\sqrt{x}=0$ or $x=0$ *i.e.* the displacement is zero.

(ii) Displacement after 6 seconds can be obtained by putting $t=6$ in the equation $t=\sqrt{x}+3$ *i.e.* $6=\sqrt{x}+3 \quad \therefore \quad x=9$ metres

Again, acceleration $f=\frac{dv}{dt}=\frac{d}{dt}\left(2\sqrt{x}\right)=2.\ \frac{1}{2\sqrt{x}}\ \frac{dx}{dt}=\frac{1}{\sqrt{x}}\times 2\sqrt{x}$ $=2$ metres/s²

(2) *A particle moves in a straight line under a retardation $a.v^{n+1}$, where v is the velocity at time t. Show that if u is the velocity of the particle at $t=0$, then*

$$a.\ t=\frac{1}{n}\left(\frac{1}{v^n}-\frac{1}{u^n}\right)$$

Find also a corresponding formula for space in terms of v.

Ans. We have $\frac{dv}{dt}=-av^{n+1}$ (minus because retardation) or, $\frac{dv}{v^{n+1}}=-a.dt$

Integrating, $\int\frac{dv}{v^{n+1}}=\int -a.dt$ or, $-\frac{1}{n.v^n}=-at+c_1$ where c_1 is a constant.

But at $t=0$, $v=u$. So, $-\frac{1}{n.u^n}=c_1$

Substituting the value of c_1, $-\frac{1}{n.v^n}=-at-\frac{1}{n.u^n}$ or, $a.t=\frac{1}{n}\left(\frac{1}{v^n}-\frac{1}{u^n}\right)$

Again, $\frac{dv}{dt}=v.\frac{dv}{ds}=-a.v^{n+1}$ (according to the problem)

or, $\frac{v.dv}{v^{n+1}}=-a.ds.$ or, $-\frac{dv}{v^n}=-a.ds$

Integrating, $\int\frac{dv}{v^n}=\int -a.ds$ or, $-\frac{1}{(n-1)}\cdot\frac{1}{v^{n-1}}=-a.s+c_2$ where c_2 is a constant.

Now, at $t=0$, $u=v$ and $s=0$; so, $-\frac{1}{(n-1)}\cdot\frac{1}{u^{n-1}}=c_2$

Substituting the value of the constant we get,

$$-\frac{1}{(n-1)}\cdot\frac{1}{v^{n-1}}=-a.s-\frac{1}{(n-1)}\cdot\frac{1}{u^{n-1}} \quad \text{or, } a.s=\frac{1}{(n-1)}\left[\frac{1}{v^{n-1}}-\frac{1}{u^{n-1}}\right]$$

Miscellaneous examples on equations of motion :

(1) *Two paper screens A and B are separated by a distance of 100 metres. A bullet pierces A and then B. The hole in B is 10 cm. below the hole in A. If the bullet is travelling horizontally at the time of hitting the screen A, calculate the velocity of the bullet when it hits the screen A. Neglect the resistance of paper and air.* $g=980\ cm/sec^2$. [*I.I.T. 1970*]

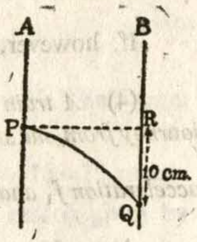


Fig. 1.15

Ans. Let t be the time required by the bullet to fall vertically downwards through 10 cm. From $S=ut+\frac{1}{2}ft^2$, we have $S=10$ cm. ; $u=0$; $f=g=980$ cm/sec².

$\therefore\ 10=\frac{1}{2}\times980\times t^2$ or $t=\frac{1}{7}$ sec.

The bullet travelled horizontally through 100 metres in $\frac{1}{7}$ sec. So, its horizontal velocity at the time of striking the screen $A=\dfrac{100}{\frac{1}{7}}=700$ metres/s.

(2) *A particle starting from rest, moves with uniform acceleration for a time t_1. Then it covers a distance a, moving with uniform velocity for time t_2. Afterwards, it moves with uniform retardation and comes to rest after time t_3. Show that the total distance travelled by the particle is* $a\left(1+\dfrac{t_1+t_3}{2t_2}\right)$.

Ans. Let f be the uniform acceleration and v the velocity acquired by the particle after t_1. Then, $v=f.t_1$. Since it travels a distance a in t_2 with uniform velocity generated at the end of time t_1, we have $a=v.t_2=f.t_1t_2\ \therefore\ f=\dfrac{a}{t_1t_2}$

Therefore, $v=\dfrac{a}{t_1t_2}\times t_1=\dfrac{a}{t_2}\ldots\ldots$(i)

Again the distance S_1 travelled by the particle during the interval t_1 is given by $S_1=\frac{1}{2}.f.t_1^2=\frac{1}{2}.\dfrac{a}{t_1t_2}\times t_1^2=\frac{1}{2}\dfrac{a.t_1}{t_2}\ldots\ldots$(ii)

If the uniform retardation at the last stage of motion be f' then, since the final velocity is zero and the initial velocity is v, we have, from $v=u+ft$, $0=v-f'.t_3$

or, $f'=\dfrac{v}{t_3}=\dfrac{a}{t_2t_3}$ [from eqn. (i)]. Now, the distance S_2 travelled during the last stage is given by $0=v^2-2f'.S_2$ or, $S_2=\dfrac{v^2}{2f'}=\dfrac{a^2}{t_2^2}\times\frac{1}{2}.\dfrac{t_2t_3}{a}=\frac{1}{2}.\dfrac{at_3}{t_2}\ldots\ldots$(iii)

So, the total distance $=S_1+a+S_2=\frac{1}{2}.\dfrac{at_1}{t_2}+a+\frac{1}{2}.\dfrac{at_3}{t_2}=a\left(1+\dfrac{t_1+t_3}{2t_2}\right)$

(3) *A constable running with a velocity of u ft/sec., saw a thief x ft ahead and started to run with an acceleration of α ft/sec². The thief, who was at rest, began to run with an acceleration of β ft/sec². Show that the constable will catch the thief* $\alpha\geq\beta$ *or* $\alpha<\beta<\alpha+\dfrac{u^2}{2x}$.

Ans. Let the constable catch the thief after a time t and suppose S is the distance run by the constable in the mean time. Then, $S=ut+\frac{1}{2}\alpha t^2$. The thief runs a distance of $(S-x)$ from rest with an acceleration β. Therefore $S-x=\frac{1}{2}\beta.t^2$

Subtracting, $x=ut+\frac{1}{2}(\alpha-\beta).\ t^2$ or $\frac{1}{2}(\alpha-\beta)t^2+ut-x=0$

$\therefore\ t=\{-u\pm\sqrt{u^2+2x(\alpha-\beta)}\}/(\alpha-\beta)$

Since x is $+ve$, $u^2+2x(\alpha-\beta)\geq 0$ i.e. $\alpha\geq\beta$.

If, however, $\alpha<\beta$, $u^2>2x(\beta-\alpha)$ or, $\frac{u^2}{2x}+\alpha>\beta$; then $\alpha<\beta<\frac{u^2}{2x}+\alpha$

(4) *A train stopping at two stations 2 miles apart takes 4 minutes on the journey from one station to the other. Assuming that its motion is first that of uniform acceleration f_1 and then that of uniform retardation f_2, show that* $\frac{1}{f_1}+\frac{1}{f_2}=4$

Ans. Let v be the velocity attained after the first part of the journey. As the train starts from rest, we have, $v^2=2f_1S_1$ and $v=f_1t_1$ where S_1 and t_1 are the distance and time for the first part. Since the train comes to rest after the second part of the journey, we get $v^2=2f_2S_2$ and $v=f_2t_2$ where S_2 and t_2 are the distance and time respectively for the second part.

From these relations, we get, $\frac{1}{f_1}+\frac{1}{f_2}=\frac{2(S_1+S_2)}{v^2}=\frac{4}{v^2}\ldots\ldots$(i)

and, $\frac{1}{f_1}+\frac{1}{f_2}=\frac{t_1+t_2}{v}=\frac{4}{v}\ldots\ldots\ldots\ldots$(ii)

From (i) and (ii) we get, $v=1$; substituting this value in any one of the above equations we get, $\frac{1}{f_1}+\frac{1}{f_2}=4$

(5) *A particle moving with uniform acceleration along a straight line passes through points A, B and C. If $AB=BC=x$ and the time taken by the particle in moving from A to B is t_1 and that in moving from B to C is t_2, prove that the acceleration of the particle is $f=2x(t_1-t_2)/t_1t_2(t_1+t_2)$*

Ans. Let u be the velocity of the particle when it arrrives at A. Since it takes time t_1 to travel a distance $AB=x$, we have,

$x=ut_1+\frac{1}{2}ft_1^2$ or, $\frac{x}{t_1}=u+\frac{1}{2}ft_1\ldots\ldots$(i)

If v be the velocity acquired by the particle when it arrives at B after time t_1, we have $v=u+ft_1$. Again since the particle covers the distance $BC=x$ in time t_2, we can write $x=vt_2+\frac{1}{2}ft_2^2$ or, $\frac{x}{t_2}=v+\frac{1}{2}ft_2=u+ft_1+\frac{1}{2}ft_2\ldots\ldots$(ii)

Subtracting (i) from (ii), we get, $\frac{x}{t_2}-\frac{x}{t_1}=\frac{1}{2}ft_1+\frac{1}{2}ft_2=\frac{1}{2}f(t_1+t_2)$

$$\therefore\ f=\frac{2x(t_1-t_2)}{t_1t_2(t_1+t_2)}$$

(6) *A mail train travelling with a speed of 108 km/hr, is 200 metres behind a goods train which is also moving on the same track in the same direction with a speed of 18 km/hr. The driver of the mail train immediately applies brakes so as to produce a constant retardation of 2 m/sec². If the goods train continues with the same constant speed, find whether there will be a collision between the two trains.*

Ans. Speed of the mail train $=108 \text{ km/hr}=\frac{108\times 1000}{60\times 60}=30 \text{ m/s}.$

" " " goods train $=18 \text{ km/hr}=\frac{18\times 1000}{60\times 60}=5 \text{ m/s}.$

Let t be the time after which the mail train comes to rest. Then from $v=u+ft$ we get, $0=30-2\times t$ or, $t=15$ sec.

During this time, the distance travelled by the goods train $=5\times 15=75$ metre.

The distance travelled by the mail train before coming to rest is given by

$0=(30)^2-2\times 2\times S$ or, $S=225$ metre.

So, the total distance covered by the goods train from the point where the mail train applies the brakes and the point where it comes to rest $=200+75=275$ metre but the mail train covers a distance of 225 metres. Hence, there will be no collision.

(7) *A car accelerates from rest at a constant rate* α *for some time after which it decelerates at a constant rate* β *to come to rest. If the total time-lapse is* t *seconds, evaluate* (*i*) *the maximum velocity reached and* (*ii*) *the total distance travelled.*

[*I.I.T. 1978*]

Ans. Let at C distant x_1 from the starting point A, the velocity becomes maximum. From C, at distance x_2 (say) the car comes to rest at B. Let the time for journey from A to C be t_1 and that from C to B be t_2. Then, $t_1+t_2=t$

For the journey from A to C, we have, $x_1=\frac{1}{2}\alpha t_1^2$ and $v_1=\alpha t_1$ where v_1 is the maximum velocity at C.

For the journey from C to B, we have, $x_2=\frac{v_1^2}{2\beta}$ [from equation $v^2=u^2+2f.S$; we have $v=0$; $u=v_1$; $f=-\beta$ and $S=x_2$] and $v_1=\beta.t_2$

$$\therefore \quad \alpha t_1=\beta t_2 \text{ or, } \frac{\alpha}{\beta}=\frac{t_2}{t_1} \text{ or, } \frac{\alpha+\beta}{\beta}=\frac{t_1+t_2}{t_1}=\frac{t}{t_1} \quad \therefore \quad t_1=\frac{t.\beta}{\alpha+\beta}$$

Substituting this value, $v_1=\frac{\alpha\beta.t.}{\alpha+\beta.}$ So, the maximum velocity $=\frac{\alpha.\beta.t.}{\alpha+\beta}$

Again total distance travelled $=x_1+x_2=\frac{1}{2}\alpha t_1^2+\frac{v_1^2}{2\beta}$

$$=\tfrac{1}{2}\alpha\left(\frac{\beta}{\alpha+\beta}\right)^2 . \quad t^2+\tfrac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)^2\frac{t^2}{\beta}=\tfrac{1}{2}.\ \frac{\alpha\beta}{(\alpha+\beta)^2}(\alpha+\beta)t^2=\tfrac{1}{2}\frac{\alpha\beta}{(\alpha+\beta)}t^2$$

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Exercises

Essay type :

1. Define acceleration. Establish the equation $v=u+f.t$.

2. Prove the equation $S=ut+\frac{1}{2}f.t^2$, algebraically and graphically.

3. Show that the instantaneous acceleration at any point in the velocity-time graph equals the slope of the tangent to the graph at that point. Which one is more important in the laws of mechanics—the average acceleration or the instantaneous acceleration ?

4. Prove the equations (i) $v^2=u^2+2.fs$ and (ii) $S_t=u+\frac{1}{2}f(2t-1)$.

Short answer type :

5. Define the terms :—(i) Speed (ii) Velocity, (iii) Acceleration (iv) Momentum.

6. Answer the following questions :—(i) What is the difference between speed and velocity (ii) Why is 'time' mentioned twice in the unit of acceleration? (iii) What is the difference between the distance travelled in 5 seconds and that travelled in 5th. second ?

7. (i) Can a body have an acceleration with zero velocity ?
(ii) Can a body have velocity without acceleration ?
(iii) Can a body moving with uniform speed have variable velocity ?
(iv) Can a body moving with uniform velocity have variable speed ?
(v) Can a body have acceleration if its speed is constant ?
(vi) A particle is moving eastward with a uniform velocity. It suddenly changes its direction and moves northward with the same uniform velocity. Does the particle possess any acceleration ?

8. (a) Answer the following, giving reason in brief : Is the time variation of position, shown in the fig 1.16 observed in nature ?

[*Hints* : No ; time decreases with the increase of x is not known to us.] [*I.I.T. 1979*]

(b) A train is moving along a straight track. Explain what would be the nature of its velocity-time graph in the following cases : (i) It is moving with uniform acceleration (ii) its acceleration is increasing. [*H. S. 1975*]

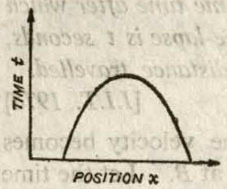


Fig. 1.16

9. State the conditions under which each of the following motions take place :

(i) a particle has a velocity but no acceleration (ii) a particle has a constant acceleration (iii) a particle has a retardation (iv) a particle has varying acceleration. Give one example in each case.

Objective type :

10. Mention the correct answer with a √ mark :—

(a) The velocity of a particle moving with uniform acceleration (i) increases (ii) decreases (iii) remains unaltered.

(b) A man goes round a square garden 50 metres long and comes back to the starting point. His displacement is (i) 50 metres (ii) 200 metres (iii) zero.

(c) A particle going in a circle with uniform velocity has an acceleration (i) directed away from the centre, (ii) directed towards the centre (iii) directed along the tangent at any point on the circle.

(d) A body moving in a curved path possesses a velocity v towards north at any instant of its motion. After t sec., the velocity of the body was found to be v towards west. The average acceleration of the body during the interval is (i) along South-West (ii) along North-East (iii) zero.

(e) A ball, moving with velocity v strikes against a wall and returns along the same path with same velocity. If m be the mass of the ball, the change of momentum of the ball is (i) mv (ii) zero (iii) $-mv$ (iv) $2mv$.

(f) A particle is moving eastward with a velocity of 5 metre/s. In 10 seconds, the velocity changes to northward. The average acceleration in this time is (i) zero (ii) $1/\sqrt{2}\ m/s^2$ towards $N-W$ (iii) $1/\sqrt{2}\ m/s^2$ towards $N-E$ (iv) $\frac{1}{2}\ m/s^2$ towards $N-W$ (v) $\frac{1}{2}\ m/s^2$ towards north.

Numerical problems :

11. The velocity of a particle changed from one foot per second to 1 mile per hour in 1 minute. If yard and minute are taken as units of length and time respectively, what would be acceleration of the particle ? [Ans. $9\frac{1}{3}$ yds/min^2.]

12. A particle starting from rest acquired a velocity of 60 miles/hr. in two minutes. What would be the acceleration of the particle in the F. P. S. system ? [Ans. 0·73 ft/sec^2]

13. If the initial velocity and the acceleration of a particle be 200 ft/sec and 10 ft/sce^2, respectively, what velocity would it acquire in $\frac{1}{4}$ minute ? [Ans. 350 ft/sec.]

14. A train, travelling with a speed of 60 miles/hr. applied brakes and, acquired a retardation of 4 ft/sec.^{2}. What would be its speed after 10 sec. ? How long after would the train come to rest ? [Ans. 48 ft/sec. ; 22 sec.]

15. A particle, travelling with a velocity of 80 cm./sec., described a distance of 1000 cm. in 10 sec. What was its acceleration ? [Ans. 4 cm./sec.^{2}]

16. A train, travelling with a speed of 30 ft./sec. gathered an acceleration of 3 ft./sec^2. How far would it go in 15 sec. ? [Ans. 787·5 ft.]

17. In describing a distance of 100 cm., the velocity of a particle increased from 20 cm./sec. to 50 cm./sec. What was its acceleration ? [Ans. 10·5 cm./sec^2]

18. A motor car, starting from rest, acquired an acceleration of 3 ft./sec.^{2} in 6 seconds. Thereafter, the car moved with uniform velocity for half a minute and applying brakes came to standstill in 5 seconds. Find the greatest velocity attained by the car in miles/hr. unit and also the distance travelled by it, in foot ? [Ans. 12·3 miles/hr. ; 639 ft.]

19. A train started from rest and moved with constant acceleration . At one time, it was travelling at the rate of 30 ft/sec and 160 ft further on, it was travelling at 50 ft/sec. Calculate (a) the acceleration (b) the time required to travel the 160 ft mentioned earlier (c) the time required to attain the speed of 30 ft/sec. [Ans. (*i*) 5 ft/s^2; (*ii*) 4 sec; (*iii*) 6 sec.]

20. A car moving with retardation took 20 sec. and 30 sec. to travel two successive quarter kilometres. How much further would it travel before coming to rest ? [Ans. 102·1 metres]

21. A train travelled 81 kilometres. During the first one third of the journey it had a velocity of 9 km/hr. and during the 2nd one third of the journey the velocity was 18 km/hr., and during the last one, the velocity was 21 km/hr. What was the average velocity of the train ? [Ans. 14 km/hr.]

22. A particle describes 72 cm. in twelfth second and 96 cm. in sixteenth second of its motion Calculate its initial velocity and acceleration. [Ans. 3 cm./sec. : 6 cm./sec.^{2}]

23. (a) The acceleration of a body is given by $f=2.t^{\frac{1}{2}}$. Apply calculus to find (i) instantaneous velocity after $t=4$ sec. and (ii) distance travelled after $t=4$ sec. [Ans. $v=10{\cdot}66$ units ; $S=17{\cdot}06$ units]

(b) The instantaneous velocity of a particle is given by $v=u+ft$. Assuming u and f to be constants, use calculus to find the equation for the instantaneous distance travelled. [Ans. $S=ut+\frac{1}{2}ft^2$]

Harder problems :

24. A particle has a displacement 12 ft. towards east and 9 ft. towards the north and then 6 ft. vertically upward. Find the magnitude of the sum of these displacements. [Ans. 14·32 ft.]

25. A shell from a naval gun barrel 3 metre long has a muzzle velocity of 820 m./s. Find (a) the average speed of the shell while being accelerated in the gun barrel (b) time taken to travel the length of the barrel (c) the acceleration. [Ans. (a) 410 m/s (b) ·0073 sec. (c) $1{\cdot}12\times10^5$m/s^2]

26. An alpha particle travels along the inside of a straight hollow tube 2 metres long. If it travels with uniform acceleration how long is the particle in the tube if it enters at a speed of 10^3 metres/s and leaves at 9×10^3 metres/s ? What is the acceleration during this interval ? [Ans. 4×10^{-4} sec ; 2×10^7 metre/s^2]

27. Two trains are approaching each other along the same straight line with velocities 60 miles/hr. and 80 miles/hr. respectively. When the distance between them is 2 miles, each driver applies brakes to produce a retardation of 3 ft./sec.^{2} in each train. Find whether there will be a collision or not. [Ans. No collision]

28. Two trains are approaching each other on the same line with velocities u_1 and u_2. When their distance apart is s, they see each other and at once pull the brakes. If the retardations of the trains be f_1 and f_2 respectively, show that it is just possible to avoid a collision if $u_1^2f_2+u_2^2f_1 =2f_1f_2$.

29. Two trains are running along the same line and with same speed (30 km./hr.). The distance apart between the trains is 5 km. What should be the speed of another train, coming from the opposite direction so that it will meet the other two trains at an interval of 4 minutes. ?

[*Hints* : Reqd. velocity$=v$ km./hr. Its velocity w.r.t. the two trains$=(v+30)$ km./hr. $\therefore\ (v+30)\times\frac{4}{60}=5$] [Ans. 45 km./hr.]

30. A train of mass M is moving with uniform velocity on a level track. A carriage of mass m suddenly got uncoupled from the rear of the train but the driver came to know of it after going a further distance l. He at once, shut off the supply of steam. Prove that when both the parts of the train come to rest, the distance apart is $\dfrac{Ml}{M-m}$ given the resistance against motion offered by the rails is proportional to the mass.

31. A bus is moving with uniform velocity. A motor-cycle, 40 metre ahead of the bus, starts from rest and moves along the same road with an acceleration 20 cm./s^2. Prove that the bus cannot overtake the motorcycle if the speed of the bus is less than 4 m/sec. Show also, that the motor-cycle and the bus will meet each other twice if the speed of the bus is greater than 4m/s.

32. A car starts from rest and is accelerated uniformly at the rate of 3 m/s^2 for 10 sec. It then maintains a constant speed for 20 sec. The brakes are then applied and the car comes to rest in 5 sec. Find the total distance travelled by the car. What is the maximum speed attained by the car ? [Ans. 825 m.; 30 m/s.]

33. A train starts from a station A, accelerates at the rate of 2 m/s^2 and reaches its maximum speed in 10 seconds. It maintains this speed for half an hour and then retards uniformly. It coms to rest at the station B after further 20 seconds. Calculate (i) the maximum speed of the train (ii) the retardation and (iii) the distance between the station *A* and *B*.

[Ans. (i) 72 km/hr (ii) 1 metre/s^2 (iii) 36·3 km]

34. A car *A* is travelling on a straight level road with a uniform speed of 60 km./hr. It is followed by another car *B* which is moving with a speed of 70 km./hr. When the distance between then is 2·5 km., the car *B* is given a retardation of 20 km./hr.^{2}. After what distance and time will *B* catch up with *A* ? [*I.I.T. 1966*] [Ans. 32·5 km. ; 30 mnt.]

35. Two trains take 3 seconds to pass one another when going in opposite directions but only 2·5 seconds if the speed of one is increased by 50%. How long would one take to pass the other when going in the same direction at their original speeds ? [Ans. 15 sec.]

36. A particle, moving with uniform acceleration describes distances s_1 and s_2 ft. in successive intervals of t_1 and t_2 seconds. Prove that the acceleration of the particle is $2(s_2t_1-s_1t_2)/t_1t_2(t_1+t_2)$.

37. A body moving with uniform acceleration travels 65 ft. in 5th second and 105 ft. in 9th second. How far will it go in 10th and in 10 seconds ? [Ans. 215 ft. ; 700ft.]

38. A rocket is fired vertically from the ground with a resultant vertical acceleration of 10 metres/s^2. The fuel is finished in 1 minute and it continues to move up. What is the maximum height reached ? $g=9{\cdot}8$ m/s^2 [*I.I.T. 1975*] [Ans. 36·4 metre.]

[*Hints* : Velocity after 1 minute, $v=f.t=10\times60=600$ m/s.

Distance covered after the fuel is finished, $v^2=u^2-2fs_1$ or $0=(600)^2-2\times9{\cdot}8\times s_1$ or $s_1=18{\cdot}4$ metre. Distance covered before the fuel is finished. $s_2=ut+\frac{1}{2}ft^2=0+\frac{1}{2}\times10\times60\times60$ or $s_2=18$ metre.]

39. The accelerated motion of a particle along straight line is given by $s=\frac{1}{2}.vt$ where s is the distance of the particle from some fixed point on the line at time t and v its velocity. Show, by the use of calculus that the particle has a constant acceleration.

40. A particle moving in a straight line is subject to a resistance which produces the retardation kv^3 where v is the velocity and k is a constant. Show that v and t are given in terms of s by the equations $v=u/(1+ksu)$ and $t=\frac{1}{2}\,ks+s/u$ where u is the initial velocity.

2

SCALAR AND VECTOR

2.1. Scalar and Vector : We are, by this time, acquainted with two kinds of quantities, viz, *scalar quantities* like speed, mass, volume, temperature etc. which have only magnitude and no direction and *vector quantities* like velocity, acceleration, force, momentum etc. which have both magnitude and direction. In this chapter we shall discuss more about these two kinds of quantities.

2.2. Representation of a scalar and a vector : Since, scalar quantities have only magnitude, they can be simply represented by a number on a suitable unit of measurement, viz, 5 hours' time, 2 kilos of rice, 20 c.c. volume etc. But vector quantities, which have both magnitude and direction, are represented by *directed line segment* with the indication of a linear scale unit.

Suppose a force of 4 dynes acting eastward is to be represented vectorially. Choosing a length *MN* of 1 cm. length representing a force of 1 dyne, we are to draw a line segment *AB* [Fig. 2.1] of length 4 cm and put an arrow head at the end of the segment pointing eastward. Such directed line segment will represent the force vector of 4 dynes. The length of the line segment represents the magnitude of the vector and the arrow represents the direction.

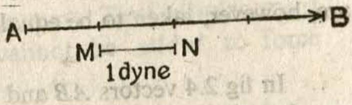


Fig. 2.1

2.3. A vector in geometry :

A vector with initial point *A* and terminal point *B* is represented as $\overrightarrow{AB}$ (Fig. 2.1). A vector can also be denoted by a single letter (Fig. 2.2). In printing, this letter is given in bold face type (**a**) but in writing it is given with an arrow $(\vec{a})$.

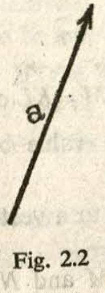


Fig. 2.2

The length of a vector is called the *absolute value* or the *modulus* of the vector. It is to be remembered that the absolute value of a vector is a *scalar quantity*.

2.4. Some facts about vectors :

(i) **Collinear vectors :** Vectors lying on parallel straight lines or on one and same the straight line are termed *collinear*. The vectors **a, b** and **c** in fig. 2·3 are collinear, as also the vectors $\overrightarrow{AC}$, $\overrightarrow{BD}$ and $\overrightarrow{CB}$. From the figure, it is evident that collinear vectors can have the same direction or they can have opposite directions. Thus, the vectors **a** and **c** are collinear and in the same direction, but the vectors **a** and **b** (and also **b** and **c**) are collinear but in opposite directions. Similarly, the vectors $\overrightarrow{AC}$ and $\overrightarrow{BD}$ are in the same direction while the vectors $\overrightarrow{AC}$ and $\overrightarrow{CB}$ are in opposite directions. But they are all collinear.

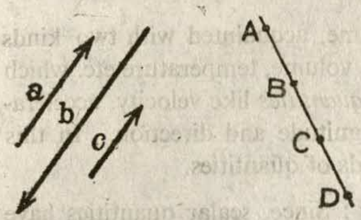


Fig. 2.3

(ii) **Null vector :** If the initial point A and the terminal point B of a line segment AB coincide, then the segment AB becomes a point and loses direction. However, for the sake of generality of the rules of vector algebra, such a pair of coincident points is regarded as a vector, called the *null vector*. It is considered collinear with any vector.

The null vector is symbolised by $\vec{0}$ (zero).

(iii) **Equality of vectors :** Two vectors **a** and **b** are said to be equal if they are in same direction and have one and the same absolute value. All null vectors are, however, taken to be equal.

In fig 2.4 vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are equal but the vectors $\overrightarrow{OM}$ and $\overrightarrow{ON}$ are not equal (although they are of same length) because they have different directions. The vectors $\overrightarrow{ON}$ and $\overrightarrow{KL}$ are likewise not equal while the vectors $\overrightarrow{OM}$ and $\overrightarrow{KL}$ are equal.

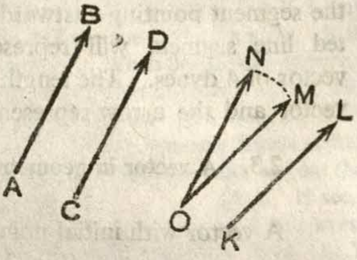


Fig. 2.4

When the vectors **a** and **b** are equal, we write **a**=**b** ; If their moduli are equal, we write $|\mathbf{a}|=|\mathbf{b}|$. Here, the vectors **a** and **b** may or may not be equal to one another.

From fig. 2.4, we can write $\overrightarrow{AB}=\overrightarrow{CD}$; $\overrightarrow{ON}\neq\overrightarrow{KL}$; $|\overrightarrow{ON}|=|\overrightarrow{KL}|$; $\overrightarrow{OM}=\overrightarrow{KL}$ etc.

(iv) **Opposite vectors :** Two vectors having the same absolute value but opposite directions are called *opposite vectors*.

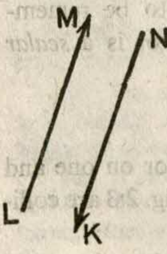


Fig. 2.5

A vector which is in a direction opposite to a vector **a** is denoted by $-\mathbf{a}$. In fig 2.5, the vectors $\overrightarrow{LM}$ and $\overrightarrow{NK}$ are in opposite directions. If the vector $\overrightarrow{LM}$ is denoted by **a**, then $\overrightarrow{NK}=-\mathbf{a}$; $\overrightarrow{ML}=-\mathbf{a}$; $\overrightarrow{KN}=\mathbf{a}$. From the definition, it follows $-(-\mathbf{a})=\mathbf{a}$ and $|-\mathbf{a}|=|\mathbf{a}|$.

2.5. Addition of scalars : The advantage of scalar quantities is that they can be added together or subtracted from each other by simply adding or subtracting the magnitudes of the quantities. If, for example, the masses of three substances are 5 kg, 10 kg and 15 kg, respectively, their total mass$=(5+10+15)=30$ kg. Again suppose, a boy takes 5 hours' time to perform an experiment as well as the result by calculation. He takes 3 hours' time only to perform the experiment. The boy therefore, needs $(5-3)=2$ hours' time for calculation.

It is to be remembered that addition or subtraction in the above manner is only possible if the scalar quantities are of same nature. Thus, mass may be added to or subtracted from mass but mass can not be added to or subtracted from volume.

2.6. Addition of vectors : The above simple method is not applicable to vectors because in the case of vectors, direction along with the magnitude is to be considered. Addition of vectors may be done with the help of two laws :—(i) *law of triangle* and (ii) *law of parallelogram.* In both cases, algebraic and geometric methods may be applied.

In this connection, it is worthwhile to mention that like scalars, addition of several vectors is only possible if the vectors are of same nature. Thus, displacement vector can be added to displacement vector but it cannot be added to force vector.

The law of triangle : (*a*) *Geometric method :*

Suppose $\overrightarrow{AB}$ is a vector, the point *A* being the initial point and *B*, the terminal point. The absolute value of the vector is $|\mathbf{a}|$.

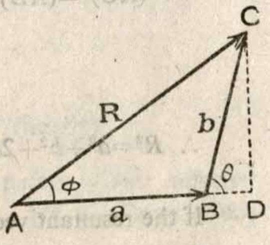


Fig.2.6

Another vector of the same nature $\overrightarrow{BC}$, having the absolute value $|\mathbf{b}|$ has the initial point at *B* and terminal point at *C*. These two vectors are to be added [Fig. 2.6].

If the triangle *ABC* is completed by joining *A* and *C*, then the resultant vector obtained by summing up the above two vectors, is represented by $\overrightarrow{AC}$. We shall get the absolute value $|\mathbf{R}|$ of the resultant $\overrightarrow{AC}$ if we measure its length by the same scale which is used to express the absolute values $|\mathbf{a}|$ and $|\mathbf{b}|$ of the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$ respectively. Measuring the $\angle BAC$ with the help of a protractor, we get the direction of the resultant vector. According to vector notation we may write, $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$ or $\vec{a}+\vec{b}=\vec{R}$.

Thus, if two sides of a triangle, $\vec{AB}$ and $\vec{BC}$, taken in order, represent two vectors, then the third side $\vec{AC}$, taken in reverse order, will represent their resultant vector. This is the law of triangle.

It is to be noted here that the absolute value of the resultant vector is not the sum of the absolute values of the vectors given.

Suppose a ship is sailing due east with a velocity 30 km/hr and a passenger on the deck of the ship is walking along north-west direction at a rate of 6 km/hr. In this case, we can find the direction and the magnitude of the velocity of the passenger relative to the sea by the vector addition. Since velocity is a vector, let us take a segment $\vec{OA}$ of a line representing the velocity of the ship in magnitude and direction [Fig 2.7]. From the point A, another segment $\vec{AB}$ is drawn along north-west direction such that it represents the velocity of the passenger. The resultant of these two vectors, according to the law of triangle is $\vec{OB}$ which represents the velocity of the passenger relative to the sea.

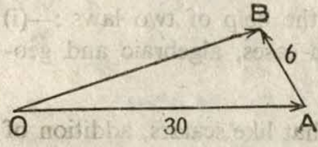


Fig. 2.7

(*b*) *Algebraic method* : Draw a perpendicular CD on the line AB from C. From the geometry of right angled triangle, we can write,

$$(AC)^2=(AD)^2+(DC)^2=(AB+BD)^2+(DC)^2$$
$$=(AB)^2+(BD)^2+(DC)^2+2AB.BD$$
$$=(AB)^2+(BC)^2+2AB.BD$$

$\therefore\ R^2=a^2+b^2+2ab\cos\theta$ [θ=angle between the vectors $\vec{AB}$ and $\vec{BC}$].

If the resultant vector $\vec{AC}$ makes an angle ϕ with the vector $\vec{AB}$ then,

$$\tan\phi=\frac{DC}{AD}=\frac{DC}{AB+BD}=\frac{b\sin\theta}{a+b.\cos\theta}$$

Special case : If the vectors $\vec{AB}$ and $\vec{BC}$ are at right angles to one another (*i.e.* $\theta=90°$) then $R^2=a^2+b^2$ and $\tan\phi=\frac{b}{a}$.

Examples : (1) *An aeroplane, while taking off, ran 12 metres towards east along the runway and then 5 metres towards the north and finally ascended 6 metres vertically upward. Find the magnitude of the sum of its displacements.*

Ans. Let $\overrightarrow{OA}$, $\overrightarrow{AB}$ and $\overrightarrow{BC}$ represent the displacements of the aeroplane along the east, the north and the vertical directions respectively (Fig. 2.8). Now, according to triangle law, $\overrightarrow{OA}+\overrightarrow{AB}=\overrightarrow{OB}$

Again, $\overrightarrow{OB}+\overrightarrow{BC}=\overrightarrow{OC}$, which is the sum of the displacement.

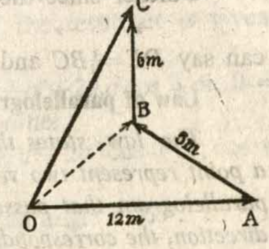


Fig. 2.8

Now, $|\overrightarrow{OB}| = \sqrt{(12)^2+(5)^2}=\sqrt{169}=13$ metre.

Also, $|\overrightarrow{OC}| = \sqrt{(13)^2+(6)^2}=\sqrt{205}=14{\cdot}32$ metre.

Hence, the magnitude of the sum of its displacements is 14·32 metres.

(2) *A person travels 10 metres due east and then changes his direction and goes 5 metres along north-east. Find the resultant displacement of the person.*

Ans. Suppose, the vector $\overrightarrow{AB}$ represents 10-metre displacement due east and $\overrightarrow{BC}$ the subsequent north-east displacement of 5 metres. According to law of triangle, $\overrightarrow{AC}$ represents the resultant vector (Fig. 2.9). If the absolute value of the resultant displacement vector be $|\mathbf{R}|$, then,

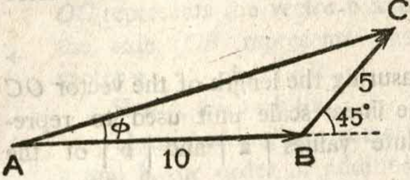


Fig. 2.9

$$R^2=(10)^2+(5)^2+2.10.5.\cos 45^\circ=125+50\sqrt{2}=195{\cdot}5 \text{ (nearly)}$$

$\therefore\ R=14$ metres (nearly)

Again, if $\overrightarrow{AC}$ makes an angle ϕ with the vector $\overrightarrow{AB}$, then,

$$\tan\phi=\frac{5\sin 45^\circ}{10+5.\cos 45^\circ}=0{\cdot}26 \quad \therefore \quad \phi=14{\cdot}6^\circ \text{ (nearly)}$$

(3) *Prove vectorially that the line joining the middle points of two adjacent sides of a triangle is parallel to and half of the third side.* [*H. S. Exam. 1983*]

Ans. Let ABC be a triangle. Let the line $\overrightarrow{BA}$ represent a vector $\overrightarrow{P}$ and the line $\overrightarrow{BC}$ a vector $\overrightarrow{Q}$. Then $\overrightarrow{DA}=\frac{1}{2}\overrightarrow{BA}=\frac{1}{2}\overrightarrow{P}$ and $\overrightarrow{EA}=\frac{1}{2}\overrightarrow{CA}$. Now, from the law of triangle, $\overrightarrow{Q}+\overrightarrow{CA}=\overrightarrow{P}$ or, $\overrightarrow{CA}=\overrightarrow{P}-\overrightarrow{Q}$. Hence $\overrightarrow{EA}=\frac{1}{2}\overrightarrow{CA}$ $=\frac{1}{2}(\overrightarrow{P}-\overrightarrow{Q})$

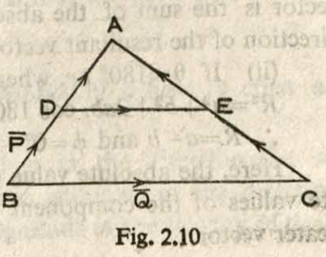


Fig. 2.10

Again, $\vec{DE}+\vec{EA}=\vec{DA}$ or, $\vec{DE}=\vec{DA}-\vec{EA}=\frac{1}{2}\vec{P}-\frac{1}{2}[\vec{P}-\vec{Q}]=\frac{1}{2}\vec{Q}=\frac{1}{2}\vec{BC}$

Further since the directions of the vectors $\vec{DE}$ and $\vec{BC}$ are the same, we can say $\vec{DE}=\frac{1}{2}\vec{BC}$ and parallel to BC.

Law of parallelogram : (i) *Geometric method* :

The law states that when two adjacent sides of a parallelogram drawn from a point represent two vectors, in magnitude and direction, then the diagonal of the parallelogram that passes through the angular point represents, in magnitude and direction, the corresponding resultant.

Consider two vectors $\vec{OA}$ and $\vec{OB}$ having the initial point O. Their terminal points are respectively A and B. Taking the line segments OA and OB as two adjacent sides, if the parallelogram $OACB$ be completed, then according to the law of parallelogram the diagonal $\vec{OC}$ represents the resultant vector (Fig. 2.11).

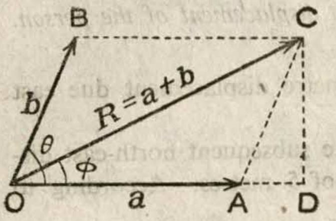


Fig. 2.11

Now, measuring the length of the vector $\vec{OC}$ accroding to the linear scale unit used to represent the absolute values $|\,\mathbf{a}\,|$ and $|\,\mathbf{b}\,|$ of the vectors $\vec{OA}$ and $\vec{OB}$, we get the absolute value $|\,\mathbf{R}\,|$ of the resultant. Also measuring the $\angle COA$ by a protractor, we get the angle ϕ made by the resultant vector with the vector $\vec{OA}$.

(ii) *Algebraic method* : Drop a perpendicular CD on the line OA produced. Suppose, the vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are inclined to one another at an angle θ. Following the procedure mentioned earlier, we get $R^2=a^2+b^2+2ab\cos\theta$ and

$$\tan\phi=\frac{b\sin\theta}{a+b.\cos\theta}$$

Special cases : (i) If $\theta=0^\circ$ *i.e.* the vectors are parallel, $R^2=a^2+b^2+2a.b\cos 0^\circ=(a+b)^2$ $\therefore$ $R=a+b$ and $\phi=0^\circ$ *i.e.* the absolute value of the resultant vector is the sum of the absolute values of the component vectors and the direction of the resultant vector is same as those of the component vectors.

(ii) If $\theta=180^\circ$ *i.e.* when the vectors are opposite and in the same line,

$R^2=a^2+b^2+2ab.\cos 180^\circ=a^2+b^2-2ab=(a-b)^2$

$\therefore$ $R=a-b$ and $\phi=0^\circ$

Here, the absolute value of the resultant vector is the difference of the absolute values of the component vectors and the resultant vector is parallel to the greater vector.

Example : *When will the magnitude of the resultant of two equal vectors added vectorially, be equal to (i) $\sqrt{2}$ times the magnitude of each (ii) $\sqrt{3}$ times the magnitude of each?* [*H. S. Exam. 1980*]

Ans. If two vectors of absolute value a and b, making an angle θ between them, are added vectorially, the absolute value R of the resultant is given by $R^2=a^2+b^2+2ab.\cos\theta$.

(i) In this case, $a=b$ and $R=\sqrt{2}.a$; So, $2a^2=a^2+a^2+2a.a\cos\theta$ or, $\theta=90°$. So, the vectors should be at right angles to each other.

(ii) In this case, $a=b$ and $R=\sqrt{3}.a$; So $3a^2=a^2+a^2+2a^2\cos\theta$ or, $\cos\theta=\frac{1}{2}$ or, $\theta=60°$.

The vectors should be inclined to each other at an angle of 60°.

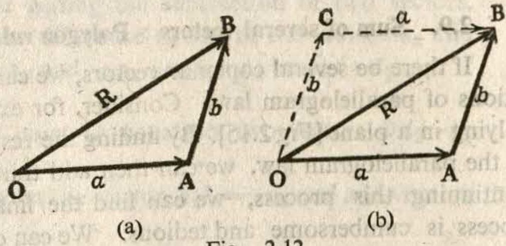
2.7. Vector addition follows commutative and associative laws :

Let $\overrightarrow{OA}$ and $\overrightarrow{AB}$ represent two vectors **a** and **b** respectively. According to the law of triangle [Fig. 2.12 (a)], **a**+**b**=**R**

Now complete the parallelogram *OABC* [Fig. 2.12 (b)]. In the triangle *OBC*, the side *OC* represents the vector **b** and the side *CB* represents the vector **a**. ∴ **b**+**a**=**R**

Hence, **a**+**b**=**b**+**a**

Thus in adding two vectors **a** and **b**, the order of addition makes no difference. This is known as *Commutative law.*

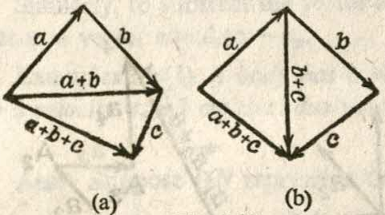


(a) (b)

Fig. 2.12

Again suppose, there are three vectors **a**, **b**, and **c** as shown in fig 2.13(a). If we first add **a** and **b**, we get **a**+**b**=(**a**+**b**). Then adding with the vector **c**, we get the resultant=(**a**+**b**)+**c** [Fig. 2.13 (a)]

Instead of adding **a** and **b** first, if we add **b** and **c** first we get **b**+**c**=(**b**+**c**). Then adding the **a**, we get the resultant=**a**+(**b**+**c**). [Fig. 2.13 (b)]



(a) (b)

Fig. 2.13

Figure shows that the two resultants are equal. Hence, (**a**+**b**)+**c**=**a**+(**b**+**c**). The resultant of three vectors can, therefore, be obtained by adding the resultant of **a** and **b** to **c** or by adding the resultant of **b** and **c** to **a**. This is known as *Associative law.*

2.8. Illustration of parallelogram law : *Motion of a boat to cross a river straight* :

Suppose the current is flowing in a river in the direction *OA* with a velocity *u* and a boat wants to cross the river straight in the direction *OC* [Fig. 2.14]. What should be the direction and magnitude of the velocity of the boat?

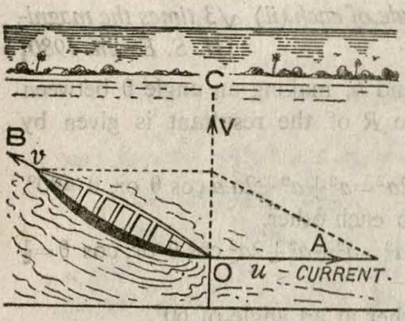


Fig. 2.14

The boat should be directed in such a direction and be given such a velocity v (say) that the resultant of the velocity vectors may be $\overrightarrow{OC}$. If the line segments $\overrightarrow{OA}$ and $\overrightarrow{OB}$ represent the velocity vectors, then OC should be the diagonal of the parallelogram they construct as shown in the figure. So, to cross the river straight, the boat should be directed along OB with a velocity v.

Suppose, the boatman can row at the rate of 4 km/hr in still water and the current is flowing at the rate of 2 km/hr. Then $u=2$ km/hr and $v=4$ km/hr. Now, $\sin BOC=\frac{u}{v}=\frac{2}{4}=\frac{1}{2}$ $\therefore$ $\angle BOC=30°$ *i.e.* the boat should be rowed upstream along OB making an angle 30° with OC.

2.9. Sum of several vectors : Polygon rule :

If there be several coplanar vectors, we can find their sum by repeated applications of parallelogram law. Consider, for example, four vectors $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ and $\mathbf{a}_4$ lying in a plane [Fig 2.15]. By finding the resultant of the vectors $\mathbf{a}_1$ and $\mathbf{a}_2$ first by the parallelogram law, we can then add third the vector $\mathbf{a}_3$ with this resultant. Continuing this process, we can find the final resultant. But obviously, this process is cumbersome and tedious. We can conveniently find the final resultant of several vectors by the application of polygon rule. The rule is as follows :

Starting from an arbitrary origin O [Fig 2.15], construct a vector $\overrightarrow{OA_1}=\mathbf{a}_1$. From the point A_1 (as origin) construct a vector $\overrightarrow{A_1A_2}=\mathbf{a}_2$. From the point A_2 again, construct a vector $\overrightarrow{A_2A_3}=\mathbf{a}_3$ and so forth. Now, join the initial point O of the first vector and the terminal point A_4 of the last vector. Here, the vector $\overrightarrow{OA_4}$ will give the sum of $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ and $\mathbf{a}_4$. Thus, $\overrightarrow{OA_4}=\mathbf{a}_1+\mathbf{a}_2+\mathbf{a}_3+\mathbf{a}_4$

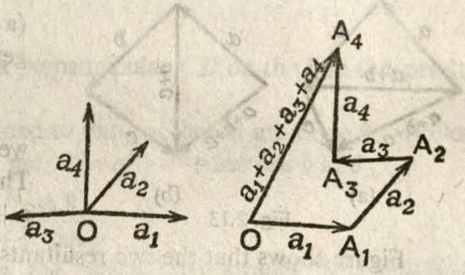


Fig. 2.15

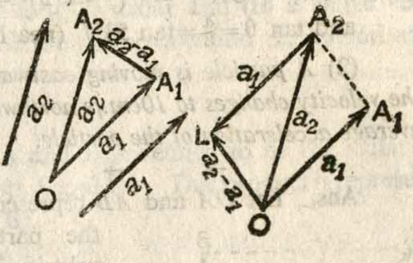
Generally, $\overrightarrow{OA_n}=\mathbf{a}_1+\mathbf{a}_2+\ldots+\mathbf{a}_n$

This is known as the Polygon rule, because the line segments representing the vectors form a polygon.

[Note : The rule is also applicable in the case of non-coplanar vectors. Further, in the addition of vectors, the terms may be grouped in any way whatsoever.]

2.10. Subtraction of vectors : Subtraction of a vector $\mathbf{a}_1$ from a vector $\mathbf{a}_2$ means to find a new vector $\mathbf{x}$ (difference) which together with the vector $\mathbf{a}_1$ yields the vector $\mathbf{a}_2$. Briefly, subtraction of vectors is the inverse operation of addition.

Suppose, we are to subtract a vector $\mathbf{a}_1$ from a vector $\mathbf{a}_2$. From an arbitrary origin O construct the vectors $\overrightarrow{OA_1}=\mathbf{a}_1$ and $\overrightarrow{OA_2}=\mathbf{a}_2$ [Fig 2.16(a)].



(a) Fig. 2.16 (b)

Now join the initial point A_1 of the vector $\mathbf{a}_1$ and the terminal point A_2 of the vector $\mathbf{a}_2$. The vector $\overrightarrow{A_1A_2}$ will give the difference. In vector notation,

$\overrightarrow{A_1A_2}=\overrightarrow{OA_2}-\overrightarrow{OA_1}$ or, $\overrightarrow{A_1A_2}=\mathbf{a}_2-\mathbf{a}_1$

There is an alternative method of getting the subtraction of two vectors. The idea is as follows : To construct the difference $\mathbf{a}_2-\mathbf{a}_1$ of the vectors $\mathbf{a}_2$ and $\mathbf{a}_1$, we can take the sum of the vectors $\mathbf{a}_2$ and $-\mathbf{a}_1$ *i.e.* $\mathbf{a}_2-\mathbf{a}_1=\mathbf{a}_2+(-\mathbf{a}_1)$.

As before, from a common initial point O, construct two vectors $\overrightarrow{OA_1}=\mathbf{a}_1$ and $\overrightarrow{OA_2}=\mathbf{a}_2$ [Fig. 2.16 (b)]. Now construct the vector $\overrightarrow{A_2L}=-\mathbf{a}_1$ and add vectors $\overrightarrow{OA_2}=\mathbf{a}_2$ and $\overrightarrow{A_2L}=-\mathbf{a}_1$ according to the law of triangle. The vector $\overrightarrow{OL}$ so obtained gives the required difference *i.e.* $\overrightarrow{OL}=\mathbf{a}_2-\mathbf{a}_1$.

Similarly, to subtract the vector $\mathbf{a}_2$ from the vector $\mathbf{a}_1$, we are to add with the vector $\mathbf{a}_1$ a vector equal to $-\mathbf{a}_2$.

Examples : (1) *A body has a velocity* $\mathbf{v}_1=4$ *cm/sec. due north and another body a velocity* $\mathbf{v}_2=3$ *cm./sec. due west. Find the difference of the velocity vectors.*

Ans. Suppose $\overrightarrow{ON}$ represents the velocity vector $\mathbf{v}_1$ due north and $\overrightarrow{OW}$ the the velocity vector $\overrightarrow{v_2}$ due west [Fig. 2.17]. Now, to find $\overrightarrow{v_1}-\overrightarrow{v_2}$, we are to add the vector $\overrightarrow{ON}$ and $-\overrightarrow{v_2}$. The line segment $\overrightarrow{OE}$ represents $-\overrightarrow{v_2}$. Hence, the diagonal $\overrightarrow{OA}$ of the parallelogram $OEAN$ gives the required difference *i.e.* $\overrightarrow{OA}=\overrightarrow{v_1}-\overrightarrow{v_2}$.

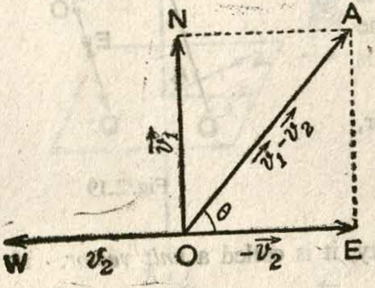


Fig. 2.17

Now, $|\overrightarrow{OA}| = \sqrt{(3)^2+(4)^2} = 5$ cm/sec.
and $\tan\theta = \frac{4}{3} = \tan 53°7'$ (nearly) $\therefore$ $\theta = 53°7'$ (nearly)

(2) *A particle is moving eastward with a velocity of 10cm./s. In 10 seconds, the velocity changes to 10cm./s northward. Find the magnitude and direction of the average acceleration of the particle.*

Ans. Let $\overrightarrow{OA}$ and $\overrightarrow{AB}$ represent the eastward and northward velocities of the particle. Although the magnitudes of the velocity are the same, yet the change of direction of the velocity will produce an acceleration. To get the acceleration we are to find the difference of velocity vectors of the particle.

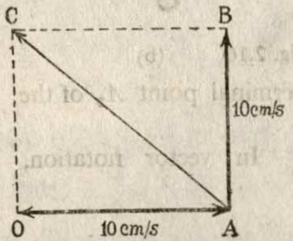


Fig. 2.18

Now, difference of $\overrightarrow{OA}$ and $\overrightarrow{AB}$ = the resultant of $\overrightarrow{AO}$ and $\overrightarrow{AB}$. (Fig 2.18)

Since $\overrightarrow{AO} = \overrightarrow{BC}$, we have, $\overrightarrow{AB} + \overrightarrow{AO} = \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

Now, $|\overrightarrow{AC}| = \sqrt{(10)^2+(10)^2} = 10\sqrt{2}$ metre.

So, the magnitude of acceleration $= \dfrac{\text{change of vel.}}{\text{time}} = \dfrac{10\sqrt{2}}{10} = \sqrt{2}$ cm/s^2

Fruther $\tan OAC = \dfrac{10}{10} = 1$ $\therefore$ $\angle OAC = 45°$

i.e. the direction of acceleration is north-west.

2.11. Multiplication of a vector by a number (*i.e. a scalar*) :

To multiply a vector **a** by a number x means to construct a new vector, the absolute value of which is obtained by multiplying the absolute value of the vector **a** by the absolute value of the number x. The direction of the product vector is same as the direction of the vector **a** or opposite to it according as the number x is positive or negative. If $x=0$, the product is the null vector. In vector notation, the product is written as $\mathbf{a}.x$ or $x\mathbf{a}$. In fig. 2.19 $\overrightarrow{OB} = \overrightarrow{OA}{\cdot}4$ or, $\overrightarrow{OB} = 4.\overrightarrow{OA}$; $\overrightarrow{OC} = 3\frac{1}{2}\ \overrightarrow{OA}$ and $\overrightarrow{OE} = -1{\cdot}5\ \overrightarrow{OA}$.

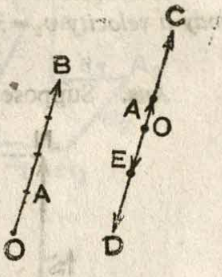


Fig. 2.19

Now, if the length of the vector $\overrightarrow{OA}$ is unity, it is called a ***unit vector.*** If the unit vector be denoted by $\mathbf{a}_1$, then we can write $\overrightarrow{OB} = 4\mathbf{a}._1$; $\overrightarrow{OC} = 3\frac{1}{2}\ \mathbf{a}_1$; $\overrightarrow{OE} =$ $= -1{\cdot}5\mathbf{a}_1$ etc.

2.12. Product of two vectors :

Two vectors can be multiplied to give (i) a scalar and (ii) a single vector. The first is known as the *scalar product* or the *dot product* and the second as the *vector product* or the *cross product.*

Scalar product :

The scalar product of two vectors **A** and **B** is defined to be the scalar $A.B \cos\theta$, where θ is the angle between the vectors **A** and **B**. This product is represented by the symbol **A. B** (pronounced **A** dot **B**). Thus according to fig. 2.20, $\mathbf{A}.\ \mathbf{B}=A.B\ \cos\theta$.

It follows that $\mathbf{A}.\mathbf{B}=\mathbf{B}.\mathbf{A}$.. The order of the factors in the case of a scalar product does not affect the product since the factor $\cos\theta$ remains the same in both the cases. That is, scalar products satisfy the *commutative law* of multiplication.

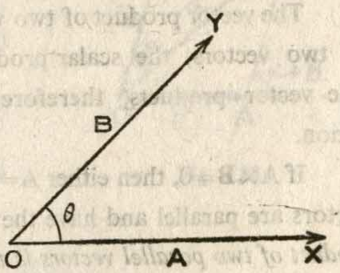


Fig. 2.20

The best example of a scalar product of two vectors is the work done by a force. If a force **F** acts in the direction OX and its point of application is displaced through a distance **S** along OY, then the work done $=F.S.\cos\theta=\mathbf{F}.\mathbf{S}$.

Further, from the definition, $\mathbf{A}.\mathbf{B}=0$ if either $\mathbf{A}=0$ or, $\mathbf{B}=0$ or, $\theta=90°$. In the latter case, the vectors **A** and **B** are mutually perpendicular. Conversely, the *scalar product of two perpendicular vectors is always zero.*

When $\theta=0$, the dot product is the same as the product of the magnitudes of the vectors because $\cos 0°=1$.

Vector product :

The vector product of two vectors **A** and **B** is defined to be the vector having a magnitude $|\mathbf{A}\times\mathbf{B}| = |\mathbf{A}| \times |\mathbf{B}| \times \sin\theta$, where θ is the angle between **A** and **B.** This product is represented by the symbol $\mathbf{A}\times\mathbf{B}$ (pronounced as **A** cross **B**).

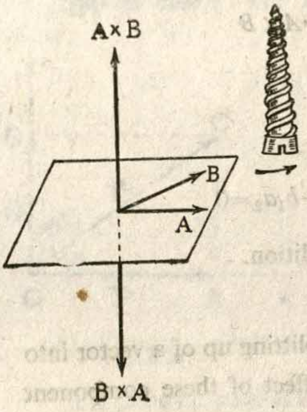


Fig. 2.21

The direction of the vector $\mathbf{A}\times\mathbf{B}$ is along the perpendicular to the plane formed by **A** and **B.** The direction can be conveniently obtained in the following way. Let a right-handed screw (Fig 2.21) whose axis is perpendicular to the plane containing the vectors **A** and **B** be turned from **A** to **B** through the angle θ between them as if driving the screw in. The direction along which the screw advances is the direction of the vector $\mathbf{A}\times\mathbf{B}$. In fig. 2.21 the product is directed upwards.

It is interesting to check whether $\mathbf{A}\times\mathbf{B}$ and $\mathbf{B}\times\mathbf{A}$ denote the same vector. A close scrutiny of the definition of the vector product shows that they are not the same. In getting the product $\mathbf{B}\times\mathbf{A}$, the rotation of the right-handed screw should be from **B** to **A** through the angle θ as if driving the screw in. Then the screw advances downwards [Fig. 2.21]. The direction of $\mathbf{B}\times\mathbf{A}$ is thus perpendicular to the plane containing the vectors **A** and **B** but directed downwards. So, the *order* of writing the vectors is important in a vector product. As the sense of rotation of a right-handed screw is reversed when the order of the vector is reversed (fig. 2.21), we can write, $\mathbf{A}\times\mathbf{B}=-\mathbf{B}\times\mathbf{A}$.

The vector product of two vectors is thus in contrast with the scalar product of two vectors, the scalar product not being affected by the order of the factors. The vector products, therefore, satisfy an *anti-commutative* law of multiplication.

If $\mathbf{A}\times\mathbf{B}=0$, then either $\mathbf{A}=0$, $\mathbf{B}=0$ or, $\theta=0$ or π. In the latter case, the two vectors are parallel and have the same or opposite senses. Conversely, *the vector product of two parallel vectors is the zero vector.*

The vector product of two vectors is useful in physics in expressing several physical quantities like torque, angular momentum, magnetic field due to a current at any point etc.

Example : *Find the condition that the vectors* $\vec{A}=a_1\mathbf{i}+b_1\mathbf{j}+c_1\mathbf{k}$ *and* $\vec{B}=a_2\mathbf{i}+b_2\mathbf{j}+c_2\mathbf{k}$ *are collinear.*

Ans. When two vectors are collinear, the angle between them is zero and in that case $\sin\theta=0$. Hence, the necessary condition that two vectors are collinear is that the cross-product of the vectors must be zero.

Now, the cross-product of the vectors $\vec{A}$ and $\vec{B}=\vec{A}\times\vec{B}$

$$=(b_1c_2-c_1b_2)\mathbf{i}+(c_1a_2-a_1c_2)\mathbf{j}+(a_1b_2-b_1a_2)\mathbf{k}$$

So, according to the condition mentioned above,

$$(b_1c_2-c_1b_2)\mathbf{i}+(c_1a_2-a_1c_2)\mathbf{j}+(a_1b_2-b_1a_2)\mathbf{k}=0$$

This gives, $b_1c_2-c_1b_2=0$; $c_1a_2-a_1c_2=0$ and $a_1b_2-b_1a_2=0$

$$\therefore\quad \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}.$$ This is the required condition.

2.13. Resolution of vectors : Resolution means splitting up of a vector into components in two given directions such that the effect of these component vectors would be the same as the original vector. Hence, the resolution of a vector is essentially a process opposite to that of the composition of vectors.

Suppose the line segment $\vec{OC}$ represents a vector **R**. Two lines OX and OY are taken inclined at angles α and β respectively to OC. From C draw CB parallel to OX and CA parallel to OY (Fig. 2.22). Here $\vec{OA}$ and $\vec{OB}$ are component vectors of $\vec{OC}$ along OX and OY respectively because, according to the parallelogram law, if we add the vectors $\vec{OA}$ and $\vec{OB}$ we get the vector $\vec{OC}$. In this way, a vector may be resolved into components in two specified directions.

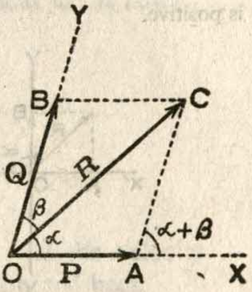


Fig. 2.22

Calculation of the components :

In ΔOAC, $\angle COA=\alpha$, $\angle OCA=\beta$, and $\angle CAO=180^\circ-(\alpha+\beta)$. From trigonometry, we can write, $\dfrac{OA}{\sin\beta}=\dfrac{AC}{\sin\alpha}=\dfrac{OC}{\sin[(180^\circ-(\alpha+\beta)]}$

$$\text{or,}\quad \frac{OA}{\sin\beta}=\frac{AC}{\sin\alpha}=\frac{OC}{\sin(\alpha+\beta)}$$

Now, if $|\vec{OA}|=P$; $|\vec{AC}|=Q$ and $|\vec{OC}|=R$, we have,

$$\frac{P}{\sin\beta}=\frac{Q}{\sin\alpha}=\frac{R}{\sin(\alpha+\beta)}$$

$$\text{Hence,}\quad P=R.\frac{\sin\beta}{\sin(\alpha+\beta)}\ \text{and}\ Q=R.\frac{\sin\alpha}{\sin(\alpha+\beta)}$$

Special case : The angles α and β can take up various values. Consequently, a vector can have a large number of components. But as soon as the directions of the components are specified, the values of the components become fixed. In the case when OX and OY are mutually perpendicular (*i.e.* $\alpha+\beta=90^\circ$ or, $\beta=90^\circ-\alpha$) the consequent components are known as **resolved parts** (Fig. 2.23).

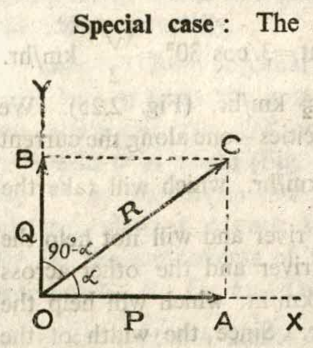


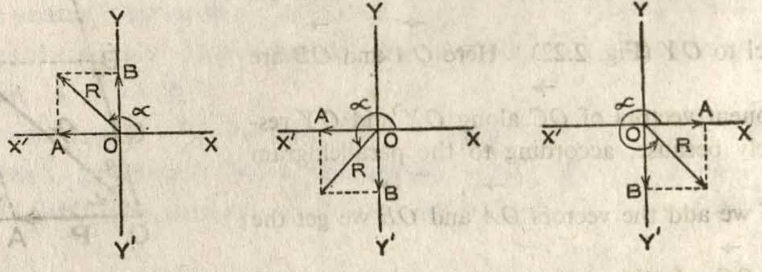
Fig. 2.23

$$\text{Here,}\quad P=R.\frac{\sin(90^\circ-\alpha)}{\sin 90^\circ}=R.\cos\alpha.$$

$$\text{and}\quad Q=R.\frac{\sin\alpha}{\sin 90^\circ}=R.\sin\alpha.$$

It is to be noted that a vector has no component in its perpendicular direction because, the perpendicular component = absolute value of the vector × cos 90° = 0.

[**Note :** In fig 2.23, the vector $\vec{R}$ makes an angle α (with the $+ve$ direction of OX) which falls in the first quadrant. But the angle α may fall in other quadrants also. In fig. 2.24(i), the angle is in the second quadrant. Here, the x-component of the vector $\vec{R}$ is along the negative direction of OX *i.e.* along OX' and hence the component is negative but the Y-component OB is positive.



(i) (ii) (iii)

Fig. 2.24

If the angle α falls in the third quadrant [fig. 2.24 (ii)], both the components of the vector $\vec{R}$ (*i.e.* $\vec{OA}$ and $\vec{OB}$) are along the negative direction of the respective axes and hence both are negative. If the angle α falls in the fourth quadrant [fig 2.24(iii)], the x-component ($\vec{OA}$) is +*ve* but the y-component ($\vec{OB}$) is negative.

Whatever may be the quadrant in which the angle α falls, the components of $\vec{R}$ are given by $R.\cos\alpha$ and $R.\sin\alpha$.]

Examples : (1) *The current is flowing in a river with a velocity 1 km/hr. A boatman is rowing his boat at an angle 30° with the current with a velocity 2 km/hr. How much time will the boatman take to cross the river from one bank to the other, if the river is 2 km. wide ?*

Ans. First, the velocity of the boat is to be resolved along the direction of the current and perpendicular to it.

The component of the velocity along the current $=3.\cos 30^\circ=\frac{3\sqrt{3}}{2}$ km/hr. and the component perpendicular to it $=3.\sin 30^\circ=\frac{3}{2}$ km/hr. (Fig. 2.25). We may now say that the boat has two simultaneous velocities—one along the current of value $\left(1+3\frac{\sqrt{3}}{2}\right)$ km/hr., which will take the boat parallel to the river and will not help the boatman to cross the river and the other across the river of value $=\frac{3}{2}$ km/hr. which will help the boatman to cross over. Since the width of the river is 2 km., the required time to cross it $=(2\div\frac{3}{2})$ $\frac{4}{3}$ hr. $=1$ hr. 20 mnt.

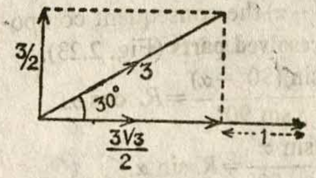


Fig. 2.25

(2) *A river of width l is flowing with a velocity u and a boatman tries to row straight across it. In what direction should he row and how long would it take him to cross the river ? How long would it take the boatman to cover a distance l up-stream and back ?*

Ans. See fig. 2.14. In order to be able to cross the river straight the boatman must row at angle θ ($\angle BOC$) with the direction OC such that the resultant V of his own velocity v and that of the river-current u is perpendicular to the latter.

Then clearly, $V=v\cos\theta$ and $u=v\sin\theta$.

$\therefore$ Time taken to row straight across the stream

$$=\frac{l}{v\cos\theta}=\frac{l}{v\sqrt{1-\sin^2\theta}}=\frac{l}{v\sqrt{1-\dfrac{u^2}{v^2}}}=\frac{l}{\sqrt{v^2-u^2}}$$

Again, resultant velocity of the boatman upstream $=v-u$, and the velocity downstream $=v+u$. So, time taken to cover a distance l upstream and back

$$=\frac{l}{v-u}+\frac{l}{v+u}=\frac{2lv}{v^2-u^2}$$

2.14. Composition of a number of vectors by resolution :

We have already seen in art. 2.9 how a number of vectors can be added with the help of polygon rule. As a geometric method, polygon rule is quite convenient but as an algebraic method, it is not so. Analytically, the composition of a number of vectors can be accomplished very easily by the process of resolution. In the case of co-planar vectors, the process is applied in the following way.

Constructing the vectors from a common initial point, two rectangular axes are to be drawn through the point as origin. Next the components of the vectors along the two axes are to be determined by resolving the vectors along the axes. Then, algebraic sums of the components along X-axis and Y-axis are to be done separately. Finally the sum along X-axis and that along Y-axis are to be added vectorially.

Consider four coplanar vectors **A**, **B**, **C** and **D** having O as the common initial point. Two rectangular axes OX and OY have been drawn with O as origin (Fig. 2.26), which obviously, lie in the same plane. Components of the vectors along X-axis are $\mathbf{A}_x$, $\mathbf{B}_x$, $\mathbf{C}_x$, and $\mathbf{D}_x$. Likewise, the components of the vectors along Y-axis are $\mathbf{A}_y$, $\mathbf{B}_y$, $\mathbf{C}_y$, and $\mathbf{D}_y$.

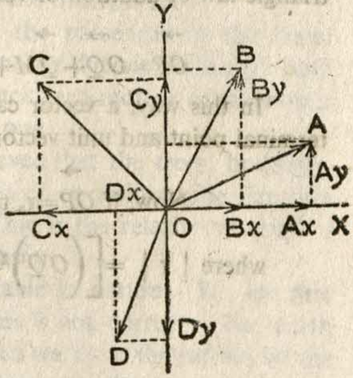


Fig. 2.26

[If the vector **A** makes an angle θ_1 with the X-axis, then $\mathbf{A}_x=A\cos\theta_1$ and $\mathbf{A}_y=A\sin\theta_1$ etc.]

Calling the algebraic sum of the components along X-axis $\mathbf{R}_x$ and that along Y-axis $\mathbf{R}_y$, we have,

$$\mathbf{R}_x=\mathbf{A}_x+\mathbf{B}_x+\mathbf{C}_x+\mathbf{D}_x+\ldots\ldots$$

$$\mathbf{R}_y=\mathbf{A}_y+\mathbf{B}_y+\mathbf{C}_y+\mathbf{D}_y+\ldots\ldots$$

[Fig. 2.26 shows that C_x and D_y are negative ; so in finding the algebraic sum, – sign should be put before C_x and D_y.]

If the final resultant be **R**, then $|\mathbf{R}|=\sqrt{R_x^2+R_y^2}$ and $\tan\phi=\dfrac{R_y}{R_x}$

2.15. Representation of a vector by co-ordinates : Position vector :

Consider a vector $\overrightarrow{OP}$, having O as the initial point and P as the terminal point. It is to be represented by co-ordinates. Draw three mutually perpendicular axes OX, OY and OZ through O. It is called right-handed cartesian co-ordinates because a right-handed screw when driven along OZ, makes the OX axis coincide with OY axis by 90° rotation.

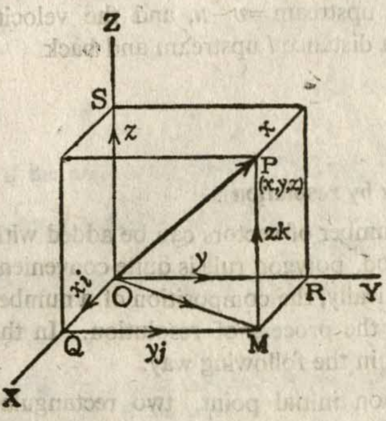


Fig.2.27

Now, if perpendiculars be drawn from P on the planes OYZ, OXZ and OXY, the intercepts on the axes are respectively OQ, OR and OS [Fig. 2.27]. Supposing the co-ordinates of P with respect to the co-ordinate system be x, y and z, then $OQ=x$, $OR=y$ and $OS=z$.

Let **i, j, k,** be the unit vectors* along the three axes respectively. From the triangle law of addition of vectors, we get,

$$\overrightarrow{OP}=\overrightarrow{OQ}+\overrightarrow{QM}+\overrightarrow{MP}=x.\mathbf{i}+y.\mathbf{j}+z.\mathbf{k}.$$

In this way, a vector can be represented with the help of co-ordinates of its terminal point and unit vectors.

Now, if $\overrightarrow{OP}=\mathbf{r}$, then $\mathbf{r}=x.\mathbf{i}+y.\mathbf{j}+z.\mathbf{k}$.

$$\text{where } |\mathbf{r}|=\left[(OQ)^2+(QP)^2\right]^{\frac{1}{2}}=\left[(OQ)^2+(QM)^2+(MP)^2\right]^{\frac{1}{2}}$$

$$=\left(x^2+y^2+z^2\right)^{\frac{1}{2}}$$

r is called the *position vector* of point P relative to O.

---

*When resolving a vector into components it is sometimes useful to introduce a vector of unit length in a given direction. Thus a vector $\vec{P}$ may be written, for example, as $\vec{P}=P.\mathbf{i}$ where **i** is a *unit vector* in the direction of $\vec{P}$. If P has components $\vec{P_x}$, $\vec{P_y}$ and $\vec{P_z}$ along three perpendicular axes, then $\vec{P}=P_x.\mathbf{i}+P_y.\mathbf{j}+P_z.\mathbf{k}$, where **i, j,** and **k** are the unit vectors along x, y and z axes respectively.

It follows from the above that if there be two points P_1 and P_2 with (x_1, y_1, z_1) and (x_2, y_2, z_2) as their respective co-ordinates and $\overrightarrow{OP_1}=\mathbf{r}_1$ and $\overrightarrow{OP_2}=\mathbf{r}_2$ as their position vectors [Fig. 2.28], we have

$\mathbf{r}_1=x_1\mathbf{i}+y_1\mathbf{j}+z_1\mathbf{k}$ and $\mathbf{r}_2=x_2\mathbf{i}+y_2\mathbf{j}+z_2.\mathbf{k}$.

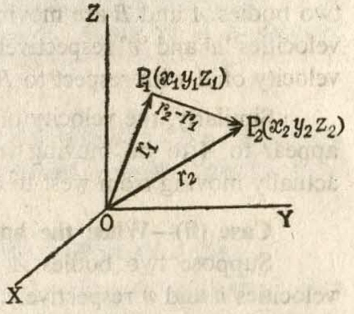


Fig. 2.28

Therefore, $\overrightarrow{P_1P_2}=\overrightarrow{P_1O}+\overrightarrow{OP_2}=-\mathbf{r}_1+\mathbf{r}_2=\mathbf{r}_2-\mathbf{r}_1$
$=(x_2-x_1)\mathbf{i}.+(y_2-y_1)\mathbf{j}+(z_2-z_1)\mathbf{k}$

Example : *If the co-ordinates of the terminal point of a vector OP be* (1, −1, 2), *represent the vector and find its absolute value.*

Ans. If the co-ordinates of P be (x, y, z), we know $\overrightarrow{OP}=x.\mathbf{i}+y.\mathbf{j}+z.\mathbf{k}$.

Here, $x=1$, $y=-1$ and $z=2$; So, $\overrightarrow{OP}=\mathbf{i}-\mathbf{j}+2\mathbf{k}$

Also, $|\overrightarrow{OP}|=(x^2+y^2+z^2)^{\frac{1}{2}}=(1+1+4)^{\frac{1}{2}}=\sqrt{6}=2{\cdot}45$ (nearly).

2.16. Relative velocity : An example will make the idea about relative velocity clear. Suppose, two trains are moving parallel to each other in the same direction with same velocity. If two passengers, sitting opposite to each other in the two trains, look to each other, it will appear to them as if none is moving, although each of them is moving forward with great speed. In this case, it is said, the relative velocity of one passenger with respect to the other, is zero.

If one train moves faster than the other, then the passenger in the faster train will see as if the other train is moving in the opposite direction although both are moving in the same direction. This apparent backward motion of the slower train is its relative velocity with respect to the faster one.

While travelling in a train, you must have observed that the trees, buildings, posts etc. near the rail-line are moving fast in a direction opposite to the direction of the train, though they are, in reality, at rest. This is the relative velocity of trees etc. with respect to a passenger in the moving train.

Illustrations of relative velocity are also available in nature. We see that the sun rises in the east and sets in the west. But this is not correct. The earth rotates about its own axis from west to east, for which we, from the surface of the earth, see that the sun rises in the east and sets in the west. So the east-west motion of the sun is its relative motion with respect to the earth.

Thus, *the relative velocity of A with respect to another Body B means the rate of change of position of A as observed by the body B.*

2.17. Calculation of relative velocity : The general rule for calculating relative velocity is as follows : *Relative velocity of a body A with respect to another body B is obtained by compounding with the velocity of A, a velocity equal and opposite to that of B.*

Case (i)—When both the bodies are moving in the same direction : Suppose two bodies A and B are moving in the same direction, say from west to east, with velocities 'u' and 'v' respectively. Applying the above rule, we can say, the relative velocity of A with respect to $B=(u-v)$.

Similarly, the velocity of B relative to $A=(v-u)$ or $-(u-v)$. Thus B will appear to A to be moving from east to west with a velocity $(u-v)$ though it is actually moving from west to east.

Case (ii)—When the bodies are moving in the opposite directions :

Suppose two bodies A and B are moving in the opposite directions with velocities u and v respectively.

Then the relative velocity of A with respect to $B=u-(-v)=u+v$.

Similarly, the velocity of B relative to $A=v-(-u)=u+v$.

Thus each will appear to move with velocity equal to the sum of the velocities of the two bodies.

Case (iii)—When the bodies are moving in two different concurrent directions : Suppose, a body A is moving in the direction OX with velocity u and another body B is moving in the direction OY with velocity v. These two velocity vectors are fully represented by the line segments OA and OB (Fig. 2.29) respectively. To determine the velocity of A relative to B, produce the line BO to B' such that $OB=OB'$. Here $\overrightarrow{OB'}$ represents a vector equal and opposite to v. Now, according to the parallelogram law, the diagonal $\overrightarrow{OC}$ of the parallelogram $OACB'$, is the rusultant of the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB'}$ and hence, denotes the relative velocity of A with respect to B.

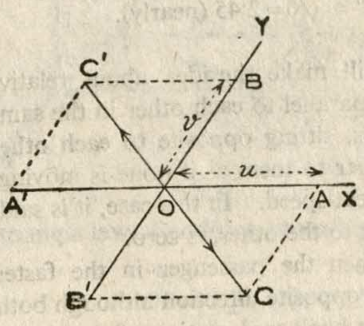


Fig. 2.29

If $|\overrightarrow{OC}|=R$, then, $R=\sqrt{u^2+v^2+2uv\cos(180-\theta)}$, where $\angle AOB=\theta$.

If $\overrightarrow{OC}$ makes an angle α with $\overrightarrow{OA}$ *i.e.* if $\angle AOC=\alpha$, then

$$\tan\alpha=\frac{v\sin(180-\theta)}{u+v\cos(180-\theta)}$$

These two equations give the magnitude and the direction of the relative velocity of A with respect to B.

Similarly, the relative velocity of B with respect to A is obtained by producing AO to A' such that $OA=OA'$ and compounding with it the velocity v of B. By constructing the parallelogram $OBC'A'$ and drawing the diagonal OC' we get the resultant and hence the relative velocity of B with respect to A.

It is to be noted that the relative velocity can also be found out by *vector subtraction* method ; for, the relative velocity of *B* with respect to *A* is obtained by subtracting the vector $\overrightarrow{OA}$ from the vector $\overrightarrow{OB}$.

Thus, if V_A and V_B be the velocities of *A* and *B* respectively, then the relative velocity of *A* with respect to *B* is $V_{AB}=V_A-V_B$. and the relative velocity of *B* with respect to *A* is $V_{BA}=V_B-V_A$.

Case (iv)—Relative velocity between rain and the motion of a man :

When rain falls vertically and a man is at rest, it is easy to understand that the man will hold his umbrella vertically above his head to protect his body from the rain. But if the man walks forward, he is to hold his umbrella inclined because in that case, the velocity of the rain relative to the man, is inclined to the vertical. The following description will make it clear.

Suppose rain falls vertically downward with a velocity v and a man is walking with a velocity u along a horizontal roadway. To find the relative velocity of rain with respect to the man, we shall apply the rule stated in the case (iii) of art 2.17. The relative velocity will be obtained by compounding with the velocity of the rain a velocity equal and opposite to the velocity of the man.

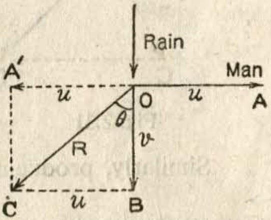


Fig. 2.30

In fig 2.30, $\overrightarrow{OA}$ represents the velocity u of the man and $\overrightarrow{OB}$ represents the downward velocity v of the rain. From *O* a segment of line *OA′* is drawn equal and opposite to $\overrightarrow{OA}$. Then $\overrightarrow{OA'}$ represents a velocity $-u$. According to the parallelogram law, the resultant of the velocities $(-u)$ and (v) represented by $\overrightarrow{OA'}$ and $\overrightarrow{OB}$ respectively is $\overrightarrow{OC}$. So, to the man, the rain will appear to come from the direction $\overrightarrow{OC}$ and the man, walking in the rain, must hold his umbrella in the direction $\overrightarrow{OC}$ to save his body.

Now, $|\overrightarrow{OC}|=\sqrt{u^2+v^2}$ or $|\overrightarrow{R}|=\sqrt{u^2+v^2}$

Knowing u and v, the magnitude of the relative velocity (R) can be found out. Further if *OC* makes an angle θ with the vertical [Fig 2.30],

$$\tan\theta=\frac{BC}{OB}=\frac{u}{v} \quad \therefore\ \theta=\tan^{-1}\left(\frac{u}{v}\right)$$

This gives the direction of the relative velocity of the rain.

The above description explains why the front wind-screen of a car gets wet whereas the hind screen remains dry when the car speeds in rain.

Examples : (1) *Two particles moving with velocities u and 2u are inclined at an angle of* 60°. *Find velocity of one with respect to the other.*

[*H. S. Exam 1979*]

Ans. Let $\vec{AB}$ and $\vec{AC'}$ represent respectively velocities $2u$ and u. To find the relative velocity of $\vec{AB}$ with respect to $\vec{AC'}$, we are to produce $C'A$ to C such that $AC'=AC$. Then $\vec{AC}=-\vec{AC'}$. Accordingly, the diagonal $\vec{AD}$ represents the relative velocity of $\vec{AB}$ with respect to $\vec{AC'}$.

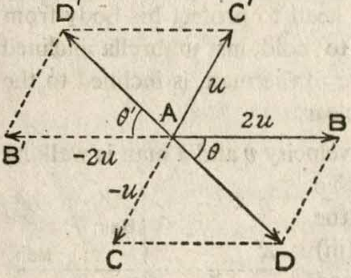


Fig. 2.31

$$\text{Now } |\vec{AD}| = \sqrt{(2u)^2+(u)^2+2.2u.u.\cos 120°}$$
$$=\sqrt{4u^2+u^2-2u^2}=\sqrt{3}.u.$$

$$\text{Also, } \tan\theta=\frac{u \sin 120°}{2u+u\cos 120°}=\frac{\sqrt{3}.u}{2(2u-u/2)}$$
$$=\frac{1}{\sqrt{3}}=\tan 30° \quad \therefore \quad \theta=30°$$

Similarly, producing BA to B' such that $AB=AB'$ and finding the diagonal $\vec{AD'}$ of the parallelogram $AB'D'C'$, we can get the relative velocity of $\vec{AC'}$ with respect to $\vec{AB}$. Needless to mention that the magnitude of the relative velocity will be $\sqrt{3}.u$ and it will make an angle 30° with AB'.

(2) *A person is driving a car at a speed of 36 km/hr. due east. A stone hits the car with a horizontal velocity of 5 metres/sec. from the north. In what direction will the stone appear to come to the driver?*

Ans. Velocity of the car $=36$ km/hr $=\dfrac{36\times 1000}{60\times 60}$ metre/sec. $=10$ metre/sec.

Suppose $\vec{OA}$ represents the velocity of the car due east and $\vec{OB}$ that of the stone. Now, draw OA' equal and opposite to OA and complete the parallelogram $OA'CB$. Draw the diagonal $\vec{OC}$ which gives the relative velocity of the stone (Fig. 2·32)

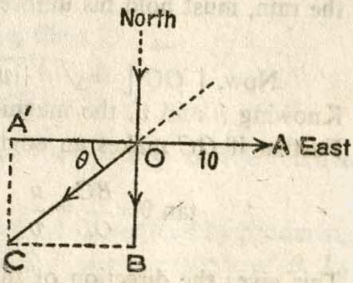


Fig. 2.32

$$\text{Now, } (OC)^2=(OB)^2+(BC)^2$$
$$=(5)^2+(10)^2=125$$

$$\therefore \quad |\vec{OC}| = 5\sqrt{5} \text{ metres/sec.}$$

The relative velocity makes an angle θ (say) with the reverse direction of the car.

Here, $\tan\theta = \frac{A'C}{OA'} = \frac{OB}{OA'} = \frac{5}{10} = \frac{1}{2}$

So, the stone appears to come at an angle $(180^\circ + \theta)$ with the direction of the car θ north of west such that $\tan\theta = \frac{1}{2}$.

(3) *A ship sailing due north with a speed of 30 km/hr. From the same place, another ship started moving with a velocity of 20 km/hr. in a direction 30° south of west. Find the relative velocity of the first ship with respect to the second and of the second ship with respect to the first.*

Ans. $\vec{OA}$ represents the velocity of the first ship and $\vec{OB}$ that of the second ship [Fig. 2·33]. To find the relative velocity of the first ship *w.r.t.* the second, we are to compound with $\vec{OA}$, a velocity equal and opposite to $\vec{OB}$ i.e. $\vec{OD}$. From the figure it is clear that $\vec{OC}$ represents the relative velocity.

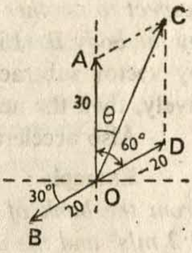


Fig. 2.33

$$\text{Now, } |\vec{OC}| = \sqrt{(30)^2+(20)^2+2\times30\times20\times\cos 60^\circ}$$
$$= \sqrt{900+400+600} = \sqrt{1900} = 43{\cdot}6 \text{ km/hr.}$$
$$\text{Again } \tan\theta = \frac{20\sin 60^\circ}{30+20\times\cos 60^\circ} = \frac{\sqrt{3}}{4} \quad \therefore\ \theta = 23^\circ 24'$$

That is, the relative velocity is 43·6 km/hr in a direction 23°24′ east of north.

Similarly, the relative velocity of the second ship *w.r.t.* the first will also be 43·6 km/hr making an angle 23°24′ west of south.

(4) *Two ships are $10\sqrt{2}$ km apart on a line running south to north. The one further north is moving west with a speed of 25 km/hr., while the other towards north with a speed of 25 km/hr. What is their distance of closest approach and how long do they take to reach it?*

Ans. Let the instantaneous position of the ships be A and B (Fig. 2.34). $AB = 10\sqrt{2}$ km. The ship at A is moving with velocity u_1 (=25 km/hr) westward and the ship at B with velocity u_2 (=25 km/hr) northward. We get the relative velocity of the ship B with respect to the ship A if we find the difference of the two velocity vectors. Let $\vec{QP}$ represent the velocity u_2 in magnitude and direction and $\vec{PR}$, a velocity $-u_1$. Summing up these two vectors, we get the resultant $\vec{QR}$ which represents the relative velocity of B *w.r.t.* A. Now,

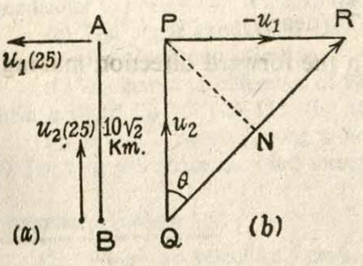


Fig. 2.34

$$|\overrightarrow{QR}| = \sqrt{PQ^2+PR^2} = \sqrt{(25)^2+(25)^2} = 25\sqrt{2}\text{ km/hr} = 35{\cdot}35\text{ km/hr.}$$

Also, $\tan\theta = \frac{PR}{QP} = \frac{25}{25} = 1 \quad \therefore\ \theta = 45°$.

The distance of closest approach is PN, the perpendicular distance of P from QR. Now, $PN = PQ \sin 45° = 10.\sqrt{2} \times \frac{1}{\sqrt{2}} = 10$ km.

Also, $ON = 10\sqrt{2}\cos 45° = 10$ km.

So, the required time $= \frac{QN}{\text{relative velocity}} = \frac{10}{35{\cdot}35}$ hr $= 17$ mnt. (nearly).

2.18. Relative acceleration : *By the relative acceleration of a body A with respect to another body B we mean the rate of change of velocity of A as observed by the body B.* Like relative velocity, relative acceleration can also be determined by vector subtraction. Thus, if $\mathbf{f}_1$ and $\mathbf{f}_2$ be the accelerations of A and B respectively, then the acceleration of A relative to B is given by $\mathbf{f}_{AB} = \mathbf{f}_1 - \mathbf{f}_2$

Also acceleration of B relative to A is given by $\mathbf{f}_{AB} = \mathbf{f}_2 - \mathbf{f}_1$

Example : *Due to sudden application of brakes in a running train, a suitcase from the bunk of a compartment slipped down. If the retardation of the train is $1{\cdot}2\ m/s^2$ and the downward acceleration of the suitcase w.r.t. the earth is $9{\cdot}8\ m/s^2$, find the direction in which the suitcase will appear to fall to the passengers of the compartment.*

Ans. In fig. 2·35 the acceleration of the train or of the passenger $\vec{f} = \overrightarrow{AO} = -\overrightarrow{AD}$. The acceleration of the suitcase $\vec{g} = \overrightarrow{AB}$.

The relative acceleration of the suitcase $\mathbf{F} = \overrightarrow{AB} - \overrightarrow{OA}$

$= \overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC}$

Now, $\tan \angle BAC = \frac{BC}{AB} = \frac{AD}{AB} = \frac{1{\cdot}2}{9{\cdot}8} = 0{\cdot}1224$

$\therefore\ \angle BAC = 7°$ (nearly)

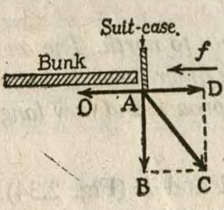


Fig. 2.35

So, the passengers will see the suitcase fall in the forward direction making an angle 7° with the vertical.

Exercises

Essay type :

1. What are scalars and vectors ? Explain with illustrations. How will you express a vector geometrically ?

2. What is the law of triangle in respect of the composition of vectors ? How will you get the addition of two vectors analytically with the help of the law of triangle ?

3. State the law of paralleogram. Two vectors P and Q are acting from a point in two

different directions. Determine the magnitude and direction of their resultant vector by the law of parallelogram. What will be the result if the vectors are mutually perpendicular ?

4. How would you get the subtraction of two vectors ? What is unit vector ?

5. What do you mean by resolution of a vector ? How will you resolve a single vector in two mutually perpendicular components ?

6. Explain the composition of a number of vectors by (i) the process of resolution and (ii) the polygon rule.

7. What is relative velocity ? Give a few illustrations. How can you determine the relative velocity of two particles when (i) they are moving along the same line, (ii) moving along two different lines ?

Short answer type :

8. Name four scalar and four vector quantities. Which of the following are scalars and which are vectors ?—(*a*) volume (*b*) energy (*c*) momentum (*d*) temperature (*e*) electric field intensity, (*f*) magnetic potential (*g*) sound frequency.

9. Work is the product of force and displacement of the point of application of the force. Both force and displacement are vectors. Is work, then, a vector or a scalar ?

10. Can two vectors of different magnitude be combined to give a zero resultant ? Can three vectors ? Explain your answer. Can a vector be zero if one of its components is not zero ?

11. A quantity has magnitude zero. Is there any sense in calling that quantity a vector ?

12. Explain (*a*) Collinear vectors, (*b*) Null vector (*c*) Opposite vectors.

13. What are (i) Scalar product and (ii) Vector product of two vectors ?

14. Distinguish between scalar addition and vector addition. [*H. S. Exam. 1980*]

15. During the rains, the rain drops falling vertically appear to come down obliquely to a person sitting in a running train. Expalin. [*H. S. Exam. 1979*]

Objective type :

16. Tick the correct answer :—

(a) Out of the following the only quantity that is vector is (i) specific heat, (ii) temperature, (iii) torque, (iv) speed.

(b) Two vectors **A** and **B** are parallel when (i) $\mathbf{A}.\mathbf{B}=0$ (ii) $\mathbf{A}\times\mathbf{B}=0$ (iii) $\mathbf{A}=0$ (iv) $\mathbf{B}=0$.

(c) Two vectors **A** and **B** are perpendicular when (i) $\mathbf{A}.\mathbf{B}=0$ (ii) $\mathbf{A}\times\mathbf{B}=0$ (iii) $\mathbf{A}=0$ (iv) $\mathbf{B}=0$.

(d) The vector product of two vectors **A** and **B** is a vector which is (i) perpendicular to both **A** and **B** and in the plane of **A**, (ii) perpendicular to both **A** and **B**, and in the plane of **B**, (iii) perpendicular to both **A** and **B** and to the plane containing **A** and **B**.

(e) The scalar product of two vectors **A** and **B** satisfies (i) commutative law, (ii) anti-commutative law of multiplication.

(f) The sum and difference of two vectors have the same magnitude. The angle between them is (i) 0° (ii) 90° (iii) 120° (iv) 60°.

(g) A car is moving along a horizontal road while rain is falling downwards vertically. (i) The rain will strike the hind screen of the car (ii) the front screen (iii) both the screens.

Numerical Problems :

17. When two velocities 3 cm/sec and 5 cm/sec are imposed simultaneously on a body, the body moves with a resultant velocity of 7 cm/sec. What is the angle between the component velocities ? [Ans. 60°]

18. Two vectors of 7 and 9 units are acting from a point inclined to each other at an angle of 60°. Find the magnitude and direction of the resultant vector. [Ans. 13·9 unit ; 34°6′]

19. A car is driven eastward for a distance of 50 miles, then northward for 30 miles and

then in a direction 30° east of north for 25 miles. Draw the vector diagram and determine the total displacement of the car from its starting point. [Ans. 81 miles ; 39·5° *N* of *E*]

20. What is the angle between two equal vectors so that their resultant is one third of any one of the vectors ? [Ans. 160°]

21. When the angle between two vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is θ, their resultant is $5\sqrt{a^2+b^2}$, but when the angle is (90° − θ), the resultant is $3\sqrt{a^2+b^2}$; show that $\tan\theta=\frac{1}{3}$

22. Show that two vectors of equal magnitude have a resultant which bisects the angle between the component vectors.

23. A steamer is moving due north with a velocity 10 km/hr. On the deck of the steamer, a passenger is walking due east with a velocity 4 km/hr. In what direction and with what velocity will the passenger appear to move to a person standing on the bank of the river ?

[Ans. 10·8 km/hr ; $\tan\theta=\frac{2}{5}$]

24. How can you represent a vector by co-ordinates ? Represent the vector $\vec{AB}$ when *A* and *B* have co-ordinates (−1, 3, 5) and (2, −1, 3) respectively. [Ans. $\vec{AB}=3\mathbf{i}-4\mathbf{j}-2\mathbf{k}$]

25. (i) $\vec{A}=5i+6j+8k$ and $\vec{B}=3i+7j-10k$ represent two vectors. Find their sum.

[Ans. $8i+13j-2k$]

(ii) Show that $|\mathbf{A}+\mathbf{B}| \leq |\mathbf{A}|+|\mathbf{B}|$ and $|\mathbf{A}-\mathbf{B}| \geq |\mathbf{A}|-|\mathbf{B}|$.

26. A steamer is going due east with a velocity 12 miles/hr and another due north with a velocity 16 miles/hr. Calculate the relative velocity of the second steamer with respect to the first. [Ans. 20 miles/hr. ; θ° west of north, $\tan\theta=\frac{3}{4}$]

27. The windscreen of a motor car is vertical. The car is travelling with a velocity *v* along a horizontal road and the rains are falling vertically, with a velocity *v*. At what angle will the rain strike the windscreen ? [Ans. 45°]

28. A car is travelling due north a velocity 32 km/hr. To a passenger in the car, the rain appears to come from east. When the speed of the car increases to 64 km/hr, the rain appears to come from north-east. What is the actual velocity of rain ? [Ans. 45 km/hr south-east]

Harder Problems :

29. A particle simultaneously possesses three velocities $(\sqrt{3}+1)$, $\sqrt{6}$ and 2 units. If the particle is at rest, find the angles at which the velocities are inclined to one another.

[Ans. 105,° 120°, 135°]

30. What is the angle between two equal vectors so that their resultant is one-third of one of the vectors ? [Ans. 160°]

31. Show that the magnitude of the resultant vector of two component vectors acting simultaneously at a point can not be greater than the sum of the magnitudes of the component vectors nor can it be smaller than the difference of their magnitudes.

32. When two vectors *P* and *Q* inclined at an angle θ act on a body, the resultant is $(2k+1)\sqrt{P^2+Q^2}$. When the vector are inclined at an angle (90° − θ), the resultant is $(2k-1)\sqrt{P^2+Q^2}$. Prove that $\tan\theta=(k-1)/(k+1)$.

33. The resultant of two vectors $\vec{a}$ and $\vec{b}$ acting at a point is $\vec{r}$. If $\vec{b}$ is doubled, the resultant also becomes doubled. If $\vec{b}$ is reversed, then also, the resultant becomes doubled. Show that $|\vec{a}| : |\vec{b}| : |\vec{r}| = \sqrt{2} : \sqrt{3} : \sqrt{2}$.

34. $\vec{P}, \vec{Q}$ and $\vec{R}$ are three coplanar vectors. Show by geometric construction that

$$\vec{P}+(\vec{Q}-\vec{R})=\vec{Q}-(\vec{R}-\vec{P}).$$

35. The position vectors of four points A, B, C and D are $\mathbf{r}_1=2\mathbf{i}+3\mathbf{j}+4\mathbf{k}$, $\mathbf{r}_2=3\mathbf{i}+5\mathbf{j}+7\mathbf{k}$, $\mathbf{r}_3=\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ and $\mathbf{r}_4=3\mathbf{i}+6\mathbf{j}+9\mathbf{k}$ respectively. Examine whether vectors $\vec{AB}$ and $\vec{CD}$ are collinear.

[*Hints* : $\vec{AB}=(\mathbf{r}_2-\mathbf{r}_1)$ and $\vec{CD}=(\mathbf{r}_4-\mathbf{r}_3)$. Now see art 2·12]

36. The resultant of two vectors $\vec{A}$ and $\vec{B}$ acting at a point is equal to $\sqrt{3}\,\vec{B}$ and makes an angle 30° with the direction of $\vec{A}$. Prove that $|\vec{A}|=|\vec{B}|$ or $|\vec{A}|=|2\vec{B}|$.

37. To a person moving due west in a train with a velocity 40 km/hr, a car appears to move due north with a velocity of $40\sqrt{3}$ km/hr. What is the actual velocity and direction of motion of the car ? [Ans. 80 km/hr ; 30° W of north]

38. A boat and a motor launch started from the same point in two different directions inclined to each other at an angle α. The speed of the motor launch is m times that of the boat. To the boatman, the launch appears to move at an angle of 45° with the direction of the boat. Show that $\sin 2\alpha=\dfrac{m^2-1.}{m^2}$

39. A boatman can row directly across a river in time t_1 and an equal distance downstream in time t_2. If u be the speed of the boat in still water and v that of the river, show that

$$t_1 : t_2=\{(u+v)/(u-v)\}^{\frac{1}{2}}.$$

40 Rain water is falling vertically downwards with a velocity V cm/sec when velocity of wind is zero and water is collected at a certain rate in a vessel in the rain. What will be the change in the rate of collection of water when the velocity of wind is W cm/s in a direction perpendicular to V ? [*Jt. Entrance 1982*] [Ans No change]

3

NEWTON'S LAWS OF MOTION

3.1. Kinetics : In the preceding chapters we discussed motion in its abstract form from a kinematical point of view *i.e.* motion had been discussed without any regard to the mass of the body that moved or the cause which produced motion.

Kinetics refer to *what moves* and *what causes it to move.* Newton's laws of motion which deal with the mass of the body and the force which causes the body to move, are considered as the pillars of kinetics.

3.2. Newton's laws of motion : In 1687, Sir Isaac Newton, in his famous treatise named 'Principia' discussed, in detail, the motion of different bodies—specially the motion of planets and other heavenly bodies. This treatise reveals the astounding talent of Newton. In the first part of the book, he established three laws known as the laws of motion.

- **First law** : *Everybody continues in its state of rest or of uniform motion in a straight line, unless it is compelled by any external impressed force to change that state.*
- **Second law** : *Rate of change of momentum of a body is proportional to the impressed force and takes place in the direction in which the force acts.*
- **Third law** : *To every action there is an equal and opposite reaction.*

3.3. Discussion of the first law : From a discussion of the first law of motion, we come to know of the following :—(i) *Inertia of matter* and (ii) *Definition of force.*

Inertia of matter : The first law states that if a body be at rest, it will continue to be at rest for ever or if a body be in motion it will, likewise, continue to be in motion in a straight line. The inability of the material bodies to change the state of rest or of uniform motion is referred to as the inertia of the material bodies and it is a fundamental property of matter. For this reason Newton's first law of motion is sometimes known as the *law of inertia.* A material body, by virtue of its inertia, will therefore oppose any external attempt to change its state. Inertia may be divided into two parts, viz (i) *Inertia of rest* and (ii) *Inertia of motion.*

It is not difficult to have a conception of inertia of rest because it is a matter of common experience that objects, at rest, do not begin to move of their own accord. If we place an object at a certain place, we expect it to remain there unless a force is applied on it. So, inertia of rest is a commonsense matter.

But it is not *immediately obvious* that a body moving with uniform velocity in a straight line tends to go on moving for ever without coming to rest. A ball rolled along the ground moves for some time but finally comes to rest. It does not move for ever. How is, then, the truth of inertia of motion defended? It is very difficult to defend because no one has yet found a means of eliminating the various outside forces which can retard a moving body. The ball does not move for ever because external forces like friction, air-resistance etc. retard the motion of the ball. We know if we roll a ball on a smooth floor or ice, it will go further than it goes on the ground because a smooth floor or ice offers less friction than ground. It would, therefore, be reasonable to suppose that if air-resistance, friction etc. could be eliminated, the ball would go on moving in a straight line for ever. In this way, we can have a conception of the inertia of motion.

Illustrations of inertia of rest : (a) If a car, carrying passengers, suddenly starts moving, the passengers have a tendency to fall back. This is an example of inertia of rest. When the car is at rest, the whole body of a passenger is at rest. With the sudden start of the car, the lower part of the body, which is in contact with the car, moves forward but the upper part, due to inertia of rest, continues to be at rest. The passenger consequently, leans back.

(b) Suppose a table cloth is laid on a smooth dinning table and some dishes and cups are kept on the table cloth. If now a sudden pull be given to the table cloth, the cloth may be removed without dislodging the dishes and cups. This is an illustration of inertia of rest. The cups and dishes tend to remain at rest because the sudden motion of the table cloth exerts no large force for any appreciable length of time on the objects kept on the table cloth.

(c) A cup is placed on a vertical stand (fig. 3.1) A small ball rests on a card placed over the cup. If, now, a spring is pulled aside and let go, the spring will flick the card away. The ball will be found to drop neatly into the cup. The card moves away so quickly that the ball cannot pertake of the motion of the card. The ball continues its state of rest and drops into the cup.

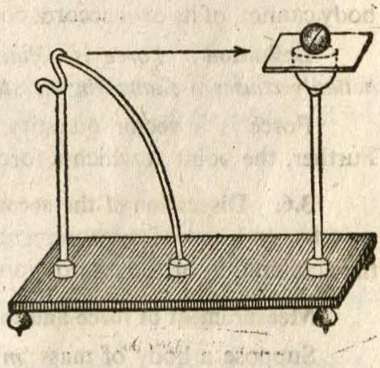


Fig. 3.1

Illustrations of inertia of motion :

(a) A person riding a bicycle along a level road does not come to rest as soon as he stops pedalling. The bicycle continues to move forward due to inertia of motion but eventually comes to rest due to retarding force of air resistance and friction.

(b) When a train or a motor car is suddenly stopped, the passengers are shot forward because their bodies tend to maintain their motion.

Similarly, if a passenger alights from a moving car absent-mindedly he is thrown forward. When inside the moving car, his whole body is in motion. But as soon as his feet touch the ground, the lower part of his body comes to rest but the upper part, due to inertia of motion, continues its forward motion.

(c) When a whirling electric fan is switched off, the blades of the fan do not stop immediately. The blades go on whirling for some time due to inertia of motion, the speed gradually becoming slower due to friction, air-resistance etc.

(d) When a railway engine pulls uncoupled wagons through a certain distance and suddenly stops, the wagons already in motion, continue to move forward and push against one another.

3·4. Mass of a body is a measure of its inertia :

The inertia of a body is expressed in terms of a very useful quantity, called the *mass* of the body. Let us try to set a car and a ball into motion by push. We know that it is much more difficult to set a car into motion rather than the ball. This indicates that, a car offers a larger resistance against the effort to put it in motion than the ball. We say that the mass or inertia of the car is larger than that of the ball. The quantity *mass (not weight) is therefore a measure of the inertia of the body.* The larger the mass of a body, the more difficult it will be to put it in motion. In the same way, if a body is in motion, even then mass is a measure of the resistance offered by the body against any effort to change its motion. Thus, a lorry and a car when travelling with same speed, will have different inertia of motion because of their different masses.

Mass and inertia are measured in the same units *viz.*, grammes or pounds.

3.5. Force : From the first law, we also know that an external force must be applied on a body in order to change its state of rest or of motion. A material body cannot, of its own accord, come to rest or accelerate or decelerate its motion.

Definition : *Force is defined as an external agency that tends to cause or actually causes a change in the state of rest or of uniform motion of a body.*

Force is a vector quantity, because it has both magnitude and direction. Further, the point at which a force acts is called its point of application.

3.6. Discussion of the second law : From a discussion of the second law, we come to know of measurement of force and of a relation between the force and the acceleration or the retardation.

Measurement of force and the equation P=mf :

Suppose a body of mass 'm' is moving with a velocity u. If a force P acts on the body for time 't', the body will have an accelerated motion and its velocity will change. Suppose, the acceleration produced on the body is 'f' and its velocity after time 't' is v.

Now, the change of momentum of the body $= mv - mu$.

And, the rate of change of momentum $= \dfrac{mv-mu}{t} = \dfrac{m(v-u)}{t}$

$$= mf. \left[\because \text{acc}^n \ f = \frac{v-u}{t} \right]$$

Now, from Newton's second law, we know, $P \propto$ rate of change of momentum $\propto mf$. So, $P = K.mf$, where K is a constant.

We now choose the unit of force in such a way that it produces unit acceleration on unit mass *i.e.*, $P=1$, when $m=1$ and $f=1$, then $K=1$. According to the above unit of force, we see that $P=mf$.

In other words, **force = mass × acceleration.** This equation gives us the measurement of force.

[*Alternative calculus method* :

If the body of mass m moves with a velocity v, its momentum $p = m.v$. If, now, a force P acts on the body for a small time dt, the rate of change of momentum of the body $= \frac{d}{dt}(mv) = m.\frac{dv}{dt}$ (m is constant). According to the second law of motion, $P \propto m.\frac{dv}{dt} \propto m.f.$ $\quad (f = \frac{dv}{dt} = \text{acceleration})$

So, $P = K.m.f.$

By choosing the unit of force, the constant of proportionality K can be made equal to 1. So, we have $P = m.f.$]

From the above equation, we know that

(i) if a force acts on a mass 'm' and produces an acceleration f in it, the magnitude of the force will be equal to mass $(m) \times$ acceleration (f).

(ii) if a force P acts on a moving mass 'm' such that the direction of the force coincides with the direction of the motion, then, the motion of the body will be accelerated, the acceleration being $f = \frac{P}{m}$.

(iii) if a force P acts on a moving mass 'm' such that the direction of the force is against the direction of the motion, then the motion of the body will be retarded, the retardation being, $f = \frac{P}{m}$.

Units of force in different systems : There are two systems of units of force *viz* (i) *Absolute system* and (ii) *Gravitational system.*

In absolute system, a *unit force is defined as that force which acting on a unit mass produces unit acceleration on it.*

Units of force in gravitational system will be taken up in Chapter 1. (General properties of matter).

(i) *Dyne* is the unit of force in the *C.G.S.* system.

Definition : *It is the force which acting on a mass of* 1 *gm. produces an acceleration of* 1 *cm/sec*^{2}.

In this system, there is a bigger unit, known as 'megadyne.'

1 megadyne $= 10^6$ dynes.

(ii) *Poundal* is the unit of force in the *F.P.S.* system.

Definition : *It is the force which acting on a mass of* 1 *lb. produces an acceleration of* 1 *ft/sec*^{2}.

(iii) *Newton* is the absolute unit of force in the *M.K.S.* system.

Definition : *It is the force which acting on a mass of* 1 *kg produces an acceleration of* 1 *metre/sec.*^{2}. The unit is named after the celebrated scientist, **Sir Isaac** Newton.

These three units, *viz.* dyne, poundal and newton, are called absolute units because they remain unaltered under all circumstances. It is to be borne in mind that the equation $P=mf$ is applicable only when the force is expressed in absolute unit.

(i) **Relation between a poundal and a dyne :**

We know, 1 poundal=1 lb×1 ft/sec^2

=453·6 gm×30·48 cm/sec^2 [1 inch=2·45 cm.]
=453·6×30·48 dynes [1 ft=30·48 cm]
=13,825·728 dynes

In round figures, 1 poundal=13,800 dynes.

(ii) **Relation between a newton and a dyne :**

1 Newton (N)=1 kg×1 metre/sec^2 =10^3 gm×10^2 cm/sec^2=10^5 dynes.

(iii) **Relation between a newton and a poundal :**

1 Newton=1 kg×1 metre/s^2=2·2 lb×3·28 ft/s^2=7·2 poundals (nearly)

Examples : (1) *A force acts on a body of mass 16 gm. for 3 sec and then ceases to act. During the next 3 sec, the body moves through a distance of 81 cm. What is the force acting on the body?* [*Tripura H.S. Exam 1982*]

Ans. The body moves through a distance of 81 cm in 3 sec with the uniform velocity generated by the force acting for the first 3 sec. Now, if this uniform velocity be v, then $81=3\times v$ or $v=27$ cm/s. Now considering the motion of the body during the first 3 sec. we have $v=f\times t$ or $27=f\times 3$ or $f=9$ cm/s^2. This acceleration is evidently produced by the force.

Now, $P=m.f=16\times 9=144$ dynes.

(2) *A waterjet issues from a nozzle of 2 sq. inch section with a velocity of 50 ft/s and strikes a plane surface at right angles to the jet. Find the force exerted on the plane. Mass of the 1 cu ft. of water=62·5 lb.*

Ans. Volume of water striking the plane per sec=area of the nozzle× velocity=$\frac{2}{144}\times 50$ cu. ft ; Mass of this water =$\frac{2}{144}\times 50\times 62{\cdot}5$ lb.

∴ The change of momentum of water per sec.=mass×velocity
=$\frac{2}{144}\times 50\times 62{\cdot}5\times 50$ units=2170·14 units.

Since force is the rate of change of momentum, here, P=2170·14 poundals.

(3) *A force of* 50 *poundals acts on a body of* 160 *lb. When will its velocity be* 80 *ft/sec?*

Ans. We know, $P=mf$.

Here, P=50 poundals ; m=160 lb. So, $50=160\times f$ or $f=\frac{50}{160}=\frac{5}{16}$ ft/sec.^{2}.

Now, if 't' be the required time, then, $v=f.t.$ or, $80=\frac{5}{16}\times t$

$$\therefore\ t=\frac{80\times 16}{5}\text{ sec}=256\text{ sec}=4\text{ mnt. }16\text{ sec.}$$

(4) *A mass of* 20 *lb is acted on by a constant force which in* 5 *seconds produces a velocity of* 15 *ft/sec. Find the force, if the mass was initially at rest.*

Ans. Here $u=0$, $v=15$ ft/sec ; $t=5$ sec ; $f=?$
we have, $v=u+ft$. So, $15=0+f.\,5$. or, $f=3$ ft/sec^2
Now, $m=20$ lb ; $f=3$ ft./sec^2 ; $P=?$
From, $P=mf$, we have, $P=20\times3=60$ poundals.

(5) *A mass of* 10 *gm. rests on a smooth horizontal ground. If a force of* 96 *dynes acts on the mass for* 10 *seconds, how far will it travel* ?

Ans. Here, $P=96$ dynes ; $m=10$ gm ; $f=?$
We know, $P=m.f$, so, $96=10f$ or $f=\frac{96}{10}$ cm/sec.^{2}
Again, $u=0$; $f=\frac{96}{10}$ cm/sec^2 ; $t=10$ sec ; $S=?$
Here, $S=ut+\frac{1}{2}ft^2=0+\frac{1}{2}\times\frac{96}{10}\times(10)^2=480$ cm.

(6) *A train of mass* 1 *metric ton acquires a velocity of* 30 *km/hr in* 25 *sec. Find the force exerted by the engine in M.K.S. unit.*

Ans. mass $(m)=1$ metric ton$=1000$ kg ; velocity $(v)=\dfrac{30\times1000}{60\times60}$ m/s.

If f be the acceleration of the train, $f=\dfrac{v}{t}=\dfrac{30\times1000}{60\times60\times25}=\frac{1}{3}$ m/s^2

Now, $P=m.f=1000\times\frac{1}{3}=333{\cdot}3$ N.

(7) *Sand drops vertically at the rate of* 200 *gm per second on to a conveyor belt moving horizontally with a velocity of* 5 *cm/s. Find the force on the belt.*

Ans. Velocity of the sand particles in the horizontal direction is zero. Rate of change of mass$=20$ gm/s.

So, change of momentum per sec$=$rate of change of mass$\times$velocity
$=200\times5=1000$ gm-cm.

So, the force on the belt$=1000$ dynes.

[*N.B.* In the above example, the velocity remains constant but mass changes. According to Newton's second law, force$=\dfrac{d}{dt}(mv)=v\times\dfrac{dm}{dt}$ where v is constant. In the above example, $\dfrac{dm}{dt}=200$ gm/s.]

3.7. Physical independance of force :

As already stated, the second part of the second law of motion furnishes us with the physical independence of force. The second part of the law states that the change of motion *i.e.* the change of momentum takes place in the direction of the straight line in which the force acts.

The above statement means that if a body be moving horizontally and a vertical force is applied on the body, the force produces a change of motion in the vertical direction independent of whether the body is at rest or moving in the horizontal direction.

As an illustration, if a ball is thrown vertically upward inside a running train, it will reach the floor of the compartment at the same spot as if the train were at rest. Although the ball is thrown upward, it has a horizontal velocity along with the train and the attractive force of the earth is in the vertically downward direction. Therefore, the change of motion of the ball in the vertically downward direction due to this attractive force is independent of the previous horizontal motion of the ball.

Arguing in the same way, *if there be a number of forces acting on a body, each force will produce a change of motion of the body in the direction in which the force acts, independent of the existence of other forces. This is known as the physical independence of force.*

3.8. Inertial mass : We have already seen that inertia of a body is measured in terms of a quantity we prefer to call the *mass* of the body. We have also seen that this quantity—the mass—appears as the constant of proportionality between force and acceleration in Newton's second law. By selecting some convenient stable object as the standard of mass we can compute numerical values of the masses of other objects.

The same force applied to different bodies produces different acceleration in them. The acceleration produced depends on the inertia of the body—the greater the inertia, the lesser the acceleration. A measure of the difficulty of accelerating a body is referred to as the *inertial mass* of the body. If the same force P acts on two bodies and produces accelerations f_1 and f_2, then $\frac{f_2}{f_1}=\frac{m_1}{m_2}$, where m_1 and m_2 are the inertial masses of the bodies.

If one of the bodies be a standard body, the above equation can be used to compute the inertial mass of the other body.

3.9. Different effects of force on a body :

The following are the effects of force on a body :

(i) To produce acceleration or deceleration in a body.

(ii) To overcome another opposing force.

(iii) To produce strain in a body.

We have already seen that a force needs be applied on a body to set it into motion if it is at rest or to move it faster or slower, if it is already in motion. In other words, force can produce acceleration or deceleration. Another function of force is to overcome or tend to overcome an opposing force. In this respect, friction and gravitation offer best illustrations. Under the action of an external force a body will not move unless and until, the external force exceeds the opposing frictional force. Even when the body is in motion, the frictional force opposes the motion and a part of the external force is always expended in overcoming the frictional force. In the same way, in lifting a body to a crtain height from the ground, a force exceeding the force of gravity should be applied on the body.

In all these cases, two forces are found to act on a body, which are unequal and opposite to each other. As a result, a single resultant force acts on the body producing its motion. So, we can say *when an unbalanced force acts on a body, it produces a motion or an acceleration in the body.*

Forces may however, act on a body in such a way that the body does not get any motion but it is strained instead. When we press a rubber ball between our fingers or pull one end of a string keeping the other end fixed, they are not set into motion bodily but are strained. In such cases, the applied force and its reaction form a system of balanced forces which do not produce any bodily motion of the mass but produce a strain in it. Elasticity offers a very good example of it (vide General properties of matter, chapter 2). So, we can say that *a system of balanced forces produces strain in a body.*

3.10. (*a*) **Impulse of a force :** *Impulse of a force on a body is the product of the magnitude of the force and the time during which the force acts on a body.*

Suppose, a force P acts on a body for a time t. In this case, the impulse of the force $=P.t.$

If under the action of the force, the velocity of the body changes from 'u' to 'v', the acceleration 'f' produced is given by $f=\frac{v-u}{t}$. Again, from Newton's second law of motion, $P=m.f.=\frac{m(v-u)}{t}$

$$\therefore \text{ Impulse}=P.t.=\frac{m(v-u)}{t}\times t=m(v-u)$$

$$=mv-mu=\text{change of momentum.}$$

So, *Impulse of a force = change of momentum of the body.*

In calculus notation, force $P=\frac{dp}{dt}$ or, $P.dt=dp$ where p denotes the momentum of the body.

The *R.H.S.* of the above equation denotes the change of momentum ; and the *L.H.S.* the impulse. It is to be noted that impulse is a vector quantity having the direction same as that of the force.

(*b*) **Impulse of a varying force :**

If a varying force acts on a body for time 't', then to determine the impulse, the duration is to be divided into a large number of small intervals such as $t_1, t_2, t_3 \ldots t_n$. If the forces acting on the body during those intervals be $P_1, P_2, P_3 \ldots P_n$ respectively, then since the intervals are very small, the forces may be supposed constant in each interval. The velocities of the body at the end of successive intervals are, suppose, $v_1, v_2, v_3 \ldots v_n$.

Then, during the 1st interval, $P_1t_1=mv_1-mu$ [u=initial velocity]

" " 2nd " $P_2t_2=mv_2-mv_1$

$$\cdot\cdot \quad \cdot\cdot \quad \cdot\cdot$$

$$\cdot\cdot \quad \cdot\cdot \quad \cdot\cdot$$

$P_n.t_n=mv-mv_{n-1}$ [v=final velocity]

Adding, total impulse $=P_1t_1+P_2t_2+\ldots+P_nt_n$

$=mv-mu$

[With the help of calculus, the total impulse can be very easily calculated in the following way :—

The impulse in a small interval $dt=P.dt$. If the force acts for time t, the total impulse

$=\int_0^t P.dt=\int_0^t \frac{dp}{dt}\,dt=p_t-p_0=mv-mu,$ where $p_t=mv$=the final momentum and $p_0=mu$= initial momentum.]

(*c*) **Impulsive force :** *An impulsive force is a large force which acts on a body for a very short period of time.*

Kicking a football, striking a ball with a bat, hammering a nail, collisions etc. are the examples of impulsive forces. The difference between an impulsive force and an ordinary force is that an ordinary force, acting on a body, produces measurable acceleration from which the force can be gauged while the impulsive force, which acts for a very short duration, produces acceleration so quickly that it can not be measured. We can measure only the change of momentum of the body and from it, the impulsive force. Thus it is easier to catch a tennis ball as compared to a cricket ball moving with the same speed. The mass of a cricket ball is more than that of a tennis ball. The change in momentum is more in the case of a cricket ball. Consequently greater force is to be applied in the case of a cricket ball.

Examples : (1) *A foot-ball player takes a spot-kick and sends the ball with a velocity 9·6 metres/sec. If the time of contact between the ball and the feet be $\frac{1}{8}$ second, what is the force applied to the ball ? The ball weighs 300 gm.*

Ans. We know, $P.t=m(v-u)$.

Here, $u=0$; $v=9{\cdot}6$ metres/sec$=960$ cm./sec ; $m=300$ gm; $t=\frac{1}{8}$ sec.

$\therefore \quad P\times\frac{1}{8}=300(960-0)=300\times 960$ or, $P=2304\times 10^3$ dynes.

(2) *A pile-driver weighing 30 lb falls through 16 ft. and is brought to rest in $\frac{1}{8}$ sec., after striking the pile. Find the force on the pile.*

Ans. At first, we are to calculate the velocity of the pile-driver just before striking the pile. We know, $v^2=u^2+2g.s.$ Here, $u=0$; $g=32$ ft./sec^2 ; $s=16$ ft.

$\therefore \quad v^2=2\times 32\times 16$ or, $v=32$ ft/sec.

So, the momentum of the driver$=mv=30\times 32$ ft. lb units.

Here, $Pt=mv$ or, $P\times\frac{1}{8}=30\times 32$

$\therefore \quad P=30\times 32\times 8$ poundals$=30\times 8$ lb. wt.$=240$ lb. wt.

This is the force due to striking. But the total force on the pile should include the weight of the driver. Hence, the total force $=240+30=270$ lb. wt.

(3) *A person of mass 50 kg jumps from a height of 6·4 metres and is brought to rest within $\frac{1}{10}$ sec without flexing his knees. Had he flexed his knees, he would have come to rest in 2 sec. What forces will be exerted on his body in the two cases ?*

Ans. Mass of the person$=50$ kg$=50\times 10^3$ gm. The velocity of the person just before touching the ground$=\sqrt{2gh}=\sqrt{2\times 980\times 640}=1120$ cm/s.

In the first case, force exerted on his body$=\dfrac{\text{Change of momentum}}{\text{time}}$

$$=\frac{50\times 10^3\times 1120}{\frac{1}{10}}=56\times 10^7 \text{ dynes.}$$

In the second case, force exerted on the body$=\dfrac{50\times 10^3\times 1120}{2}$ dynes.

$$=28\times 10^6 \text{ dynes.}$$

So, if he does not flex his knees, a force about 20 times will be exerted on him.

[*N.B.* The above example shows that less damage is done to a person if he jumps to the ground from a certain height and flexes his knees in landing.]

(4) *A cricket ball of mass 150 gm is moving with a velocity of 12 metres/s and is hit by a bat so that the ball is turned back with a velocity of 20 metres/s. The force of the blow acts for 0·01 sec. Find the average force exerted on the ball by the bat.* [*I.I.T. 1974*]

Ans. Forward momentum of the ball=forward velocity×mass $=12\times10^2\times150=18\times10^4$ gm-cm/s.

Backward momentum =backward velocity×mass

$=20\times10^2\times150=30\times10^4$ gm-cm/s

Change of momentum $=18\times10^4-(-30\times10^4)=48\times10^4$ gm-cm/s

We know, Impulse =change of momentum

or, $P\times t=48\times10^4$

or, $P=\dfrac{48\times10^4}{t}=\dfrac{48\times10^4}{0{\cdot}01}=48\times10^6$ dynes. = 480 Newton.

3.11. Composition of forces :

Force is a vector quantity, having both magnitude and direction. So forces can be combined according to the law of composition of vectors.

Suppose, a force is acting on a body due south. The body will move towards south in the direction of the force. If, now, another force due east, acts simultaneously on the body, the resultant force will, evidently, act in a particular direction, depending upon the magnitudes of the two forces. To obtain the magnitude and direction of the resultant force, we shall have to apply the law of parallelogram of forces.

Law of parallelogram of forces : *The law states that when two adjacent sides of a parallelogram, drawn from a point, represent two forces, in magnitude and direction, then the diagonal of the parallelogram drawn from the angular point, represents in magnitude and direction, the corresponding resultant.*

Suppose, two forces P and Q act simultaneously at O on a body and they are represented, in magnitude and direction by the vectors $\vec{OA}$ and $\vec{OB}$. If now, the parallelogram $OACB$ is constructed and the diagonal OC drawn, then, the vector $\vec{OC}$ represents fully the resultant of the forces, P and Q (Fig. 3.2)

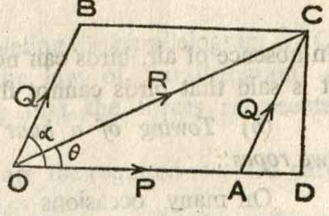


Fig. 3.2

Let $\angle AOB=\alpha$. Produce the line OA to D and draw CD perpendicular on OD. From trigonometry, we have,

$$\begin{aligned}|OC|^2&=OA^2+AC^2-2\,OA.AC.\cos OAC\\&=OA^2+AC^2-2OA.\,AC.\cos(180-\alpha)\\&=OA^2+AC^2+2OA.\,AC\cos\alpha\quad[\because\ \cos(180-\alpha)=-\cos\alpha]\\&=OA^2+OB^2+2.\,OA.\,OB\cos\alpha\end{aligned}$$

[∵ OB and AC are equal and parallel]

But $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ represent P, Q and the resultant force R (say) respectively. So, $R^2 = P^2 + Q^2 + 2\,P.Q.\cos\alpha$.

From this equation, we get the magnitude of the resultant force. To get its direction, let OC be inclined to the line OA at an angle θ. In this case.

$$\tan\theta = \frac{CD}{OD} = \frac{CD}{OA+AD} = \frac{AC.\sin\alpha}{OA+AC.\cos\alpha} = \frac{Q\sin\alpha}{P+Q\cos\alpha}$$

Special cases of composition :

(i) If the two forces act in the same direction *i.e.* if $\alpha=0$, then $\cos\alpha = \cos 0 = 1$ and in that case, $R^2 = P^2 + Q^2 + 2\,P.Q$ or, $R = P + Q$.

(ii) If the two forces act in the opposite directions *i.e.*, if $\alpha = 180°$, then $\cos\alpha = \cos 180° = -1$ and in that case, $R^2 = P^2 + Q^2 - 2PQ$ or, $R = P - Q$.

(iii) If the two forces act at right angles to each other *i.e.* if $\alpha = 90°$, than $\cos\alpha = \cos 90° = 0$ and in that case, $R^2 = P^2 + Q^2$

$$\text{or, } R = \sqrt{P^2+Q^2} \text{ and } \tan\theta = \frac{Q}{P}.$$

Illustrations : (*a*) *Soaring of a bird* : When a bird wants to soar up, it gains wind force in its favour along DO and EO by flapping its wings against air as shown in the fig. 3.3 (*a*). As a reaction, air exerts equal forces on the wings along OD and OE. If these forces are represented by $\overrightarrow{OB}$ and $\overrightarrow{OA}$ respectively, then according to the law of composition of forces, the diagonal $\overrightarrow{OC}$ of the parallelogram $OACB$ will represent the resultant force. Hence the bird will soar up high in the direction of $\overrightarrow{OC}$. In absence of air, birds can not get the reaction-forces and hence cannot fly. So, it is said that birds cannot fly in vacuum.

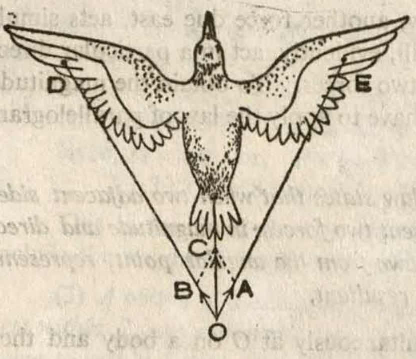


Fig. 3.3(*a*)

(*b*) *Towing of a boat with two ropes* :

On many occasions a boat is towed along a canal by two ropes. It is an illustration of composition of forces. Two boatmen exert equal pull on the boat along two ropes tied to the boat [Fig. 3.3(b)]. If $\overrightarrow{OB}$ and $\overrightarrow{OA}$

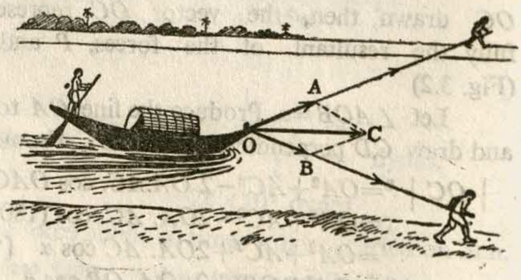


Fig. 3.3(*b*)

represent the equal force vectors, the resultant $\overrightarrow{OC}$ is obtained according to the parallelogram law of composition of forces. The boat, therefore, moves straight in the direction $\overrightarrow{OC}$ along the canal.

(*c*) *Action of a sling* : Action of a sling affords a good illustration of the composition of forces. Slings are very often used by children for shooting practice.

A stone is held at O on a rubber strap attached to two ends of the sling [Fig 3.3(c)]. The rubber strap is pulled inwards producing tensions $\overrightarrow{OA}$ and $\overrightarrow{OB}$ on the strings. According to the parallelogram law, a resultant force $\overrightarrow{OC}$ acts on the stone when the strap is released. The stone is hurled forward in the direction of $\overrightarrow{OC}$.

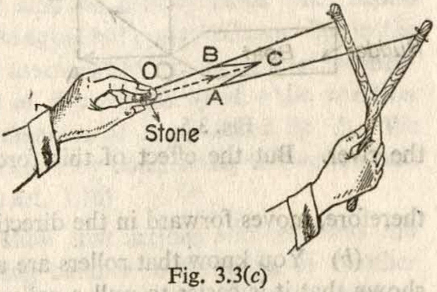


Fig. 3.3(*c*)

3.12. Resolution of forces : Resolution of a force is the reverse process of composition of forces. By resolution we mean splitting up of a single force into two components. The special case where the components are at right angles to each other is very important. Such components are called *resolved parts* of a force. We can find, in the following way, the resolved parts of a force.

Suppose, the vector $\overrightarrow{OC}$ represents, in magnitude and direction, a single force R. A straight line OA is drawn making an angle θ with OC. OB is a straight line drawn perpendicular to OA. It is required to find the resolved parts of the force R along OA and OB [Fig. 3·4].

Now, completing the parallelogram $OACB$ and applying the law of parallelogram of forces, it is clear that the forces represented by $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are the required resolved parts.

Now, $\cos\theta = \dfrac{OA}{OC} = \dfrac{P}{R}$ $\therefore$ $P = R.\cos\theta.$

And, $\sin\theta = \dfrac{CA}{OC} = \dfrac{OB}{OC} = \dfrac{Q}{R}$ $\therefore$ $Q = R.\sin\theta.$

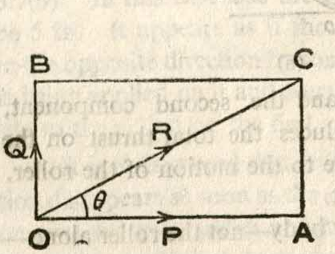


Fig. 3.4

Illustrations : (*a*) *Towing of a boat by a single rope* : It is often seen that a boat is towed with a single rope. This affords a good illustration of the resolution of a single force in actual practice.

Let OC be the direction in which the pull is exerted along the rope and the length $\overrightarrow{OC}$ represent the force in magnitude and direction (Fig. 3·5). If a parallelogram $OACB$ is drawn with OC as its diagonal, then according to the law of parallelogram of vectors, the vector $\overrightarrow{OC}$ may be said to have been resolved into the component vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$. The force $\overrightarrow{OB}$ being perpendicular to the river, will try to take the boat across the river. But the effect of this force is neutralised by the rudder. The boat, therefore, moves forward in the direction of $\overrightarrow{OA}$.

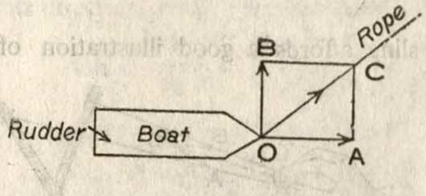
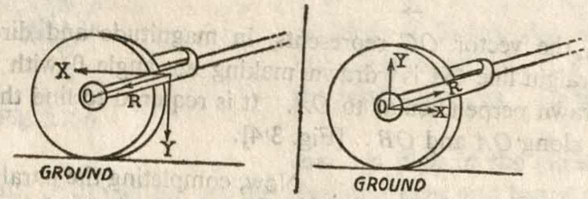


Fig. 3.5

(*b*) You know that rollers are used to level an uneven ground. It can be shown that it is easier to pull a roller along the ground than to push it.

Suppose, first, that the roller is being pushed and the force R is given in the direction shown in fig. 3.6 (i). Resolving this force into components parallel and perpendicular to the ground, we get two forces, one along X which helps the roller to move forward and another along Y vertically downward which increases the weight of the roller. This increased weight of the roller produces, in its turn, an increased reaction of the ground which offers greater resistance to the motion of the roller.

On the other hand, when the roller is pulled, a force R is applied in a direction shown in fig. 3.6 (ii). Resolving this force again, we get two components —one along X parallel to the ground and another along Y vertically upward. The first component helps the roller to move forward and the second component, acting in opposition to the weight of the roller, reduces the total thrust on the ground. Reduced reaction offers much less resistance to the motion of the roller. It thus becomes easier to pull a roller than to push it.



(i) Fig. 3.6 (ii)

As a matter of fact, it is easier to pull any heavy body—not the roller alone—than to push it.

3.13. Discussion of the third law of motion :

We take two bodies A and B. If A exerts a force on B, then according to the third law of motion, B will exert an equal and opposite force on A. The force exerted by *A*, in this case, is called the *action*, while that exerted by *B* is called the reaction. The law is applicable to two bodies irrespective of their motion, rest, contact or separation by a distance. In nature, forces always come in pairs. A single isolated force is an impossibility. These pairs are equal in magnitude but exactly opposite in direction. In symbols we can write $F_{AB}=-F_{BA}$, F_{AB} denoting the *action* of *A* on *B* and F_{BA} the equal and opposite *reaction* of *B* on *A*.

In the case of an ordinary force which acts for a measurable time, action means the aforesaid force and reaction means an equal but opposite force. But in the case of impulsive force, the effect of which is measured by the change of momentum, action means the change of momentum of *B* produced by *A* while reaction means an equal but opposite change of momentum of *A* produced by *B*. We can establish the *law of conservation of momentum* by considering the action and reaction in the case of impulsive forces. (See art. 3.16)

A simple experiment may be done to show that actions and reactions are equal and opposite. Fix the hook of a spring balance to the hook of another and stretch the two spring balances equally in the opposite directions by hand as shown in the fig. 3.7(*a*). The pointers of the balances will show equal readings. Let the reading be 5 lb in each balance. Now, attach one spring balance against a rigid support, say a wall, and pull the other equally as before as shown in fig. 3.7(*b*). In this case also the readings of both the spring balances will be found to be 5 lb. It appears as if the spring balance attached to the wall is being pulled in the opposite direction by someone. What is actually happening is that a reaction is being applied on it and equal readings show that this reaction is equal and opposite to the action on the first spring balance.

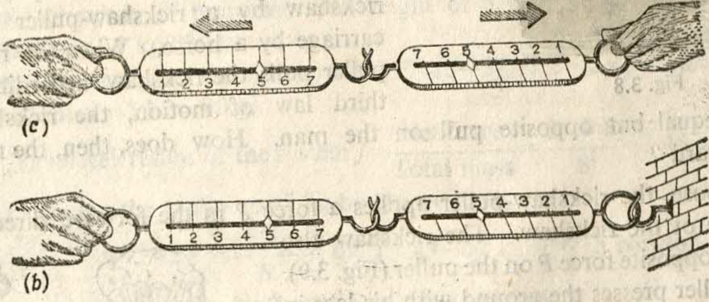


Fig. 3.7

It is to be noted that reaction lasts only as long as there is action. The reaction disappears as soon as the action ceases to act. Besides, action and reaction act on two *separate* bodies and hence cannot produce equilibrium. For equilibrium, it may be mentioned, two equal and opposite forces must act on the *same* body.

We come across many illustrations of third law of motion. Some of which are mentioned below :

(*a*) When a man jumps from a boat to the shore of a river, the boat is pushed back. The backward motion of the boat is due to the force exerted by the man on the boat and the reaction that boat exerts on the man is responsible for the forward motion of the man.

(*b*) When a ball is hit by a bat, the ball moves forward due to the action of the bat on it. But the bat also moves backward due to the reaction of the ball on the bat.

(*c*) When a man fires a shot, he feels a backward push. This is due to the reaction of the shot on the gun.

(*d*) When two bodies come in contact and try to move one along the surface of the other, a force of reaction comes into play between them, retarding the motion. This force of reaction is called friction which becomes predominant when the surfaces in contact are rough.

(*e*) It is a common experience that a boat is often plied by pressing one end of a bamboo pole against the earth. In this case the bamboo pole presses the earth and the earth gives a reaction along the pole (ground reaction in fig. 3.8). A part of this reaction is communicated to the boat through the boatman and the boat moves forward.

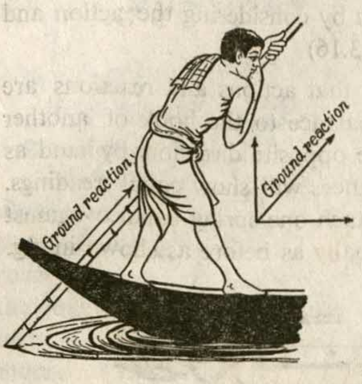


Fig. 3.8

(*f*) Another interesting example of action and reaction is the pulling of a rickshaw by a rickshaw-puller or of a carriage by a horse. When the rickshaw-puller pulls the rickshaw, according to the third law of motion, the rickshaw also exerts an equal but opposite pull on the man. How does, then, the rickshaw move forward ?

Suppose, the rickshaw-puller applies a force P in the forward direction on the handle of the rickshaw. The rickshaw also applies an opposite force P on the puller (Fig. 3.9). But the puller presses the ground with his legs in an inclined direction and the ground, in its turn, exerts an equal and opposite reaction. Let the force of reaction be Q. If it is resolved into horizontal and vertical directions, the vertical component V neutralises the weight of the puller. If the horizontal component R is greater than the backward reaction P of the rickshaw, the rickshaw-puller moves forward under a force$=(R-P)$.

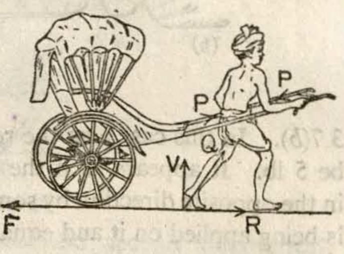


Fig. 3.9

Now, consider the motion of the rickshaw. The frictional force F offered by the ground opposes the forward motion of the rickshaw. When the force P exerted by the puller is greater than F, the rickshaw starts to move and the net force on it is$=(P-F)$.

At the start of the motion, therefore, $R>P>F$; when the rickshaw moves with uniform speed, $R=P=F$.

If m=mass of the rickshaw puller ; M=mass of the rickshaw and f their common acceleration, $R-P=mf$ and $P-F=M.f$ $\therefore$ $f=\dfrac{R-F}{M+m}$

(*g*) Consider a body of weight W suspended by a rope which passes over a smooth pulley P. When the body is stationary, the weight W of the body exerts a pull downwards on the rope and at the same time, the rope exerts an equal and opposite force T on the body, which is due to the force exerted by the person holding the free-end of the rope. When the force F applied by the person at the free end exceeds the tension T required to balance W, the body moves upward. Under stationary condition, however, the pull of the body on the rope is balanced by the pull of the rope exerted on the body. Also, on the other side of the pulley, the pull exerted by the person at the free end of the rope balanced by the pull of the rope exerted on the hand. This is, therefore, an illustration of third law of motion.

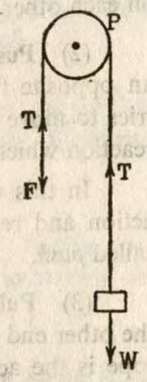


Fig. 3.10

Examples : (1) *Two blocks of wood of mass 3 kg and 5 kg respectively are connected by a light rope passing over a smooth pulley. Find the acceleration of the system and the tension in the rope.* $g=9{\cdot}8\ m/s^2$.

Ans. Suppose W represents the weight of 5 kg block [Fig. 3.10] and F represents that of 3 kg block.

Net force acting on the system$=W-F=(5-3)\times9{\cdot}8=2\times9{\cdot}8$ newton.

Total mass$=5+3=8$ kg.

$\therefore$ The acceleration of the system $f=\dfrac{\text{Net force}}{\text{Total mass}}=\dfrac{2\times9{\cdot}8}{8}=2{\cdot}45\ \text{m/s}^2$.

Considering the motion of 5 kg block, we have,

$f=\dfrac{W-T}{5}$ or, $2{\cdot}45=\dfrac{5\times9{\cdot}8-T}{5}$ or, $T=49-12{\cdot}25=36{\cdot}75$ Newton.

Also, considering the motion of 3 kg block, we get,

$f=\dfrac{T-3\times9{\cdot}8}{3}$ or, $2{\cdot}45=\dfrac{T-3\times9{\cdot}8}{3}$ or, $T=36{\cdot}75$ Newton.

(2) *Two blocks of wood of mass 200 gm and 50 gm are placed side by side on a smooth frictionless table. A force of 5 gm-wt is applied on the 200 gm block parallel to the table as shown in fig. 3.11. What force does the first block exert on the second ?*

Ans. The force applied on 200 gm. block will produce a common acceleration to both the blocks. If f be the common acceleration, then considering the motion of the blocks, we have $P=mf$ or, $5\times980=(200+50)f$

$\therefore$ $f=19{\cdot}6\ \text{cm/s}^2$

If P_1 be the force exerted on the second block, then $P_1=50\times19{\cdot}6$ dynes $=\dfrac{50\times19{\cdot}6}{980}=1$ gm-wt.

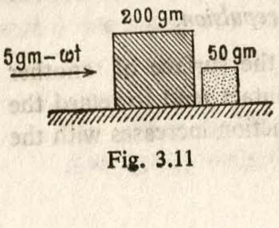


Fig. 3.11

3.14. Different kinds of action and reaction :

(1) **Thrust** : Consider a book lying on a table. The book, by virtue of its weight, exerts a downward force on the table. According to the third law, the table will apply an equal and opposite force on the book. If the former force be called the action, the latter force is the reaction. Such action and reaction are called *thrust*. As a matter of fact, any body placed on another, will exert thrust on each other.

(2) **Push** : If you press the table with your hand, the table will also apply an opposite force on your hand. If your pressure be called the action which tries to move the table away from you, then, the force exerted by the table is the reaction which also tries to keep you away from the table.

In this way, when two bodies coming in contact with each other produce action and reaction which try to keep the two bodies separated, the forces are called *push*.

(3) **Pull or tension** : Consider a rope tied to a tree and a man pulling at the other end of the rope. The force exerted by the man on the tree through the rope is the action. The tree will also exert an equal and opposite reaction on the man through the rope. By the action, the man tries to draw the tree nearer to him and by the reaction, the tree also tries to draw the man nearer to it.

In this way, when two bodies coming in contact with each other (through another body), produce action and reaction which try to draw them closer to each other, the forces are called *pull* or *tension*.

In a tug-of-war, if two teams pull the tug equally the tug remains steady. In this case, every part of the tug transmits two forces which are equal and opposite. As a result, the tug does not get any motion. If the force exerted by each team is 500 kg-wt. (say), the tension in the tug is also 500 kg-wt.

If the tug be cut at the middle and the two cut-ends be joined to the two ends of a spring balance the spring balance will show a reading of 500 kg-wt. when the two ends of the tug are pulled apart with 500 kg. wt. force.

(4) **Attraction and Repulsion** : A magnet attracts a piece of iron from a distance. Similarly, a magnetic north pole attracts a magnetic south pole or a positively charged body attracts a negatively charged body from afar.

Opposite things happen between two similar magnetic poles or between two similarly charged bodies. They repel each other from afar.

In this way when two bodies due to action and reaction *at a distance*, try to come closer to each other, the forces are called the forces of *attraction*. On the other hand, due to action and reaction *at a distance*, if two bodies try to move away from each other, the forces are called the forces of *repulsion*.

(5) **Friction** : When a body tries to move along the surface of another body, the action and reaction between the surfaces in contact tend to retard the motion. Such forces are called the forces of *friction*. Friction increases with the roughness of the surfaces in contact.

3.15. Reaction in a lift :

Consider a man standing on lift. What will be the reaction on the lift if it moves up or goes down? We can solve the problem in the following way.

(i) When the lift is ascending with an acceleration 'f' :

If 'm' be the mass of the man, mg is his thrust on the lift. If R be the reaction of the floor, then since the floor is ascending with an acceleration 'f' the resultant force acting on the lift$=R-mg$ in the upward direction. According to Newton's second law of motion, this resultant force is producing the acceleration f.

Hence, $R-mg=mf$. or, $R=m\,(g+f)$.

In this case, since the reaction is greater than the weight of the person, he will feel himself very heavy or pressed down on the floor of the lift. Everyone experiences this feeling when a lift starts upward from rest.

(ii) When the lift is descending with an acceleration 'f' :

In this case, the weight mg must be greater than the reaction R because the lift is going downward. Here, the resultant force $=mg-R$ and according to Newton's second law of motion,

$$mg-R=mf \quad \text{or, } R=m(g-f) \quad [f<g]$$

In this case, the person will feel himself lighter or pressed down on the floor with less force than when he and the lift are at rest.

When $f=g$. *i.e.* the lift descends with an acceleration g, $R=0$; for this reason, when a man jumps from a height with a heavy load on his head, he does not feel the weight of the load as long as he is in air. As a matter of fact, our sense of weight arises from the reaction of the ground on which we stand. If this reaction is zero, the weight disappears and we feel weightless. For this reason, when a man jumps from a height, he feels weightless as long as he is in air.

When $f>g$, *i.e.*, the lift descends with an acceleration greater than the acceleration due to gravity. If the man in the lift stands without holding anything, then his feet become detached from the floor of the lift as soon as the lift begins to move. The person then moves upward relative to the lift till his head touches the roof of the cage. Afterwards, he moves downward along with the lift with an acceleration f and the roof of the lift exerts a reaction R on his head such that $R+mg=mf$ or, $R=m(f-g)$ where m is the mass of the man. According to Newton's third law, the man also presses the roof upward with a force$=m(f-g)$.

(iii) When the lift is at rest :

Here, $f=0$, so $R=mg$ *i.e.* the reaction on the floor of the lift is same as the weight of the man.

Examples : (1) *A man weighing 60 kg stands on a lift. What will be the reaction of the floor on the man when* (*i*) *the lift is at rest* (*ii*) *the lift ascends with an acceleration 490 cm/s*^{2} (*iii*) *the lift ascends with a uniform velocity and* (*iv*) *the lift ascends with a retardation 490 cm/s*^{2}. *g=980 cm/s*^{2} [*Jt. Entrance 1982*]

Ans. (i) When the lift is not moving, the reaction $R=$weight of the man $=60$ kg-wt.

(ii) When the lift ascends with an acceleration,

$R=m(g+f)=m(g+\frac{g}{2})=\frac{3}{2}mg=\frac{3}{2}\times 60=90$ kg-wt. [$f=490$ cm/s$^2=\frac{g}{2}$]

(iii) Since there is no acceleration, $f=0$, hence $R=mg=$weight of the man $=60$ kg-wt.

(iv) Since there is retardation, $f=-\frac{g}{2}$, So,

$$R=m(g-\tfrac{g}{2})=\tfrac{1}{2}mg=\tfrac{1}{2}\times 60=30 \text{ kg-wt.}$$

(2) *A man stands on a weighing machine kept on the floor of a lift. When the lift ascends with a uniform velocity, the machine reads 150 lb. What will be the reading of the machine when the lift descends* (*i*) *with an acceleration 10 ft/sec*^{2} *and* (*ii*) *with an acceleration 32 ft/sec*^{2}.

Ans. When the lift ascends with uniform velocity, the reaction $R=$the weight of the man ; So, the weight of the man$=150$ lb or, the mass of the man $=\frac{150}{32}$ lb.

(i) When the lift descends with 10 ft/sec^2 acceleration,

$$R=m(g-f)=\frac{150}{32}\,(32-10)=\frac{150\times 22}{32}=103{\cdot}1 \text{ lb. The machine reads } 103{\cdot}1 \text{ lb.}$$

(ii) When the lift descends with 32 ft/sec^2 acceleration,

$$R=m\,(g-f)=\frac{150}{32}\,(32-32)=0. \quad \text{So, the machine reads zero.}$$

3.16. A problem in connection with the application of the laws of motion :

Two bodies of masses m_1 *and* m_2 $(m_1>m_2)$ *are connected by a light inextensible string which passes over a smooth fixed pulley. To find the resulting motion of the system, the tension of the string and the pressure on the pulley.*

Since the string is *inextensible*, the acceleration of the system will be uniform and equal for both the masses. Further, since the string passes over a *smooth* pulley, the tension of the string will be same everywhere. Let us consider the motion of m_1 first [Fig. 3.12]. The weight of the mass m_1g acts downwards and the tension T acts upward along the string. As the mass m_1 moves downwards, the resultant force acting on it$=m_1g-T$. If f be the common acceleration of the masses, then from the second law of motion,

$$m_1f=m_1g-T \quad .. \ (1)$$

Similarly, considering the motion of m_2 which is moving upward, and for which T is greater than m_2g, we have,

$$m_2f=T-m_2g \quad .. \ (2)$$

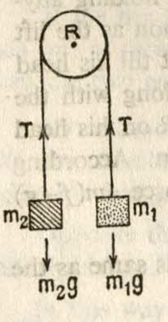


Fig. 3.12

Adding equations (1) and (2), we get, $(m_1+m_2)f=(m_1-m_2)g$

$$\therefore \quad f=\frac{m_1-m_2}{m_1+m_2}.g. \qquad .. \ (\text{i})$$

Also, from equation (2) ,we have, $T=m_2(f+g)$

$$=m_2\left(\frac{m_1-m_2}{m_1+m_2}+\ \right).g.=\frac{2.m_1m_2}{m_1+m_2}.g. \quad (\text{ii})$$

As the string exerts pressure on the pulley downwards on both sides, the total pressure (P) on the pulley $=2.T$

$$\therefore \quad P=\frac{4.m_1m_2}{m_1+m_2}.g. \qquad \text{.. (iii)}$$

Example : *Two masses m and 2m are connected by a massless string which passes over a light frictionless pulley as shown in fig. 3.12(a). The masses are held initially with equal lengths of the strings on the either side of the pulley at a height 13·08 metre from the ground. Find the velocity of the masses at the instant the lighter mass moves up a distance 6·54 metre. The string is suddenly cut at that instant. Calculate the time taken by each mass to reach the ground. g=981 cm/sec².*

[I.I.T. 1977]

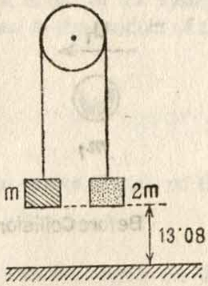


Fig. 3.12(a)

Ans. Both the masses will have a common acceleration which, suppose, is f. Then from eqn. (i) above

$$f=\frac{2m-m}{2m+m}.\ g=\frac{g}{3}$$

If v be the velocity of the lighter mass when it moves up a distance 6·54 metre,

then from $v^2=2.f.s.$ we have $v=\sqrt{2.f.s}=\sqrt{2\times\frac{g}{3}\times 6{\cdot}54}=\sqrt{2\times\frac{9{\cdot}81}{3}\times 6{\cdot}54}$

$=6{\cdot}54$ metre/sec.

The larger mass will, evidently have the same velocity at that instant but in the downward direction.

Now, the distance of the smaller mass from the ground now $=13{\cdot}08+6{\cdot}54=19{\cdot}62$ metre and that of the larger mass $=13{\cdot}08-6{\cdot}54=6{\cdot}54$ metre. The string at this instant, is cut. The masses will now move downwards with acceleration due to gravity. Considering the downward motion of the larger mass, we have $S=6{\cdot}54$ metre ; $g=9{\cdot}81$ metre/sec² ; $u=6{\cdot}54$ metre/s ; $t=?$ From $S=ut+\frac{1}{2}gt^2$, $6{\cdot}54=6{\cdot}54\ t+\frac{1}{2}\times 9{\cdot}81\times t^2$ or, $2=2t+\frac{3}{2}\times t^2$

or, $3t^2+4t-4=0$ or, $(3t-2)(t+2)=0$

$\therefore \quad t=\frac{2}{3}$ or, -2

Since negative time has no meaning, the larger mass will take $\frac{2}{3}$ sec. to reach the ground after the string is cut.

Regarding the smaller mass, it has an upward velocity when the string is cut. For it, we have, $S=19{\cdot}62$ metre ; $g=9{\cdot}81$ m/s², $u=-6{\cdot}54$ m/s ; $t=?$

From, $S=ut+\frac{1}{2}gt^2$, we get $19{\cdot}62=-6{\cdot}54\times t+\frac{9{\cdot}81}{2}.t^2$ or, $6=-2t+\frac{3}{2}.t^2$

or, $3t^2-4t-12=0 \qquad \therefore \quad t=\frac{4\pm\sqrt{16+144}}{6}=2{\cdot}7$ or, $-1{\cdot}44$

Since negative time has no meaning, the smaller mass takes 2·7 sec. to reach the ground after the string is cut.

3.17. Law of conservation of linear momentum :

The law states that *the total momentum of an isolated system * of bodies in a given direction is not altered by the actions and reactions between the bodies constituting the system, provided no external force acts on the system.*

Let us consider the case of direct collision between two balls (Fig. 3.13). Before collision, the ball of mass m_1 was moving with a velocity u_1 and its momentum was m_1u_1. Similarly, the ball of mass m_2 was moving with a velocity u_2 in the same direction so that its momentum was m_2u_2. The total momentum of the system before collision $= m_1u_1 + m_2u_2$.

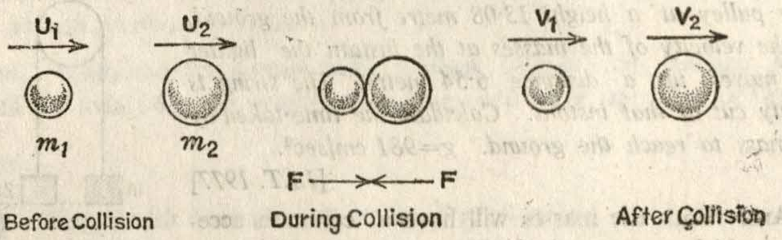


Fig. 3.13

After collision, if the velocity of the masses be v_1 and v_2 respectively in the same direction as before, the total momentum of the system after collision $= m_1v_1 + m_2v_2$. According to the principle of conservation of momentum,

Total momentum before collision $=$ total momentum after collision.

$$\text{i.e. } m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \ldots \quad \ldots \quad \text{(i)}$$

Proof : The principle follows from Newton's laws of motion. During collision, two equal and opposite forces are developed between the bodies—one force exerted by the mass m_1 on the mass m_2 and the other by the mass m_2 on the mass m_1. They are evidently action and reaction. Since collision produces impulsive force, according to Newton's laws of motion, equal but opposite impulsive forces will produce equal but opposite change of momentum in the two bodies.

The impulse of the force on the first body $=$ its change of momentum $= m_1u_1 - m_1v_1$.

The impulse of the force on the second body $=$ its change of momentum $= m_2u_2 - m_2v_2$.

Impulses of the two bodies being equal and opposite, we have,

$$m_1u_1 - m_1v_1 = -(m_2u_2 - m_2v_2)$$

$$\text{or, } m_1u_1 + m_2u_2 = m_2v_2 + m_1v_1$$

Hence, the total momentum of the system remains unaltered or conserved.

---

*By an isolated system we mean a system of bodies surrounding which no other bodies exist. In discussing anything about the isolated system we do not consider the action and reaction of the surrounding bodies.

[*A formal proof* :

Suppose there are n particles of mass $m_1, m_2 .. m_n$ in an isolated system. Suppose also that they are moving with velocities $v_1, v_2 .. v_n$ etc. respectively. Then their total linear momentum P is given by the vector sum of the linear momenta of the particles.

$\vec{P}=m_1\vec{v_1}+m_2\vec{v_2}+ \ldots m_n\vec{v_n}$ But we have, $M\vec{V}=m_1\vec{v_1}+m_2\vec{v_2}+ \ldots +m_n\vec{v_n}$

where M=total mass of the particles and $\vec{V}$=the velocity of the centre of mass* of the system. In other words, total linear momenta of the particles of the system is equal to the product of the total mass of the system and the velocity of its centre of mass.

$$\therefore \ \vec{P}=M.\vec{V}$$

Differentiating, $\frac{d\vec{P}}{dt}=M.\frac{d\vec{V}}{dt}=M.\vec{f}$ where f is the acceleration of the centre of mass of the system. Now, Newton's second law of motion gives $\frac{d\vec{P}}{dt}$=Resultant external force on the system. If the resultant external force is zero, $\frac{d\vec{P}}{dt}=0$ or $\vec{P}$=constant *i. e.* the total linear momenta (vector sum) of the particles is constant when no external force acts on the system. This is the principle of conservation of momentum.]

3.18. Newton's third law of motion from the principle of conservation of momentum :

We have seen in art. 3.17 that the principle of conservation of momentum is a direct consequence of Newton's laws of motion. It may now be proved in the following way that Newton's third law of motion directly follows from the principle of conservation of momentum.

Let m_1 and m_2 be the masses of two bodies in an isolated system moving along the same line. Their velocities and hence momentum will change when one body interacts with the other. Let δp_1 and δp_2 be the change in momentum of m_1 and m_2 respectively in a short interval of time δt. According to the law of conservation of momentum, the total linear momentum will remain constant or the net change in the linear momentum must be zero.

$$\therefore \ \delta p_1+\delta p_2=0 \quad \text{or,} \quad \delta p_1=-\delta p_2$$

Dividing by the time-interval δt, we have, $\frac{\delta p_1}{\delta t}=-\frac{\delta p_2}{\delta t}$. In the limit when $\delta t \to 0$, this gives $\frac{dp_1}{dt}=-\frac{dp_2}{dt}$

i.e. Rate of change of momentum of m_1 = −(Rate of change of momentum of m_2) or Force on m_1 = −(Force on m_2) or Action = −Reaction. This is third law of motion.

---

*See art. 5.9

Examples : (1) *A body of mass 5 gm travelling with a velocity 20 cm./sec collided against another body of mass 10 gm which was travelling in the same direction and in the same straight line with velocity 10 cm/sec. After collision, the first body continued to move in the same direction with a velocity 8 cm/sec. What would be the velocity of the second body after collision ?*

Ans. From eqn. (i) of the art. 3.17. we can write,

$$5\times20+10\times10=5\times8+10\times v_2$$

or, $$200=40+10.v_2$$

or, $$10\times v_2=160 \quad \therefore \quad v_2=16 \text{ cm/sec.}$$

So, the second body would move in the same direction (as v_2 is $+ve$) with a velocity 16 cm/sec after collision.

(2) *A projectile of mass 50 kg is shot vertically upwards with an initial velocity of 100 metres/s. After 5 seconds, it explodes into two fragments, one of which, having mass 20 kg travels vertically up with a velocity of 150 metres/s. What is the velocity of the other fragment at that instant ? $g=9{\cdot}8$ m/s^2* [*I.I.T. 1973*]

Ans. Let v be the velocity of the projectile after 5 seconds. Then from the equation, $v=u+gt$. We get $v=100-9{\cdot}8\times5=51$ metre/s.

After explosion, let v_1 and v_2 be the velocities of the fragments. During explosion, the forces that act are the internal forces. No external force acts on the system in this case. Applying the principle of conservation of momentum, we have,

Total momentum before explosion = Total momentum after explosion

or, $$50\times51=20\times150+30\times v_2$$

or, $$2550=3000+30\times v_2$$

or, $$v_2=-15 \text{ m/s.}$$

Negative sign shows the fragment travels vertically downward.

(3) *A ball A moving with a speed of 9 metres/s strikes an identical stationary ball B such that after collision the direction of each ball makes an angle 30° with the original line of motion. Find the speeds of the two balls after the collision.*

[*I.I.T. 1975*]

Ans. Let the ball A of mass m be moving with a velocity 9 m/s and collide against the stationary ball B, also of mass m. Let, after collision, A and B move with velocities v_1 and v_2 respectively [Fig. 3.14]. Momentum is a vector quantity and since it is conserved in a collision, its components must also be conserved.

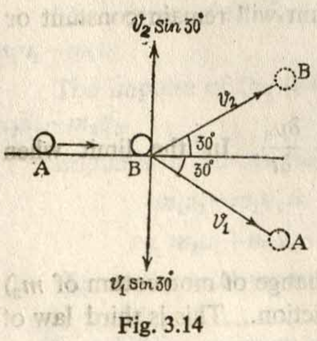


Fig. 3.14

Applying the principle of conservation of linear momentum along the original line of motion, we get, $m\times9+0=mv_1\cos30°+m.v_2\cos30°$

or, $$9=\frac{\sqrt3}{2}(v_1+v_2) \quad .. \quad .. \quad \text{(i)}$$

Applying the principle of conservation of linear momentum perpendicular to the original line of motion, $0=mv_2\sin30°-mv_1\sin30°$ or, $v_1=v_2$ .. (ii)

[The ball A originally had no velocity in the perpendicular direction ; hence its momentum in that direction before collision was zero].

Substituting the values of eqn (ii) in eqn (i) we get,

$$9=\frac{\sqrt{3}}{2}\times 2v_1 \quad \text{or} \quad v_1=3\sqrt{3}\text{ m/s}$$

Since, $v_2=v_1$, $\therefore$ $v_2=3\sqrt{3}$ m/s.

(4) *A body of mass m moving with velocity V in the x-direction collides with another body of mass M moving in the y-direction with a velocity v. They coalesce into one body during collision. Calculate the direction and magnitude of the velocity of the combined masses.* [*I.I.T. 1977*]

Ans. After collision let the combined masses $(M+m)$ move with a velocity u in a direction making angle θ with the x-direction. According to the principle of conservation of momentum, the total momentum of the colliding bodies along x-direction is equal to the momentum of the composite body along the same direction.

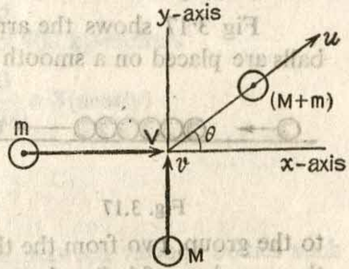


Fig. 3.15

Hence, $m.V+0=(M+m)u\cos\theta$ .. (i)

[The mass M, moving along y-direction has no component of velocity in the perpendicular x-direction.]

Similarly, considering momentum of the system in y-direction,

$$0+M.v=(M+m)u\sin\theta \quad .. \quad \text{(ii)}$$

[The mass m, moving along x-direction has no component velocity in the perpendicular y-direction.]

Squaring eqns (i) and (ii) and then adding, we get

$$m^2V^2+M^2.v^2=(M+m)^2.u^2$$

$$\therefore \quad u=(\sqrt{m^2V^2+M^2.v^2})/(M+m)$$

Further, dividing (ii) by (i), $\tan\theta=\dfrac{M.v}{mV}$ or $\theta=\tan^{-1}\dfrac{Mv}{mV}$.

(5) *A 20 gm bullet pierces through a plate of mass $M_1=1$ kg and then comes to rest inside a second plate of mass $M_2=2{\cdot}98$ kg. It is found that the two plates initially at rest, now move with equal velocities. Find the percentage loss of initial velocity of the bullet when it is between m_1 and m_2.* [*I.I.T. 1979*]

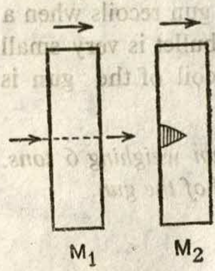


Fig. 3.16

Ans. Let m=mass of the bullet, v=its initial velocity and v_1=its velocity after the penetration of the mass M_1. Let the velocity of the mass M_1 or $M_2=V$.

Applying the principle of conservation of momentum between the bullet and M_1 we have, $mv+0=mv_1+M_1V$ or $m(v-v_1)=M_1V$ .. (i)

Final velocity of the bullet in the mass M_2 is zero. Applying the principle of conservation of momentum between the bullet and M_2, we get,

$$mv_1+0=M_2V+0 \quad \text{or} \quad mv_1=M_2V \quad .. \quad \text{(ii)}$$

Dividing (i) by (ii), we get, $\frac{v-v_1}{v_1}=\frac{M_1}{M_2}$ or $\frac{v}{v_1}=\frac{M_1+M_2}{M_2}$ or $\frac{v_1}{v}=\frac{M_2}{M_1+M_2}$

Now percentage loss in initial velocity $=\left(\frac{v-v_1}{v}\right)\times 100=\left(1-\frac{v_1}{v}\right)\times 100$

$$=\left(1-\frac{M_2}{M_1+M_2}\right)\times 100=\frac{M_1}{M_1+M_2}\times 100=\frac{1}{1+2\cdot 98}\times 100=25\%\text{ (nearly)}$$

3.19. Experimental demonstration :

(1) An interesting experiment may be done to demonstrate the principle of conservation of momentum. Many of you might have done this experiment while playing carrom without, of course, bothering about the underlying principle.

Fig 3·17 shows the arrangement. Six identical large glass balls or pingpong balls are placed on a smooth table in contact with each other. When another ball is rolled up to them it will be stopped by the collision against the group but one on the extreme right-hand end of the group will roll out with same velocity as shown in the figure. If two balls are rolled up to the group, two from the the right-hand end will roll out. As a matter of fact, the number of balls that roll out from the right-hand end is same as the number rolled to the group to conserve the momentum.

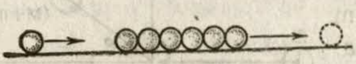


Fig. 3.17

(2) **Recoil of a gun** : Firing a shot from a gun or shell from a cannon means explosion which produces internal impulsive force. In those cases, the principle of conservation of momentum is applicable because no external force acts on the system. Before firing, both the gun and the shell being at rest, the total momentum of the system is zero.

Let the mass of the gun $=M$ and the mass of the shell $=m$. After the shot is fired, let the velocities of the gun and the shot be V and v respectively. So, after firing, their total momentum $=M.V+m.v$.

According to the principle of conservation of momentum,

$$M.V+m.v=0 \quad \therefore\ V=-\frac{m}{M}.v$$

The negative sign shows that the direction of motion of the gun is opposite to the direction of motion of the shell. This explains why the gun recoils when a shell is fired from it. Further, since the mass m of the bullet is very small compared to the mass M of the gun, the velocity V of recoil of the gun is very small compared to the velocity of the bullet.

Examples : (1) *A shell weighing 150 lb is fired from a gun weighing 6 tons. If the velocity of the shell is 1200 ft/sec, find the velocity of recoil of the gun.*

Ans. We know, the velocity of recoil V is given by,

$$V=-\frac{m}{M}.v=-\frac{150}{6\times 2240}\times 1200=-13\cdot 4\text{ ft/sec.}$$

(2) *A body of mass 5 gm travelling with a velocity 10 cm/sec overtakes another body of mass 2 gm travelling with a velocity 3 cm/sec. After that the two bodies coalesce into one. What will be the velocity of the combined single body ?*

Ans. Let V be the velocity of the single body. Since the momentum is not altered due to the collision, we have,

$$(5+2)V=5\times10+2\times3=56 \quad \therefore \quad V=\tfrac{56}{7}=8 \text{ cm/sec.}$$

(3) *A hunter can fire a bullet of mass 0·06 kg with a velocity 900 metres/s from his gun. If a tiger weighing 40 kg jumps to the hunter with a velocity 10 metres/s, how many bullets should the hunter fire in order to counteract the tiger ?*

Ans. Momentum of the tiger $=40\times10$ kg-metre/s.

The number of bullets should be such that their total momentum is equal to the momentum of the tiger. Let the number be n.

The total momentum of the bullets $=n\times0{\cdot}06\times900$ kg-metre/s

$$\therefore \quad n\times0{\cdot}06\times900=40\times10 \quad \text{or} \quad n=\frac{40\times10}{0{\cdot}06\times900}=8\,(\text{nearly})$$

The hunter should fire, at least, 8 bullets.

3·20. Inelastic and Elastic collisions :

We see around us many examples of collision between various bodies such as collision between two cars, collision between two pingpong balls, collision between molecules of a gas or between two atomic particles etc. In such cases of collision, the velocities of the colliding bodies alter after the collision. By the application of the conservation laws, the velocities or energies of the bodies after collision may be calculated provided no external force acts on the system.

In reality, when two bodies collide, the total momentum of the system remains unaltered but *the system loses some kinetic energy*. The reason for this loss of energy is that due to the collision, some heat is produced. The heat energies are dissipated causing a loss of kinetic energy to the system.

Collisions where total momentum of the colliding bodies remains unaltered but there is a loss of kinetic energy, are characterised as *inelastic collisions*. All practical cases of collisions that we see around us are inelastic collisions. If a body sticks to another body after collision it is perfectly inelastic.

On the other hand, collisions where total momentum and total kinetic energy of the colliding bodies remain unaltered, are characterised as perfectly *elastic collisions*. Collisions between atoms and molecules or collisions between fundamental particles like electrons, protons, neutrons etc are assumed to be perfectly elastic collisions.

3·21. Elastic collision between two particles moving in the same straight line :

We have already mentioned that collisions where total momentum and total kinetic energy of the colliding bodies remain conserved are characterised as elastic collisions.

Suppose two particles of masses m_1 and m_2 travelling along the same straight line and in the same direction with velocities u_1 and $u_2 (u_1>u_2)$ collide and after collision they travel, as before, in the same line and in the same direction with velocities v_1 and v_2 $(v_2>v_1)$. Here, the momentum will be reckoned positive if the velocity is directed from left to right along the line of motion and negative if the velocity is directed oppositely.

From the principle of conservation of momentum, we get

$$m_1u_1+m_2u_2=m_1v_1+m_2v_2 \quad .. \quad .. \quad \text{(i)}$$

Also, since the kinetic energy of the system is conserved, we have,

Total K.E. of the bodies before collision = Total K.E. after collision

$$\text{or,} \quad \tfrac{1}{2}m_1u_1^2+\tfrac{1}{2}m_2u_2^2=\tfrac{1}{2}m_1v_1^2+\tfrac{1}{2}m_2v_2^2 \quad .. \quad .. \quad \text{(ii)}$$

From the eqn. (i), we get, $m_1(u_1-v_1)=m_2(v_2-u_2)$ .. .. (iii)

" " " (ii), we get, $m_1(u_1^2-v_1^2)=m_2(v_2^2-u_2^2)$ .. (iv)

Dividing (iv) by (iii) we have, $u_1+v_1=v_2+u_2$

or, $u_1-u_2=v_2-v_1$ .. .. (v)

From this equation, we know that the relative velocity of approach of two bodies before collision is equal to the relative velocity of their separation after the collision.

Now, $v_2=u_1-u_2+v_1$ [From eqn. (v)]

Substituting in eqn. (iii) we get,

$$m_1(u_1-v_1)=m_2(u_1-u_2+v_1-u_2)$$

$$\text{or,} \quad v_1(m_1+m_2)=u_1(m_1-m_2)+2m_2u_2$$

$$\therefore \quad v_1=\frac{m_1-m_2}{m_1+m_2}.\,u_1+\frac{2m_1}{m_1+m_2}.u_2 \quad .. \quad .. \quad \text{(vi)}$$

$$\text{Similarly,} \quad v_2=\frac{2m_1}{m_1+m_2}.\,u_1+\frac{m_2-m_1}{m_1+m_2}.u_2 \quad .. \quad .. \quad \text{(vii)}$$

Special cases (i) When the masses are equal *i.e.* $m_1=m_2$. From eqns. (vi) and (vii), we get $v_1=u_2$ and $v_2=u_1$. This shows that *the particles exchange their velocities after collision.*

(ii) The particle of mass m_2 is initially at rest *i.e.* $u_2=0$.

In this case, $v_1=\dfrac{m_1-m_2}{m_1+m_2}.u_1$ and $v_2=\dfrac{2m_1}{m_1+m_2}.u_1$.

(iii) When $m_1=m_2$ and $u_2=0$; in this case, $v_1=0$ and $v_2=u_1$. This shows that *after collision the first particle comes to rest and the second moves with the velocity of the first.*

(iv) When $m_2>>m_1$ and $u_2=0$; In this case, $v_1\cong-u_1$ and $v_2=0$. This shows that *if a light body collides against a heavy body at rest, the heavy body remains at rest but the light one rebounds in the opposite direction with almost the same velocity.*

For example, suppose that we drop a tennis ball vertically on to a floor. This is in effect a collision between the ball and the earth. If the collision is elastic, the ball will rebound with a reversed velocity and will reach the same height from which it fell.

(v) When $m_1>>m_2$ and $u_2=0$; In this case, $v_1\cong u_1$ and $v_2\cong 2u_1$. This means that when a very massive particle collides against a very light particle at rest, the velocity of the massive particle is virtually unchanged but the light particle is thrown forward with approximately twice the velocity of the massive particle.

Examples : (1) *A ball moving on a horizontal, frictionless plane hits an identical ball at rest with a velocity of 50 cm/s. If the collision is elastic, calculate the speed imparted to the target ball if the speed of the projectile after the collision is 30 cm/s. Show that the two balls will move at right angles to each other after the collision.* [*I.I.T. 1972*]

Ans. Let the ball A, moving with velocity 50 cm/s, collide with the stationary ball B. Let after collision they fly away with velocities v_1 and v_2 respectively in direction making angles θ_1 and θ_2 respectively with the original direction of motion of A [Fig 3.18]. The balls being identical, their masses are equal.

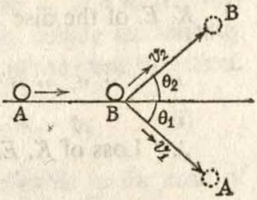


Fig. 3.18

Since the collision is elastic, the total energy of the system is conserved.

So, $\frac{1}{2}m\times(50)^2=\frac{1}{2}m\times v_1^2+\frac{1}{2}m\times v_2^2$

or $(50)^2=(30)^2+v_2^2$ or, $v_2=40$ cm/s. $[v_1=30 \text{ cm/s}]$

Applying the principle of conservation of momentum of the system along the original direction of motion of A, we have,

$$m\times 50+0=mv_1\cos\theta_1+mv_2\cos\theta_2$$

$$\text{or}\quad 50=v_1\cos\theta_1+v_2\cos\theta_2=30\times\cos\theta_1+40\times\cos\theta_2$$

$$\text{or,}\quad 3\cos\theta_1+4\cos\theta_2=5 \quad .. \quad \text{(i)}$$

Applying the principle of conservation of momentum perpendicular to the original direction of motion of A, we have,

$$0=mv_1\sin\theta_1-mv_2\sin\theta_2 \quad \text{or} \quad v_1\sin\theta_1=v_2\sin\theta_2$$

$$\text{or,}\quad 30\times\sin\theta_1=40\times\sin\theta_2 \quad \text{or} \quad 3.\sin\theta_1=4\sin\theta_2 \quad .. \quad \text{(ii)}$$

Squaring (i) & (ii) and adding we get.

$$9(\sin^2\theta_1+\cos^2\theta_1)=16\sin^2\theta_2+25+16\cos^2\theta_2-40.\cos\theta_2$$

$$\text{or}\quad \cos\theta_2=\tfrac{4}{5} \quad \therefore \quad \sin\theta_2=\sqrt{1-\cos^2\theta_2}=\sqrt{1-\tfrac{16}{25}}=\tfrac{3}{5}$$

Substituting the value of $\sin\theta_2$ in eqn (ii), $\sin\theta_1=\frac{4}{3}\times\frac{3}{5}=\frac{4}{5}$

$$\therefore \quad \cos\theta_1=\sqrt{1-\sin^2\theta_1}=\sqrt{1-\tfrac{16}{25}}=\tfrac{3}{5}$$

Now, $\sin(\theta_1+\theta_2)=\sin\theta_1\cos\theta_2+\cos\theta_1\sin\theta_2$

$$=\tfrac{4}{5}\times\tfrac{4}{5}+\tfrac{3}{5}\times\tfrac{3}{5}=\tfrac{16}{25}+\tfrac{9}{25}=1=\sin 90^\circ \quad \therefore \quad \theta_1+\theta_2=90^\circ$$

This shows that after collision A and B move at right angles to each other.

(2) *Two perfectly elastic flat discs A and B, one k times as massive as the other rest on smooth horizontal table. The disc A is made to move towards B with a velocity u and makes a head-on collision. Apply the conservation laws to find out an expression for the fraction of kinetic energy of A transferred to B. Also show that the value of this fraction is the same whether B is k times as massive as A or vice versa.* [*Joint Entrance 1978*]

Ans. Suppose the mass of $B=m$; then the mass of the disc $A=km$. From the principle of conservation of momentum, we get,

$$k.m.u+0=kmv_1+mv_2 \quad \text{or,} \quad ku=kv_1+v_2 \quad .. \quad .. \quad \text{(i)}$$

The *K.E.* of the discs before collision $=\frac{1}{2}k\,m\,u^2+0=\frac{1}{2}k\,m\,u^2$

" " " " " after " $=\frac{1}{2}\,k\,m\,v_1^2+\frac{1}{2}\,m\,v_2^2$

From the principle of conservation of energy we get,

$$\frac{1}{2}\,k\,m\,u^2=\frac{1}{2}\,k\,m\,v_1^2+\frac{1}{2}\,m\,v_2^2 \quad \text{or,} \quad k.u^2=k\,v_1^2+v_2^2 \quad .. \quad .. \quad \text{(ii)}$$

From eqn (i) we get, $v_2^2=k^2(u-v_1)^2$

From eqn (ii) we get, $v_2^2=k(u^2-v_1^2)$

$$\text{Dividing} \quad 1=\frac{k^2(u-v_1)^2}{k(u^2-v_1^2)}=\frac{k(u-v_1)}{u+v_1} \quad \therefore \quad v_1=\left(\frac{k-1}{k+1}\right).u.$$

K. E. of the disc *A* before collision $=\frac{1}{2}k.mu^2$

$$\text{"} \quad \text{" "} \quad \text{" "} \text{ after} \quad \text{"} \quad =\frac{1}{2}kmv_1^2=\frac{1}{2}km\left(\frac{k-1}{k+1}\right)^2.u^2$$

$$\therefore \quad \text{Loss of } K.\ E. \text{ of the disc } A=\frac{1}{2}kmu^2-\frac{1}{2}km\left(\frac{k-1}{k+1}\right)^2.u^2$$

$$=\frac{1}{2}kmu^2\left[1-\frac{(k-1)^2}{(k+1)^2}\right]=\frac{1}{2}k.m.u^2\left[\frac{4k}{(k+1)^2}\right]$$

The gain of energy by *B*=loss of energy by *A*

∴ The fraction of the energy transferred to *B*

$$=\frac{1}{2}kmu^2\left[\frac{4k}{(k+1)^2}\right]\div\frac{1}{2}kmu^2=\frac{4k}{(k+1)^2}$$

Now, if $k=\frac{1}{k}$, the mass of *B* becomes *k* times the mass of *A*. In the case the fraction of the energy transferred $=\dfrac{4.\frac{1}{k}}{\left(\frac{1}{k}+1\right)^2}=\dfrac{4k}{(k+1)^2}$

This shows that the fraction of energy transferred is the same.

3.22. Co-efficient of restitution : Resilience :

It has been mentioned earler that almost all the collisions that we see around us are inelastic ; in all these collisions, some energy is always lost. In art 3.21 we have seen that when two bodies moving with velocities u_1 and u_2 in the same direction and along the same line collide, the relative velocity of their approach $=u_1-u_2$ and the relative velocity of separation $=v_2-v_1$. In the case of inelastic collision, *Newton's law* states that the ratio of the relative velocity of separation to the relative velocity of approach is equal to the co-efficient of restitution (*e*) between the bodies. That is,

$$e=\frac{\text{Relative velocity of separation}}{\text{" \quad " \quad " approach}}=\frac{v_2-v_1}{u_1-u_2}$$

The co-efficient of restitution between two materials is almost constant. In the case when $e=0$, the collision is *perfectly inelastic* ; the two colliding bodies stick together after the collision. When $e=1$, the collision is *perfectly elastic* and the two colliding bodies are extremely hard. In other cases, e may have values between 1 and 0.

Bodies, for which $e=1$, are called perfectly *resilient*. They rebound from each other very rapidly. Soft bodies are less resilient and rebound very slowly. *Resilience is defined as the ability of a body to undergo a compression or deformation without the development of permanent deformation.* The vigour with which a body returns to the original shape after a deformation is known as *restitution*.

3.23. Inelastic collision between two spheres :

Let two spheres of mass m_1 and m_2 move with velocities u_1 and u_2 $(u_1>u_2)$ in the same line and in the same direction. Let the two bodies collide inelastically and move with velocities v_1 and v_2 respectively after collision in the same direction.

According to Newton's law, $e=\dfrac{v_2-v_1}{u_1-u_2}$ or $e(u_1-u_2)=v_2-v_1$ .. (i)

As the principle of conservation of momentum is applicable in the case of inelastic collision, we have, $m_1u_1+m_2u_2=m_1v_1+m_2v_2$ .. (ii)

Multiplying eqn (i) by m_1 and adding with (ii) we get,

$$v_2=\frac{(1+e)m_1u_1+u_2(m_2-e.m_1)}{m_1+m_2} \quad .. \quad \text{(iii)}$$

In the same way, $$v_1=\frac{(1+e)m_2u_2+u_1(m_1-em_2)}{m_1+m_2} \quad .. \quad \text{(iv)}$$

Loss of energy : In inelastic collision, some energy is always lost. It can be shown in the following way. Total energy before collision$=\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2$ and total engrgy after collision $=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2$

Difference in energy$=(\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2)-(\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2)$

$$=\frac{1}{2}\frac{1}{(m_1+m_2)}\left[(m_1+m_2)(m_1u_1^2+m_2u_2^2)-(m_1+m_2)(m_1v_1^2+m_2v_2^2)\right]$$

But, $(m_1+m_2)(m_1u_1^2+m_2u_2^2)=m_1^2u_1^2+m_2^2u_2^2+m_1m_2(u_1^2+u_2^2)$

$$=(m_1u_1+m_2u_2)^2+m_1m_2(u_1-u_2)^2$$

Similarly, $(m_1+m_2)(m_1v_1^2+m_2v_2^2)$

$$=(m_1v_1+m_2v_2)^2+m_1m_2(v_1-v_2)^2$$

$$=(m_1u_1+m_2u_2)^2+m_1m_2e^2(u_1-u_2)^2$$ [From eqn. (i) & (iii)]

$\therefore$ Diff. in energy$=\dfrac{1}{2(m_1+m_2)}[(m_1u_1+m_2u_2)^2+m_1m_2(u_1-u_2)^2$

$$-(m_1u_1+m_2u_2)^2-m_1m_2e^2(u_1-u_2)]^2$$

$$=\frac{1}{2}.\frac{m_1m_2}{m_1+m_2}(1-e^2)(u_1-u_2)^2 \; .. \text{(v)}$$

Since $e<1$ and $u_1>u_2$, the above expression is a $+ve$ one which means that there is a loss of energy.

[**Note** : (i) When $e=1$, *i.e.* for perfectly elastic collision, the difference in energy is zero which means the total energy remains constant.

(ii) when $e=0$ *i.e.* when the colliding bodies stick together, the loss of energy is maximum.]

Example : *An ivory ball with a mass 0·35 kg moving with a speed of 5 m/s collides head-on with another ivory ball of the same mass and size at rest. If the co-efficient of restitution is 0·6, find the velocity of each ball after collision. Find also the loss of energy due to the impact.*

Ans. Using eqn (iii) and (iv) of art 3.23, and remembering that $m_1=m_2=0{\cdot}35$ kg, $u_1=5m/s$; $u_2=0$; $e=0{\cdot}6$, we have,

$$v_1=\frac{(1+e)m_2u_2+u_1(m_1-e.m_2)}{m_1+m_2}=\frac{(1+0{\cdot}6)\times0{\cdot}35\times0+5(0{\cdot}35-0{\cdot}6\times0{\cdot}35)}{2\times0{\cdot}35}=1 \text{ m/s.}$$

$$\text{and } v_2=\frac{(1+e)m_1u_1+u_2(m_2-e.m_1)}{m_1+m_2}=\frac{(1+0{\cdot}6)\times0{\cdot}35\times5+0\times(0{\cdot}35-0{\cdot}6\times{\cdot}35)}{2\times0{\cdot}35}=4\text{m/s}$$

$$\text{Using eqn (v), the loss of energy}=\tfrac{1}{2}\times\frac{m_1m_2}{m_1+m_2}(1-e^2)(u_1-u_2)^2$$

$$=\tfrac{1}{2}\times\frac{0{\cdot}35\times0{\cdot}35}{2\times0{\cdot}35}(1-0{\cdot}6\times0{\cdot}6)(5-0)^2=1{\cdot}4\text{ joule}$$

3.24. Direct collision between a sphere and a fixed plane :

Let a sphere be dropped from a height h on to a fixed plane, which is horizontal. If u be the velocity of the sphere just before striking the plane, then $u^2=2gh$ or $u=\sqrt{2gh}$.

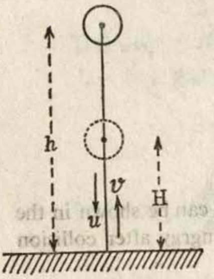


Fig. 3.19

If v be the vertically upward velocity with which the sphere rebounds, then the height H attained by the sphere is given by $0=v^2-2gH$ [Fig 3.19.]

$$\therefore\quad v=\sqrt{2gH}.$$

Since both u and v are perpendicular to the horizontal fixed plane, the collision is direct. Here, the relative velocity of approach between the sphere and the plane $=u-0=u$ and the relative velocity of separation$=v-0=v$. If e be the co-efficient of restitution between the material of the sphere and the plane,

$$e=\frac{v}{u}=\frac{\sqrt{2gH}}{\sqrt{2gh}} \quad\text{or}\quad H=e^2.h \quad .. \quad .. \quad \text{(i)}$$

Noting the heights h and H, the co-efficient of restitution can be evaluated.

Example : *A ball falls from a height of 9·8 metres on a horizontal fixed plane. After rebounding twice, it reaches a height* $\frac{1}{16}$ *of the initial height of fall. Calculate the co-efficient of restitution. Assuming that the total number of rebounds to be infinite before it stops rebounding, calculate the total distance described.*

Ans. Applying the formula (i) of the art 3.24, the height attained after first rebound is given by $H=e^2.h=e^2\times9{\cdot}8$. metres

For the second rebound, the ball falls from a height H and ascends a height $\frac{9{\cdot}8}{16}$ metre. So, $\frac{9{\cdot}8}{16}=e^2.H=e^2\times9{\cdot}8\times e^2 \quad\therefore\quad e=\frac{1}{2}$

After the first rebound, the ball ascends a height e^2h and drops through the same height. So, just before the second rebound, the total distance travelled $=h+2e^2h$.

After the second rebound, the balls ascends $e^2(e^2h)=e^4h$ and then drops through the same height. Similarly, after the third rebound, the ball ascends $e^2(e^4h)=e^6h$ and drops through the same height & so on before it stops rebounding.

Hence, the total distance travelled $=h+2e^2h+2e^4h+2e^6h+\dots\infty$

$$=h[2+2e^2+2e^4+\dots\infty]-h$$

$$=2h[1+e^2+e^4+e^6+\dots\infty]-h$$

$$=\left(2h\times\frac{1}{1-e^2}\right)-h=h\times\frac{1+e^2}{1-e^2}$$

According to the problem, $h=9{\cdot}8$ metre.

$\therefore$ Total distance travelled $=\dfrac{1+(\frac{1}{2})^2}{1-(\frac{1}{2})^2}\times 9{\cdot}8$ metres $=16{\cdot}3$ metres

3·25. Rocket : The flight of rockets is an example of the principle of conservation of momentum. A rocket contains some solid chemical fuel. When the fuel is fired, a hot blast of gas with high velocity is produced. In rockets designed to travel in space, there are tanks full of liquid fuel and for combustion of the fuel, liquid oxygen or liquid chemicals producing oxygen are supplied.

As a result of the combustion of the fuel, a blast of high pressure gas comes out through a nozzle at one end of the rocket. Although the mass of gas that comes out is not big, yet due to its high velocity, its momentum is very large. The main body of the rocket gets an equal and opposite momentum which gives the rocket a high velocity in spite of its heavy weight.

Let a rocket of mass M be moving with a velocity v at any instant. When a mass m of the fuel is ejected, the mass of the rocket becomes $(M-m)$. As a result, the velocity of the rocket increases. Let the velocity increase to $(v+\Delta v)$. Suppose the fuel is always ejected at a constant speed u relative to the rocket. Then the velocity of the mass m of the fuel $=\left(v+\dfrac{\Delta v}{2}-u\right)$ in the direction of the rocket, because the initial velocity of the rocket is v and the final velocity is $v+\Delta v$, giving an average $=v+\dfrac{\Delta v}{2}$

We may now apply the principle of conservation of momentum to the rocket and fuel. Before the fuel is ejected, the momentum of the rocket and the fuel inside $=M.v$. After a mass m of the fuel is ejected, the momentum of the rocket and the remaining fuel inside $=(M-m)(v+\Delta v)$ and that of the fuel ejected $=m(v+\dfrac{\Delta v}{2}-u)$

$$\therefore\quad (M-m)(v+\Delta v)+m(v+\frac{\Delta v}{2}-u)=M.v. \quad\text{or}\quad M.\,\Delta v-mu=0$$

or, $M.\,\Delta v=mu$ or, $\dfrac{m}{M}=\dfrac{\Delta v}{u}$ [$m.\Delta v$ being very small is neglected]

Now, $m=$mass of the fuel ejected $=-\,\Delta M$ (say) $\therefore\ -\dfrac{\Delta M}{M}=\dfrac{\Delta v}{u}$ ...(i)

Negative sign shows that the direction of motion of the rocket is opposite to the direction of ejection of the fuel.

[**N.B.** According to differential notation, the eqn. (i) can be written as $-\frac{dM}{M}=\frac{dv}{u}$ or $dv=-u.\frac{dM}{M}$. Integrating with respect to t, $\int\frac{dv}{dt}.dt=-u\int\frac{dM}{Mdt}.dt$ or $v=-u\log_e M+K$, where K is a constant. If Mo be the mass of the rocket and v_0 its velocity at $t=0$, we have $v_0=-u\log_e M_0+K$ or $K=v_0+u\log_e M_0$. ∴ $v=-u\log_e M+v_0+u\log_e M_0=v_0+u\log_e\frac{M_0}{M}$

This gives the value of the velocity v of the rocket at time t in terms of the initial mass and the mass after time t of the rocket.]

Examples : (1) *A rocket consumes 100 kg of fuel per second. The gas produced due to the combustion of fuel is ejected from the rocket at a speed of* 5×10^3 *metres/s. What thrust is exerted on the rocket ?*

Ans. Here the mass is changing. Thrust $P=-\frac{dM}{dt}\times v$

The rate of consumption of fuel$=\frac{dM}{dt}=100$ kg/s and the velocity of ejection of the gas$=5\times10^3$ metre/s. ∴ Thrust$=-100\times5\times10^3=-5\times10^5$ Newton.

Negative sign shows that the rocket moves in a direction opposite to the direction in which the gas is ejected.

(2) *During the first second of its upward journey, a rocket loses* $\frac{1}{100}$ *part of its mass. If the velocity of gas coming out from the rocket be 2000 metres/s, what is the acceleration of the rocket ?*

Ans. We know, $\frac{\Delta M}{M}=\frac{\Delta v}{u}$ (Ignoring the $-ve$ sign)

Here, $\frac{\Delta M}{M}=\frac{1}{100}$ and $u=2000$ metres/s ; ∴ $\frac{1}{100}=\frac{\Delta v}{2000}$ or $\Delta v=20$ metre/s.

Since this change of velocity occurs in the first second, the acceleration of the rocket$=\frac{20}{1}=20$ metre/s^2.

(3) *The gas produced due to the combustion of fuel is ejected from the rocket at a speed of 2 km/sec. After a time t, the ratio of the original mass to the subsequent mass of the rocket is found to be 10. With what velocity is the rocket speeding at that instant ?*

Ans. We know $v=v_0+u\log_e\frac{M_0}{M}$; Here $v_0=0$ (the rocket is at rest just at the moment of launching); $M_0/M=10$. So $v=0+2\log_e{}^{10}=2\times2{\cdot}3026=4{\cdot}6052$ km/sec$=4605{\cdot}2$ metres/s.

3·26. Jet : Jet propulsion system has been developed in Germany and Italy mainly with the idea of devising a powerful means of propulsion for high-speed aircraft. It utilises the same principle as rockets. Fig. 3.20 shows a sketch of a turbo-jet.

It consists of a compressor on the left and a gas turbine T on the right connected to a common shaft S. A mixture of air and fuel is sucked in and compressed by the compressor and then ignited in the combustion space A so as to generate

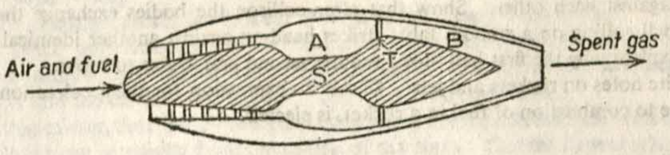


Fig. 3.20

high pressure. This high pressure gas traverses the turbine where it gets accelerated and the pressure falls. The gas suffers further expansion in B and escapes at the rear of the machine with jet velocity. The machine, on account of the reaction thus furnished moves forward (towards the left of diagram) with tremendous velocity.

Now-a-days such jet engines are being used for air-craft propulsion. Modern jet planes are now flying from one continent to another with speeds more than that of sound. *The essential difference between a jet aircraft and a rocket is that the former obtains oxygen from air and therefore, cannot go beyond the atmosphere while the latter carries its own oxygen supply and thus can go into space.*

Exercises

Essay type :

1. State Newton's laws of motion and cite illustrations of first and third laws, Explain how from the first law, we get a definition of force and from the second law a measure of force. [*H. S. Exam 1980 ; Tripura H. S. 1978*]

2. State Newton's second law of motion. Show how this law gives us a unit of force when the unit of mass is 1 kg and the unit of acceleration is 1 metre/sec^2. What is the name of this unit of force ? [*H. S. Exam 1978*]

3. What do you understand by inertia of matter ? Explain with illustrations. [*Tripura H. S. 1980*]

4. Establish the equation $P=mf$ and explain the units of force in the two systems. What is 'Newton' ? Express it in dynes. [*Tripura H. S. 1981*]

5. Define absolute unit of force from second law of motion. Establish the relation between dyne, newton and gm. wt. Which of these are absolute units of force?

6. What are action and reaction ? Do they act on the same body ? 'A system of unbalanced forces produces acceleration while a system of balanced forces causes deformation'. Explain the statement fully.

7. What is the impulse of a force? Prove that the impulse of a force is equal to the change of momentum of a body. What is 'impulsive force'? How will you measure an impulsive force?

8. Show that the first law of motion can be derived from the second law.

[**Hints :** From second law we get $P=mf$. Now, if $P=0, f=0$ because m cannot be zero. Zero acceleration means either the body will move with uniform velocity if it is in motion or will remain at rest if it is initially at rest. So, if no external force is applied (*i.e.* $P=0$), the body will continue to move with uniform velocity or will continue to be at rest. This is the essence of first law.]

9. State the laws of parallelogram of forces. How can you combine, with the help of this law, two forces acting at different directions from a point?

10. (i) Define momentum and impulse. State the principle of conservation of momentum. When a ball is thrown up, the magnitude of its momentum decreases and then increases. Does this violate the conservation of momentum principle? [*H. S. Exam. 1979*]

(ii) Mass is a measure of inertia of a body : Explain. [*Tripura H. S. 1980*]

11. Two perfectly elastic bodies of eqal mass, travelling in the same direction along a straight line, collide against each other. Show that after collison the bodies exchange their velocities.

12. A ball rolling on a smooth table strikes head-on against another identical ball, which is at rest. Explain why the first ball stops and the second ball begins to move.

13. Write notes on rockets and jets. Obtain an expression for the acceleration of a rocket when gas, due to combustion of fuel in a rocket, is ejected.

Short answer type :

14. A one kilogram mass is suspended by a piece of thread *A*. From the bottom of the mass, another piece of identical thread *B* is fixed. If a slow and steady pull *P* is given at the end of the thread *B*, the thread *A* breaks. But a quick pull *P*, breaks the thread *B*, Why ?

[**Hints :** In the case of steady pull, the tension in the thread *A* is greater and is equivalent to the force *P* plus the weight of the mass *B*. In the case of a quick pull, the force *P* is momentarily very large on the thread *B*. The inertia of the mass does not allow the force to be transmitted to the upper thread.]

15. A boy in a railway carriage moving with constant velocity throws a ball straight up into the air. Will the ball fall behind him? in front of him? into his hand? What happens when the carriage is (i) accelerated forward (ii) goes round a curve when the ball is in the air?

[**Hints :** If the carriage is accelerated forward or goes round a curve when the ball is in the air, the ball will not drop into the hands of the boy.]

16. Why do you fall forward when a moving train decelerates to a stop and fall backward when the train accelerates? What would happen if the train rounded a curve at constant speed?

17. When a man jumps from a certain height, he feels more pain if he does not flex his knees on landing than when he flexes his knees. Why?

18. When a cart puller is asked to pull a cart, he replied, "If I pull the cart, the cart, according to the third law of motion, will also pull me equally ; so what is the use of pulling a cart?" —Express your views on the statement.

19. A man stands on the floor of a lift. If the lift starts descending with an acceleration f, what will happen when (i) $f<g$ (ii) $f>g$ (iii) $f=g$, where g is the acceleration due to gravity?

20. A body *A* rests on another body *B*. State and explain the conditions in which the action and reaction between *A* and *B* will be (i) equal to the weight of *A* (ii) greater than the weight of *A* (iii) less than the weight of *A* (iv) zero.

21. When a body falls freely in air, it is weightless. Explain the statement fully.

22. A sand glass is weighed twice in a sensitive balance—once when the sand particles are slowly falling from the upper compartment to the lower and then when all the sand is collected in the lower compartment. Will the two weighings be equal?

[**Hints :** We know that weight of a body is felt only when it produces a thrust on some other body and an opposite reaction is produced thereby. Falling bodies do not produce thrust. As a result, falling bodies are weightless. When the sand particles fall slowly from upper compartment they become weightless as long as they are in air. So, the weight will be greatest when all sand particles are collected in the lower compartment. The weight at other times will be less because some sand particles will then be in air.]

23. When two bodies collide, there is generally a rapid change of their velocity and direction. Why is then a collision between two fast moving cars so fatal? If the change of momentum of a bullet is the same as that of the gun when a bullet is fired, why can we not face a bullet although we can bear the recoil of the gun?

24. Why is it easier to pull a lawn-roller than to push it?

25. A meteorite burns in the atmosphere before it reaches the earth's surface. What happens to its momentum? [*I. I. T 1975*]

[Hints : The momentum is transferred to the atmosphere. As the meteorite passes through the atmosphere, it collides with air particles and during collision it transfers its momemtum to the air particles. Due to air friction, a part of its kinetic energy is, however, converted into heat energy and it burns away.]

26. Can a passenger sitting inside a car move it by pushing it from inside?

[Hints : When the passenger pushes the car, the car also exerts an equal and opplsite reaction. As a result of this internal action and reaction, the momentum of the system consisting of the passenger and the car does not change. So, the car will not move.]

27. In a tug-of-war, the rope is being pulled by three people on each side with equal forces. A weight of 5 lb is hung vertically from the centre of the rope. Can the men now keep the rope straight and horizontal ?

28. (a) Both the following statements are true. Explain them :

(i) Two teams having a tug-of-war must always pull equally hard on one another.

(ii) The team that pushes harder against the ground wins.

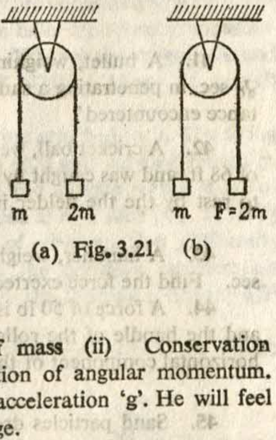
(b) Two persons are pulling at the two ends of a rope in opposite directions with a force F each. Next one end of the rope is fixed to a rigid support and a person is pulling the rope at the other end with a force $2F$. In which case is the tension in the rope greater?

29. Distinguish between elastic and inelastic collision. What type of collision are the following :—(a) Collision between a motor car and a lorry (b) collision between a neutron and an atomic nucleus (c) collision between two ping-pong balls.

30. Two identical balls B and C lie in a smooth horizontal straight groove so that they are touching. A third identical ball A moves at a speed u along the groove and collides with B. Assuming that the collisions are all perfectly elastic ; explain why it is impossible for (i) A to come to rest while B and C move off together at speed $u/2$. (ii) A to come to rest while B and C move off at speed $u/\sqrt{2}$.

[Hints : (i) Energy is not conserved (ii) Momentum is not conserved.]

31. The pulley arrangements in Fig. 3.21 are identical. The mass of the rope is negligible. In (a) the mass m is lifted up by attaching a mass $2m$ to the other end of the rope. In (b) m is lifted up by pulling the other end of the rope with a constant downward force $F=2mg$. Is the acceleration of m the same in both cases? [*I.I.T. 1984*]



(a) Fig. 3.21 (b)

Objective type :

32. Mark the correct answer in the following cases :—

(a) A jet engine works on the principle of (i) Conservation of mass (ii) Conservation of energy (iii) Conservation of linear momentum (iv) Conservation of angular momentum.

(b) A man is standing on a lift which is descending with an acceleration 'g'. He will feel himself (i) heavier (ii) lighter (iii) weightless (iv) with no change.

(c) A whirling electric fan on being switched off, the blades of the fan continue to whirl for some time because of (i) inertia (ii) circulating air (iii) angular momentum.

(d) A body that is moving with uniform velocity is (i) acted upon by a net force (ii) acted upon by a net zero force (iii) having a uniform acceleration.

(e) A constant force acts on a free body placed on a smooth table. Then all will change except (i) acceleration (ii) velocity (iii) K.E. (iv) momentum (v) P.E.

(f) A boy sitting in a railway carriage which is moving at a constant velocity throws a ball straight up into the air. The ball will drop (i) behind the boy (ii) into the hand (iii) in front the boy.

(g) The coefficient of restitution between two bodies is zero. When these two bodies collide (i) they stick together (ii) they move in the opposite direction (iii) both will come to rest.

(h) A man throws a ball straight up. The ball has a momentum in the vertically upward direction. The principle of conservation of momentum demands that the earth has [i) a momentum in the vertically upward direction [ii] zero momentum [iii] a momentum in the vertically downward direction.

Numerical Problems :

33. A force of 100 dyne acts on a body of mass 25 gm. for 5 seconds. What will be the velocity produced? *[Tripura H. S. 1981]* [Ans. 20 cm/s]

34. What force, acting on a mass of 10 lb will produce an acceleration of 15 ft/sec^2? [Ans. 150 poundals]

35. A force acts on a mass of 120 lb for 3 sec. and then ceases to act. During the next 3 sec the mass moves through a distance of 108 ft. What was the force which acted on the mass? *[Tripura H. S. 1980]* [Ans. 45 lb]

36. A motor-car weighing 400 lb. was running at a speed of 30 miles/hr. By applying brakes, the car was stopped within a distance of 40 ft. What force acted on the car? [Ans. 9680 poundals]

37. A hose-pipe, whose nozzle area is 100 cm^2, is throwing water at the rate of 20 cm/s against a rigid wall. If the water be falling normally on the wall and the velocity of water perpendicular to the wall after striking be zero, find the force exerted on the wall. [Ans. 4×10^4 dynes]

[**Hints :** mass of water falling per sec$=100 \times 20 \times 1=2000$ gm. Change of momentum per sec$=(20-0) \times 2000$ gm cm/s]

38. A force of 800 dynes is applied against a body of mass 5 gm. moving with a velocity 40 cm/s. Within what distance and time will the body be brought to rest? [Ans. 5 cm ; $\frac{1}{4}$ sec]

39. A force of 50 gm-wt. acts on a mass of 10 gm. for 10 sec. What velocity will be produced? [Ans. 49,000 cm/sec]

40. Two blocks, mass m_1 and m_2 are connected by a light spring on a smooth horizontal surface. Find the ratio of the accelerations f_1 and f_2 after they are pulled apart and then released. $\left[\text{Ans. } \frac{f_1}{f_2}=\frac{m_2}{m_1}\right]$

41. A bullet, weighing $\frac{1}{4}$ oz, and travelling with a velocity 50ft/sec. was brought to rest in $\frac{1}{20}$ sec. in penetrating a sandbag. How far did the bullet penetrate and what was the average resistance encountered? [Ans. 1·25 ft., 15 poundal]

42. A cricket ball, weighing $5\frac{1}{2}$ oz. having been hit by a batsman, went up straight to a height of 68 ft. and was caught by a fieldsman when it was 4 ft. above the ground. The ball was brought to rest by the the fielder in 0·1 second. What force did the fielder exert in stopping the ball? [Ans. 6·875 lb-wt]

43. A hammer, weighing 50 lb, fell from a height of 12 ft. on a peg and came to rest in $\frac{1}{8}$ sec. Find the force exerted by the hammer? [Ans. 266·25 lb-wt]

44. A force of 50 lb is applied on the handle of a roller. If the weight of the roller is 150 lb and the handle of the roller is inclined at an angle of 60° with the horizontal, calculate (i) the horizontal component of the force and (ii) the force exerted by the roller on the ground. [Ans. (i) 25 lb (ii) 193·3 lb]

45. Sand particles drop vertically at the rate of 150 gm/sec on to a conveyor belt moving horizontally with a velocity 2 metre/s. Calculate the force needed to maintain the motion of the belt. [Ans. 3×10^4 dynes]

46. A lift is ascending with an acceleration of 5 ft/sec.^{2} What reaction a man weighing 160 lb will produce on the floor of the lift when (i) the lift just starts moving (ii) the lift ascends with a constant vetocity of 12 ft/sec. [Ans. (i) 185 lb-wt (ii) 160 lb-wt]

47. A body of mass 2 kg is suspended from the book of a spring balance which is hanging from the roof of a lift. Find the readings on the spring balance when (i) the lift is ascending with 20 cm/s^2 acceleration. (ii) descending with 10 cm/s^2 acceleration (iii) ascending with a uniform velocity 15 cm/s. [Ans. (i) $20{\cdot}4 \times 10^5$ dynes (ii) $19{\cdot}8 \times 10^5$ dynes (iii) 20×10^5 dynes]

48. A bullet of mass 0·02 lb fixed from a gun of mass 8 lb, gets an acceleration 10^5 ft/s^2 as long as the bullet is in the barrel. (a) What force acts on the bullet? (b) Does any force act on the gun? If so, what force and in which direction does it act? (c) If the bullet is accelerated for 0·007 sec, with what velocity does it leave the barrel?

[Ans. (a) 2×10^3 poundal (b) 2×10^3 poundal backward (c) 700ft/s]

49. A gun, weighing 1 ton fires a shot weighing 2 cwt, which leaves the gun with a velocity of 1200 ft/sec. What will be the initial velocity of recoil of the gun and what force must be applied to bring it to rest within a distance of 1 yard? [Ans. 120 ft/sec. ; 75 ton-wt]

50. A bullet of mass 8 gm. is fired from a gun weighing 5 kg with a velocity 400 metre/s. Find the recoil velocity of the gun [*H. S. Exam. 1979*] [Ans. 64 cm/s]

51. A 10 gm bullet is fired from 1 kg. gun suspended to move freely. The bullet now enters a block of wood of mass 990 gm. If the speed of the bullet is 500 metres/sec., find the speed of of the recoil of the gun and the velocity imparted to the block

[Ans. 5 metres/sec ; 5 metres/sec]

52. For elastic collision of a body of mass m with another mass km the energy lost by the former for a head-on collision is maximum when $k=1$. Prove it by applying the laws of conservation of energy and momentum. [*Jt. Entrance, 1979*]

53. A particle of mass m_1 travelling with velocity u_1 struck head-on against another particle of mass m_2 at rest. After collision the particles move with velocity v_1 and v_2 respectively. If the collision is perfectly elastic, show that $v_2=2u_1\Big/\left(1+\frac{m_2}{m_1}\right)$. Also show that the fractional decrease of kinetic energy of the mass m_1 is given by $\frac{4m_1m_2}{(m_1+m_2)^2}$.

Harder Problems :

54. A compartment of mass 500 kg suddenly got uncoupled from a train which was travelling at the rate of 72 km/hr. If the detached part of the train came to standstill in 10 seconds, calculate the resistive force due to friction in Newton. [Ans. 1000N]

55. Two balls A and B of masses 100 gm and 250 gm respectively are connected by a stretched spring of negligible mass and placed on a smooth table. When the balls are released simultanously the initial acceleration of the ball B is 10 cm/sec^2 westward. What is the magnitude and direction of the initial acceleration of the ball A? [*I. I. T 1975*] [Ans. 25 cm/s^2]

[**Hints :** Resultant force is zero ; So, $m_1f_1=m_2f_2$]

56. A mass A (500 gm) is placed on a smooth table with a string attached to it. The string goes over a frictionless pulley and is connected to another mass B (200 gm) hanging vetrically. At $t=0$, the mass A is at a distance of 200 cm from the pulley end and moving with a speed of 50 cm/s towards the opposite end. What will be its position and speed at $t=1$ sec? [*I. I. T. 1975*]

[Ans. 90 cm. to the right of initial position ; 230 cm/s]

[**Hints ;** For A : $T=m.f$; For B, $mg-T=mf$ where f is the common acceleration towards the left. For the motion of A, $u=-50$ cm/s ; Apply the formula ; $S=ut+\frac{1}{2}ft^2$ and $v=u+ft$]

57. Two cubes of masses m_1 and m_2 lie on two frictionless slopes of block A which rests on a horizontal table [Fig. 3·22]. The cubes are conneced by a string which passes over a pulley as shown. To what horizontal acceleration f should the whole system (*i.e.* block and cubes) be subjected so that the cubes do not slide down the planes? What is the tension in the string in this situation? [*I.I.T. 1978*]

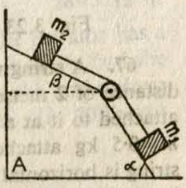


Fig. 3.22

[Ans. $f=\left(\frac{m_1 \sin\alpha+m_2 \sin\beta}{m_1 \cos\alpha+m_2 \cos\beta}\right).g$

$T=[m_1\, m_2 \sin(\alpha-\beta)g]/(m_1 \cos\alpha+m_2 \cos\beta)$

58. A boy whose mass is 50 kg stands on a spring weighing machine inside a lift. The lift starts to ascend with an acceleration of 2·45 m/s^2. What is the reading on the machine. What will be the reading when in its ascent the lift moves (a) with uniform velocity (b) with the retardation of 4·9 m/s^2? [Ans. 62·5 kg ; 50 kg ; 25 kg]

59. A roller of mass 500 kg is attached by a light horizontal chain to a tractor of mass 1000 kg. The backward force of friction exerted by the ground is 1000N. If the system has a forward acceleration of 2·5 m/s^2, calculate (i) the forward force on the tractor and (ii) the tension in the chain. [Ans. (i) 4750 N (ii) 2250 N]

60. A rocket of mass M travelling at a speed of v, ejects some gas at any instant. Its mass then becomes Mo and speed increases to v_0. If the velocity of ejection of gas relative to the rocket be u, then prove $v=v_0-u \log_e M/M_0$

61. A shot of mass 28 lb is fired with a velocity of 800 ft/s from a cannon of mass 1 ton. The recoil of the cannon was stopped within a distance of 5 ft by the application of force. Calculate the magnitude of the force. [Ans. 700 lb-wt.]

62. A body of mass 1 kg, initially at rest, explodes and breaks into three fragments of masses in the ratio 1 :1 :3. The two pieces of equal mass fly off perpendicular to each other with a speed of 30 metre/s each. What is the velocity of the heavier fragment? [*I. I. T. 1981*] [Ans. $10\sqrt{2}$ metre/s]

[Hints : Let m, m and $3m$ be the masses of the fragments. Suppose, the heavier fragment flies off with a velocity V, making an angle θ with the direction of any of the smaller fragments. Since linear momentum is conserved, $m\times 30=3m\times V\cos\theta$ and $m\times 30=3m\times V\sin\theta$ $\therefore$ $\theta=45°$ and $V=10\sqrt{2}$]

63. A cricket ball of mass 150 gm came with a velocity of 12 metre/s and was hit by a bat. As a result, the ball returned along the same path with a velocity of 20 metres/s. If the ball remained in contact with the bat for 0.01 sec., find the force exerted by the bat on the ball. [Ans. 48×10^6 dynes]

64. Two bodies of equal mass are, at rest, side by side. One of the bodies starts moving under a constant force F while the other, at the same instant, receives an impulsive force I. Show that the bodies again come side by side after a time $\frac{2I}{F}$.

65. Two bodies of equal mass m are at rest at a distance x from each other. One body is driven towards the other with an impulsive force I while the latter is driven in the same direction with a uniform force F. Show that the two bodies will never meet if $I^2<2xmF$.

66. (a) Two bodies A and B of masses 100 gm and 400 gm respectively are moving towards each other with speeds 100 cm/s and 10 cm/s respectively. They suffer a head-on collision and stick together. In which directon will the combined mass move after the collision? What will be the distance travelled by the combined mass in 10 seconds after the collision? [*I. I. T. 1971*]

[Ans. In the direction of 100 gm mass ; 120 cm]

(b) Three particles A, B and C of equal mass move with equal speed V along the medians of an equilateral triangle as shown in Fig. 3·23. They collide at the centroid G of the triangle. After collision A comes to rest, B retraces its path with speed V. What is the velocity of C? [*I.I.T. 1982*] [Ans. V along $\overrightarrow{BG}$]

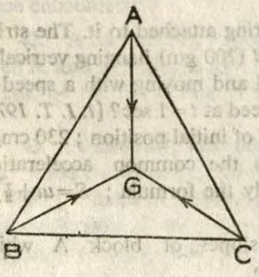


Fig. 3.23

67. A string with one end fixed on a rigid wall, passing over a fixed frictionless pulley at a distance of 2 metres from the wall has a point mass $M=2$ kg attached to it at a distance of 1 metre from the wall. A mass $m=0{\cdot}5$ kg attached at the free end is held at rest so that the string is horizontal between the wall and the pulley and vertical beyond the pulley. What will be the speed with which the mass M will hit the wall when the mass m is released? [*I.I.T. 1985*] [Ans. 25·60 cm/sec]

[Hints : $T-mg=mf$ and $T=Mf$; so $f=\frac{9.8}{3}$m/s².

Now apply the equation $v^2=u^2+2f.s$]

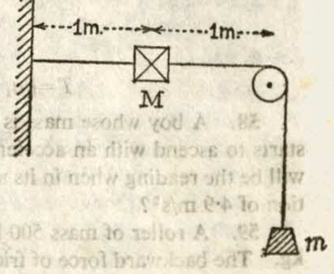


Fig. 3.24

68. A bomb falling vertically downwards with a velocity of 150 metres/s, suddenly bursts into two pieces, whose masses are in the ratio of 3 :1. The velocity of the bigger chunk immediately after the explosion was 240 metres/s in the vertically upward direction. What will be the velocity of the smaller chunk immediately after explosion? [Ans. 120 metres/s upward.]

[**Hints :** $M \times 150 = \frac{3}{4} \times 240 + \frac{1}{4} \times v$]

69. A nucleus of mass M is at rest. A neurton of mass m, moving with a velocity v collides head-on with the nucleus. The neucleus is driven in a direction at an angle ϕ to the original direction of motion of the neutron. Assuming that the collision is perfectly elastic, show that the velocity of the nucleus is $\frac{2m.v}{M+m} \cos \phi$.

70. Show that the fractional decrease in kinetic energy of neutron in the above case is $4Mm/(M+m)^2$.

71. The velocities of two particles moving along a straight line are u and v. Their masses are m_1 and m_2 respectively. If the collision between them is perfectly elastic, show that the second particle will gain a momentum of $2m_1m_2(u-v)/(m_1+m_2)$ from the first particle.

72. A steel sphere of mass 1 lb is attached to a thread 27 inches long. Keeping the thread horizontal, the sphere is let go. It strikes a block of mass 5lb, at rest, at the lowset point of its trajectory. Assuming the collision to be elastic and the block resting on a frictionless plane, find the velocities of the sphere and the block after the collision. [Ans. 4ft/s ; 8ft/s]

[**Hints :** See fig 3.25. If u_1 be the velocity of the sphere due to a fall through 27 inches, $u_1^2 = 2gh = 2 \times 32 \times \frac{27}{12} = 144$ $\therefore$ $u_1 = 12$ ft/s. After collision, let the recoil velocity of the sphere be v_1 and the forward velocity of the block be v_2.

Then, $m_1u_1 + 0 = -m_1v_1 + m_2v_2$.

Also from Newton's law, $u_1 = v_1 + v_2$; Solve for v_1 and v_2]

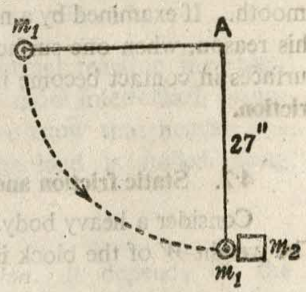


Fig. 3.25

73. A small sphere is dropped from a height h on a horizontal plane. If the coefficient of restitution between the sphere and the plane be e, then show that the total distance travelled by the sphere before it stops rebounding is given by $\frac{(1+e^2)}{(1-e^2)} h$. and the time taken by it is $\sqrt{\frac{2h}{g}} \frac{(1+e)}{(1-e)}$

74. A cannon is mounted on a railway wagon which stands on a straight section of the track. The mass of the wagon with the cannon, projectiles and men is 2000 kg and the mass of a projectile is 25 kg. The cannon is fired in a horizontal direction along the track, the projectile having an initial velocity of 1000 metres/s with respect to the cannon. Find (a) the speed of the cannon after the first shot (b) speed after the third. [Ans. (a) 12·66 m/s ; 26·15 m/s].

75. A lamp hangs vertically from a chord in a descending elevator. The elevator has a deceleration of 8 ft/s² before coming to a stop. If the tension in the chord is 20 lb. find the mass of the lamp. What is the tension in the cord when the elevator ascends with an acceleration of 8 ft/s² ? [Ans. (i) 0·5 lb. (ii) 20 lb-wt.]

4

FRICTION

4·1. What is friction? If you know bi-cycle riding, you must have seen that it is easier to ride a cycle on a smooth asphalted road than on a rough meadow. Set a ball rolling on the ground and note the distance it travels. Set it rolling again similarly on a smooth floor. It will now go a greater distance. Your feet will slip if you try to walk on a smooth floor with a pair of new shoes with polished sole. Why do all these things happen? The explanation is as follows :

When one surface slides over another or tends to do so, a force is called into play which opposes the motion. This force is called the *force of friction.* If the surfaces are smooth, the friction is less ; if the surfaces are rough and uneven, the the friction is greater. A surface, though apparently smooth, cannot be perfectly smooth. If examined by a microscope, its undulations will be readily visible. For this reason, when one surface tends to slide over another, the undulations of the surfaces in contact become interlocked and the motion is retarded. This is called **friction.**

4·2. Static friction and limiting friction :

Consider a heavy body *A*, say, a wooden block resting on a horizontal table. The weight *W* of the block is acting vertically downwards and the reaction *R* of the table in the vertically upward direction [Fig. 4.1 (i)]. These two forces, being equal and opposite, constitute a system of balanced forces under which the block is at rest. Hence no force of friction acts between the surfaces in contact. If now, we apply a small horizontal force *P* to the body, [Fig. 4.1 (ii)], it may not move. The forces *R* and *W* being perpendicular to the plane of the table and there being no other horizontal force except *P*, the block should have moved as soon as the force *P* is applied on it. But in actual practice, it is seen that the block does not move even if the force *P* is slowly increased to a particular value. This clearly shows that the force *P* was so long opposed by the force of friction (*F*) between the table and the block. Further, the force of friction increases as we increase *P* till *P* exceeds a certain limiting value when the block just starts moving. The force of friction evidently reaches the maximum value just before the motion ensues.This is called the *limiting friction.*

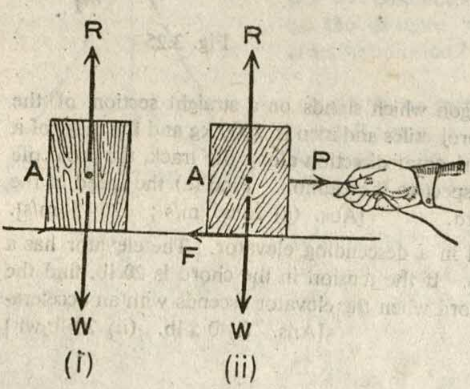


Fig. 4.1

As long as the block is at rest *i.e.* there is no relative motion between the surfaces in contact, the friction that acts between them, is known as *static friction.* The force of static friction increases from zero value to the limiting value.

4·3. Co-efficient of friction : Consider, as before a block A resting on a horizontal table whose normal reaction on the block is R (Fig. 4·2). *Normal reaction is the perpendicular force that acts between two surfaces in contact.* If, now, a force P is applied horizontally to the block so that the block is on the point of moving, the force of friction will attain the limiting value and will just oppose the force P. In the limiting condition, the force of friction (F) is found to be proportional to the normal reaction (R). A little consideration will make it clear as to why the force of friction is proportional to normal reaction. If the normal reaction increases, the undulations of the surfaces in contact become much more interlocked, bringing in greater friction. From our everyday experience we know that heavier the load, greater is the frictional force encountered when the load is pulled along the floor of a room. So,

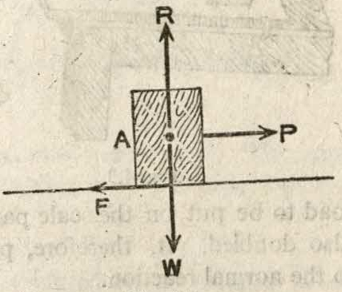


Fig. 4.2

$$F \propto R \text{ or } F=\mu.\ R.\ (\mu=\text{a constant})$$

The constant is known as the *coefficient of friction.* It depends on the materials and roughness of the surfaces in contact. Its value is always found to be less than 1. Hence.

$$\mu=\frac{F}{R}=\frac{\text{limiting value of the frictional force}}{\text{normal reaction}}$$

It is to be noted that the coefficient μ is the ratio of two forces ; hence it is a pure number and has no unit.

4·4. Laws of static friction : The static friction that acts between two surfaces obeys certain laws, known as the laws of static friction. They are as follows :

(i) The force of friction always opposes the motion.

(ii) The limiting value of the static friction is proportional to the normal reaction.

(iii) For a given normal reaction, the coefficient of friction depends upon the nature of the surfaces in contact but not upon the area of contact.

Experimental verification : The first law may be verified by the experiment described in art 4·2. In that experiment, we have seen that the block is on the point of moving in the direction of the force P but the force of friction, acting in opposition to the force P, prevents the motion.

To verify the second and the third laws, the following procedure may be adopted :

A string passing over a pulley *P* connects a movable wooden block *B* with a scale pan *W* on which weights may be placed (Fig. 4·3). For small weights on the scale pan, the block *B* may not move. At a certain load, it just starts moving. Note the load ; it gives the limiting value of the friction. The normal reaction is the weight of the block *B*. If another block of similar type is placed on the block *B*, the normal reaction will be doubled. It will be seen that the load to be put on the scale pan to just commence the motion of the blocks is also doubled. It, therefore, proves that the limiting friction is proportional to the normal reaction.

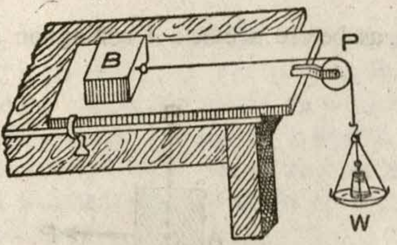


Fig. 4.3

If, now, the second block be tied behind the block *B*, so that the area of contact is doubled, it will be seen that the load necessary for commencement of motion is the same as the load that commenced motion when the blocks were placed one above the other. It means that the limiting friction does not increase with the increase of area of contact between the surfaces.

4·5. Angle of friction and cone of friction :

Suppose *R* is the normal reaction of the table on a wooden block *A* (Fig. 4·4). *P* is such a force applied on the block parallel to the table that the block is just on the point of moving. In this condition, the force *F* gives the limiting value of the static friction. If the forces *F* and *R* are combined to give a single resultant force *S*, then the force *S* is called the resultant reaction. The angle λ between the resultant *S* and the normal reaction *R* is called the *angle of friction* (fig. 4·4). It is usually denoted by λ.

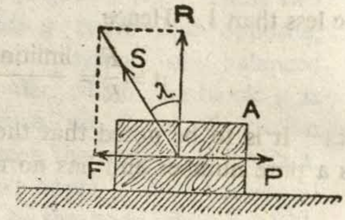


Fig. 4.4

Now, $R = S.\cos\lambda$ and $F = S.\sin\lambda$

Again, we know, the co-efficient of friction $\mu = \dfrac{F}{R} = \dfrac{S.\ \sin\lambda}{S.\ \cos\lambda} = \tan\lambda$

i.e. the tangent of the angle of friction is equal to the co-efficient of friction.

Also, $S = \sqrt{F^2 + R^2} = \sqrt{\mu^2 R^2 + R^2} = R\sqrt{\mu^2 + 1}$.

[N.B. For a certain normal reaction since *F* represents the limiting value of the friction, it may be said that the limiting angle between the normal reaction and the resultant reaction is λ. For this reason this angle is also known as the *limiting angle of friction.*]

In the fig. 4·4, the block A is just on the point of motion under the force P and the force of limiting friction F. The same thing will happen, whatever may be the direction of P parallel to the table; but with the change of direction of P, the direction of F (which always opposes the force P) and hence the direction of S will change. Considering the various directions that the force P can take up in a plane, we see that the force S, due to its consequent change of direction, describes a cone (Fig. 4·5). The apex of the cone is the point of contact O between the two bodies and the axis is the direction (OR) of the normal reaction R. The semi-vertical angle of the cone is equal to the angle of friction λ. This cone is known as the *cone of friction*.

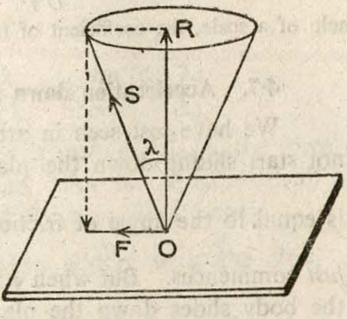


Fig. 4.5

4·6. Angle of repose : Consider a block A placed on an inclined plane. The inclination of the inclined plane with the horizontal can be varied at will. If the weight of the block be W, the frictional force F and θ the inclination of the inclined plane at any time [Fig. 4·6 (a)], then considering the equilibrium of the block, we can write,

$$R = W.\cos\theta \text{ and } F = W.\sin\theta.$$

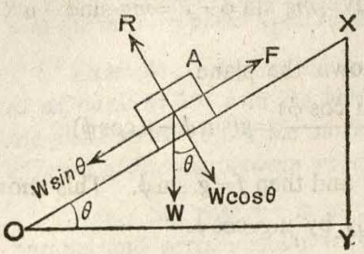


Fig. 4.6(a)

Now, suppose the inclination of the plane OX with the horizontal is slowly increased until at an inclination ϕ [Fig. 4·6 (b)] the block A is just on the point of sliding down the plane. In this limiting condition we know, the co-efficient of friction

$$\mu = \frac{F}{R} = \frac{W.\sin\phi}{W.\cos\phi} = \tan\phi$$

Here, the angle ϕ is called the *angle of repose*. But we know $\mu = \tan\lambda$, where λ is the angle of friction. So, $\tan\phi = \tan\lambda$ or $\phi = \lambda$

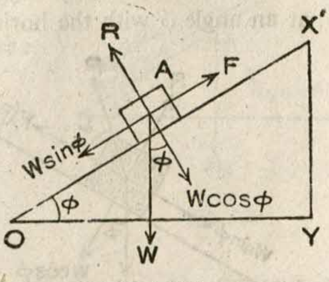


Fig. 4.6(b)

i.e. the angle of friction is equal to the angle of repose.

[N.B. (i) Though the angle of friction is same as the angle of repose, yet it is to be borne in mind that they are not identical. Angle of repose arises in connection with an inclined plane only, whereas the angle of friction may arise in the cases of inclined as well as horizontal planes.

(ii) We have seen that when a block is just on the point of sliding down an inclined plane $\mu = \tan\phi$, where ϕ is the inclination of the plane at the limiting condition, But $\tan\phi = \frac{X'Y}{OY}$

[From fig. 4.6(b)] $\therefore\ \mu=\frac{X'Y}{OY}$; now measuring the height $X'Y$ and the distance OY with the help of a scale, the coefficient of friction μ can be determined].

4·7. Acceleration down an inclined place :

We have just seen in art 4·6 that a body placed on an inclined plane will not start sliding down the plane until the angle of incliaction ϕ of the plane is equal to the angle of friction λ. When $\phi=\lambda$ or $\tan\phi=\tan\lambda=\frac{F}{R}$, the sliding *just* commences. But when $\phi>\lambda$ *i.e.* $\tan\phi>\tan\lambda$ and hence greater than F/R, the body slides down the plane with accelerated motion. We are to calculate this acceleration.

Resolving weight W of the body into two perpendicular components, the component along the plane $=W\sin\phi=mg\sin\phi$ [Fig 4.6(b)] and the component perpendicular to the plane $=W\cos\phi=mg\cos\phi$, where $m=$ mass of the body. There being no motion perpendicular to the plane $mg\cos\phi=R$, the normal reaction. If F be the frictional force acting upward along the plane, the resultant force responsible for the downward sliding of the body $=mg\sin\phi-F=mg\sin\phi-\mu R=mg\sin\phi-\mu mg.\cos\phi=mg(\sin\phi-\mu\cos\phi)$

$\therefore$ Acceleration of the body down the plane

$$f=\frac{\text{force along the plane}}{\text{mass of the body}}=\frac{mg\ (\sin\phi-\mu\cos\phi)}{m}=g(\sin\phi-\mu\cos\phi)$$

If the plane be perfectly smooth, $\mu=0$ and then $f=g.\sin\phi$. This shows that friction reduces the acceleration of the body by $\mu.g.\cos\phi$.

4·8. Equilibrium of a body on a rough inclined plane :

Consider a body of weight W resting on a rough inclined plane inclined at an angle ϕ with the horizontal (Fig. 4.7). A force P is acting on the body at an angle α with the inclined plane as shown in the figure. It this case, two limiting conditions may arise : (i) When the body is on the point of *sliding down* the inclined plane and (ii) When the body is on the point of *moving up* the inclined plane.

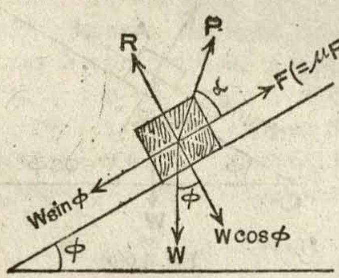


Fig. 4.7

(i) When the body is about to slide down the inclined plane, the frictional force $F(=\mu R)$ acts up the plane. The forces that act on the body, in this condition, are shown in the fig. 4.7. Resolving the forces along and perpendicular to the inclined plane, we can write, $P\cos\alpha+\mu R=W.\sin\phi$ and $P\sin\alpha+R=W.\cos\phi$

Multiplying the second equation by μ and then subtracting from first equation, we get, $P(\cos\alpha-\mu\sin\alpha)=W(\sin\phi-\mu\cos\phi)$

$$\text{or, } P=W.\frac{\sin\phi-\mu\cos\phi}{\cos\alpha-\mu\sin\alpha}$$

But we have $\mu=\tan\lambda$, where λ is the angle of friction. So,

$$P=W.\frac{\sin\phi-\tan\lambda\cos\phi}{\cos\alpha-\tan\lambda\sin\alpha}$$

$$=W.\frac{\sin\phi\cos\lambda-\cos\phi\sin\lambda}{\cos\alpha\cos\lambda-\sin\alpha\sin\lambda}=W.\frac{\sin(\phi-\lambda)}{\cos(\alpha+\lambda)}$$

(ii) When the body is about to move up the inclined plane, the force of friction $F(=\mu R)$ will act down the inclined plane. For the equilibrium of the body, we can write, $P.\cos\alpha-\mu R=W.\sin\phi$ and $P\sin\alpha+R=W.\cos\phi$

Multiplying the second equation by μ and then adding with the first, we get, $P.(\cos\alpha+\mu\sin\alpha)=W(\sin\phi+\mu\cos\phi)$

$$\therefore P=W.\frac{\sin\phi+\mu\cos\phi}{\cos\alpha+\mu\sin\alpha}=W.\frac{\sin\phi+\tan\lambda\cos\phi}{\cos\alpha+\tan\lambda\tan\alpha}=W.\frac{\sin(\phi+\lambda)}{\cos(\alpha-\lambda)}$$

For a particular inclined plane and for two given bodies, ϕ and λ are constants. So, in order to find the minimum force P which will make the body just move up the inclined plane, $\cos(\alpha-\lambda)$ should be maximum *i.e.* should be equal to 1. This means that $\alpha=\lambda$ *i.e. the force P will be minimum if it makes an angle with the inclined plane equal to the angle of friction.*

Example : *A piece of iron of mass* 10 *kg rests on a wooden inclined plane, at an angle of* 30° *with the horizontal. It is found that the least force parallel to the plane which causes the block to slide up the plane is* 100 *N. Calculate the coefficient of friction between wood and iron.* $g=10\ m/s^2$.

Ans. Suppose m=mass of the piece of iron. Its weight$=mg$ which resolved parallel and perpendicular to the plane gives $mg\sin\theta$ and $mg\cos\theta$ respectively [Fig. 4.8]. As the piece slides up, the force of friction F acts downwards along the plane. From the figure, the normal reaction $R=mg\cos\theta=10\times10\times\cos 30°$ $=100\times\frac{\sqrt{3}}{2}=50\sqrt{3}N$. Considering the force along the plane, we get,

$$100=mg\sin\theta+\mu R=10\times10\times\sin 30°+\mu\times50\sqrt{3}$$
$$=100\times\tfrac{1}{2}+50\sqrt{3}\times\mu=50+50\sqrt{3}.\mu.$$
$$\therefore\ \mu=\frac{1}{\sqrt{3}}$$

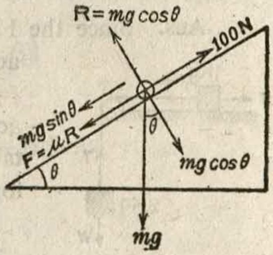


Fig. 4.8

4·9. Sliding or kinetic friction : The friction that we have been discussing so long is called static friction because the moving condition of the body has not been considered.

The force of friction acts even when the body is in motion. It has been found by experiment that when a body moves over a fixed surface uniformly, the frictional force that comes into play is less than that offered by the surfaces when the body is on the point of sliding. In other words, *sliding friction is less than static friction.*

Sliding friction, like static friction, is guided by several laws, known as the *laws of sliding or kinetic friction.* The laws are as follows :

(i) The force of sliding friction is proportional to the normal reaction and it is less than the force of static friction.

If R be the normal reaction and F_s be the frictional force between two surfaces when steady sliding is attained, then $F_s \propto R$ or $F_s = \mu_s . R$ where μ_s is a constant, known as the *coefficient of sliding friction.* As sliding friction is less than static friction, $\mu_s < \mu$.

(ii) The sliding friction does not depend on the area of contact between the surfaces but depends on the nature, condition and material of the surfaces.

(iii) The sliding friction does not depend on the relative velocity of the surfaces, provided the velocity is not too large.

When the relative velocity of the surfaces becomes too large, considerable amount of heat is produced due to friction. This heat alters the condition of the surfaces in contact and the law does not hold good.

Let a force P, acting on a body of mass m produce an acceleration f. If μ_s be the coefficient of sliding friction between the surfaces in contact, a frictional force $F_s = \mu_s . R$ will oppose the motion of the body, R being the normal reaction.

Hence, in this case, $f = \dfrac{P - \mu_s . R}{m}$

Examples : (1) *A body of mass 2kg is hanging over the edge of a horizontal table, being connected to another body of mass 1kg resting on the table by a light thread passing over a smooth pully at the edge of the table. The body resting on the table travels a distance of 9·6 metre in 2 seconds. Find the coefficient of sliding friction between the body and the table.*

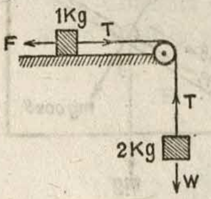


Fig 4.9

Ans. Since the 1 kg mass on the table covers 9·6 metre in 2 seconds, its acceleration is given by the equation $S = \frac{1}{2} f t^2$ or $9{\cdot}6 \times 100 = \frac{1}{2} f . (2)^2$ $\therefore$ $f = 480$ cm/s². This will be the common acceleration for both the masses. Now, the mass on the table is acted on by two forces (i) the tension T and the force of friction F. So, the resultant force $= T - F$.

$$\therefore \quad T - F = 1 \times 10^3 \times 480 \quad .. \quad \text{(i)}$$

[mass $= 1 \times 10^3$ gm]

Considering the motion of the mass hanging, we have,

$W - T = 2 \times 10^3 \times 480$ [mass $= 2 \times 10^3$ gm]

or, $2 \times 10^3 \times 980 - T = 2 \times 10^3 \times 480$ $\therefore$ $T = 2 \times 10^3 \times 500 = 10^6$ dynes.

Putting this value in eqn (i), we get $10^6 - F = 1 \times 10^3 \times 480 = 0{\cdot}48 \times 10^6$

$\therefore$ $F = 52 \times 10^4$ dynes.

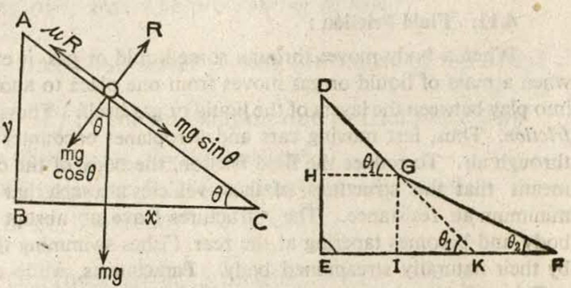
If μ_s be the coefficient of sliding friction, then,

$$\mu_s = \frac{\text{Force of friction}}{\text{Normal reaction}} = \frac{52 \times 10^4}{10^3 \times 980} = 0.53 \text{ (nearly)}$$

(2) *In fig.* 4.10 (*a* and *b*) *AC, DG and GF are fixed inclined planes. BC=EF=x and AB=DE=y. A small block of mass m is released from rest from*

the point A. It slides down AC and reaches C with a speed V_C. The same block is released from rest from the point D. It slides down DGF and reaches the point F with a speed V_F. The coefficient of kinetic friction between the block and both the surfaces AC and DGF is μ. Calculate V_C and V_F. [*I.I.T. 1980*]

Ans. See fig. 4.10 (*a*). Normal reaction $R=mg\cos\theta$ and force of friction $=\mu R=\mu mg\cos\theta$. Considering the motion of the block down the plane *AC*, we have, $mg\sin\theta-\mu R=m.f$, where f is the acceleration of the block.



(a) Fig. 4.10 (b)

$$\therefore\ f=g.\sin\theta-\frac{\mu R}{m}$$

$$=g.\sin\theta-\frac{\mu\, mg\cos\theta}{m}$$

$$=g.\sin\theta-\mu g.\cos\theta=g.(\sin\theta-\mu\cos\theta)\quad\ldots\quad\text{(i)}$$

Now, $V_C^2=0+2f.AC.=2g\,AC\,(\sin\theta-\mu\cos\theta)$

$$=2g.(y-\mu x)\quad[AC\sin\theta=y\text{ and }AC\cos\theta=x]$$

$$\therefore\ V_C=\sqrt{2g\,(y-\mu x)}\quad\ldots\quad\ldots\quad\ldots\quad\text{(ii)}$$

Now, see fig. 4.10 (*b*). We can look upon this plane as a combination of two inclined planes—one *DGH* inclined to the horizon at an angle θ_1 and the other *GFE* inclined at an angle θ_2. Velocity of the block at *G* can be written from eqn. (ii) as :

$$V_G=[2g(DH-\mu HG)]^{\frac{1}{2}}=[2g(DH-\mu DH\cot\theta_1)]^{\frac{1}{2}}=[2g.DH(1-\mu\cot\theta_1)]^{\frac{1}{2}}$$

Similarly, the velocity of the mass at *F* is given by :

$$V_F^2=V_G^2+2g[\sin\theta_2-\mu\cos\theta_2]GF\qquad[\text{see eqn. (i)}]$$

$$=2gDH(1-\mu\cot\theta_1)+2gGI-\mu.2gIF$$

$$=2g[DH-DH\ \mu\cot\theta_1+GI-\mu.IF]$$

$$=2g[y-\mu IF-DH\mu\cot\theta_1]\quad[\because\ DH+GI=DE=y]$$

$$=2g[y-\mu(IF+DH\cot\theta_1)]=2g(y-\mu x)$$

$$[\because\ IF+DH\cot\theta_1=IF+GH=x]$$

$$\therefore\ V_F=\sqrt{2g(y-\mu.x)}$$

So, the velocity at *C* is same as the velocity at *F*.

4.10. Rolling friction : Rolling friction is the resistance to motion caused by the deformation produced when a rolling body like a wheel or a cylinder pushes against the surface on which it rolls. The corresponding coefficient is called the coefficient of rolling friction (μ_r). μ_r is found to be much less than μ_s, the coefficient of sliding friction or μ the coefficient of static friction. When a motor car moves along a road, the deformation of the tyre is readily visible. Even in the case of a steel wheel rolling on a steel rail, some deformation of the surfaces takes place. Internal friction in the two bodies is called into play due to the deformation of the two surfaces.

The theory of rolling friction is very complicated but the fact remains that the rolling friction is always less than the sliding friction. This is why, whenever possible, sliding friction is replaced by rolling friction. Use of ball bearing in cycles, roller bearing in heavy machines etc. are illustrations of an attempt to replace sliding friction by rolling friction.

4.11. Fluid Friction :

When a body moves through some liquid or gas, it experiences an opposing force. Even when a mass of liquid or gas moves from one place to another, such opposing forces are called into play between the layers of the liquid or gas itself. These forces are known as forces of *fluid friction*. Thus, fast moving cars and aeroplanes encounter a lot of fluid friction while moving through air. To reduce the fluid friction, the body of the car or aeroplane is streamlined. This means that the structures of these vehicles are such that while speeding through air, they face minimum air resistance. The structures leave no abrupt bend or curvature anywhere in the body and becomes tapering at the rear. Fishes swimming through water overcome fluid friction by their naturally streamlined body. Parachutists, while descending from an aeroplane, get sufficient fluid resistance and can avail himself of a soft landing.

4.12. Effect of friction and its removal : Friction is found to be of advantage to us in many cases. You can hold a book in your hand because there is friction as otherwise the book would have slipped through your hands. We can walk along a road because there is friction between the road and the shoes. You know how difficult it is to walk on a slippery ground where friction is much less. Nails and screws hold two pieces of boards together by means of friction. When brakes are applied to a bicycle in motion, the brake-blocks press against the wheels and the force of friction between the wheels and brake-blocks reduces the speed of the bi-cycle. It is due to friction again that we drive a car, strike a match and do hosts of other things.

In some cases, however, friction is found to be of disadvantage. Due to friction more work is necessary causing a wastage of energy. Further, it causes wear and tear to different parts of a machine and dissipation of energy due to production of heat.

Friction can be reduced or minimised by using suitable surfaces and lubricating oils. Rolling friction, it has already been mentioned is much less than the sliding friction. In order to take advantage of this fact in the moving parts of the machinery, hard surfaces are used and hard rollers or balls are kept between the surfaces. Friction will increase if the metal surfaces of ball bearings are soft. This is why hard steel is used in making ball bearings.

A steel surface moving over another steel surface produces greater friction that a steel surface moving over the surface of an alloy made of lead and antimony. Such alloys are known as *antifriction* alloys. Process of reducing friction by the use of antifriction alloys is known as *babbiting*.

In some big machines, arrangement is made for a continuous supply of lubricating oil to prevent the moving parts of the machine from getting unduly heated. Surfaces face greater resistance in sliding one over the other if they are dry. Continuous supply of lubricant offers grease or oil film which makes the functioning of the machine smooth and reduce unnecessary wear and tear.

Compressed air is also used as a lubricant. It forms an elastic cushion between the moving parts and reduces friction and heating considerably. Ejection of air from the machine also prevents entry of dust particles in the machine.

Examples : (1) *A body travelling with a velocity 36 km/hr on rough ground finally comes to a standstill. If the coefficient of friction between the body and the ground be 0.2, how far does the body travel before coming to rest?*

Ans. We know, $\mu=\frac{F}{R}$, here $\mu=0{\cdot}2$ and $R=mg$ (m=mass of the body)

$\therefore\ F=\mu.R=0{\cdot}2\times mg$ If f be the retardation due to friction, $F=mf$

$\therefore\ mf=0{\cdot}2\times mg=0{\cdot}2\times 980\times m$ or, $f=196$ cm/sec^2

Now, $u=36$ km/hr$=1000$ cm/sec : $f=196$ cm/sec^2 ; $v=0$; $S=?$

We know, $v^2=u^2-2.f.S.$ or, $0=(1000)^2-2{\cdot}196.S.$

$\therefore\ S=\frac{(1000)^2}{2\times 196}=2550$ cm. (nearly)$=25{\cdot}5$ metres.

(2) *A 100 lb wooden box is being pushed across a rough floor with a horizontal force of 45 lb. How long will it take for the box, starting from rest, to travel 200 ft? Coeff^n of friction=0.4.*

Ans. Force of friction$=\mu R=0{\cdot}4\times 100=40$ lb. The resultant force on the box $P=45-40=5$ lb. Let f be the acceleration produced in the box.

From $P=m.f.$ we have, $5\times 32=100\times f$ or, $f=\frac{5\times 32}{100}$ ft/sec^2

Again, $S=\frac{1}{2}\ f.t^2$ or $t^2=\frac{2S}{f}=\frac{2\times 200\times 100}{5\times 32}=250\ \therefore\ t=5\sqrt{10}=15{\cdot}8$ sec.

(3) *A bullet of mass 10 gm travelling with a velocity 300 m/s struck a wooden block of mass 290 gm. After striking, the bullet and the block moved together over a wooden table and came to rest after describing a distance of 15 metre. Calculate the coefficient of friction between the block and the table.*

Ans. From the principle of conservation of momentum we get $10\times 300=(290+10)\times v$ or $v=10$ m/s. If f the retardation produced while the block moves over the table, then from $v^2=u^2+2\,f.s$, we get $0=(10)^2-2.f.\times 15$ or $f=\frac{100}{30}$ m/s^2 The force acting on the block $P=m.f=\frac{300}{1000}\times\frac{100}{30}=1$ newton. If R be the normal reaction, $R=\frac{P}{\mu}$; But the normal reaction R=wt of the block=300 gm-wt=$\frac{300}{1000}\times 9{\cdot}8$ N $\therefore\ \mu=\frac{1000}{300\times 9{\cdot}8}=0{\cdot}34.$

(4) *A 60 kg. block is placed on an inclined plane making an angle 30° with the horizontal. If the resultant force down the plane on the block be 14.34 kg. wt, find the value of the coeff^n of kinetic friction.*

Ans. See the fig 4.6 (b). Weight of the block W when resolved perpendicular to and along the plane, yields components $W\cos\phi$ and $W\sin\phi$ respectively.

The normal reaction, $R=W.\cos\phi=60\times\cos 30^\circ=60\times 0{\cdot}87=52{\cdot}2$ kg. wt. If μ be the coeff of friction, the force of friction $F=\mu R=\mu\times 52{\cdot}2$ kg. wt.

The resultant force down the plane $=W.\ \sin\phi-F=60\times\sin 30^\circ-\mu\times 52{\cdot}2$ $=60\times\frac{1}{2}-\mu\times 52{\cdot}2=(30-\mu\times 52{\cdot}2)$ kg wt.

According to the problem, the resultant force $=14{\cdot}34$ kg. wt.

$\therefore\ 30-\mu\times 52{\cdot}2=14{\cdot}34$ or, $\mu=0{\cdot}3$

(5) *A particle is falling down a rough inclined plane. Assuming the frictional resistance to be 0·2 times the normal reaction and the angle of inclined plane to be 30°, calculate the acceleration of the particle. Calculate also the change of the sum of the kinetic and the potential energies as the particle falls through a length of 1 metre along the inclined plane. The mass of the particle is 1 gm.*

[*Jt. Entrance 1982*]

Ans. With reference to art. 4·7, we have seen that the acceleration of a body down an inclined plane is given by $f=g\ (\sin\phi-\mu\cos\phi)$.

Here, $\phi=30^\circ$ and $\mu=\frac{F}{R}=0{\cdot}2$;

So, $f=980\ (\sin 30^\circ-0{\cdot}2\cos 30^\circ)=980\left(0{\cdot}5-0{\cdot}2\times\frac{\sqrt{3}}{2}\right)=980\ (0{\cdot}5-0{\cdot}173)$

$=980\times{\cdot}327=320{\cdot}46\ \text{cm/s}^2$

Further, change of the sum of K.E. and P.E.

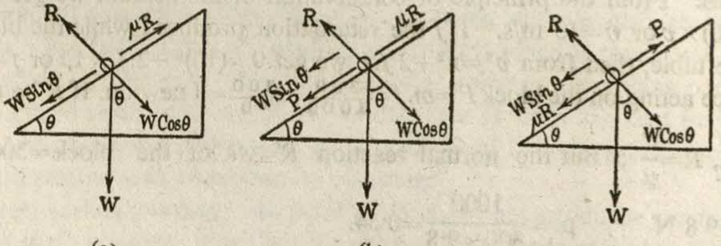
=Work done by the body against the frictional force.

Now, resultant force$=mf=1\times 320{\cdot}46=320{\cdot}46$ dynes

Work done$=320{\cdot}46\times 100=32046$ erg$=3{\cdot}2\times 10^4$ erg.

(6) *A block of mass 2 kg slides on an inclined plane which makes an angle of 30° with the horizontal. The coefficient of friction between the block and the surface is $\sqrt{\frac{3}{2}}$. Show that* (*i*) *the block will not slide down due to its own weight only. Calculate also,* (*ii*) *the force needed to move the block down without any acceleration and* (*iii*) *the force needed to move the block up without any acceleration.* [*I. I. T. 1976*]

Ans. (i) Suppose the inclination of the plane is θ and the block is in limiting equilibrium. Considering the forces on the block [Fig. 4.11 (*a*)] we can write, $R=W\cos\theta$ and $\mu\ R=W\sin\theta\ \ \therefore\ \tan\theta=\mu.=\sqrt{\frac{3}{2}}$



(a) (b) (c)

Fig. 4.11

This gives θ greater than 45°. But as the inclination is 30°, the block will not slide down due to its weight alone.

(ii) Let the force applied on the block to slide it down be *P*. [Fig 4.11 (b)]. In this case.

$$R=W\cos\theta=2\times 9{\cdot}8\times\cos\ 30^\circ=2\times 9{\cdot}8\times\frac{\sqrt{3}}{2}=\sqrt{3}\times 9{\cdot}8\ N.$$

Also, $\mu R = P + W \sin\theta$. or $P = \mu R - W \sin\theta$

$$\text{or, } P=\sqrt{\tfrac{3}{2}}\times\sqrt{3}\times 9{\cdot}8-2\times 9{\cdot}8\times\tfrac{1}{2}=9{\cdot}8\left(\frac{3}{\sqrt{2}}-1\right)=\frac{9{\cdot}8\times(3-\sqrt{2})}{\sqrt{2}}\,N$$

$$=10{\cdot}9N.$$

(ii) Let P be the upward force. Here the frictional force μR will act down the plane [Fig. 4.11 (c)]. We have, $R = W\cos\theta = \sqrt{3}\times 9{\cdot}8N$

and $P=\mu R + W.\sin\theta. = \sqrt{3}\times 9{\cdot}8\times\sqrt{\tfrac{3}{2}}+2\times 9{\cdot}8\times\tfrac{1}{2}$

$$=9{\cdot}8\left(\frac{3}{\sqrt{2}}+1\right)=\frac{9{\cdot}8\,(3+\sqrt{2})}{\sqrt{2}}=30{\cdot}5\ N.$$

(7) *Two blocks A and B are connected to each other by a string and a spring. The string passes over a frictionless pulley as shown in the fig 4.12. The block B slides over the horizontal top surface of a stationary block C and the block A slides along the vertical side of C both with the same uniform speed. The coefficient of friction between the surface of blocks is 0·2. Force constant of the spring is 1960 Newton/metre. If the mass of block A is 2 kg, calculate the mass of the block B and the energy stored in the spring.* [*I.I.T. 1982*]

Ans. Let T be the tensions in the blocks A and B. Considering the forces acting on B, T=frictional force$=\mu\times$normal reaction $=\mu\times mg$ where m=mass of the block B.

Considering the forces on A, we have, $T=2\times g$ newton

$$\therefore\quad \mu\times mg=2\times g \text{ or } m=\frac{2}{\mu}=\frac{2}{0{\cdot}2}=10\text{ kg}$$

Again, if x be the extension of the spring under the weight of the mass A, then $x=\dfrac{2\times g}{1960}$ metre. So, the energy stored in the spring$=\tfrac{1}{2}\times$force constant$\times x^2$

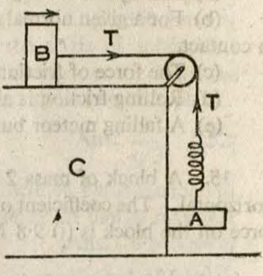


Fig. 4.12

$$=\tfrac{1}{2}\times 1960\times\frac{(2\times g)^2}{(1960)^2}=\tfrac{1}{2}\times\frac{4\times(9{\cdot}8)^2}{1960}=0{\cdot}098\text{ joule [See art 7.14]}$$

Exercises

Essay type :

1. What is friction ? What do you understand by 'coefficient of friction' ? State the laws of static friction and describe suitable experiments to verify them.

2. State the laws of friction and establish a relation between the angle of repose and the coefficient of friction.

3. In some cases, friction is advantageous and in some other it is disadvantageous. Cite some examples in support of the above statement. What steps are taken to obviate friction ?

4. Show that the acceleration of a body sliding down an inclined plane is reduced by an amount $\mu g\cos\phi$ due to friction, where μ is the coefficient of friction between the body and the plane and ϕ, the inclination of the plane to the horizontal.

Short answer type :

5. A chair is resting on the floor of a room. When will the force of friction act between them ? Where does this force act ? Is the magnitude of the force constant ?

6. Is it unreasonable to expect a coefficient of friction to exceed unity?

[*Hints* : $\mu=\frac{F}{R}$; *F* cannot be>*R* ; so μ is always less than 1].

7. Machine bearings are often made of one metal while their rotating shafts are made of another. Why ?

[*Hints* : Surfaces of same material show greater friction than surfaces of different materials.]

8. When a person walks on a rough surface, the frictional force exerted by the surface on the man is opposite to the direction of his motion. Is the statement true? Give reason in brief. [*I.I.T. 1981*]

9. What are angle of friction and cone of friction ?

10. Show that the tangent of the angle of friction is equal to the coefficient of friction.

11. Why are ball-bearings employed in some machines in place of sleeve-bearings ?

[*Hints* : Sliding friction is many times greater than rolling friction.]

12. Space-crafts are so designed as to have a special heat shield. Why ?

13. Why are tyres of motor car, bus, lorry etc made rough rather than smooth ?

Objective type :

14. Fill up the gaps with suitable words given in the parenthesis :

(a) Sliding and rolling friction are usually confined to —. (liquids, solids, gases)

(b) For a given normal reaction, the coefficient of friction depends upon the — of the surfaces in contact. (area, nature, smoothness.)

(c) The force of friction always — the motion of a body. (favours, opposes, stops)

(d) Rolling friction is always — than the sliding friction. (greater, less, equal to)

(e) A falling meteor burns in the atmosphere due to heat produced by —. (change of energy, friction, change of momentum)

15. A block of mass 2 kg rests on a rough inclined plane, making an angle 30° with the horizontal. The coefficient of static friction between the block and the plane is 0·7. The frictional force on the block is (i) 9·8 *N* (ii) $9{\cdot}8\times\sqrt{3}N$ (iii) $0{\cdot}7\times9{\cdot}8\times\sqrt{3}N$ (iv) 0·7×9·8 *N*. [*I.I.T. 1979*]

Numerical Problems :

16. A body, weighing 50 gm., rests on a rough surface, having a coefficient of friction equal to 0·5 ; what is the minimum force required to set the body in motion along the surface ? [Ans. 25 gm. wt.]

17. A wooden block weighing 40 gm. rests on an inclined plane, inclined at angle 30° to the horizontal. If the coefficient of friction between the plane and the wood be 0·2, find the resultant force on the block. [Ans. 13·12 gm-wt]

18. A block weighing 5 kg is being pulled by means of a thread tied to it along a horizontal table. The thread passes over a frictionless pulley attached to the edge of the table such that the portion of the thread over the table is horizontal. A weight of 3 kg. hangs from the other end of the thread. Starting from rest, if the block travels 20 cm. in the first second, calculate the coefficient of sliding friction. [Ans. 0·53]

19. A car is moving along a straight horizontal road with a speed v_0. If the coefficient of static friction between the tyres and the road is μs, show that the shortest distance in which the car can be stopped is $v_0^2/2g\,\mu s$.

20. (a) A body of weight *W* rests on a rough inclined plane and a force *P* acting at an angle α with the inclined plane just prevents the body from sliding down. If the inclined plane makes an angle ϕ with the horizontal, prove that, $P=W.\dfrac{\sin(\phi-\lambda)}{\cos(\alpha+\lambda)}$.

(b) When the body just starts moving up under a force P, prove that the force P will attain minimum value when the angle of inclination of the force with the inclined plane is equal to the angle of friction.

21. A rough inclined plane is inclined to the horizontal at an angle ϕ where ϕ is less than the angle of friction λ and a body of weight W is placed on the plane. Calculate the minimum *horizontal* force required to make the body move (i) down the plane and (ii) up the plane.

22. A wooden carriage of mass 100 lb is drawn along a floor by a horizontal force of 45 lb. How long will the carriage take to travel a distance of 80 ft from rest ? Coefficient of friction of the floor$=0{\cdot}4$. [Ans. 10 sec]

23. A box weighing 50 kg is placed on an inclined plane inclined to the horizontal at angle of 30°. The effective force on the box along the plane is 12 kg-wt. Calculate the coefficient of friction. [Ans. 0·3]

24. An iron chain, 1 metre long, is resting on a stone table with a portion hanging out. What is the maximum length that can hang out without the chain slipping out of the table ? Coefficient of friction between the stone and iron$=0{\cdot}4$. [Ans. 28·6 cm (approx)]

[*Hints* : Let l be the maximum length. Its weight$=l.\rho.g$. For limiting equilibrium $l.\rho.g=(100-l)\mu.\rho.g$].

25. The coefficient of friction between rubber and road is 0·25. Calculate the minimum distance within which a lorry of mass 1 ton may be brought to rest by the application of brakes ? The speed of the lorry is 45 miles/hr. [Ans. 272·25 ft]

26. A 20 kg wooden crate falls from a truck when it is travelling along a straight level road at 25 metre/sec. If the crate slides over a distance of 80 metres in coming to rest, find (i) the force of friction and (ii) the coeff*n*. of sliding friction. [Ans. (i) 78·11N (ii) 0·4]

27. What force parallel to an inclined plane of height 3 ft and base 4 ft.. can just support a 1 kg. mass on the plane ? $\mu=0{\cdot}3$. [Ans. 360 gm-wt.]

28. A box is placed on a plank which is gradually raised. When the angle of inclination of the plank with the horizontal reaches 30°, the box starts to slip and slides 4 metres down the plank in 4 sec. Find the coefficients of static and sliding friction between the box and the plank.

[Ans. $\frac{1}{\sqrt{3}}$; 0·52]

29. A body of mass 10 kg. is just on the point of sliding down an inclined plane inclined at an angle 20° to the horizontal. What force should be applied on the body so that it may just try to move up ? $\sin 20°=0{\cdot}3420$. [Ans. 6·84 kg-wt]

30. A piece of wood of mass 150 gm rests on an inclined plane. The coefficient of friction between the surfaces in contact is 0·3. To what maximum extent the plane may be inclined without allowing the piece to slip down ? [Ans. 17°]

31. A man holds a book weighing 2 lb wt between his hands and keeps it from falling by pressing both hands against the book with a force of 5 lb wt each. Find the coefficient of friction between the book and the hand. [*H. S. Exam 1981*] [Ans. 0·2]

Harder Problems :

32. A disc is sliding down an inclined plane, inclined at an angle θ with the horizon, with uniform velocity. If the disc be pushed up the plane with a velocity u, prove that the disc will come to rest after describing a distance $u^2/4g \sin \theta$ and will not come down the plane again.

33. A box of mass 6 kg rests upon an inclined plane. The inclination of the plane to the horizontal direction is gradually increased. It is found that when the slope of the plane is 2 in 3, the box starts sliding down the plane. Find the coefficient of friction between the box and the plane. What force applied to the box parallel to the plane will just make it move up the plane ? $g=9{\cdot}8$ m/s^2. [Ans. 0·88 ; 76·4N]

34. A block slides with uniform speed down an inclined plane which is inclined at an angle of 20° to the horizontal. If the same block is projected up the plane with an initial velocity of 2 metre/s, how far up the plane would it travel before it comes to rest ? Will the block slide down the plane again? [Ans. 29 cm.]

35. A block of wood of mass 150 gm. rests on an inclined plane. If the coefficient of static friction between the surfaces in contact is 0·3 find the force parallel to the plane necessary to prevent slipping when the angle of the plane with the horizontal is 30°. [Ans. 0·36 N]

36. A body can move up through a distance S in time t along an inclined plane which is inclined at an angle θ to the horizon. How long will the same body take to slide down the same length along an equally rough inclined plane? [Ans. $t\sqrt{1-\mu \cot \theta}$]

(Coeff of friction between the body and the plane $=\mu$)

37. How high can a particle rest inside a hollow sphere of radius r, if the coefficient of friction be $\frac{1}{\sqrt{3}}$?

[*Hints* : Let the particle move to the point A at a height h and rest there [Fig. 4.13]. Weight W of the particle acts vertically downward. Normal reaction R at A acts along the radius AO. Resolving the weight W, we get $R=W.\cos\theta$; the component $W\sin\theta$ pulls the particle downward but is counterbalanced by the frictional force $F=\mu R$. For the equilibrium, $W\sin\theta$

$$=\mu R=\mu W\cos\theta \text{ or } \tan\theta=\mu=\frac{1}{\sqrt{3}}=\tan 30^\circ$$

or $\theta=30^\circ$ From ΔOAB, $OB=OA\cos 30^\circ=\frac{\sqrt{3}}{2}r$

$$\therefore\ h=r-\frac{\sqrt{3}r}{2}.=\frac{r}{2}(2-\sqrt{3})=0{\cdot}134\,r]$$

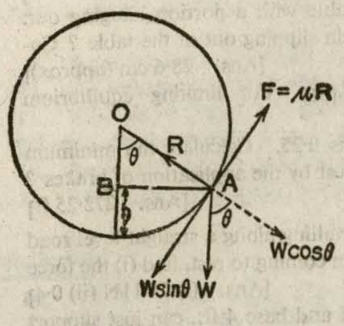


Fig. 4.13

38. A piece of iron rests at the bottom of a hemispherical bowl of radius r. The coefficient of friction between iron and the surface of the bowl is μ. The bowl is rotated with an angular velocity ω about a vertical axis. The radius of the bowl joining the greatest height attained by the piece with the centre of the bowl, makes an angle α with the vertical. Show that,

$$\frac{r\omega^2}{g}=\frac{\sec\alpha-\mu\operatorname{cosec}\alpha}{1+\mu\tan\alpha}$$

39. Masses M_1 M_2 and M_3 are connected by strings of negligible mass which pass over massless and frictionless pulleys P_1 and P_2 as shown in fiig 4.14(a). The masses move such that the portion of the string between P_1 and P_2 is parallel to the incline and the portion of the string between P_2 and M_3 is horizontal. The masses M_2 and M_3 are 4.0 kg each and the coefficient of kinetic friction between the masses and the surfaces is 0·25 The inclined plane makes an angle of 37° with the horizontal. If the mass M_1 moves downwards with a uniform velocity, find (i) the mass of M_1 and (ii) the tension in the horizontal portion of the string. $g=9{\cdot}8$ cm/s^2 and $\sin 37^\circ \cong \frac{3}{5}$ [*I. I. T. 1981*]

[Ans. 4·2 Kg ; 9·8N]

[*Hints* : See fig. 4.14(b) for the mass M_1, we have $M_1g-T=0$ (no acceleration)\.....(i) $T_1-T_2-\mu M_2g\cos 37^\circ-M_2g \sin 37^\circ=0$.....(ii) $T_2-\mu R_3=0$.....(iii) $\mu R_3=\mu\times M_3g$].

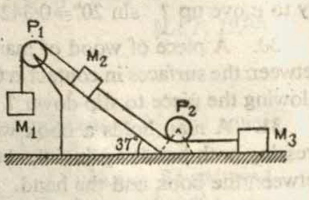


Fig. 4.14(a)

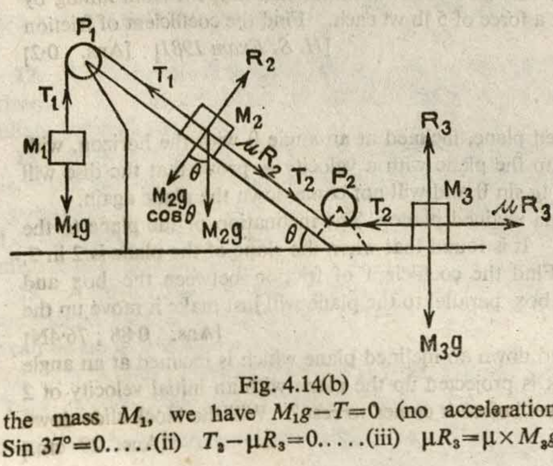


Fig. 4.14(b)

40. Two blocks connected by a massless string slide down an inclined plane having an angle of inclination of 37°. The masses of two blocks are $M_1=4$ kg and $M_2=2$ kg respectively and the coefficient of friction of M_1 and M_2 with the inclined plane are 0·75 and 0·25 respectively. Assuming the string to be taut, find (i) the common acceleration of the two masses and (ii) the tension in the string. Sin 37°=0·6; cos 37°=0·8

[*I. I. T. 1979*] [Ans. 1·31 m/s^2 ; 0·53 Kg-wt]

41. In the diagram no 4.15, the blocks *A*, *B* and *C* weigh 3 kg, 4 kg and 8 kg respectively. The coefficient of sliding friction between any two surfaces is 0·25. *A* is held at rest by a massless rigid rod fixed to the wall while *B* and *C* are connected by a light flexible cord passing around a fixed frictionless pulley. Find the force *F* necessary to drag *C* along the horizontal surface to the left at constant speed. Assume that the arrangement shown in the diagram, *B* on *C* and *A* on *B*, is maintained all through.

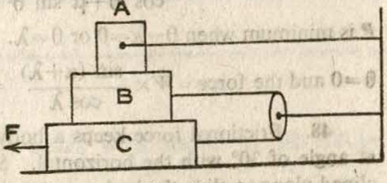


Fig. 4.15

[*I. I. T. 1978*] [Ans. 6·25 kg. wt].

[*Hints* : See fig. 4.15(a). Considering the motion of *B* over *C* and *A* over *B*, we get $T=3\mu+7\mu=10\mu$. Again considering the motion of *C*, we get, $F-T=15\mu$; Now find *F*].

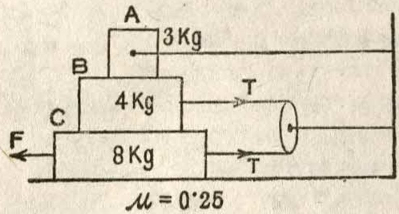


Fig. 4.15 (a)

42. A 0·5 kg block slides from the point *A* (Fig. 4.16) on a horizontal track with an initial speed of 3 m/s towards a weightless horizontal spring of length 1 metre and force constant 2 *N*/*m*. The part *AB* of the track is frictionless and the part *BC* has the coefficient of static and kinetic friction as 0·22 and 0.2 respectively. If the distances *AB* and *BD* are 2m and 2·14 metre respectively, find the total distance through which the block moves before it comes to rest completely.

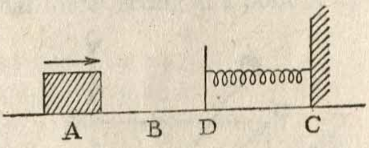


Fig. 4.16

[*I.I.T. 1983*] [Ans. 4·24 m]

43. A pull *P* is exerted on a lawn-roller of weight *W* in a direction making an angle θ with the horizontal. If μr be the coefficient of rolling friction between the roller and the ground, then show that $P=\dfrac{W.\mu r}{\mu r \sin\theta+\cos\theta}$

44. A chain of length *L* is kept on a table with a portion *l* of its length hanging. If μ be the coefficient of friction between the chain and the table, show that the maximum value of *l* is $\frac{\mu L}{1+\mu}$.

45. A chain lies on a table with some part of it hanging below the table. The chain is found to slide when the length of the part hanging is 25% of the entire length. Calculate the cofficient of friction between the chain and the table. [Ans. 0·33]

46. A uniform rod rests in limiting equilibrium in contact with a horizontal floor and a vertical wall, the rod being in a vertical plane, which is perpendicular to the wall. If the wall and the floor be equally rough prove that the angle between the rod and the wall is twice the angle of friction.

47. The minimum force which can move up a body along an inclined plane is *P*. Show that the force required to be applied parallel to the inclined plane to move the body up the plane is $P\sqrt{1+\mu^2}$, where μ=coefficient of friction.

[*Hints* Let the force *P* act at an angle θ to the inclined plane which is inclined at an angle α with the horizontal. Since the body moves up, the frictional force acts down the plane. So, $P \cos \theta - \mu R = W. \sin \alpha$..(i) and $P. \sin \theta + R = W. \cos \alpha$...(ii) Multiplying (ii) by μ and then adding with (i), we have, $P(\cos \theta + \mu. \sin \theta) = W(\sin \alpha + \mu \cos \alpha)$

$$\text{or} \quad P = W \times \frac{\sin \alpha + \mu \cos \alpha}{\cos \theta + \mu \sin \theta} = W \times \frac{\sin \alpha + \tan \lambda \cos \alpha}{\cos \theta + \tan \lambda \sin \theta} = W \times \frac{\sin(\alpha + \lambda)}{\cos(\theta - \lambda)}$$

P is minimum when $\theta - \lambda = 0$ or $\theta = \lambda$. When the force is applied parallel to the inclined plane, $\theta = 0$ and the force $= W \times \dfrac{\sin(\alpha + \lambda)}{\cos \lambda} = P \sec \lambda = P\sqrt{1 + \tan^2 \lambda} = P\sqrt{1 + \mu^2}$.]

48. Frictional force keeps a body in the limiting equilibrium on an inclined plane inclined at angle of 30° with the horizontal. Show that the minimum force required parallel to the inclined plane to slide the body up the plane is equal to the weight of the body.

5

STATICS

5.1. Introduction :

Statics is that branch of mechanics which deals with bodies at rest under the action of a system of forces.

When a body is at rest under the action of two or more than two forces, the forces are said to be in *equilibrium*. It is to be remembered that when forces act on a body in such a way that the body moves with uniform velocity, even then the forces are said to be in equilibrium. For example, when a motor car runs on a level road with uniform velocity, the force exerted by the engine of the car is in equilibrium with frictional and other opposing forces.

A body, in equilibrium, therefore, does not necessarily mean that the body is at rest. It means that the body has no linear and rotational acceleration.

5.2. Composition of any number of co-planar forces acting at a point :

Let $P_1, P_2, P_3 \ldots$ etc. be a number of co-planar forces acting at a point O in different directions as shown in fig. 5.1. OX and OY are two mutually perpendicular lines drawn from O. Let the forces P_1, P_2, P_3 etc. make angles $\alpha_1, \alpha_2, \alpha_3 \ldots$ etc. respectively with OX. If now the forces are resolved along OX and OY, the components of the forces along OX will be $P_1 \cos\alpha_1, P_2 \cos\alpha_2, P_3 \cos\alpha_3 \ldots$ etc and those along OY will be $P_1 \sin\alpha_1, P_2 \sin\alpha_2, P_3 \sin\alpha_3 \ldots$ etc.

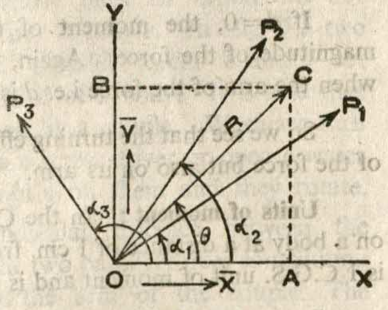


Fig. 5.1

Suppose, the resultant of the components along OX be $\overline{X}$ and that along OY be $\overline{Y}$. Then, $\overline{X}=P_1\cos\alpha_1+P_2\cos\alpha_2+P_3\cos\alpha_3+\ldots$ and, $\overline{Y}=P_1\sin\alpha_1+P_2\sin\alpha_2+P_3\sin\alpha_3+\ldots$

If these resultants are represented by OA and OB respectively, then the final resultant R will be given by the diagonal OC of the parallelogram $OACB$. Here, $R^2=(\overline{X})^2+(\overline{Y})^2$ or $R=\sqrt{(\overline{X})^2+(\overline{Y})^2}$

If the final resultant R makes an angle θ with the direction OX, then,

$$\tan\theta=\frac{\overline{Y}}{\overline{X}}.$$

5.3. Rotation of a body and moment of a force :

From our everyday experience we know that in order to rotate a body about a point or an axis, we shall have to apply a force on the body in such a way that the line of action of the force does not pass through the point or the axis about which the body is to be rotated. The further is the line of action of the force from the point or the axis, the easier it is to rotate the body.

Take the case of a heavy wheel capable of rotating in a vertical plane about a horizontal axis passing through its centre. To set the wheel rotating, a force is to be applied and it is clear from the adjoining figure (Fig. 5.2) that further is the point of application of the force from the axis of rotation, the easier it is to rotate the wheel. Thus, it is easier to rotate the wheel if the force is applied at Y than at X.

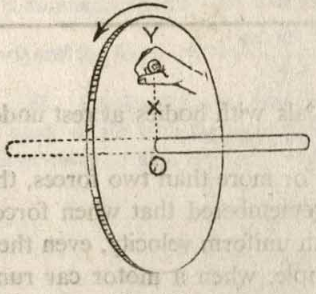


Fig. 5.2

Moment of a force : From the above discussion it is clearly understood that the rotational effect of a body under the action of a force depends upon (i) *the magnitude* (P) *of the force* and (ii) *the perpendicular distance* (d) *of the line of action of the force from the axis of rotation.* This perpendicular distance is known as the arm of the force.

Definition : The moment of a force may, therefore, be defined as the tendency of the force to turn the body about an axis and is measured by the product of the force and its arm. Thus, $G = P \times d$

If $d = 0$, the moment of the force is also zero, whatever may be the magnitude of the force. Again, the moment of a given force will be maximum when the arm of the force i.e. d is maximum.

So we see that the turning effect of a force depends not only on the magnitude of the force but also on its arm.

Units of moment : In the C.G.S. system when a force of 1 dyne is applied on a body at a distance of 1 cm. from the axis of rotation, the moment of the force is 1 C.G.S. unit of moment and is expressed as 1 dyne-cm.

In the F.P.S. system, when a force of 1 poundal is applied on a body at a distance of 1 ft. from the axis of rotation, the moment of the force is called 1 F.P.S. unit of moment and is expressed as 1 poundal-ft.

In the M.K.S. system, when a force of 1 Newton is applied on a body at a distance of 1 metre from the axis of rotation, the moment of the force is called 1 M.K.S. unit of moment and is expressed as 1 Newton-metre.

Positive and negative moment : Suppose a body is capable of rotating about a point O. If an upward force F is now applied at A, the body will rotate in an anticlockwise direction. Again, if the force is applied at A in the downward direction, the body will rotate in the clockwise direction (Fig. 5.3). The moment is considered positive in the former and negative in the latter case.

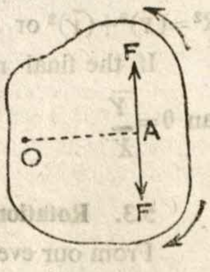


Fig. 5.3

Principle of moments : Suppose two forces F_1 and F_2 are acting on a body (Fig. 5.4) The perpendicular distances of their lines of action from the axis of rotation are respectively d_1 and d_2. If in this condition, $F_1d_1=F_2d_2$ then the forces are said to be equivalent to each other in respect of the rotation of the body.

This is known as the principle of moments.

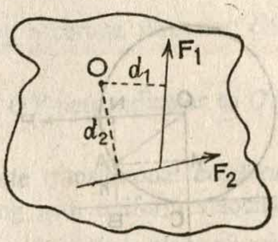


Fig. 5.4

5.4. Moment of any number of co-planar forces about a point:

If a system of co-planar forces acts on a rigid body tending to rotate the body about a point, the algebraic sum of the moments of the forces about the point is equal to the moment of the resultant of the forces about the same point.

Let P_1, P_2, P_3... etc. be the system of co-planar forces at respective distances d_1, d_2, d_3... etc. from the point O. Let R be the resultant of these forces and D its distance from the same point. Then,

$$R\times D=P_1d_1+P_2d_2+P_3d_3+\dots$$
$$=\Sigma P.d$$

This is true when the forces are parallel to one another.

5·5. Couple :

Two equal, unlike and parallel forces whose lines of action are not the same, constitute a *couple*. In fig. 5·5 two equal, unlike and parallel forces F and F act at the points A and B of a body. It is said that the body is acted on by a couple. When we turn a door knob, drive a screw driver or open a water tap, we apply couples on them and they rotate.

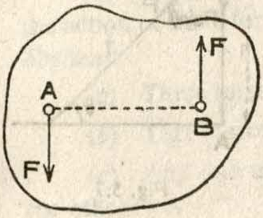


Fig. 5.5

The perpendicular distance between the lines of action of the two parallel forces constituting the couple is called the arm of the couple. The product of the arm of the couple and the magnitude of any of the forces is called the *moment* or the *torque* of the couple. So, *the moment of the couple or the torque* $G=F\times AB$.

The moment of a couple, like the moment of a force, may be positive or negative according to the same rule stated earlier. Torque, it may be pointed out, is a vector quantity. Further, the units of torque are same as the units of moment.

Examples : (1) *A heavy wheel of radius r rests against an obstacle of height h* ($h<r$). *A horizontal force P is applied through the centre of the wheel. Show that the wheel will overcome the obstacle if* $P>W\dfrac{\sqrt{2hr-h^2}}{r-h}$, *where W is the weight of the wheel.*

Ans. The horizontal force P acts through the centre O of the wheel. Weight W acts vertically downwards [Fig. 5.6]. AB is the obstacle of height h. Drop perpendiculars AN and AM on the lines of action of the force P and W respectively. Now, the wheel will overcome the obstacle if the moment of the force P about A is greater than the moment of the force W about the same point *i.e.*

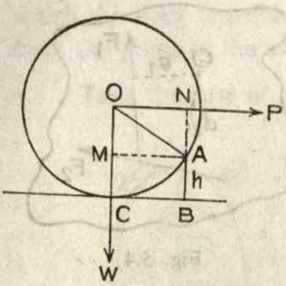


Fig. 5.6

$$P \times AN > W \times AM \text{ or } P > W.\frac{AM}{AN}$$

Now, $AN = MO = OC - MC = r - h$

and $AM = \sqrt{(AO)^2 - (OM)^2} = \sqrt{r^2 - (r-h)^2} = \sqrt{2rh - h^2}$

$$\therefore \quad P > W\frac{\sqrt{2hr - h^2}}{r - h}.$$

(2) *At what height from the bottom of a vertical pole should one end of a rope of length l be fastened so that a person standing on the ground and pulling the other end of the rope with a force P may have the best chance of overturning the pole?*

Ans. Let the required height be h [Fig. 5.7]. Let the rope be inclined at an angle θ with the horizon. The force P may be resolved into two components $P\cos\theta$ and $P\sin\theta$ perpendicular and parallel respectively to the pole AB. The component $P.\sin\theta$ has no effect on the pole. The moment of the effective component of the force P about the foot A of the pole $= P.\cos\theta \times h. = P.\cos\theta \times l.\sin\theta$ $= \frac{P.l}{2}\sin 2\theta$. The person will have the best chance of overturning the pole when the above moment is maximum *i.e.* when $\sin 2\theta$ is maximum.

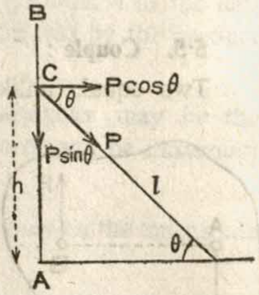


Fig. 5.7

For this $2\theta = 90°$ or $\theta = 45°$

$$\therefore \quad h = l.\sin 45° = \frac{l}{\sqrt{2}}.$$

5·6. General conditions of equilibrium of any number of co-planar forces :

A body is said to be in equilibrium if it has no acceleration either translational or rotational. When a system of co-planar forces, either parallel or non-parallel, act on a body, it may either reduce to a single force, R (say) or to a single couple of moment G. (say).

Now, when the system reduces to a single force R, let the components of this resultant force be X and Y in the two rectangular directions OX and OY. For the single force R to vanish, the components X and Y should separately be zero.

Again when the forces reduce to a single couple of moment G, the system of forces will not produce translatory motion of the body but can produce rotation.

Hence, for the body to have no rotation, G should be equal to zero. Thus, the conditions of equilibrium of co-planar forces are :

(1) Sum of the resolved parts of the forces along a certain direction OX (fig. 5.1) should be zero *i.e.* $\bar{X}=0$.

(2) Sum of the resolved parts of the forces along OY perpendicular to OX should be zero *i.e.* $\bar{Y}=0$.

These two conditions, when fulfilled, will eliminate translational acceleration of the body. The body is either at rest or moving with uniform velocity.

(3) Sum of the moments of the forces about any point in their plane should be zero, *i.e.* $G=0$.

This condition, when fulfilled, eliminates rotational acceleration of the body. Hence, to produce equilibrium, the body must satisfy simultaneously all the conditions stated above.

If the forces are co-planar and *concurrent i.e.* if they act at a point, first two conditions are sufficient for keeping the system in equilibrium.

5·7. (i) **Equilibrium under two forces :** If two forces, acting on a body, keep the body in equilibrium, then from the above conditions of equilibrium, it follows that (*a*) the forces must act along the same line but in the opposite directions and (*b*) the magnitudes of the forces must be equal.

If the forces do not act along the same line, then they (being equal and opposite) will produce a couple causing angular acceleration.

(ii) **Equilibrium under three forces :** When a body is in equilibrium under the action of three forces, we get the following from the general conditions of equilibrium :

(*a*) Three forces must be coplanar.

(*b*) Three forces must be concurrent.

(*c*) Any one of the forces must be equal and opposite to the resultant of the other two.

(*d*) The three forces can be represented, in magnitude and direction, by the three sides of a triangle, taken in order. This is known as the *triangle of forces*. Also, each force is proportional to the sine of the angle between the other two.

Suppose, the forces in equilibrium are P, Q and R and they are represented by the sides of the ΔABC, taken in order [Fig. 5.8]. Then,

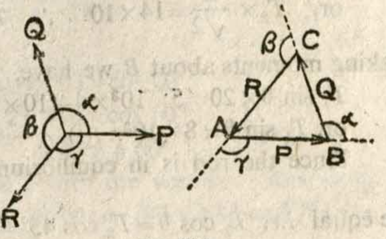


Fig. 5.8

$$\frac{P}{\sin\beta}=\frac{Q}{\sin\gamma}=\frac{R}{\sin\alpha}$$

This is known as *Lami's theorem*.

Examples : (1) *A 5 kg. weight is supported by a thin cord attached to a hook in the ceiling. Another cord is attached to the ring of the weight and is pulled horizontally until the supporting cord makes an angle of 30° with the vertical. Find the tensions in both the strings.*

Ans. The two tensions and the weight have been represented by the sides of the ΔABC (Fig. 5.9). According to Lami's theorem,

$$\frac{T_1}{\sin 90^\circ}=\frac{T_2}{\sin\ 150^\circ}=\frac{5}{\sin\ 120^\circ}$$

$$\text{or,}\quad \frac{T_1}{1}=\frac{T_2}{\cos 60^\circ}=\frac{5}{\cos 30^\circ}$$

$$\therefore\ T_2=5.\frac{\cos 60^\circ}{\cos 30^\circ}=\frac{5}{\sqrt{3}}=2{\cdot}88\text{ kg.}$$

$$\text{and } T_1=\frac{5}{\cos 30^\circ}=\frac{5\times 2}{\sqrt{3}}=5{\cdot}8\text{ kg.}$$

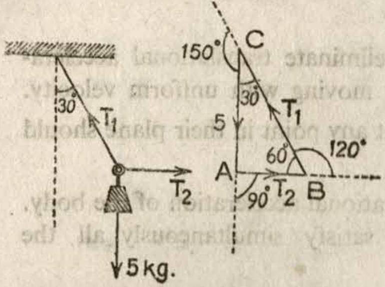


Fig. 5.9

(2) *A thin bar AB, whose weight can be neglected is suspended by strings from two points P and Q. The bar carries masses of 10 kg. and 5 kg as shown. The threads connected to P and Q are inclined at angles θ and 45° respectively with the horizontal. Find the tensions in the strings and the angle θ, if the system is in equilibrium.* [*I. I. T. 1977*]

Ans. AB is the rod. Weight 10 kg and 5 kg are suspended from C and D at distances 6 cm from A and 4 cm from B respectively. Let T_1 and T_2 be the tensions at P and Q respectively (Fig. 5.10). Resolving the tensions parallel and perpendicular to the rod AB, and taking moments about A, we have,

$$T_2 \sin 45^\circ\times 20=10\times 10^3\times 6+5\times 10^3\times 16$$

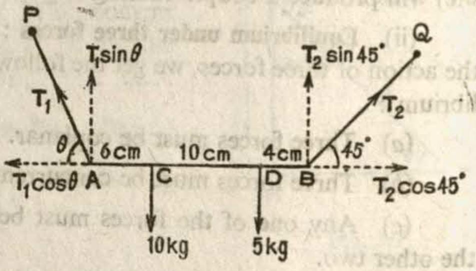


Fig. 5.10

$$\text{or,}\quad T_2\times\frac{20}{\sqrt{2}}=14\times 10^4\quad\therefore\ T_2=7\sqrt{2}\times 10^3\text{ gm-wt.}=7\sqrt{2}\text{ kg.-wt.}$$

Taking moments about B we have,

$$T_1 \sin\theta\times 20=5\times 10^3\times 4+10\times 10^3\times 14=16\times 10^4$$

$$\text{or, } T_1 \sin\theta=8\times 10^3\ldots\text{(i)}$$

Since the rod is in equilibrium, the horizontal components of the tensions are equal *i.e.* $T_1 \cos\theta=T_2 \cos 45^\circ=7\sqrt{2}\times 10^3\times\frac{1}{\sqrt{2}}=7\times 10^3\ldots\ldots\text{(ii)}$

Dividing (i) by (ii), we get, $\tan\theta=\frac{8}{7}\quad\therefore\quad \theta=\tan^{-1}\frac{8}{7}$

Squaring (i) & (ii) and then adding, $T_1^2=113\times 10^6$

$\therefore\ T_1=10{\cdot}63\times 10^3$ gm-wt. $=10{\cdot}63$ kg-wt.

(3) *Two uniform beams AB and BC each of length L and mass M are hinged smoothly together at B and a light in extensible string of lenght $L/\sqrt{2}$ is tied between their mid-points D and F. The beams stand in a vertical plane with A and C resting on a smooth horizontal plane. Calculate the tension in the string* [*I. I. T. 1971*].

Ans. The forces acting on the beams are as follows [Fig. 5·11] :

(i) the weights Mg of the rods acting vertically downwards through the mid-points D and F of the beams.

(ii) Equal reactions R_1 and R_1 vertically upward by the plane at A and C.

(iii) Equal reactions R_2 and R_2 acting horizontally outwards at B in the opposite directions.

(iv) Tensions T, T, in the string acting horizontally.

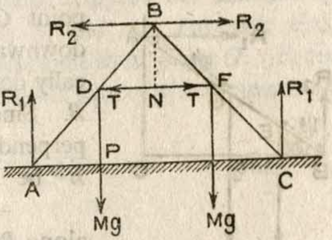


Fig. 5.11

Since the system is in equilibrium,

$2R_1=2Mg$ or, $R_1=Mg$ and $2R_2=2T$ or $R_2=T$.

Taking moments about D, we get, $R_1\times AP=R_2\times BN$

or, $R_1\times AD.\cos 45°=R_2\times BD\sin 45°$ $[\angle DBF=90°]$

or, $R_1=R_2$ $[AD=BD]$.

$\therefore\ T_1=Mg$.

i.e. the tension in the string is equal to the weight of any beam.

(4) *A ladder, 600 cm long is inclined at 60° to the horizontal floor and rests on the floor and against a smooth wall. The ladder weighs 2 kg. Find the reactions at the wall and the floor.*

Ans. Since the wall is smooth, the reaction R at the wall will be perpendicular to the wall [Fig. 5·12]. The weight W of the ladder acts vertically downward through the centre of gravity G of the ladder. These two forces meet at O. Since the ladder is at rest, the reaction F on the floor at A will act along $\overrightarrow{AO}$. As the forces are in equilibrium, the algebraic sum of the moments of the forces about any point is zero. Taking moments about the point A,

$R.a-W.AC=0$ [a=distance from A of the line of action of the force R]

$\therefore\ R.a=W.AC$

or $R\times 600\times\sin 60°=2\times 300\times\cos 60°$

$$\therefore\ R=\frac{2\times 300\times\cos 60°}{600\times\sin 60°}=\frac{1}{\sqrt{3}}\text{kg}$$

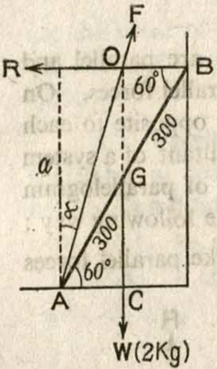


Fig. 5.12

Suppose the reaction F is making an angle α with the vertical. Resolving F along horizontal and vertical directions, we get, $F.\cos\alpha=W=2$ kg and $F.\sin\alpha$ $=R=\frac{1}{\sqrt{3}}$ $\therefore\ F^2\cos^2\alpha+F^2.\sin^2\alpha=(2)^2+\left(\frac{1}{\sqrt{3}}\right)^2$

$$\text{or}\ F^2=\frac{13}{3}\ \therefore\ F=\sqrt{\frac{13}{3}}\ \text{kg}.$$

(5) *A uniform ladder stands with one end resting against a smooth vertical wall and other end on the rough ground. A man weighing 80 kg. stands on the ladder at a distance equal to one fourth the length of the ladder from the bottom. If the inclination of the ladder with the horizontal be 30°, find (i) the reaction at the wall and (ii) the co-efficient of friction at the ground. The ladder weighs 40 kg.*

Ans. $AB \rightarrow$ ladder ; $AC \rightarrow$ wall and $BC \rightarrow$ ground [Fig. 5·13]. From mid-point G of the ladder, the weight 40 kg acts vertically downward. The weight of the man, 80 kg. acts vertically downward through the point E at a distance $l/4$ from B. Since the wall is smooth, the reaction R_1 will act perpendicular to the wall. Let the normal reaction at B be R_2. Then, the frictional force μR_2 will act along $\overrightarrow{BC}$, because the end B tends to slide along $\overrightarrow{CB}$.

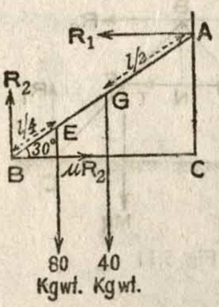


Fig. 5.13

Since the ladder is in equilibrium, $R_1 - \mu R_2 = 0$ and $R_2 - 80 - 40 = 0$.

$\therefore\ R_1 = \mu R_2$ and $R_2 = 120$ kg.-wt.

Taking moments of the forces about B, we get,

$$40 \times \frac{l}{2} \cos 30° + 80 \times \frac{l}{4} \cos 30° - R_1 l. \cos 30° = 0$$

$$\text{or, } 40 \times \frac{l}{2} \times \frac{\sqrt{3}}{2} + 80 \times \frac{l}{4} \times \frac{\sqrt{3}}{2} = \tfrac{1}{2} R_1\, l.$$

$$\text{or, } \tfrac{1}{2}.R_1 = 20\sqrt{3} \text{ or } R_1 = 40\sqrt{3} \text{ kg.-wt.}$$

$$\text{Again, } \mu = \frac{R_1}{R_2} = \frac{40\sqrt{3}}{120} = \frac{1}{\sqrt{3}}$$

5·8. Like and unlike parallel forces : When the forces are parallel and their directions are the same, they constitute a system of like parallel forces. On the other hand, if the forces are parallel but their directions are opposite to each other, they constitute a system of unlike parallel forces. The resultant of a system of parallel forces cannot, however, be found by the application of parallelogram law as they are not concurrent. The resultant can be found in the following way :

(A) **Resultant of two like parallel forces :** Consider two like parallel forces P and Q acting at A and B respectively [Fig. 5.14 (*a*)]. Through their points of applications and at right angles to their lines of action, a straight line OX has been drawn. OY is another line perpendicular to OX and hence parallel to the forces. From general conditions of equilibrium of forces, we know, that the algebraic sum of the components of the forces along any direction and along a direction perpendicular to it, is separately zero. As the forces P and Q, in the present case, have no components in the

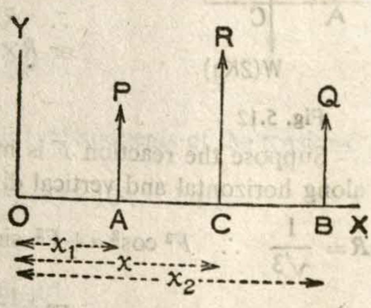


Fig. 5.14(a)

direction OX, their resultant will also have no component in this direction. Hence, taking components along OY, we get, $R=P+Q$, where R is the resultant force.

So, the sum of the magnitudes of the like parallel forces gives the magnitude of the resultant which acts parallel to the component forces.

To find the point of application of the resultant, suppose the distance of A and B from an arbitrary point O are respectively x_1 and x_2. Suppose, also, that the resultant R acts at C which is at a distance x from the same point O. From the conditions of equilibrium, we know that the algebraic sum of the moments of the forces about any coplanar point is zero.

So, $P.x_1+Q.x_2=R.x=(P+Q).x$, or, $P(x-x_1)=Q(x_2-x)$

or, $P.AC=Q.BC$. $\therefore \dfrac{P}{Q}=\dfrac{BC}{AC}$

Hence, the point of application of the resultant force divides the line AB *internally* in the inverse ratio of the forces.

(B) **Resultant of two unlike parallel forces :** Consider two unlike parallel forces P and Q acting at A and B respectively ($P>Q$). Following the procedure mentioned earlier, it will be seen that the resultant $R=P-Q$ and it will act parallel to and in the same direction as the greater force *viz*, P. [Fig. 5·14 (*b*)].
Its point of application C will lie on the line AB such that $\dfrac{P}{Q}=\dfrac{BC}{AC}$
i.e. the point of application of the resultant divides the line AB *externally* in the inverse ratio of the forces.

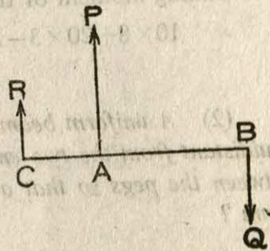


Fig. 5.14(b)

(C) **Resultant of any number of parallel forces, like or unlike :** Consider a number of parallel forces P_1, P_2, P_3. . .etc. acting on a body. Their resultant may be found out in the following way :

If R_1 be the resultant of the forces P_1 and P_2, then $R_1=P_1+P_2$. Then combining R_1 and P_3 we get a resultant R_2 which is given by $R_2=R_1+P_3$ $=P_1+P_2+P_3$. Continuing in this way, the final resultant R is given by,

$$R=P_1+P_2+P_3+\ldots \quad \ldots \quad \ldots$$
$$=\Sigma P.$$

i.e. the magnitude of the resultant force is the algebraic sum of the magnitudes of the individual forces.

If the forces given are coplanar, the point of application of their resultant may be determined in the following way :

Suppose, the distances of the lines of action of the forces P_1, P_2, P_3 . . . etc. from an arbitrary point O lying on the same plane be respectively x_1, x_2, x_3. . . etc. and the distance of the line of action of their resultant R from the same point is $\bar{x}$. The algebraic sum of the moments of all the component forces about the point O is given by, $P_1x_1+P_2x_2+P_3x_3+\ldots\ldots$etc.

The moment of the resultant R about the same point $=R.\bar{x}$

$$\therefore\ R.\bar{x}=P_1x_1+P_2x_2+P_3x_3\ldots\ldots$$

$$\text{or,}\quad \bar{x}=\frac{P_1x_1+P_2x_2+\ldots\ldots}{R}=\frac{P_1x_1+P_2x_2+\ldots\ldots}{P_1+P_2+P_3+\ldots\ldots}=\frac{\Sigma P.x}{\Sigma P}$$

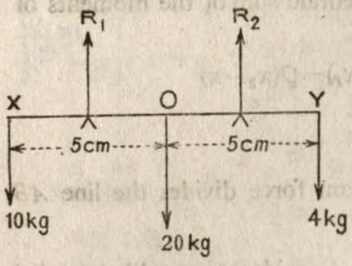


Fig. 5.15

Examples : (1) *A beam XY of weight 20 kg. rests on two supports at distance 2 cm and 8 cm from the end X. Weights of 10 kg and 4 kg are suspended from the ends X and Y respectively. Find the reactions at the two supports. The length of the beam is 10 cm.*

Ans. $XO=5$ cm. Distance of 1st support from $X=2$ cm and the distance of second support from $X=8$ cm. Let the reactions be R_1 and R_2 respectively. For equilibrium, $R_1+R_2=10+20+4=34$ kg-wt.

Taking moment of the forces about the second support,

$$10\times8+20\times3-R_1\times4-4\times2=0 \quad \text{or,}\ 4\times R_1=132 \quad \therefore\ R_1=33 \text{ kg-wt.}$$

$$\therefore\ R_2=1 \text{ kg-wt.}$$

(2) *A uniform beam of length 8 metre and of weight 8 kg rests on two pegs equidistant from the two ends of the beam. What should be the minimum distance between the pegs so that a boy weighing 16 kg may stand safely anywhere on the beam ?*

Ans. Let AB be the beam and C and D be the pegs on which the beam rests [Fig. 5·16]. The weight (8 kg) of the beam acts vertically downward through the C.G. which is the mid-point G of the beam. Let the distance of each peg from the respective end be x metre. The beam has the greatest chance of toppling over when the boy stands at any one end of the beam. Suppose the boy stands at the end A and the beam is in limiting equilibrium. In this condition, the beam will be just detached from the contact at D, where the reaction is, therefore, zero. Let the reaction at the peg C be R. Since the beam is in limiting equilibrium the algebraic sum of the moments of the forces acting on the system about any point is zero. Taking moment about C, we get.

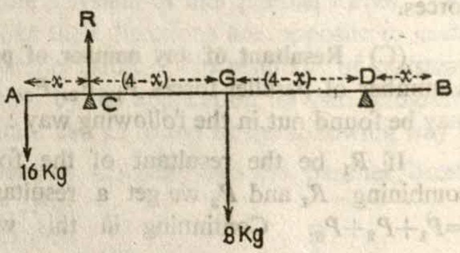


Fig. 5.16

$$16\times x=8\times(4-x) \text{ or } x=\tfrac{4}{3} \text{ metre.}$$

Hence the minimum distance between the pegs $=8-2x=8-\dfrac{2\times4}{3}=5\tfrac{1}{3}$ metre.

(3) *P and Q are two like parallel forces. If P is displaced parallel to itself through x, then show that the resultant is displaced through P.x/(P+Q)*

[*Jt. Entrance 1980*]

Ans. Suppose P and Q at first act through A and B respectively and their resultant $(P+Q)$ through E. Then $\frac{P}{Q}=\frac{EB}{AE}$

$$\text{or } \frac{P+Q}{Q}=\frac{AE+EB}{AE}=\frac{AB}{AE}$$

$$\therefore\ AE=Q.\frac{AB}{P+Q} \quad \text{.... (i)}$$

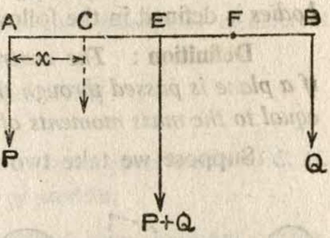


Fig. 5.17

In the second case, let the force P act through C, so that $AC=x$. Let the resultant now acts through the point F.

$$\therefore\ \frac{P}{Q}=\frac{FB}{CF} \text{ or } \frac{P+Q}{Q}=\frac{FB+CF}{CF}=\frac{CB}{CF} \text{ or } \frac{P+Q}{Q}=\frac{AB-x}{AF-x}$$

$$\text{or } (P+Q)AF-(P+Q)x=Q.AB-Q.x$$

$$\text{or } (P+Q)\ AF-P.x=Q.AB \text{ or } AF=\frac{Q.AB+P.x}{P+Q}$$

$$\therefore\ AF-AE=\frac{Q.AB+P.x}{P+Q}-\frac{Q.AB}{P+Q}=\frac{P.x}{P+Q} \quad \text{[From eqn. (i)]}$$

Hence, the displacement of the resultant $=\frac{P.x}{P+Q}$

5.9 Centre of mass : In our discussion of kinematics or dynamics, we did not consider the shape or size of a body. By the mass of a body, we so long meant a point mass. We are, however, entitled to consider the mass of a body as a point mass as long as our discussion is not influenced by the shape or size of the body. But this does not always happen. In some cases, it becomes necessary to consider the shape or size of a body although, Newton's laws of motion in their simplest forms, may be applied without any error. Two important terms—the centre of mass and the centre of gravity—arise in connection with the consideration of shape and size of a body.

Let us consider a smooth uniform rod lying on a frictionless horizontal surface. If the rod is struck by a force near any one end, the rod will roll and move forward *i.e.* it will have both rotational and translational motion. If, however, the rod is struck at the centre, it will not roll but simply slide forward *i.e.* it will have only translational motion.

The *centre of mass* of a body may thus be defined as the point at which an applied force produces only translational motion but no rotational motion. For this reason, while discussing the translational motion in dynamics (chapter one), we always considered a particle, instead of a rigid body, representing the whole mass of the body as concentrated at the centre of mass of the body. The centre of mass of a body is sometimes referred to as *centre of inertia* and called the *centroid* of the body.

With two separated masses m_1 and m_2 connected rigidly, experiment shows that the centre of mass is at a distance r_1 from m_1 and r_2 from m_2 given numerically by $m_1r_1=m_2r_2$. For this reason, the centre of mass of a *system of bodies* is defined in the following way :

Definition : *The centre of mass of a system of bodies is such a point that if a plane is passed through the point, the mass moments of one side of the plane are equal to the mass moments of the other side.*

Suppose we take two spheres A and B of masses m_1 and m_2 respectively [Fig. 5.18]. Their centre of mass P is situated on the line AB obtained by joining their centres A and B, such that $m_1\times r_1=m_2\times r_2$. If a vertical plane is imagined to be drawn perpendicular to the horizontal line AB, through the point P, then $m_1\times r_1$ will give the mass moment of the sphere A, while $m_2\times r_2$ will give the mass moment of the sphere B.

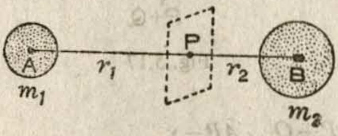


Fig. 5.18

The mass moment of a body with respect to a given plane is the product of the mass of the body and its perpendicular distance from the plane.

5.10. Position of C.M. of a number of particles :

Suppose there are a number of particles of masses m_1, m_2 ...etc rigidly connected together. If (x_1, y_1), (x_2, y_2)...etc are the coordinates of the particles respectively relative to the axes OX and OY [Fig.5.19], then generally the co-ordinate $\bar{x}$ of the centre of mass C is given by,

$$\bar{x}=\frac{m_1x_1+m_2x_2+\dots\dots}{m_1+m_2+\dots\dots}=\frac{\Sigma mx}{M}$$

where M=total mass of the particles.

Similarly, the distance $\bar{y}$ of the centre of mass C from OX axis is given by,

$$\bar{y}=\frac{m_1y_1+m_2y_2+\dots}{m_1+m_2+\dots\dots}=\frac{\Sigma my}{M}$$

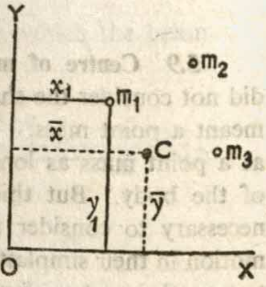


Fig. 5.19

This shows that the centre of mass of a system of particles depends only on the masses of the particles and the positions of the particles relative to one another.

Motion of C.M. : We have seen that if two particles of masses m_1 and m_2 be at distances x_1 and x_2 from the C.M. then $m_1x_1+m_2x_2=(m_1+m_2)\bar{x}$. Since velocity $v=\frac{dx}{dt}$, we have, $m_1v_1+m_2v_2=(m_1+m_2)\bar{v}$, (i) where $\bar{v}$ is the velocity of C.M. Now, $(m_1v_1+m_2v_2)$=total momentum of the particles and $m_1+m_2=M$, the total mass of the particles. Hence $M\bar{v}$=total momentum of the particles.

Thus, we can imagine that while the particles move, the total mass is concentrated at the C.M. and the velocity $\bar{v}$ of C.M. is always given by the total momentum.

Differentiating eqn. (i) $m_1\frac{dv_1}{dt}+m_2\frac{dv_2}{dt}+\ldots=(m_1+m_2+\ldots)\frac{d\bar{v}}{dt}$

or $m_1f_1+m_2f_2+\ldots=(m_1+m_2+\ldots)f$ where f_1, f_2 ..etc. are the accelerations of different particles and f the acceleration of C.M.

Now, from Newton's second law, $m_1f_1=P_1$, $m_2f_2=P_2$ ..etc. Thus,

$$(m_1+m_2+\ldots)f=P_1+P_2+P_3+\ldots.$$

Hence, *the product of the total mass of the group of particles and the acceleration of C. M. is equal to the vector sum of all the forces acting on the group of particles.*

If *internal forces* act on the particles, while they move, their resultant on the whole body is zero because actions and reactions are equal and opposite. So, the total momentum and hence the velocity of C.M. is not affected. If, however, *external forces* act on the particles, the total momentum changes and hence the velocity of C.M. is also changed.

Take the example of a shell fired from the gun. The C.M. of the shell, due to force of gravity, follows a parabolic path. If the shell explodes in mid-air, the fragments fly off in different directions. Numerous internal forces which take place due to explosion have zero resultant, since every action has an equal and opposite reaction. Consequently, the C.M. of all fragments continues to follow the same parabolic path. [Fig. 5.20].

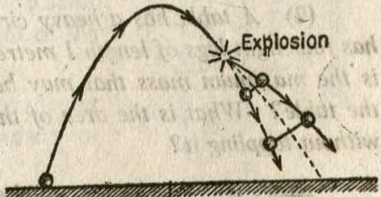


Fig. 5.20

C.M. of bodies with regular shpe :

The centres of mass of bodies with regular shape as shown in fig. 5.21 are situated at their respective geometric centres. A plane passing through those points divides the body into two equal parts.

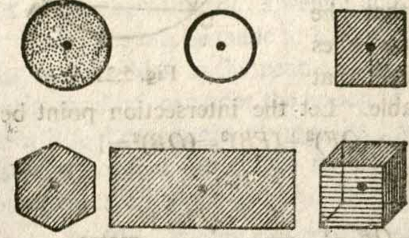


Fig. 5.21

It is to be noted that if the attraction of the earth on every part of a body is same, the centre of mass of the body coincides with its centre of gravity. If, however, the attraction is different at different parts, the C.M. will not coincide with C.G.. For a body of very large size, the gravitational attraction is not same at all parts of the body. So, its C.M. will not coincide with its C.G.

Examples : (1) *A cylindrical can is made of a material of mass 20 gm/sq. cm and has no lid. The diameter of the can is 10 cm and its height 40 cm. Find the position of the centre of mass when the can is half full of water.*

Ans. The area of the base $=\pi r^2=\pi(\frac{10}{2})^2$ sq. cm ; hence the mass of the base $=\pi(5)^2\times 20$ gm. and it acts at P, the centre of the base [Fig. 5·22].

The mass of the curved surface of the cylinder $=2\pi rh\times 20=2\pi(\frac{10}{2})\times 40\times 20=2\pi\times 5\times 40\times 20$ gm. and acts at R, half-way along the axis.

The mass of water $=\pi r^2 h=\pi(\frac{10}{2})^2\times 20=\pi(5)^2\times 20$ gm and acts at Q, the mid-point of PQ.

Thus the resultant mass $=(\pi\times 25\times 20+2\pi\times 5\times 40\times 20+\pi\times 25\times 20)$ gm $=9000\pi$ gm.

Taking moments about P, we get, $9000\pi\times x=2\pi\times 4000\times PR+\pi\times 500\times PQ$

[$x=$ the distance of C.M. from P].

or, $9000\times x=165{,}000$ or $x=18{\cdot}3$ cm (nearly)

i.e. the C.M. is 18·3 cm. from the base of the cylinder.

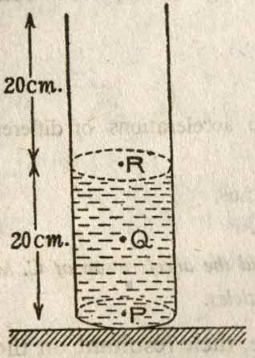


Fig. 5.22

(2) *A table has a heavy circular top of radius 1 metre and mass 20 kg. It has four light legs of length 1 metre fixed symmetrically on its circumference. What is the maximum mass that may be placed anywhere on this table without toppling the table? What is the area of the table-top over which any weight may be placed without toppling it?* [*I.I.T.* 1974]

Ans. (i) As the legs of the table are light, the centre of mass of the table is at the centre O of the circular top [Fig. 5·23]. The table will not topple as long as the centre of mass of the table together with the weight remains within the area enclosed by the four legs. But if the heavy weight be of such magnitude and is placed at such a position that the centre of mass goes outside the area $ABCD$, the table will topple. The point like E on the circumference is the furthest from the line BC and along OE. If the weight placed at E is such that the centre of mass falls on the intersection of the lines BC, and OE, then that is the maximum weight that can be placed at E without toppling the table. Let the intersection point be called F. From geometry we have, $OF=BF$; also $(OF)^2+(FB)^2=(OB)^2=1$

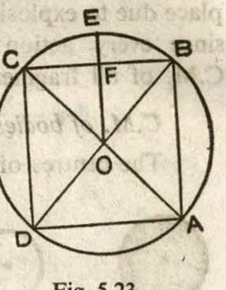


Fig. 5.23

[$OB=$radius$=1$ m]

$$\therefore\ OF=\frac{1}{\sqrt{2}}\text{ metre.}\quad\text{So,}\quad EF=OE-OF=1-\frac{1}{\sqrt{2}}=\frac{\sqrt{2}-1}{\sqrt{2}}\text{ metre}$$

Now, suppose the maximum mass placed at E be M. Since the table is not toppling, the moment of M about $F=$moment of the weight of the table about F.

$$i.e.\ M.g\times EF=20\times g\times OF \text{ or } M=\frac{20\times OF}{EF}=\frac{20\times\sqrt{2}}{\sqrt{2}(\sqrt{2}-1)}=48{\cdot}3\text{ kg. (nearly)}$$

(ii) The area of the table-top over which any weight may be placed without toppling is evidently the area of the square $ABCD$ determined by the four supporting legs.

Now, $AB=\sqrt{(OA)^2+(OB)^2}=\sqrt{r^2+r^2}=\sqrt{2}r=\sqrt{2}$ metre

$\therefore$ the area of the square $ABCD=(\sqrt{2})^2=2$ sq. metre.

5·11. Centre of gravity : Any particular body, a stone, for example, may be regarded as being made up of a very large number of tiny particles. Each such tiny particle has some mass and consequently some weight. In fig 5·24 ω_1, ω_2, ω_3...etc are the weights of those tiny particles. We have already seen that the weight of a body is defined as the force with which the earth attracts it towards its centre. Since the centre of the earth is far away from the particles of the body, the attractive forces of the earth on them may be regarded as parallel forces. Hence, the forces $\omega_1, \omega_2, \omega_3$ etc. constitute a system of large number of like parallel forces. The resultant obtained by combining the forces, will evidently be parallel to and in the same direction as the forces. This resultant is the weight of the body W and $W=\omega_1+\omega_2+\omega_3+\ldots\ldots=\Sigma\omega$.

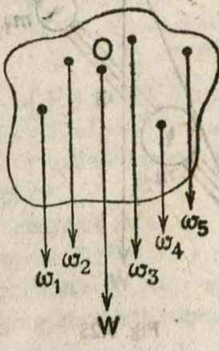


Fig. 5.24

The resultant acts downwards through a particular point (O in the figure) called the *centre of gravity* of the body.

In whatever position the body may be kept, the weights of the elementary particles will remain unchanged in magnitude and direction and hence their resultant *i.e.* the weight of the body will also remain unchanged. We may, therefore, say that whatever be the position of a body, its centre of gravity remains unaltered.

Definition : *The centre of gravity of a body is defined as the point of application of the resultant force due to the earth's attraction on it.*

In simpler terms, we may regard the centre of gravity of a body as the point at which the whole of its weight acts.

A disc can be made to balance on a finger-tip if the finger is placed just below its centre which is the centre of gravity of the disc. In this condition, the disc is in equilibrium under the action of two forces,—its weight acting vertically downwards and an equal and opposite reaction of the finger. Should the disc be held at any position other than its centre, the weight of the disc will have a turning moment about the finger and the disc will topple over.

Position of centre of gravity of some laminae (*i. e.* thin flat) of symmetrical shape :

(*a*) The C.G. of a triangular lamina is at the intersection point of the medians of the triangle.

(*b*) The C. G. of a rectangular lamina is at the intersection point of the diagonals of the rectangle.

(*c*) The C. G. of a circular disc is at the centre of the disc.

Position of centre of gravity of some symmetrical figures :

(*a*) The C.G. of a thin uniform rod is at the middle point of the rod.

(*b*) The C.G. of a uniform sphere is at the centre of the sphere.

(*c*) The C. G. of a right circular cylinder is at the middle point of its axis.

(*d*) The C. G. of a circular uniform ring is at the centre of the ring. It shows that the C. G. may be situated outside the material of a body.

Determination of C. G. by easy means :

The centre of gravity of a body or a system of bodies may be easily determined by the method of suspension. In fig. 5·25, a system has been shown which consists of two spheres connected by a uniform rod. The whole system is suspended vertically at the point Q by a thread tied at the ends of the rod. A plumb line is also suspended vertically from the same point. The point (G) where the plumb line intersects the rod gives the centre of gravity of the system. In whatever manner the system is suspended from the point Q (the manner of suspension may be changed simply by sliding the thread over the point Q) the plumb line will always intersect the rod at the point G, showing that the centre of gravity does not depend upon the orientation of the system.

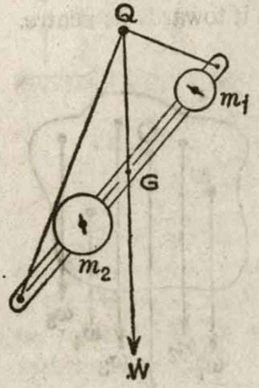


Fig. 5.25

5.12. Position of C. G. of a number of coplanar particles :

Let a number of coplanar particles of weights ω_1, ω_2, ω_3...etc. be placed at points A_1, A_2, A_3..... respectively (Fig. 5.26). Let the co-ordinates of the points A_1, A_2 etc. are respectively (x_1, y_1) (x_2, y_2)...etc with reference to OX and OY as axes. Also, suppose $G(\bar{x}, \bar{y})$ is the position of the C.G.

Taking moments about the axes, it can be shown that,

$$\bar{x}=\frac{\omega_1x_1+\omega_2x_2+\ldots}{\omega_1+\omega_2+\ldots}=\frac{\Sigma\omega.x}{\Sigma\omega}$$

and $$\bar{y}=\frac{\omega_1y_1+\omega_1y_2+\ldots}{\omega_1+\omega_2+\ldots}=\frac{\Sigma\omega.y}{\Sigma\omega}$$

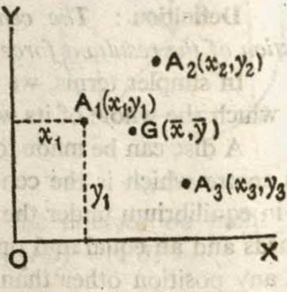


Fig. 5.26

Examples : (1) *Three particles of masses 1, 4, and 5 gm. are respectively placed at the corners of an equilateral triangle of side 10 cm. Find the distance of the C.G. of the system from the corner where the particle of mass 1 gm. is placed.*
[*I.I.T. 1970*]

Ans. Let the masses 1, 4 and 5 gm. be placed at A, B, and C respectively. Consider A as the origin (0, 0), AB as the X-axis and a line AY perpendicular to AB as Y-axis [Fig. 5.27]. Co-ordinates of B are (10,0) and of C are $(5, 5\sqrt{3})$

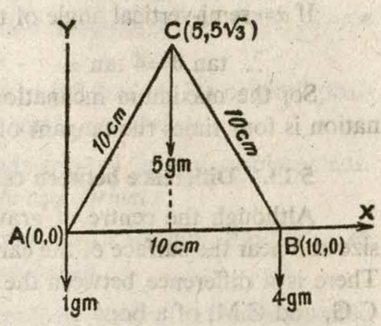


Fig. 5.27

Let $(\bar{x}, \bar{y})$ be the co-ordinates of the C.G. Then from art 5.12.,

$$\bar{x}=\frac{1\times0+4\times10+5\times5}{1+4+5}=\frac{65}{10}=6{\cdot}5 \text{ cm.}$$

$$\text{and } \bar{y}=\frac{1\times0+4\times0+5\times5\sqrt{3}}{1+4+5}=\frac{25\sqrt{3}}{10}=\frac{5\sqrt{3}}{2}$$
$$=4{\cdot}32 \text{ cm.}$$

i.e. the co-ordinates of C. G. are (6·5, 4·32)

Distance of C. G. from $A=\sqrt{(\bar{x})^2+(\bar{y})^2}$
$=\sqrt{(6{\cdot}5)^2+(4{\cdot}32)^2}=7{\cdot}81$ cm.

(2) *A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of diameter 42 cm. is remioved from one edge of the plate. Find the centre of gravity of the remaining portion.* [*I. I. T.* *1980*]

Ans. G is the centre and hence the position of C. G. of the whole plate. Let A be the centre of the cut-out portion. Hence A is the C.G. of the cut-out portion. Suppose B is the C.G. of the remaining portion.

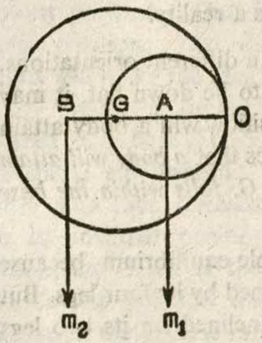


Fig. 5.28

Now, M =mass of the whole plate
$=\pi(28)^2\rho$, where ρ=mass/unit area
m_1=mass of the cut out portion$=\pi(21)^2.\ \rho.$
$\therefore\ m_2$=mass of the remaining portion$=M-m_1=\pi\rho\{(28)^2-(21)^2\}=\pi\rho\times49\times7.$ Now taking moments about G, we have, $m_1.g\times AG=m_2.g\times BG.$

or, $\pi(21)^2\times\rho\times g\times AG=\pi\rho\times49\times7\times g\times BG.\ \therefore\ \dfrac{BG}{AG}=\dfrac{9}{7}$

Now, $AG=OG-OA=28-21=7$ cm. $\therefore\ BG=\frac{9}{7}\times7=9$ cm.

This gives the position of the C.G. of the remaining portion.

(3) *A solid cone of height h has a circular base of radius r. It rests on an inclined plane without slipping. What is the maximum angle of the inclined plane at which the cone is just on the point of toppling ?*

Ans. Let $OA=r$, the radius of the circular base and $OP=h$, the height of the cone. [Fig. 5.29]. Let θ be the maximum angle of inclination at which the cone is just on the point of toppling. In this condition, the weight W of the cone acting vertically downward through the centre of gravity G of the cone will pass through B, the extreme end of the base.

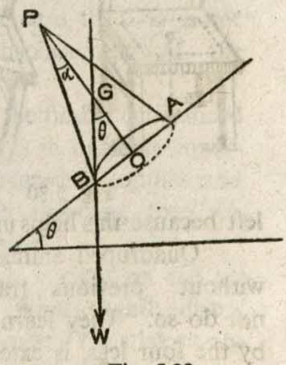


Fig. 5.29

Now, inclination $\theta=\angle OGB$.

$$\therefore\ \tan\theta=\tan\angle OGB=\frac{OB}{OG}=\frac{r}{\frac{1}{4}h}=\frac{4r}{h}.$$

[the C. G. of a cone is $\frac{3}{4}\,h$ below the vertex of the cone].

A cone resting on its base [Fig. 5·33 (a)] is in stable equilibrium, but when resting on its side and apex, [Fig. 5·33 (b) & (c)] it is in neutral and unstable equilibrium respectively. Sectional figures show how they attain the three different states of equilibrium.

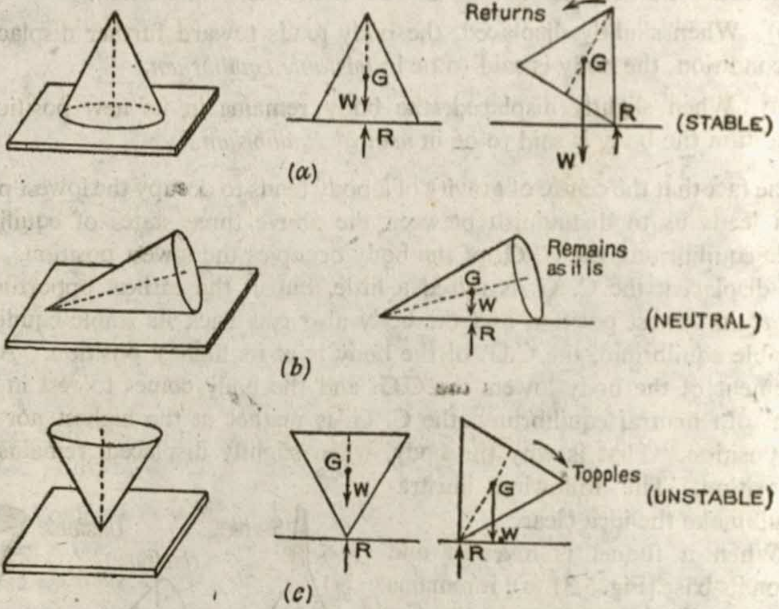
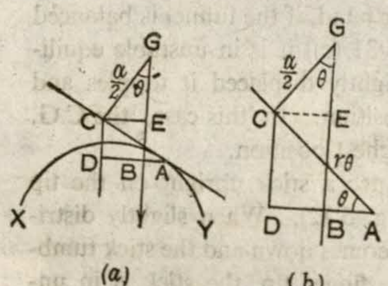


Fig. 5.33

From the above discussions, it is clear that when the C.G. occupies the highest position, the body becomes 'top-heavy' and there is a risk of toppling over. For this reason, passengers are not allowed to stand on the upper deck of a double-decker bus because in that case, the bus becomes 'top-heavy' and may overturn while taking a bend.

Examples : (1) *A rectangular beam of thickness a is balanced on the curved surface of a rough cylinder of radius r. Show that the beam will remain in stable equilibrium if* $r>\frac{a}{2}$.

Ans. *XAY* is the curved surface of the cylinder [Fig. 5·34]. Suppose the beam is tilted through a small angle θ. The point of contact then moves from *C* to *A*, the radius (*r*) turns through an angle θ and the vertical line *GC* through the centre of gravity *G* of the beam moves to *GB*, turning through an angle θ. The change has been shown in magnified form in fig. 5·34 (b).



(a) (b)

Fig. 5.34

Now, $AC=r.\theta$. If the vertical line through *G* lies on the left of *A*, a restoring moment will act on the beam and the beam will be a in stable equilibrium. In other words, for stable equilibrium $AD>DB$, where CD is the vertical through C.

Now, $AD=AC.\cos\theta=r.\theta.\cos\theta$ and $DB=CE=\frac{a}{2}\sin\theta$.

[The distance between the C.G. and the first point of contact $C=CG=\frac{a}{2}$]

So, for stable equilibrium, $r.\theta.\cos\theta>\frac{a}{2}\sin\theta$.

When θ is very small, $\cos\theta=1$ and $\sin\theta=\theta$. $\therefore\ r.\theta>\frac{a}{2}.\theta$ or $r>\frac{a}{2}$.

(2) *A wheel of mass M can revolve in a vertical plane and slide down an inclined plane inclined at an angle ϕ. Show that if a mass m be attached to the rim of the wheel, it will attain stable equilibrium if* $\sin\phi\leq\frac{m}{M+m}$. *The slipping along the plane may be neglected.*

Ans. Suppose R=radius of the wheel and the mass m is attached at the point B of the rim [Fig. 5·35]. The weight (Mg) of the wheel acts vertically downward through the C.G. of the wheel *i.e.* through the centre O and the weight (mg) of the mass m through the point B. The weight of the wheel will tend to slide it along the inclined plane while the weight mg will oppose it. In the stable equilibrium, the moment of Mg about A (the contact point between the wheel and the inclined plane) should be equal to the moment of mg about the same point.

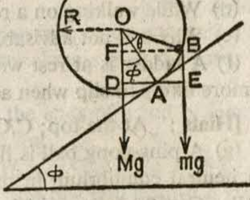


Fig. 5.35

Now join A and O. It is clear that $\angle AOD=\phi$. Join B and O. Suppose the line OB makes an angle θ with the vertical line OD. In this case, the moment of Mg about $A=M.g.\times AD=M.g\times R.\ \sin\phi$. The moment of $m.g$ about $A=mg\times AE$

Now, $DE=FB=OB.\sin\theta=R\sin\theta$. Again, $AD=R\sin\phi$.

$\therefore\quad AE=DE-AD=R(\sin\theta-\sin\phi)$.

Hence, $M.g\times R\sin\phi=mg\times R(\sin\theta-\sin\phi)$

$$\text{or}\quad \sin\theta=\frac{m+M}{m}\sin\phi.$$

It is clear that for stable equilibrium of the wheel, $180°>\theta>0°$ *i.e.* $\sin\theta\leq1$. So, the condition necessary for stable equilibrium of the wheel is

$$\frac{m+M}{m}\sin\phi\leq1\ \text{ or }\sin\phi\leq\frac{m}{M+m}.$$

Exercises

Essay type :

1. What is the significance and measure of the moment of a force about a point? What are positive and negative moments?

2. What is a couple? What is the torque of a couple? What is the effect of a couple on a body?

3. What do you mean by the term 'equilibrium of a body'? State and explain the conditions of equilibrium of a body acted upon by a number of coplanar forces.

4. State the conditions of equilibrium of a body under three forces. What is Lami's theorem?

5. Explain how the C.G. of a body distinguishes the stable, neutral and unstable equilibrium of a body.

6. What are like and unlike parallel forces? How can you find the resultant of a number of unlike parallel forces?

Short answer type :

7. What is centre of gravity? Where is the C.G. situated in the following cases :—(a) a circular disc (b) a triangular sheet (c) a sphere (d) a cylinder.

8. Explain the following carefully :—

(a) Why is it easier to stand on two legs than on one leg?

(b) Why is it convenient to walk with two equal loads in two hands than one load in one hand?

(c) Quadrupeds can walk without previous training but bipeds can not. Why?

(d) While walking on a rope in a circus, the man swings to and fro. Why?

(e) Why is it not advisable to stand on the upper deck of a double decker bus?

(f) A ladder is at rest with its upper end against a wall and the lower end on the ground. Is it more likely to slip when a man stands on it at the bottom or at the top?

[Hints : At the top; C.G. is raised making the equilibrium unstable].

(g) A ping-pong ball is floating on the top of a vertical water jet. Is it in stable, unstable or in neutral equilibrium in the vertical direction? [*I.I.T. 1973*]

9. Can there be a single force whose effect on a body is equivalent to the combined effect of two equal and unlike parallel forces?

10. If a body is not in translational equilibrium, will the torque about any point be zero if the torque about some particular point is zero ?

11. A circular portion touching the edge of a circular disc is cut out. Will the C.G. of the remainder coincide with the C.G. of the whole disc ?

12. Can the C.G. of a body in any case, lie outside the body? If so, give an example.

13. Define centre of mass and centre of gravity. Are these two points always identical? [*H. S. Exam. 1981*]

14. Distinguish between centre of gravity and centre of mass. Must there necessarily be any mass at the centre of mass of a system?

Objective type :

15. Some statements are made below. Point out which one is correct and which one is incorrect.

(a) If a number of coplanar forces act on a body, then for translational equilibrium, the sum of the moments of the forces about any point in their plane should be zero.

(b) For equilibrium under three concurrent coplanar forces, each force is proportional to the cosine of the angle between the other two.

(c) A body freely suspended from a fixed point rests with its C.G. vertically below the point of suspension.

(d) The position of centre of gravity of a moving body changes with its motion.

(e) In unstable equilibrium of a body, the C.G. of the body is at its highest position.

(f) The centre of mass and the centre of gravity coincide for a building.

16. Two particles A and B initially at rest move towards each other under a mutual force of attraction. At the instant when the speed of A is v and the speed of B is $2v$ the speed of the centre of mass of the system is (i) zero (ii) v (iii) $1{\cdot}5\ v$ (iv) $3v$. Which is correct? [*I.I.T. 1982*]

[Hints : zero]

Numerical Problems :

17. Forces P, $3P$, $\sqrt{3}P$ and $\sqrt{3}P$ are acting in a plane along OA, OB, OC and OD. If $\angle AOB=60^\circ$, $\angle BOC=90^\circ$ and $\angle COD=120^\circ$, find the magnitude and direction of the resultant force. [Ans. $2P$ along OB]

18. Two men are carrying a load of 45 kg. placed on a rod whose ends are resting on the shoulders of the men. If the load be 3 metres away from one man and 5 metres from the other, find the weight carried by each man. [Ans. 16·9 kg ; 28·1 kg]

19. The mass of a body is 2 kg. and the mass of another is 5 kg. If they are 14 metres apart, where is their centre of mass? [Ans. 10 metres from 2 kg mass]

20. A uniform plank 4 ft. long weighing 40 lb, is supported at each end. Weights of 12 lb and 20 lb are placed at distances of 8 inches and 3 ft. respectively from one end. What weight is taken by each support? [Ans. 35 lb, 37 lb]

21. The two ends of a uniform pole 12 ft. long and weighing 40 lb., rest on the shoulders of a man and his son. Where should a 200 lb load be suspended from the pole if the man is to carry twice the load carried by the boy? [Ans. 3·6 ft. from the man]

22. A metre stick balances on a knife edge at the 50 cm mark. When two small loads are placed on 12 cm mark, the loaded stick is found to balance at the 45.5 cm mark. Mass of each load is 5 gm. What is the mass of the metre stick ? [Ans. 74.4 gm]

23. A hollow metallic cylinder of height 200 cm, has a circular base of diameter 35 cm. It is filled with water to a height of 50 cm. Find the height of the centre of mass from the base. The cylinder has no lid and the mass per sq. cm. of the metal=2 gm. [Ans. 6 cm.]

24. A body of mass 2 kg and another of 5 kg are placed at the ends of a rod of length 10 cm. Find the position of the balance, neglecting the weight of the rod.
[*H. S. Exam. 1981*] [Ans. $2\frac{6}{7}$ cm from the 5 kg. end]

25. A uniform ladder, 600 cm. long, weights 2 kg. It rests with one end against a smooth vertical wall and the other on the smooth ground with an inclination of 60° with the horizon. Calculate the reactons at the wall and the ground. [Ans. $\frac{4}{\sqrt{3}}$ kg wt; $\frac{2}{\sqrt{3}}$ kg wt]

26. A body of mass 80 lb. is suspended by two threads 6 ft. and 8 ft. long. The free ends of the threads are tied to the two ends of a rod 10 ft long. If the rod is held in such a position that the body is suspended vertically below the mid-point of the rod, calculate the tensions in the thread. [Ans. 48 lb-wt ; 64 lb-wt]

27. A horizontal roadway passes over a bridge, which rests on two pillars. The bridge is 36 metre long and weighs 5700 kg. A lorry, weighing 3360 kg travels a distance equal to $\frac{2}{3}$ of the length of the bridge from the end. What weights do the pillars carry in this position?
[Ans. 3970 kg wt and 5090 kg-wt]

28. A uniform plank, 6 metre in length and 60 kg in weight rests with one end against a smooth vertical wall and the other end on a cement floor, in an inclined position. The coefficient of friction between the plank and the floor is 0·3. How far should the foot of the plank be situated from the wall so that the plank is just on the point of slipping?
[*Jt. Entrance 1979*] [Ans. 3·08 metre]

29. A beam weighs 100 lb. It stands inclined to the horizon at an angle 30° with one end resting against a smooth vertical wall and the other end on the ground. The lower end of the beam is tied to the wall by a horizontal rope so that it may not slip. If the C.G. of the beam divides the length of the beam in the ratio 3 :5, calculate the tension in the rope.
[Ans. 21·65 lb-wt]

Harder Problems :

30. The angle of inclination between two forces P and Q is α. If P and Q interchange positions, show that the resultant will turn through an angle θ such that

$$\tan\frac{\theta}{2}=\frac{P-Q}{P+Q}\tan\frac{\alpha}{2}.$$

31. Two forces P and Q—one parallel to the length and the other parallel to the base of an inclined plane can separately keep a body of weight W at rest on the inclined plane. Show that $\frac{1}{P^2}-\frac{1}{Q^2}=\frac{1}{W^2}$

32. Two like paralled forces P and Q act at A and B respectively of a rigid body. If the forces interchange their points of application, the resultant is displaced through a distance d. Show that $d=\frac{(P-Q)}{P+Q}.AB$

33. A rod, 16 cm long, rests on two pivots 10 cm apart situated equally on two sides of the midpoint of the rod. The rod just tends to topple if a 4 kg weight is suspended at one end or a 6 kg weight is suspended at the other. Calculate the weight of the rod and the point of application of the weight. [Ans, 3 kg ; 7 cm from 4 kg end]

34. A uniform rod, 10 ft, long and weighing 40 lb rests with its upper end against a point on a smooth vertical wall, 8 ft. above the ground and its lower end on a rough floor, Find (i) the reaction of the wall (ii) the force of friction due to the floor (ii) the reaction of the floor.
[Ans. (i) 15 lb (ii) 15 lb (iii) 42·72 lb]

35. A uniform ladder of weight W, rests with one end resting against a rough vertical wall and the other on a rough floor. It is inclined at an angle 45° with the horizon. If μ and μ' be the coefficient of friction at the floor and the wall respectively, show that the minimum horizontal force that can move the lower end of the ladder towards the wall is given by $\frac{W(1+2\mu-\mu\mu')}{2(1-\mu')}$

36. A light horizontal rod AB is 2 ft long and is supported at its ends ; the reaction at A acts at 30° to the vertical. Find both reactions if a load of 3 lb wt. is placed on the rod at 8 inches from A. [Ans. 2·309 lb wt ; 1·527 lb wt at 49°6′ to the vertical]

37. A balance is made up of a rigid rod free to rotate about a point not at the centre of the rod. It is balanced by unequal weights placed in the pans at each end of the rod. When an unknown mass m is placed on the left-hand pan it is balanced by a mass m_1 placed on the right hand pan. Similarly when the mass m is placed in the right-hand pan, it is balanced by a mass m_2 in the left hand pan. Show that $m=\sqrt{m_1.m_2}$

38. A cube of uniform side a and mass m rests on a rough horizontal table. A horizontal force P is applied normal to one of the faces at a point that is directly above the centre of the face at a height $\frac{3}{4}a$ above the base. For what minimum value of P, the cube begins to tip about an edge? [Ans. $\frac{2}{3}mg$]

[**Hints :** Moment of force P about an edge=moment of the weight of the cube about the same edge].

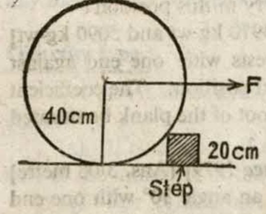


Fig. 5.36

39. A wheel of radius 40 cm. rests against a step of height 20 cm as shown in the figure 5·36 What is the minimum horizontal force F, which if applied perpendicular to the axle will make the wheel climb up the step. The mass of the wheel is 2 kg. [*I. I. T. 1976*] [Ans. 3·46 kg-wt]

40. A uniform cylinder of radius r rests on an inclined plane inclined at an angle θ with the horizon. Prove that the cylinder will not topple over as long as the height of the cylinder is less than 2 r. cot θ. Possibility of the cylinder slipping down may be ignored.

41. Calculate the maximum height of a uniform cylinder of diameter 12 cm. that can be placed on its base on an inclined plane, inclined at an angle 60° to the horizon, without the cylinder toppling over. [Ans. 6·92 cm.]

42. Three masses 2 gm., 3 gm. and 5 gm. are situated at the corners of an equilateral triangle with sides 5 cm. long. Find the distance of the centre of gravity from the corner where 2 gm. is kept. [Ans. 3·5 cm.]

[Hints : See Example 1, page 122]

43. If G_1 and G_2 are the centres of gravity of two portions of a body of weights W_1 and W_2 respectively and G the centre of gravity of the whole body, show that $GG_1=\dfrac{W_2}{W_1+W_2}G_1G_2$ and

$$GG_2=\frac{W_1}{W_1+W_2}G_1G_2.$$

44. A square hole has been punched out of a circular sheet in such a manner that the radius of the circle is a diagonal of the square. If a be the diameter of the circle, show that the C.G. of the remaining position of the sheet is at a distance of $\dfrac{a}{8\pi-4}$ from the centre of the circle.

6

DYNAMICS OF ROTATIONAL MOTION

6.1. Rotational motion of a particle :

It has been mentioned earlier that a particle may have (i) *a translational motion* and (ii) *a rotational motion.* The first type of motion has already been discussed in detail. The second type of motion is frequently met with. When we ride a cycle or go in a car, we very often keep to a circular path or take bend. In atheletic sports, competitors are asked to run in circular tracks. The earth goes round the sun in an approximately circular orbit. Rotational motion, being thus very much common, is worth while to study. We, now, propose to discuss this type of motion.

When a particle or different particles of a rigid body move in concentric circles round a common centre, the motion is called *rotational motion.* The line passing through the centre of these circles perpendicular to the plane of the circles is called the *axis of rotation.*

The radius of the circle along which a given particle moves during its rotational motion, is known as the *radius vector.* The radius vector rotates along with the rotation of the particle and sweeps out different angles at the centre of the circle at different intervals of time.

In translational motion, the principal quantity involved is the linear distance travelled by the particle. In rotational motion, however, the principal quantity involved is the angle swept out by the radius vector at the centre of the circle. Generally, this angle is called *angular displacement* and is measured in *radians.* The angular displacement is generally considered *positive* if the rotation is anti-clockwise and *negative* if clockwise. It is found convenient to express angles in radians rather than in degrees.

Definition : *Radian is defined as the angle subtended at the centre of a circle by an arc equal in length to the radius of the cricle.*

According to this system, angle $=\dfrac{\text{arc}}{\text{radius}}$

Remember, 1 radian $=\dfrac{360}{2\pi}=57{\cdot}32^\circ$ $[\pi=3{\cdot}14]$

Radian, being the ratio of two lengths, has no unit. Radian measure of angle is, therefore, satisfactory for all systems of units.

6·2. Angular velocity :

The angular velocity of a rotating particle is defined as the time rate change of the angle described by the particle at the centre of the circular path. If θ be the angular displacement turned through by the radius vector at the centre of the circle, in time t, the angular velocity (ω) given by, $\omega=\dfrac{\theta}{t}$ [Fig. 6.1]

Angular velocity in increment notation may be written as $\omega = \frac{\theta_2 - \theta_1}{t}$

[*Note* : θ/t gives the average value of the angular velocity if the particle has variable angular velocity. If the angular velocity is not variable, θ/t gives the constant value of the angular velocity]

In calculus notation, $\omega = d\theta/dt$. Now, if $t=1$, then, $\omega=\theta$, *i.e.* the angular velocity of a body is measured by the angular displacement in unit time.

It is to be noted that the above equation of the angular velocity is similar to the equation $v=\frac{s}{t}$ of linear motion. Angular velocity ω corresponds to linear velocity v and angular displacement θ corresponds to linear displacement s. With θ measured in radians and t in seconds, the angular velocity ω has the unit of radians per second (*rad/s*).

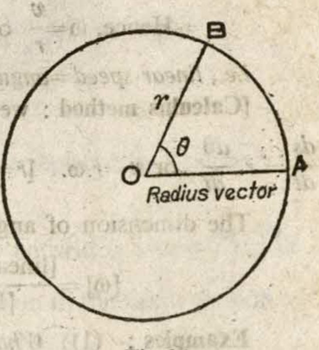


Fig 6.1

If the body takes T seconds for one complete revolution round the circle, its angular velocity ω will be equal to $\frac{2\pi}{T}$ radians/sec., because in one complete revolution, the radius vector turns through 2π radians at the centre. In this case, T is called the *time-period* of revolution of the body. Again, if the rotating body makes n complete revolutions in 1 second, $\omega=2\pi n$ and 'n' in this case denotes the *frequency* of the rotating body. It measures the rotational speed of the body.

A flywheel, for example, might be said to have a rotational speed of 10 revolutions per second (10 *r.p.s.*). This is equivalent to a rotational speed or frequency of 600 revolutions per minute (600 *r.p.m.*) and to a speed or frequency of 36,000 revolutions per hour (36,000 *r.p.h.*).

Relation between r.p.m. and radians/sec. : Usually the rotational speed of a wheel is expressed in revolution per minute, abbreviated as r.p.m. if N be the r.p.m. of a wheel then in 1 minute it performs N complete revolutions. So, for one complete rotation it takes $\frac{60}{N}$ sec and that is its time-period T. If ω be the angular velocity in radian per sec, the $\omega = \frac{2\pi}{T} = \frac{2\pi \times N}{60} = \frac{\pi \times N}{30} = \frac{3{\cdot}14\,N}{30}$ rad/s.

6·3. Relation between angular velocity and linear speed :

we know, $\omega = \frac{\theta}{t}$. Now, according to circular measure,

$$\theta = \frac{\text{the length of the arc } AB}{\text{the radius}} = \frac{s}{r} \quad \text{[Fig. 6.1].}$$

$$\text{So,} \quad \omega = \frac{s}{r} \times \frac{1}{t} = \frac{s}{t} \times \frac{1}{r}$$

Now, if the linear speed of the body be 'v', then $v=\frac{s}{t}$

Hence, $\omega=\frac{v}{r}$ or, $v=\omega\times r$

i.e., linear speed=angular velocity×radius. This is a very useful relation.

[**Calculus method** : we have seen $s=r.\theta$. Differentiating *w.r.t.* time, we get $\frac{ds}{dt}=r.\frac{d\theta}{dt}$ or $v=r.\omega$. [r=constant]

The dimension of angular velocity is given by

$$[\omega]=\frac{[\text{linear velocity}]}{[\text{Radius}]}=\frac{[LT^{-1}]}{[L]}=T^{-1}$$

Examples : (1) *What is the angular velocity of the second's hand and minute's hand of a clock?*

Ans. Second's hand makes a complete circle in 60 sec. So, its angular velocity $\omega=\frac{2\pi}{60}=0{\cdot}104$ rad/sec.

Minute's hand makes a complete circle in 60 min. So, its angular velocity $\omega=2\pi/60\times60=0{\cdot}17\times10^{-2}$ rad/sec.

(2) *A particle is revolving along a circle of radius 10 cm. with an angular velocity 0·3 radian/sec. Calculate the linear speed of the particle and the time taken by it for a complete revolution.*

Ans. Here, r=10 cm and ω=0·3 rad/s.

We know, $v=\omega\times r$. So, $v=0{\cdot}3\times10=3$ cm/s.

Again, $T=\frac{2\pi}{\omega}=\frac{2\times3{\cdot}14}{0{\cdot}3}=20{\cdot}93$ sec. ($\pi=3{\cdot}14$)

(3) *A stone is whirled at the end of a string 50 cm. long and it makes 8 complete revolutions in 2 seconds. Find the angular velocity and the linear speed of the stone.*

Ans. If θ be the angle made by the stone at the centre of the circle in 2 sec., then $\theta=2\pi\times8=50{\cdot}2$ radians. ($\pi=3{\cdot}14$)

Hence, $\omega=\frac{\theta}{t}=\frac{50{\cdot}2}{2}=25{\cdot}1$ rad/s.

Also, $v=\omega\times r=25{\cdot}1\times50=1255$ cm/s.

6.4. Angular acceleration :

When the velocity of body moving in a straight line changes, we say that the body has a linear acceleration. In the same way, when the speed of a particle moving in a circular path changes, it has an angular acceleration. Just as the acceleration of a body in linear motion is defined as the time rate change of linear velocity, so *the angular acceleration of a rotating body is defined as the time rate change of angular velocity.* If ω be the change of angular velocity in time

t, then, the angular acceleration $\alpha=\frac{\omega}{t}$. The corresponding equation in linear motion is $f=\frac{v}{t}$. The unit of angular acceleration is radian/sec². (rad/s²).

[According to calculus $\alpha=\frac{d\omega}{dt}=\frac{d}{dt}\left(\frac{d\theta}{dt}\right)=\frac{d^2\theta}{dt^2}$].

It is clear that s, v and f in the linear motion are replaced by θ, ω and α respectively in rotational motion.

The angular acceleration in increment notation may be written as, $\alpha=\frac{\omega_2-\omega_1}{t}$ wherefrom we get, $\omega_2=\omega_1+\alpha t$. Corresponding equation in the linear motion : $v=u+f.t$

The dimension of angular acceleration is $[\alpha]=\frac{LT^{-1}}{T}=LT^{-2}$

Examples : (1) *A flywheel starting from rest acquires a speed of 250 r.p.m. in 15 sec. What is its angular acceleration?*

Ans. Here, $\omega_1=0$ and $\omega_2=\frac{2\pi\times 250}{60}=26{\cdot}2$ rad/s.

Hence, $\alpha=\frac{26{\cdot}2}{15}=1{\cdot}74$ rad/s².

(2) *A wheel starts from rest and acquires a speed of 300 rev/min in 40 seconds. Find the angular acceleration. How many revolutions did the wheel make during the 40-second interval ?*

Ans. We know, angular acceleration $\alpha=\frac{\omega_2-\omega_1}{t}$. Here $\omega_1=0$ (the wheel was initially at rest) $\omega_2=\frac{300\times 2\pi}{60}=10\pi=31{\cdot}41$ rad/sec ; $t=40$ sec.

$$\therefore\quad \alpha=\frac{31{\cdot}41}{40}=0{\cdot}785 \text{ rad/s}^2$$

Now, average angular velocity$=\frac{\omega_1+\omega_2}{2}=\frac{300}{2}=150$ rev/min.

$$=\frac{150}{60}=2{\cdot}5 \text{ rev/sec.}$$

$\therefore$ No. of revolutions made by the wheel$=2{\cdot}5\times 40=100$.

6·5. Relation between angular and linear accelerations :

Suppose a particle changes its linear speed from v_1 to v_2 in time t while moving along a circular path. Its linear acceleration, therefore, is $f=\frac{v_2-v_1}{t}$. If, now. the angular velocity of the particle changes from ω_1 to ω_2 during the same interval t, then, $v_1=\omega_1\times r$ and $v_2=\omega_2\times r$ (r=radius vector)

So, $f=\dfrac{\omega_2\times r-\omega_1\times r}{t}=r.\ \dfrac{\omega_2-\omega_1}{t}=r.\alpha$ [α=angular acc^n]

Hence, *linear acc^n =angular acc^n × radius.*

[**Calculus method :** we know, $f=\dfrac{dv}{dt}=\dfrac{d}{dt}(r.\omega.)=r.\dfrac{d\omega}{dt}=r.\alpha$] [$r$=const]

The similarity in the following equations is striking :

$s=\theta.\ \ v=r.\ \omega\ ;\ f=r.\alpha$

These transformations are well worth remembering, for they are found to be quite useful in solving many problems.

6·6. Kinematical equations of linear and rotational motions :

The following kinematical equations relating to linear motion have already been proved : $v=u+f.t$

$S=u.t+\frac{1}{2}f.t^2$

$v^2=u^2+2f.S.$

In article 6.4 we have proved in the case of rotational motion :

$\omega_2=\omega_1+\alpha.t$

Similarly, it may be proved, $\theta=\omega_1 t+\frac{1}{2}.\alpha t^2$

and $\omega_2{}^2=\omega_1{}^2+2.\alpha.\theta.$

Again note that angular quantities θ, ω and α take the place of the corresponding linear quantities S, v and f.

Example : *An automobile engine running at 210 r.p.m. is given an angular acceleration of 25 rad/sec² for 10 seconds. Find (a) the angle turned through and (b) the total number of revolutions.*

Ans. Here, $\omega_1=\frac{210}{60}\times 2\pi=22$ rad/sec [$\pi=\frac{22}{7}$]

Now, we know, $\theta=\omega_1 t+\frac{1}{2}\alpha.t^2=22\times 10+\frac{1}{2}.25\times 100=1470$ radians.

Again, if n be the number of revolutions, $2\pi.n.=\theta$.

$$\therefore\quad n=\frac{\theta}{2\pi}=\frac{1470}{2\pi}=234 \text{ (nearly)}$$

6·7. Rotation of a rigid body ; Moment of inertia :

In the treatment of angular velocity or angular acceleration in the preceding articles, neither the torques causing the acceleration, nor the mass of the rotating body entered in the calculations. When these two factors are introduced in the equations, the treatment is referred to as the *dynamics of rotation of a rigid body.*

A *rigid body* means a body in which the distance between each pair of particles remains invariant. If the body undergoes some displacement, every particle in it suffers the same displacement. If the body turns through a certain angle about an axis, every particle in it rotates through the same angle about the axis in the same time. This implies that every particle of a rigid body has the same angular velocity as the rigid body. Further, we regard a rigid body as an agglomeration of particles held together by cohesive forces such that action and reaction between any pair of particles are equal and opposite.

Consider a rigid body of mass M rotating about an axis AB with a uniform angular velocity ω (Fig 6.2). Due to rotational motion, the body possesses some kinetic energy, known as *rotational kinetic energy.* Suppose, m_1, m_2, m_3...etc. are the masses of the numerous particles of the rigid body and they are at distances r_1, r_2, r_3...etc. from the axis AB respectively. Since the particles are rigidly fixed to the body, they have the same angular velocity ω but as they are at different distances from the axis of rotation, their *linear velocities are different.*

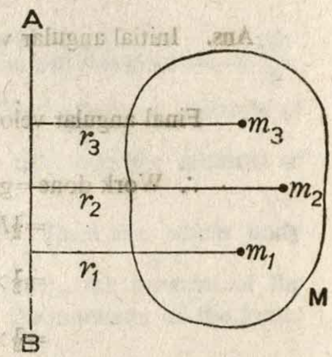


Fig. 6.2

Now, the linear velocity of the particle of mass m_1 is given by $v_1=\omega.r_1$.

Hence, its kinetic energy$=\frac{1}{2}m_1v_1^2=\frac{1}{2}m_1\omega^2.r_1^2$

Similarly, the linear velocity of the second particle, $v_2=\omega.r_2$

So, its kinetic energy$=\frac{1}{2}m_2v_2^2=\frac{1}{2}m_2\omega^2r_2^2$

In the same way, we can find the kinetic energy of all particles which constitute the rigid body. Hence, the kinetic energy of the whole rigid body

$$=\tfrac{1}{2}m_1\omega^2r_1^2+\tfrac{1}{2}m_2\omega^2.r_2^2+\tfrac{1}{2}.m_3\omega^2r_3^2+..+$$
$$=\tfrac{1}{2}\omega^2[m_1r_1^2+m_2.r_2^2+m_3r_3^2+...]$$
$$=\tfrac{1}{2}\omega^2\Sigma mr^2=\tfrac{1}{2}I.\omega^2,\text{ where }I=\Sigma mr^2$$

The quantity I is called *moment of inertia* of the rigid body about the axis AB. It denotes the sum of the products of the mass of every particle of the body and the square of its distance from the axis of rotation *i.e.*, equal to Σmr^2.

The moment of inertia of a body about a given axis is constant. For the same body, however, the moment of interia is different for different axes of rotation. Its dimension is ML^2 and units are *gm.cm*^{2} in the C.G.S. and *Kg-m*^{2} in the M.K.S. systems.

It is to be noted that the moment of inertia and mass are somewhat similar quantities. The latter is a measure of inertia for translatory motion while the former is a measure of the rotational inertia about the axis of rotation.

[N. B. (i) Moment of inertia of a thin uniform rod of length l and mass M rotating about an axis passing through its centre and perpendicular to its length$=Ml^2/12$.

(ii) Moment of inertia of a thin circular ring of mass M and radius r rotating about an axis passing through its centre and perpendicular to its plane $=M.\ r^2$.

(iii) Moment of inertia of a thin circular disc of mass M and radius r rotating about an axis passing through its centre and perpendicular to its plane$=\frac{1}{2}M.r^2$. For calculations see appendix]

Radius of gyration : If the whole mass M of a rigid body be supposed concentrated at a point such that the moment of inertia of the rigid body about an axis is same as the moment of inertia of the point mass about the same axis, the distance of the point from the axis is known as the radius of gyration of the body.

Mathematically, $K^2=\Sigma r^2$, where K is the radius of gyration,

Example : *A flywheel increases its speed from 200 r.p.m. to 250 r.p.m. in 15 sec. If the moment of inertia of the flywheel about the axis of rotation be* 25×10^3 *gm-cm²*, *calculate the work done.*

Ans. Initial angular velocity $\omega_1=\frac{2\pi\times200}{60}=\frac{20\pi}{3}$ rad/s.

Final angular velocity $\omega_2=\frac{2\pi\times250}{60}=\frac{25\pi}{3}$ rad/s.

$\therefore$ Work done = gain in rotational K.E.

$$=\tfrac{1}{2}I(\omega_2{}^2-\omega_1{}^2)$$

$$=\tfrac{1}{2}\times25\times10^3\left\{\left(\frac{25\pi}{3}\right)^2-\left(\frac{20\pi}{3}\right)^2\right\}$$

$$=\tfrac{1}{2}\times25\times10^3\times25\times\pi^2 \text{ ergs.}$$

$$=\tfrac{1}{2}\times6{\cdot}25\times10^6 \text{ ergs.} \qquad [\pi^2=10]$$

$$=0{\cdot}3125 \text{ joule.}$$

6.8. Relation between the torque and the angular acceleration :

Let us suppose that a small mass *m* fastened to the end of a string is set into rotation as shown in fig. 6.3

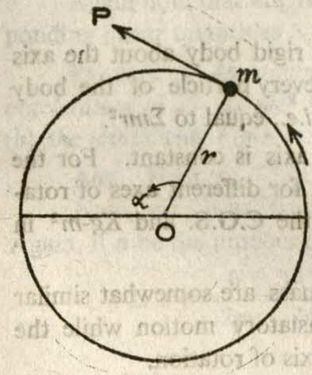


Fig. 6.3

By Newton's second law of motion, the linear acceleration *f* of the mass around the circumference of the circle, is given by, $P=m.f.$ where *P* is the force acting on the particle along the tangent to the circle.

Multiply both sides of this equation by the radius *r* of the circle. Then $P.r=m.f.r.$

The product *P.r* on the left side represents the applied torque *G*. If we replace the linear acceleration *f* on the right by $r.\alpha$ where α is the angular acceleration, we get, $G=mr^2.\alpha.$

But $mr^2=I$, the moment of inertia of the mass *m* about an axis passing through *O*. Hence, $G=I.\alpha.$

If the body has some shape and size, instead of being a particle, its moment of inertia depends upon the distribution of its mass with respect to the axis of rotation.

So, *torque applied=moment of inertia of the body×angular acceleration.*

[*Alternative treatment :* Consider a rigid body of mass *M* rotating about an axis *AB* with a uniform angular velocity ω. [Fig. 6.2]. When a couple is applied on the body, its motion of rotation is accelerated. The angular acceleration is same for all the particles of the rigid body but as the particles are at different distances from the axis of rotation, their linear accelerations are different.

If $\frac{d\omega}{dt}$ be the angular acceleration of the particles, then the linear acceleration of the particle of mass $m_1 = r_1 \times \frac{d\omega}{dt}$.

So, the force acting on the particle = mass × linear acceleration $= m_1 r_1 \frac{d\omega}{dt}$.

The moment of the force about the axis of rotation = force × distance of the particle from the axis $= m_1 r_1 \frac{d\omega}{dt} \times r_1 = m_1 r_1^2 \frac{d\omega}{dt}$. In this way, the moment of the force acting on the particle of mass $m_2 = m_2 r_2^2 \frac{d\omega}{dt}$. Since the whole body is made up of numerous particles of mass m_1, m_2 m_3...etc., the moment of the couple on the rigid body (or the torque) is the sum of the moments of the forces acting on these particles.

$$\text{Hence, the torque } G = m_1 r_1^2 \frac{d\omega}{dt} + m_2 r_2^2 \frac{d\omega}{dt} + m_3 r_3^2 \frac{d\omega}{dt} + \ldots\ldots$$

$$= \frac{d\omega}{dt} \times \Sigma m r^2 = I . \frac{d\omega}{dt} \Bigr].$$

Torque becomes very often necessary to change the angular velocity of a rotating body. Thus when a football approaches a player with a spin, it becomes difficult for him to shoot it back in the desired direction because of the necessity to apply suitable torque to change the orientation of the axis of rotation of the spinning football. Every cricket batsman knows how difficult it is to play a ball from a spinner. If he fails to apply suitable torque on the ball, it may go straight into the hands of a fieldsman.

Example : *A uniform circular disc of mass 50 gm. and of radius 10 cm. is acted upon by a torque consisting of two equal and opposite forces applied tangentially to the rim from the extremities of a diameter. The disc is free to rotate about an axis passing through its centre and perpendicular to its plane. What should be the magnitude of each force so that the disc may rotate with an angular acceleration of 20 rad/s²? What is the k.E. transferred to the disc after 2 seconds ?*

Ans. We know, torque, $G = I \times \alpha$

In this case, $I = \frac{1}{2} M . r^2 = \frac{1}{2} \times 50 \times (10)^2 = 2500$ gm-cm².

$\therefore\ G = I \times \alpha = 2500 \times 20 = 5 \times 10^4$

If P be one of the forces applied at one extreme end of a diameter [Fig. 6.4], then $G = P \times 2r$.

$\therefore\ P \times 2 \times 10 = 5 \times 10^4$ or $P = 2{\cdot}5 \times 10^3$ dynes.

Angular velocity ω at the end of 2 sec. $= 20 \times 2 = 40$ rad/s.

$\therefore$ K.E. transferred $= \frac{1}{2} I . \omega^2 = \frac{1}{2} \times 2500 \times (40)^2 = 0{\cdot}2 \times 10^7$ ergs $= 0{\cdot}2$ joule

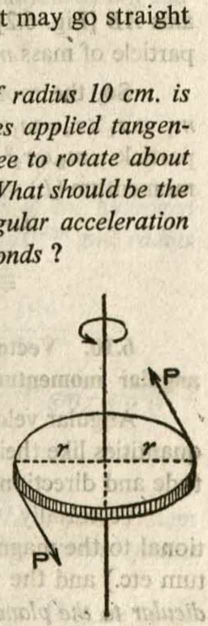


Fig. 6.4

6.9. Angular momentum or moment of momentum and its relation with torque:

In the preceding article, we have seen, $G = I.\alpha$.

If now, the body attains the above angular acceleration in time t during which its angular velocity changes from ω_1 (say) to ω_2 then we know, $\alpha = \frac{\omega_2 - \omega_1}{t}$

$$\therefore \quad G = \frac{I(\omega_2 - \omega_1)}{t} = \frac{I\omega_2 - I\omega_1}{t}$$

In the case of linear motion, we have seen that the force P is given by the equation, $P = \frac{mv_2 - mv_1}{t}$

Comparing these two equations we may say that if $m \times v$ is the linear momentum, then $I \times \omega$ will be the angular momentum or the moment of momentum of the body.

Hence, *the angular momentum of a rotating body may be defined as the product of its moment of inertia about the axis of rotation and its angular velocity.* In other words, angular momentum $L = I.\omega$. The dimensions of L are ML^2T^{-1}

Again, we may write, from the above equation, $G.t = I.\omega_2 - I\omega_1$

i.e., torque × time = change in angular momentum

The analogous equation in linear motion is $P.t = mv_2 - mv_1$

In linear motion, $P.t$ is called the impulse. By analogy, therefore, $G.t$ may be called the *angular impulse*.

[*Alternative treatment* :

Consider a rigid body rotating with a uniform angular velocity ω about the axis AB [Fig. 6.2]. We have already seen (art. 6·7) that the linear velocity of the particle of mass $m_1 = \omega \times r_1$; hence its linear momentum $= m_1 \times \omega \times r_1$.

So, the moment of momentum of the particle about the axis of rotation $= m_1 . \omega . r_1 \times r_1 = m_1 \omega . r_1^2$. Similarly the moment of momentum of the second particle $= m_2 . \omega r_2^2$ and so on. Hence the moment of momentum or the angular momentum (L) of the whole rigid body,

$$L = m_1.\omega r_1^2 + m_2.\omega r_2^2 + m_3.\omega r_3^2 + \ldots\ldots\ldots$$
$$= \omega(m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots\ldots\ldots)$$
$$= \omega.\Sigma mr^2 = I.\omega.]$$

6.10. Vector representation of angular velocity, angular acceleration and angular momentum :

Angular velocity, angular acceleration, angular momentum etc. are all vector quantities like their corresponding quantities in linear motion. They have magnitude and direction.

Vectorially, they are represented by a straight line whose length is proportional to the magnitude of the vector quantity (*i.e.*, velocity, acceleration, momentum etc.) and the straight line is drawn *parallel to the axis of rotation or perpendicular to the plane of rotation.* It is directed towards the observer for clockwise rotation and away from him for anti-clockwise rotation.

In Fig. 6.5, *OX, OY* and *OZ* are three mutually perpendicular axes, *OZ* being perpendicular to the plane of the paper. A disc is rotating in the plane *YOZ* about the axis *XOX'*, which passes through its centre and is perpendicular to its faces. It is rotating from *OZ* to *OY* [fig. 6.5 (a)], *i.e.*, clockwise when looked from the right *i.e.*, from *X* towards *O* and therefore, its angular velocity (or angular acceleration or momentum) is represented vertorially by the line $\overrightarrow{OA}$ drawn along *OX*. For anticlockwise rotation [Fig. 6.5 (b)], the line to represent these vectors is $\overrightarrow{OA'}$ in the direction *OX'*.

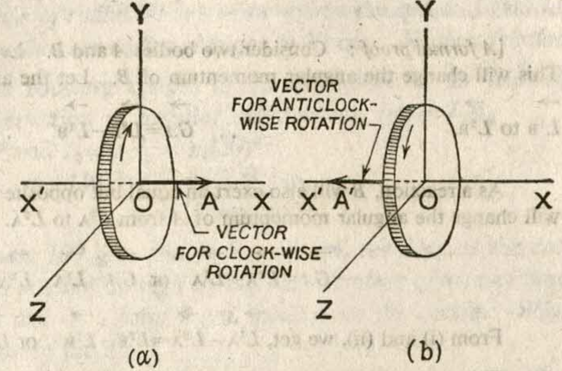

Y
Y
VECTOR FOR ANTICLOCK-WISE ROTATION
X'
O
A
X
X'
A'
O
X
VECTOR FOR CLOCK-WISE ROTATION
Z
Z
(a)
(b)

Fig. 6.5

Similarly, for rotation in the plane *XOY*, from *OY* to *OX*, the line would be drawn in the direction *OZ* to represent the vector quantities, while its direction will be *OY* if *XOZ* were the plane of rotation and the disc rotated from *OX* to *OZ*.

It is clear that the rule for finding the direction of the vectors in the aforesaid case is the same as that for the *cross product* of two vectors (art 2·12). This fact, along with the magnitude enables us to write the vector equation of angular momentum in the following way :

$$L=I.\omega=mr^2\omega=m.r^2.\frac{v}{r}=r.m.v \quad [v=\text{linear velocity}]$$

Vectorially, $\vec{L}=\vec{r}\times\vec{mv}$

Example : *Calculate the angular momentum of the electron of the hydrogen atom about its own nucleus, given mass of the electron* $(m)=9{\cdot}10\times10^{-28}$ *gm, radius of orbit*$=0{\cdot}6\times10^{-8}$ *cm and time for one revolution*$=1{\cdot}5\times10^{-16}$ *sec.*

Ans. Angular momentum, $L=mr^2\omega$. Here $\omega=\frac{2\pi}{T}=\frac{2\times3{\cdot}14}{1{\cdot}5\times10^{-16}}$

$$\therefore\quad L=9{\cdot}10\times10^{-28}\times(0{\cdot}6\times10^{-8})^2\times\frac{2\times3{\cdot}14}{1{\cdot}5\times10^{-16}}=1{\cdot}37\times10^{-27}\ gm-cm^2/s.$$

6.11. Principle of conservation of angular momentum :

The principle states that *if no external torque acts upon a body or a system of bodies already in rotation or at rest, the angular momentum of the system remains constant.*

In the preceding article, we have seen that $G.t=I\omega_2-I\omega_1$. If there be no torque acting upon the body, $G.t=0$. Hence $I.\omega_1=I.\omega_2$.

In other words, the final angular momentum is always equal to the initial angular momentum. This is the principle of conservation of angular momentum.

[*A formal proof*: Consider two bodies A and B. Let A exert a torque $\vec{G}$ on B for a time t. This will change the angular momentum of B. Let the angular momentum of B change from $\vec{L}^1_B$ to $\vec{L}^2_B$. $\therefore\ \vec{G}.t = \vec{L}^2_B - \vec{L}^1_B$ .. (i)

As a reaction, B will also exert an equal but opposite torque *i.e.* $-\vec{G}$ on A for time t which will change the angular momentum of A from L^1_A to L^2_A. For this, we get

$$-\vec{G}.t = \vec{L}^2_A - \vec{L}^1_A \quad \text{or} \quad \vec{G}.t = \vec{L}^1_A - \vec{L}^2_A \qquad \text{.. (ii)}$$

From (i) and (ii), we get, $\vec{L}^1_A - \vec{L}^2_A = \vec{L}^2_B - \vec{L}^1_B$ or $\vec{L}^1_A + \vec{L}^1_B = \vec{L}^2_B + \vec{L}^2_A$

This shows that the vector sum of the initial angular momenta of A and B is equal to the vector sum of the final momenta, when no external torque acts on the system. This is the principle of conservation of angular momentum.]

From the above principle, it follows that if the moment of inertia of the rotating body decreases, its angular velocity increases and *vice versa*. An interesting experiment illustrating the above may be performed in the following way.

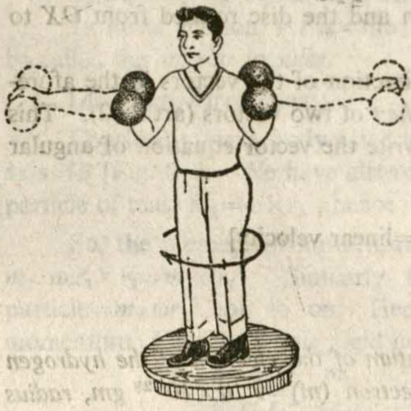


Fig. 6.6

A person stands on a turntable with weights in each hand. With the arms extended fully in a horizontal direction, he is first set rotating slowly (Fig. 6.6). Upon drawing his hands and weights in toward the chest as shown, the angular velocity is considerably increased. If the person again extends his arms, the angular velocity will decrease. This change of angular velocity is best appreciated by the turning person who feels himself speeded up or slowed down by what seems to be a mysterious force but actually by the increase and decrease of moment of inertia of the person.

This principle is used by expert figure skaters on the ice. They start into a whirl with their arms outstretched and perhaps one leg extended and then upon drawing the arms and leg in, obtain a greatly increased angular velocity. When a diver jumps from a diving platform, his moment of inertia can be decreased by curling his body more, in which case his angular velocity is increased. He may then turn more somersaults before touching the water.

Angular momentum and the principle of conservation of angular momentum are of great importance in physics. They are applied to the circular motion of enormous rotating masses like the earth as well as very minute spinning particles in the atomic and sub-atomic regions.

Examples : (1) *A small body is rotated along a horizontal circle at the end of a thread which passes down through a hole at the centre of the table top. If the body rotates at 3 rev/sec in a circle of radius 40 cm, what will be the speed of rotation when the thread is pulled down so that the radius changes to 20 cm. Neglect friction.*

Ans. The torque on the rotating system is nil and there is no friction. Applying the principle of conservation of angular momentum, $I_1\omega_1=I_2\omega_2$

Now, $I_1=mr_1^2=m\times(40)^2$ and $I_2=m.r_2^2=m(20)^2$

Also, $\omega_1=3$ rev./sec. $\therefore\ m\times(40)^2\times 3=m(20)^2\times\omega_2$

or, $\omega_2=12$ rev./sec.

(2) *A circular disc of mass 100 gm. and radius 10 cm. revolves at the rate of 30 r.p.m. about a vertical axis passing through its centre. A piece of wax of mass 20 gm. is slowly dropped on the disc at a point 8 cm. away from the centre. What will be the present rate of rotation of the disc?*

Ans. As no external torque acts on the system, the principle of conservation of angular momentum may be applied. The angular momentum of the disc before the piece of wax is dropped $=I.\omega$.

Now, the moment of inertia of the disc about a vertical axis passing through its centre, $I=\dfrac{M.r^2}{2}=\frac{1}{2}\times100\times(10)^2=5\times10^3$ gm-cm.^{2}

So, angular momentum $=I.\omega=5\times10^3\times\omega$, where ω is its angular velocity.

After the piece of wax is dropped, the total angular momentum of the disc+wax $=I\omega_1+mx^2\omega_1=5\times10^3\times\omega_1+20\times(8)^2\times\omega_1=6{\cdot}28\times10^3\times\omega_1$ [ω_1=present angular velocity]

$$\therefore\ 5\times10^3\times\omega=6{\cdot}28\times10^3\times\omega_1$$

$$\text{or, } \frac{\omega_1}{\omega}=\frac{500}{628} \text{ or, } \frac{2\pi n}{2\pi\times40}=\frac{500}{628} \quad [\text{new rate of rotation}=n \text{ r.p.m.}]$$

$$\therefore\ n=\frac{500\times40}{628}=32 \text{ r.p.m.}$$

6.12. Motion in a circular path : Radial or Normal acceleration :

From the first law of motion, we have come to know that when a particle is set into motion it will continue to move in a straight line if it is not acted on by some external force. So, whenever a particle is found to move in a circular path with uniform speed, the conclusion is that there must be some external force acting on it. Now, for a particle moving in a circular path, the direction of motion at any point is along a tangent drawn to the circle at that point. So, if the external force acts along this tangential direction, the speed of the particle can not remain uniform. It is, therefore, clear that this external force must be acting in a direction which will not affect the uniform speed in the tangential direction. This implies that the force must be acting at every point of the circular path in a direction at right angles to the tangent at that point, *i.e. the force must be a radial one.*

Illustrations of tangential motion at every point of the circular path are not far to seek. It is a common experience that mud-particles are thrown off tangentially from the wheels of a cycle when the cycle moves very rapidly in a muddy road.

The existence of a radial force when a body moves round a circular path is well experienced when one whirls a stone tied to a piece of thread (Fig. 6.7]. The hand automatically exerts a force always on the stone along the string. If the string snaps at some position, the stone, instead of moving in the circular path, shoots out straight along the tangential path.

Since a force acting on a body produces an acceleration, the body moving in a circular path, has also an acceleration. Thus, a particle rotating along the cirumference of a circle, always feels an acceleration directed towards the centre of the circle. This acceleration is called *radial* or *normal acceleration.*

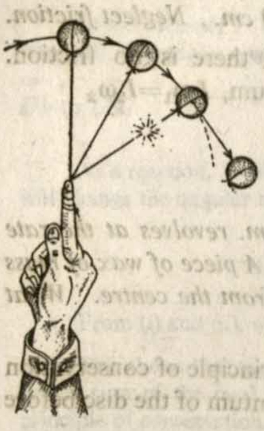


Fig. 6.7

Magnitude of normal acceleration :

Suppose a particle is revolving along a circle of radius r with a linear speed v and moves from A to B in a small interval of time t. [Fig. 6.8]. The angular velocity of the particle is, therefore, $\omega=\frac{\theta}{t}$, where θ is the angular displacement in time t.

The linear speed of the particle at any instant is directed along a tangent drawn to the circle at that instant. So, the linear speed of the particle at A is in the direction of the tangent AP and that at B is in the direction of the tangent BQ. In this case, $AP=BQ=v$, because, the magnitude of the linear speed is always constant.

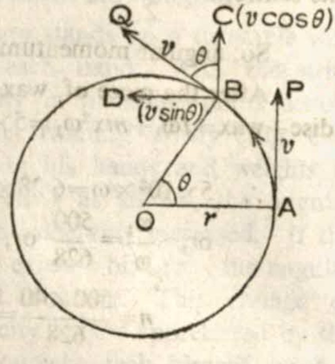


Fig. 6.8

Now, the linear speed at A of the particle is wholly directed along AP ; it has no component in the direction AO, because AO and AP are mutually perpendicular. The linear speed of the particle at B which is acting along BQ, when resolved in a direction parallel to AO, will give a component $BD=v\sin\theta$ and a direction perpendicular to AO will give a component $BC=v.\cos\theta$. If θ be very small, $\sin\theta=\theta$ (in circular measure) and $\cos\theta=1$.

So, the component BD parallel to $AO=v.\sin\theta=v.\theta$.

And the component BC perpendicular to $AO=v.\cos\theta=v$.

So, it is seen that for two very near points A and B, the linear speed of the particle perpendicular to AO remains unchanged while there is a velocity change along AO. Since this change of velocity takes place in time t, the acceleration of the particle $=\frac{v\theta}{t}=v.\omega.=v.\frac{v}{r}=\frac{v^2}{r}$ $\left[\because\ \omega=\frac{\theta}{t} \text{ and } \omega=\frac{v}{r}\right]$

Since, the magnitude of the speed remains same, the acceleration will be everywhere perpendicular to the speed, *i.e.*, the acceleration is directed towards the centre of the circle. Hence, the radial or the normal acceleration, $f_n = \frac{v^2}{r}$.

[This acceleration is sometimes called *centripetal acceleration* as it always acts towards the centre of the circle.]

Again, if *n* be the frequency of the rotating particle,

$$f_n = \frac{v^2}{r} = \omega^2 \times r = 4\pi^2 n^2 r \quad [\text{because } \omega = 2\pi n.]$$

[Note : It is to be noted that in circular motion, the velocity v and the acceleration f_n are always at right angles to each other. But there is no fixed relation, in general, between the velocity and the acceleration. In the parabolic path of a projectile, the velocity and acceleration take up different directions at different times. For a free fall of a body the velocity and the acceleration have the same direction while for the vertically upward motion, the velocity is directed opposite to the acceleration. Only in uniform circular motion, they are at right angles to each other. Further in uniform circular motion, although the particle has an acceleration, it may not have any motion in the direction of the acceleration.]

[Alternative proof by vector :

Suppose a particle is moving in a circular path of radius *r* with uniform velocity *v*. The direction of its velocity $\vec{V_A}$ at *A* is along the tangent *AC* drawn to the circle at *A* ; after a small interval of time δt, when the particle arrives at *B*, its velocity $\vec{V_B}$ is directed along the tangent *BD* drawn to the circle at *B* [Fig. 6.9(a].

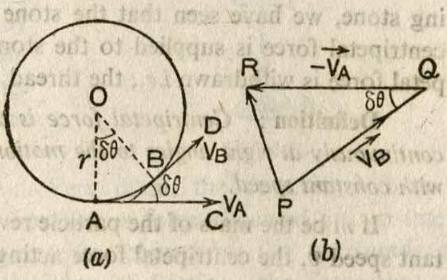


Fig. 6.9

Although $|\vec{V_A}| = |\vec{V_B}| = v$, yet their directions at *A* and *B* are different and hence the particle may be said to have a change of velocity in going from *A* to *B i.e.* it has an acceleration.

Now, in going from *A* to *B*, the change of velocity of the particle $= \vec{V_B} - \vec{V_A} = \vec{V_B} + (-\vec{V_A})$. In fig. 6.9 (b), the line segment *PQ* represents the velocity $\vec{V_B}$ in magnitude and direction and the line segment *QR* represents in the same way, the velocity $(-\vec{V_A})$. Here, the change of velocity $= \vec{V_B} + (-\vec{V_A}) = \vec{PR}$.

$$\text{Acceleration } |\vec{f_n}| = \frac{\text{change of velocity}}{\text{time}} = \frac{|\vec{PR}|}{\delta t} = \frac{v.\delta\theta}{\delta t}$$

$$[\because RP = PQ \times \delta\theta = v\delta\theta]$$

when $\delta t \to 0$, $\frac{\delta\theta}{\delta t} = \frac{d\theta}{dt} = \omega$, the angular velocity,

$$\therefore \; |\vec{f_n}| = v.\omega = v.\frac{v}{r} = \frac{v^2}{r} \quad [\because v = \omega.r]$$

The direction of acceleration may be ascertained in the following way. When $\delta t \to 0$, $\vec{V_B}$ and $\vec{V_A}$ become almost parallel to each other. Then $\vec{PR}$ will be perpendicular to the velocity vector. Hence, the acceleration is perpendicular to the velocity vector. Since the direction of velocity at any point on the circle is along the tangent to the circle at that point, the acceleration will be along the radius through the point towards the centre.]

6.13. Centripetal force :

We have seen in the preceding article, that a rotating body always has an acceleration towards the centre of the circle. According to Newton's second law, a force must be exterted on the body to produce this radial acceleration and the direction of this force must be the same as the direction of the acceleration *i.e., it is a radial force.* This force is known as the *centripetal force* ("Centripetal" means seeking a centre) which is equal to the product of the mass of the body and its centripetal acceleration. In the illustration of the whirling stone, we have seen that the stone moves in the circular path as long as the centripetal force is supplied to the stone by the fingers. As soon as the centripetal force is withdrawn *i.e.*, the thread snaps, the stone flies off tangentially.

Definition : *Centripetal force is defined as that constant force which, acting continuously at right angles to the motion of a particle, causes it to move in a circle with constant speed.*

If m be the mass of the particle revolving in a circle of radius r with a constant speed v, the centripetal force acting on it is given by

$$F = m \times f_n = \frac{mv^2}{r} = 4\pi^2 n^2 m.r.$$

Depending upon the circumstances, the centripetal force may arise from mechanical, gravitational, magnetic or electrical forces. When a stone is whirled at the end of a string, the centripetal force acting on the stone is a mechanical force. The planets moving round the sun, get their centripetal force from the gravitational attraction of the sun. The electrons of an atom revolve round the nucleus due to the centripetal force arising out of the electrical force of attraction between the electrons and the nucleus.

By Newton's third law of motion, centripetal force has a *reaction force.* This oppositely directed force is called the *centrifugal reaction.* Referring to the case of a whirling stone, the centrifugal reaction acts on the finger which holds the string. It should be emphasized, however, that the centrifugal reaction does not act on the stone. One force and only one force acts on the stone and that is centripetal force. If the string snaps at any moment, the stone does not fly off *radially.* ; it flies off *tangentially*, because the centripetal force ceases to act then and the particle in its bid to maintain the straight line motion at every point of the circle, ejects out tangentially.

6.14. Centrifugal force :

Sometimes, the centrifugal reaction is loosely referred to as the centrifugal force. As a matter of fact, centrifugal force arises due to rotational inertia of the body and *it acts radially outwards on the rotating body.* The centrifugal reaction, it may be mentioned, does not act on the rotating body.

From Newton's first law of motion, we know that to overcome the inertia of a body, force is to be applied on it. Similarly, in rotational motion, application of force is necessary to overcome its *directional inertia.* This force is centrifugal force and it is always in dynamical equilibrium with the centripetal force. This is why they are equal in magnitude but opposite in direction.

Centrifugal force can be measured by means of the arrangement shown in the fig. 6.10. The device can be fitted in a hollow rapidly rotating axis. When the frame is rotated, the ball, which is capable of movement along a horizontal rod. *PQ* moves away from the axis towards the end *Q* of the rod being driven by the centrifugal force. In so doing, the ball stretches a thread passing over a pulley and then through the axis to a scale pan *S* ; the tension of the thread is measured by putting weights on the hanging scale pan so that the ball is prevented from moving. The weights placed on the scale pan measure the centrifugal force.

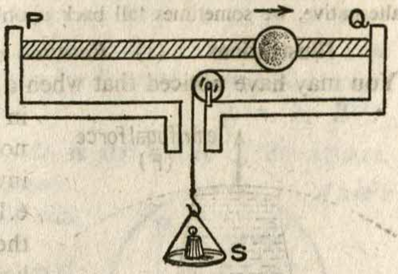


Fig. 6.10

Centrifugal force is a pseudo-force : Very often, the centrifugal force is called a '*pseudo-force*'. A 'real force' is, however, a force obtained due to interaction of bodies and is associable with specific objects in the surroundings. Such forces as the tension in a rope, the force of friction, the force that we exert on a wall by pushing, are all real forces. Centripetal force is in this sense, a real force but centrifugal force is not. This force (centrifugal) is only apparent in a rotating frame of reference ; it has no existence in a stationary frame of reference.

A frame of reference is called *inertial frame* if it is at rest or is moving at a constant velocity with respect to some fixed object. It is a frame of reference defined by Newton's first law. On the other hand, a *non-inertial frame* is one that is rotating (and therefore accelerating) with respect to some fixed object. We generally choose inertial frame in our analysis of motion of bodies. Sometimes, however, it becomes necessary to select a non-inertial frame of reference.

If an observer, remaining in a stationary frame of reference, (*i.e.* inertial) observes another rotating body, he will experience only the centripetal force. But if the observer himself remains in a frame of reference rotating with the same angular velocity as the body, (*i.e.* non-intertial) he will observe the body to be at rest. As an explanation, he will perhaps say that some force must be acting on the body which balances the 'real' centripetal force and keeps the body at rest. This force is the centrifugal force and this is why it is called a 'pseudo-force.'

When an omnibus takes a bend, the passengers inside the bus seem to be thrown outwards. Here, the bus is the non-inertial frame of reference and the force experienced by the passengers inside the bus is the 'pseudo-force'. A person standing on the road-way, which is a stationary frame of reference, will however explain the force as due to rotational interia of the passengers.

As a matter of fact, whenever a body is set to rotate in a circular path but is not supplied with necessary centripetal force, the body exhibits the action of centrifugal force and is radially thrown outwards.

[Note : The above discussion on pseudo-force leads us to belive that in mechanical problems we have two alternatives : (i) choice of an *intertial frame* as a reference frame where only 'real' forces *i.e.* forces which are associable with definite bodies in the surrounding are to be considered or (ii) choice of *non-inertial frame* as a reference frame where not only the real forces but suitably defined pseudo-forces are to be considered. Although we usually choose the first alternative, we sometimes fall back upon the second alternative too.]

Illustrations : (1) *Rotation of a bucket full of water in a vertical plane* : You may have noticed that when a bucket full of water is made to rotate quickly in a vertical plane, the water in the bucket does not fall from the bucket even when the bucket is inverted at the topmost point of the circle [Fig. 6.11]. In this case, the centripetal force acts on the bucket and not on the liquid. But as the liquid rotates along with the bucket, centrifugal force (F) acts on it and presses the liquid against the bottom of the bucket.

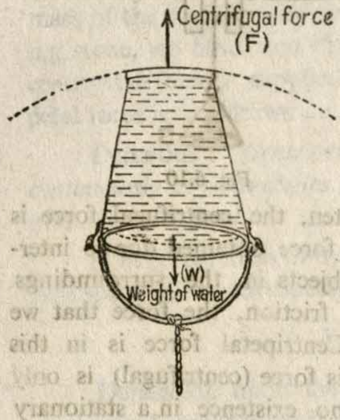


Fig. 6.11

Parts of rotating machineries, which keep some external parts attached to the axis of rotation, always feel a tension due to centrifugal force acting outward. If the centripetal force increases sufficiently, the tension also increases and the parts of machinery may crumble. This is why a limit is always set to the angular velocity of such rotating machines.

(2) On a brass frame a stout brass rod is stretched. Two ivory balls are slid into the rod. The balls can move freely along the rod and they are arranged as shown in the fig. 6.12 The frame is rapidly rotated by means of a whirling table. The balls will be found to strike the ends of the frame, being driven by the centrifugal force.

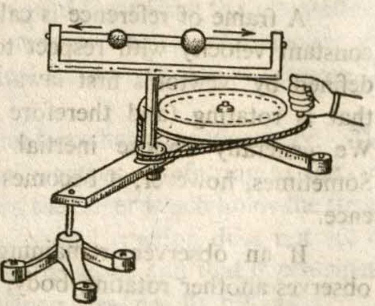


Fig 6.12

(3) In a circus show, a spherical cage is erected for performing a motor-cycle feat of going in a vertical circle. When the rider goes to the topmost position of the cage, he does not fall down. His weight, acting downward, is balanced

by the centrifugal force just as water from the bucket does not fall when the bucket is inverted at the topmost point of the circle.

Examples : (1) *A stone weighing 100 gm, is whirled on the end of a thread 80 cm. long in a circle. If the stone makes 2 complete revolutions in a second, calculate the tension of the string.*

Ans. We know, tension=centripetal force$=\frac{mv^2}{r}$

Here, $m=100$ gm. $r=80$ cm. ; $v=\frac{2\pi r}{t}=\frac{2\times 3{\cdot}14\times 80}{\frac{1}{2}}$

$$\therefore\ \text{tension}=\frac{100}{80}\times\left(\frac{2\times 3{\cdot}14\times 80}{\frac{1}{2}}\right)^2=100\times 80\times(4\times 3{\cdot}14)^2=1262\times 10^3\ \text{dynes.}$$

(2) *Four balls of mass 5 kg each are placed on the top of a horizontal turn-table and fastened together with four strings each 1 metre long to form a square of side 1 metre. The axis of rotation passes through the centre of the square. Find the tension in the strings when the turn-table is rotated at the rate of 30/π revolutions per minute.* [*I. I. T. 1970*]

Ans. A, B, C and D are the four balls. O is the centre of the square, through which the axis of rotation passes. Now, when the turn-table rotates, each ball will experience a centrifugal force$=\frac{mv^2}{r}=m\omega^2.r\ [v=\omega.r]$

where ω is the angular velocity and $r=OA$

Due to this, each string feels a tension T, which in the figure, is given by $T=m\omega.^2r\cos 45°$.

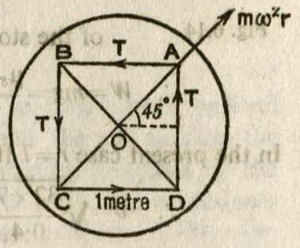


Fig. 6.13

Now, $r=OA=\frac{1}{2}\times\sqrt{(CD)^2+(AD)^2}$

$$=\tfrac{1}{2}\sqrt{(100)^2+(100)^2}=\frac{100}{\sqrt{2}}\ \text{cm. and}\ \omega=2\pi\times\frac{30}{\pi\times 60}\ \text{rad/s}=1\ \text{rad/s.}$$

$$\therefore\ T=5\times 10^3\times(1)^2\times\frac{100}{\sqrt{2}}\times\frac{1}{\sqrt{2}}=2{\cdot}5\times 10^5\ \text{dynes.}$$

(3) *A stone of mass 100 gm. is whirled in a horizontal circle on the end of a string 50 cm. long. The string can withstand a maximum tension of 288 dynes. At what maximum speed can the stone be whirled without snapping the string ? What is the greatest number of revolutions made by the stone in a minute ?*

Ans. Suppose, the maximum speed of the stone$=v$ cm/sec.

The centripetal force of the stone$=\frac{mv^2}{r}=\frac{100\times v^2}{50}=2v^2$ dynes.

Maximum tension=288 dynes. $\therefore\ 2v^2=288$ or $v=12$ cm./sec.

Again, the distance travelled by the stone in one complete revolution$=2\pi r=2\times 3{\cdot}14\times 50=314$ cm.

Maximum velocity attained by the stone$=12$ cm/sec.$=12\times 60$ cm/min.

∴ the greatest number of revolutions done by the stone in 1 minute $= \frac{v}{2\pi r} = \frac{12\times 60}{314} = 2{\cdot}3$ *i.e.*, the greatest number of complete revolutions $=2$.

(4) *A hollow cylinder of radius 7ft is revolving in a vertical plane about its own axis. Both ends of the cylinder are open. A stone is in contact with the wall of the cylinder. What should be the minimum speed of the cylinder so that the stone remains stuck to the cylinder and does not fall down? The coefficient of static friction between the wall and the stone is 0·4.*

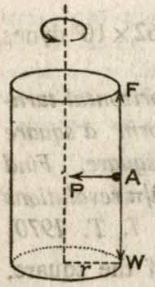


Fig. 6.14

Ans. The forces acting on the stone A are shown in Fig. 6·14. W is the weight of the stone, F is the force of static friction between the stone and the wall and P is the centripetal force exerted by the wall on the stone necessary to keep it moving in a circle. Let the radius of the cylinder be r and v the minimum speed of the cylinder.

Since the stone does not fall vertically but experiences a radial force we have, $W-F=0$, or $W=F=\mu_s.P$ where μ_s is the coefficient of static friction,

Now, the centripetal force $P=\frac{mv^2}{r}$ where m is the mass of the stone.

$$\therefore\ W=mg=\frac{\mu_s.m.v^2}{r} \quad \text{or} \quad v=\sqrt{\frac{gr}{\mu_s}}$$

In the present case $r=7$ ft. $g=32$ ft./s^2 and $\mu_s=0{\cdot}4$.

$$\therefore\ v=\sqrt{\frac{32\times 7}{0{\cdot}4}}=23{\cdot}7 \text{ ft./sec. (nearly)}$$

[*N.B.* Note that the result does not depend on the weight of the stone].

(5) *A road overbridge has the form of a vertical circular arc of radius r ft. What is the greatest speed at which a car can cross the bridge without leaving the ground at the crest of the road? If the radius of the arc is 18 ft., what is the maximum permissible speed of cars negotiating the bridge?*

Ans. At the crest of the road, the acceleration is g towards the centre but the acceleration away from the centre $=\frac{v^2}{r}$, where v is the speed of the car. If $\frac{v^2}{r}\leqq g$, the car will not leave the ground, The necessary condition, therefore, is $\frac{v^2}{r}\leqq g$ or $v^2\leqq gr$ or $v\leqq\sqrt{gr}$ Hence the greatest speed is $v_{max}=\sqrt{gr}$

When $r=18$ ft, $v_{max}=\sqrt{32\times 18}=24$ ft/sec.

(6) *A sphere of mass 200 gm. is attached to an inextensible string of length 130 cm. whose upper end is fixed to a ceiling. The sphere is made to describe a*

horizontal circle of radius 50 cm. Calculate (i) the time-period of one revolution and (ii) the tension in the string. [*I.I.T. 1974*]

Ans. Tension T acts along the thread BA. Resolving the tension horizontally and vertically, we get $T\cos\theta = mg$, the weight of the sphere and $T\sin\theta = \frac{mv^2}{r}$, the centrifugal force on the sphere [Fig. 6.15]. Dividing one by the other, we get, $\tan\theta = \frac{v^2}{r.g}$ or, $\frac{OB}{OA} = \frac{v^2}{r.g}$

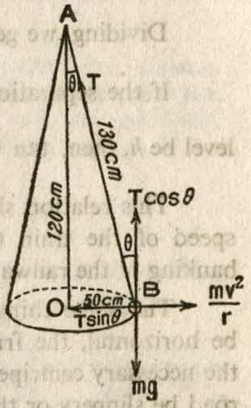


Fig. 6.15

Now, $OA = \sqrt{(AB)^2 - (OB)^2} = \sqrt{(130)^2 - (50)^2}$
$= 120$ cm.

$\therefore \frac{50}{120} = \frac{v^2}{50 \times 980}$ or, $v = \frac{350}{\sqrt{6}}$ cm/s.

(i) Time-period $t = \frac{2\pi r}{v} = \frac{2\pi \times 50 \times \sqrt{6}}{350} = 2{\cdot}2$ sec.

(ii) Tension $T = \frac{mg}{\cos\theta} = \frac{200 \times 980 \times 130}{120} = 2{\cdot}12 \times 10^5$ dynes

6.15. Some illustrations of centripetal force :

(a) *Banking of railway lines* : When a trian tries to take a bend on level railway lines, the force between the rim of the wheel and the rail supplies the ncecessary centripetal force to the trian. The normal reaction produced by the rails is completely used up in overcoming the weight of the train. This produces enough grinding between the rails and the wheels to cause unnecessary damage. There is also a danger of derailment particularly when the speed is high. To remove this difficulty, the outer rail near the bend is placed at a slightly higher level than the inner rail. This is known as the '*banking*' of the railway lines. The angle made by the banked line with the horizontal is called the '*banking angle.*'

In the case of railway lines suitably banked, the reaction R of the rail acts perpendicular to the line AB joining the banked lines and will be inclined to the vertical at the banking angle θ [Fig. 6.16]. On resolving this reaction along horizontal and vertical directions, the horizontal component will provide the necessary centripetal force to the train for its bending. This will obviate unnecessary grinding between the rails and the wheels and hence the consequent damage.

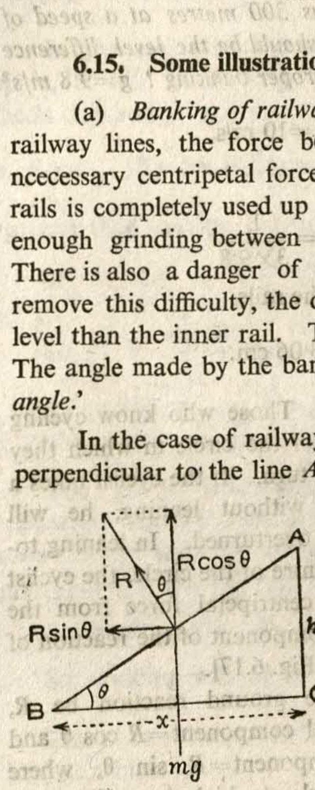


Fig. 6.16

Now, the vertical component of the reaction $= R.\cos\theta$. and the horizontal component $= R.\sin\theta$. The first component neutralises the weight of the train while the latter supplies the centripetal force. If the speed of the train be v and the radius of the bend 'r', then,

$$R.\sin\theta=\frac{mv^2}{r} \text{ and } R.\cos\theta=mg$$

Dividing, we get, $\tan\theta=\frac{v^2}{r.g}$.

If the separation between the lines be x (*i.e.*, $BC=x$) and the difference of level be h, then, $\tan\theta=\frac{h}{x}$ $\therefore$ $\frac{h}{x}=\frac{v^2}{r.g}$ or, $h=\frac{v^2.x}{r.g}$.

This relation shows that the banking of the lines *i.e.*, h depends on (i) the speed of the train (v) and (ii) the radius (r) of the bend of the track. Hence, banking of the railway lines at the bend is guided mainly by the speed of the train.

The same thing happens when an automobile takes a bend. If the road be horizontal, the frictional force between the motor tyre and the road supplies the necessary centripetal force. In such cases, the car may skid or overturn if the road be slippery or the speed is high. To remove this difficulty, the road-surface is inclined to the horizontal at the bend, the slope being towards the centre of the bend.

Example : *A train is rounding a curve of radius 300 metres at a speed of 36 km/hr ; if the separation of the lines be 90 cm, what should be the level difference between the inner and the outer rail in order to secure proper banking ?* $g=9{\cdot}8\ m/s^2$

Ans. Speed of the train $=36$ km./hr. $=\frac{36\times1000}{60\times60}=10$ m/s.

The radius of the curve $=300$ metres.

If θ be the banking angle, $\tan\theta=\frac{v^2}{r.g}=\frac{100}{300\times9{\cdot}8}=\frac{1}{3\times9{\cdot}8}$

Again if h metre be the level difference between the rails,

$\frac{h}{x}=\tan\theta$. or, $\frac{h}{0{\cdot}9}=\frac{1}{3\times9{\cdot}8}$ $\therefore$ $h=\frac{0{\cdot}9}{3\times9{\cdot}8}$ metre $=3{\cdot}06$ cm.

(*b*) *Turning of a bi-cycle on an unbanked curve* : Those who know cycling must have noticed that they lean towards the centre of the circle in which they want to take turn. If the cyclist takes a quick turn without leaning, he will probably be overturned. In leaning towards the centre of the circle, the cyclist derives the centripetal force from the horizontal component of the reaction of the ground [Fig. 6.17].

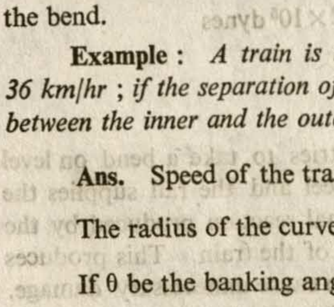


Fig. 6.17

If the ground reaction be R, its horizontal component $=R\cos\theta$ and vertical component $=R\sin\theta$, where θ is the angle at which the cyclist leans towards the horizontal road-way.

Hence, $R. \sin\theta = mg$ and $R. \cos\theta = \frac{mv^2}{r}$ $\therefore$ $\tan\theta = \frac{r.g}{v^2}$

So, if the cyclist wants to take turn with a speed v, he will have to lean towards the road at an angle θ given by the above relation.

[N. B. When the cyclist inclines, friction of the road prevents the cyclist from slipping. if μ be the coefficient of static friction between the tyre and the road, the maximum possible value of $R \cos\theta = \mu.m.g.$ If the cyclist inclines still more he will slip. So, for no slipping,

$\mu\, mg > \frac{mv^2}{r}$ or $\mu > \frac{v^2}{r.g}$ or $v^2 < \mu.g.r$]

Turning of a car on an unbanked curve :

Similar conditions hold good in the case of an automobile or any other vehicle about to take a turn. If we imagine the car to be turning to the right [Fig. 6.18], the forces acting on it are (i) normal reactions of the ground R_1 and R_2 (ii) the frictional forces F_1 and F_2 (iii) the weight mg of the car and (iv) the centripetal force$=\frac{mv^2}{r}$.

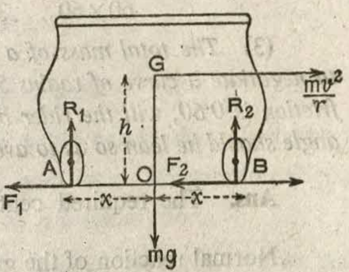


Fig. 6.18

In the event of the car being about to topple, it will be moving on the right hand wheels only so that the normal reactions on the left hand wheels will be zero *i.e.* $R_1=0$. The car will topple over as soon as the moment of $\frac{mv^2}{r}$ about B is greater than the opposite moment of mg about B.

i.e. as soon as $\frac{mv^2}{r} \times GO > mg \times OB$

i.e. when $\frac{v^2 \times h}{r} > g.x$ or, when $v^2 > \frac{g.r.x}{h}$

where h is the height of the C.G. of the car (G) from the ground and $2x$ the distance between the wheels.

The above relation shows that for stability of the car, the distance $(2x)$ between the wheels should be large and the C.G. of the car should be low. It also shows that the safe maximum speed at which the car should travel along an unbanked curve is $v=\sqrt{g.r.x/h}$.

Again, the maximum value of the total frictional force $F_1+F_2=\mu.mg$

So, skidding will occur when $\frac{mv^2}{r} > \mu mg$ or when $v^2 > \mu rg$.

This shows that a car is likely to skid when it takes a sharp turn (where r is less) at a high speed.

Examples : (1) *A cyclist, going 12 km/hr, rounds a curve of 100 cm. radius. At what angle should he lean towards the road?*

Ans. The speed of the cyclist $(v)=\dfrac{12\times1000\times100}{60\times60}=\dfrac{1000}{3}$ cm/sec

Radius of the curve $(r)=100$ cm. If θ be the required angle,

$$\tan\theta=\frac{r.g}{v^2}=\frac{100\times980\times9}{1000\times1000}=\frac{882}{1000}=0{\cdot}882 \quad \therefore\ \theta=42^\circ \text{ (nearly).}$$

(2) *A cyclist, going 12 km/hr. wants to round on the opposite side, If the coefficient of friction between the tyre and road be 0·4, what is the shortest radius of curve along which he can turn round? $g=9{\cdot}8\ m/s^2$.*

Ans. Shortest radius is given by $r_{min}=\dfrac{v^2}{\mu.g}$ [see note art. 6.15]

Now $v=\dfrac{12\times1000}{60\times60}=\dfrac{10}{3}$ m/s ; $\therefore\ r=\dfrac{100}{9\times0{\cdot}4\times9{\cdot}8}=2{\cdot}83$ metre (nearly)

(3) *The total mass of a motor cycle and the rider is 130 kg. The rider has to negotiate a curve of radius 50 metre at a speed of 15 m/s. If the co-efficient of friction is 0·60, will the rider be able to go round the bend successfully? At what angle should he lean so as to avoid toppling? $g=9{\cdot}8\ m/s^2$*

Ans. The required centripetal force$=\dfrac{mv^2}{r}=\dfrac{130\times(15)^2}{50}=585$ N

Normal reaction of the ground=wt of the cycle+ rider$=130\times9{\cdot}8$N

$\therefore$ The maximum force of friction$=\mu\times$normal reaction

$=0{\cdot}60\times130\times9{\cdot}8=764{\cdot}4$N

As the frictional force is greater than the centripetal force, the rider will be able to negotiate the bend. If θ be the angle at which he leans towards the ground, then $\tan\theta=\dfrac{r.g}{v^2}=\dfrac{50\times9{\cdot}8}{(15)^2}=2{\cdot}18$ $\quad\therefore\ \theta=65^\circ\,20'$ (nearly)

(4) *The radius of curvature of a railway line at a place is 240 yards, the distance between the rails being 5 ft. If the C.G. of a train is at a height of 9 ft from the level of the lines, find the safe minimum speed with which the train can take a turn at the curved path. $g=32\ ft/s^2$.*

Ans. The safe speed limit for turning on an unbanked curve, is given by, $v^2=\dfrac{r.g.x}{h}$ Here, $r=240$ yds$=240\times3$ ft; $x=\frac{5}{2}$ ft and $h=9$ ft.

$$\therefore\ v^2=\frac{240\times3\times32\times5}{2\times9}=400\times16 \quad\text{or } v=80 \text{ ft/s.}$$

(5) *A horizontal disc rotating freely about a vertical axis passing through its centre makes 120 r.p.m. Find the maximum distance from the centre at which a small piece of substance may be put so that it may have no motion relative to the disc. The coefficient of friction between the piece and the disc=0·8*

Ans. Due to rotation of the disc, the piece will try to fly off but frictional force will oppose it. It is clear that at the maximum distance, the centrifugal force acting on the piece will be just equal to the frictional force. If r be the maximum distance and v the velocity of the disc, the centrifugal force $=\frac{mv^2}{r}=m.\omega^2 r$ $=m\times\frac{4\pi^2 r}{T^2}$. The force of friction $=\mu.mg$. Hence, $\frac{4\pi^2 r}{T^2}\times m=\mu.m.g$

$$\text{or}\quad r=\frac{\mu.g.T^2}{4\pi^2}=0{\cdot}8\times980\times\frac{(\frac{120}{60})^2}{4\times(3{\cdot}14)^2}=79{\cdot}5\text{ cm (nearly)}$$

(c) The planets move round the sun and the satellites round their respective planets. If their orbits be supposed circular (actually elliptical) the centripetal forces in all these cases are supplied by the gravitational attraction of the sun for the planet or of the planet for its satellite as the case may be.

6·16. Some effects and applications of centrifugal force :

(a) *Loss of weight of terrestrial bodies due to the earth's diurnal motion* : The earth moves round the sun as well as about its own axis. Its rotation about its axis, which produces day and night, is called its diurnal motion.

Suppose there is a body of mass 'm' on the surface of the earth. Its weight mg will act towards the centre of the earth (Fig. 6.19). With the diurnal motion of the earth, the body also rotates about the axis of the earth with the same angular velocity and experiences a centrifugal force ($m\omega^2 r$) due to which the body tends to fly off along the radius of the circle over which the body moves. But a part of its weight always acts in a direction opposite to this force and neutralises it. As a result the body loses some of its weight.

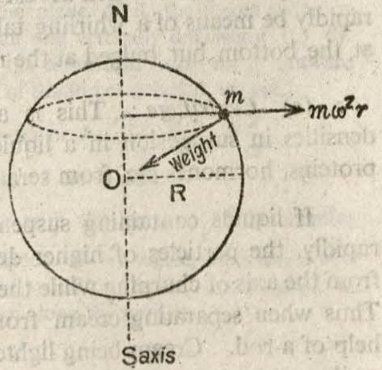


Fig. 6.19

It is easy to understand that the more the body moves towards the equator from the pole, the greater becomes the radius of its circular path and hence greater is the centrifugal force ($m\omega^2 r$) on it. At the equator, the radius (R) is the greatest and hence, the loss of weight is the greatest. On the other hand, the radius is nil at the poles and hence the loss of weight is also nil there.

[N.B. For further discussion, see chapter on 'Gravitation']

(*b*) *Reason for the flattening of the earth at the poles* :

Although the earth is often said to be spherical, it is in reality an oblate spheroid ; that is, a slightly flattened sphere. This flattening is due to the centrifugal force caused by the diurnal motion of the earth.

When the earth was first created, it was believed to have been composed of fused matter. Due to the diurnal motion of the earth, the particles on the surface of the earth felt centrifugal force which was greatest at the equator and nil at the poles. Consequently, the particles at the equatorial region had the greatest tendency to fly off. Although the terrestrial matter was in a fused condition at the first stage, yet there was sufficient cohesion among the particles of the matter. As a result the equatorial region, in its attempt to bulge out due to centrifugal force, pulled the polar regions inwards, which, therefore, got flattened. With the progress of time, the earth crust became hardened and this irregularity in the shape became a permanent feature.

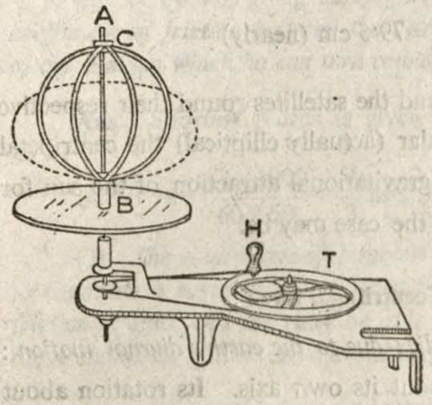


Fig. 6.20

The flattening of the earth at the poles is well illustrated by several circular metal strips forming a model of the earth (Fig. 6.20). The strips are all connected to a spindle *AB* at the bottom and to a movable collar *C* at the top. The collar can move up and down along the axis of the spindle. The strips are round when at rest (shown by full line). But when they are rotated rapidly be means of a whirling table *T*, the rings become flattened at the top and at the bottom but bulged at the middle (shown by broken line).

(*c*) *Centrifuge* : This is a device which separates particles of different densities in suspension in a liquid. It is used for separating cream from milk, proteins, hormones etc. from sera or other liquid media.

If liquids containing suspended particles of different densities be churned rapidly, the particles of higher density than the liquid are found to move away from the axis of churning while the particles of lower density gather round the axis. Thus when separating cream from milk, milk is churned quickly with the help of a rod. Cream being lighter than milk, gathers round the rod whence it is easily removed.

In another machine known as '*centrifugal drying machine*', the same process is adopted for drying wet clothes. The machine consists of a cylinder having perforations on its walls and is capable of rotating rapidly about the axis of the cylinder. Wet clothes are put in the vessel which is set into rapid circular motion. Water particles being driven off by the centrifugal force come out of the perforations and the clothes gradually become dried up.

(*d*) *Speed governor* : The rotational speed of a machine is often controlled by governors. Automobile engines, locomotive engines etc. are provided with speed governors for controlling speed. It was devised by James Watt, the inventor of steam engine. For this reason it is sometimes known as

Watt's speed governor. Its working principle is based on the use of centrifugal force.

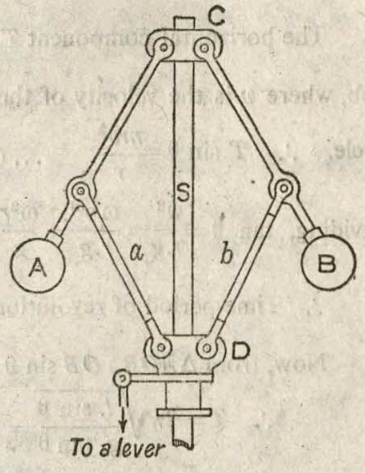


Fig. 6.21

A and *B* represent two heavy metallic spheres supported by rods which are pivoted at both ends [Fig. 6.21. The pivots at *C* are fixed to the central shaft which rotates when the engine moves. The pivots at *D* are attached to a movable collar. As the shaft rotates faster, the centrifugal forces on *A* and *B* cause them to fly outwards and so raise the collar *D*. The collar by its motion operates a lever (or a set of levers) which controls the supply of fuel to the engine.

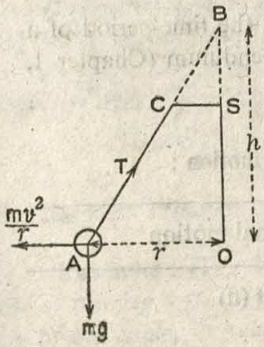


Fig. 6.22

Principle of action : Let the sphere *A* be pivoted at *C* by a hinge [Fig. 6.22]. The line *AC* meets the vertical spindle *OS* at *B*. When the sphere *A* rotates in a circular path, *AB* describes a cone whose apex is *B* and the radius of the base is *AO*. Let *AO* be equal to *r* and the height *OB*=*h*. Now, considering the plane *AOB*, the forces acting on the sphere *A* are (i) its weight *mg* vertically downward (ii) the tension *T* along *AC* and (iii) the centrifugal force mv^2/r outward along *OA*. Taking moments of the forces about *B*, we get,

$$\frac{mv^2}{r}\times h=mg\times r \text{ or } h=\frac{r^2g}{v^2}=\frac{r^2g}{\omega^2r^2}=\frac{g}{\omega^2}$$

where ω is the angular velocity of the spindle.

It is seen that $h\propto\frac{1}{\omega^2}$ *i.e.* with the increase of the angular velocity of the spindle or the speed of the engine, *h* decreases and the collar *D* rises up along the spindle. On the other hand, as the engine slows down, *h* increases *i.e.* the collar comes down along the spindle.

(*e*) *Conical pendulum* :

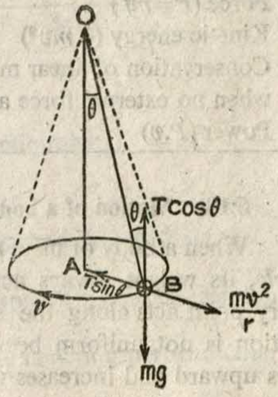


Fig. 6.23

We have already discussed the motion of a whirling stone. The thread attached to the stone, under the action of gravity, describes the curved surface of a cone, instead of moving in a horizontal plane. Such a whirling body is called a *conical pendulum.*

B is the bob of the pendulum describing a horizontal circle with centre at *A*, while the thread of length *l* describes the conical surface whose semi-vertical angle is θ.

Let the tension along the thread *BO* be *T*.

The vertical component of the tension, $T\cos\theta$ balances the weight mg of the bob *i.e.*, $T\cos\theta = m.g.$.... (i)

The horizontal component $T\sin\theta$ balances the centrifugal force $\frac{mv^2}{r}$ of the bob, where v is the velocity of the bob along the circumference of the horizontal circle. $\therefore\ T\sin\theta = \frac{mv^2}{r}$ ... (ii)

Dividing, $\tan\theta = \frac{v^2}{r.g} = \frac{\omega^2 r^2}{r.g} = \frac{\omega^2 r}{g}$, where ω is the angular velocity of the bob.

$\therefore$ Time-period of revolution of the bob, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{r}{g.\tan\theta}}$

Now, from ΔAOB, $OB\sin\theta = AB = r.$ or $l.\sin\theta = r$ $[OB = l]$

$$\therefore\ T = 2\pi\sqrt{\frac{l.\sin\theta}{g.\tan\theta}} = 2\pi\sqrt{\frac{l.\cos\theta}{g}}$$

If θ is very small, $\cos\theta = 1$, So, $T = 2\pi\sqrt{\frac{l}{g}}$

It is to be noted that for small angular displacement the time-period of a conical pendulum is same as the time-period of a simple pendulum (Chapter 1, General properties of matter) of same length.

6·17. Comparison between linear motion and rotational motion :

Linear motion	Rotational motion
1. Displacement (s)	Angular displacement (θ)
2. Velocity (v)	Angular velocity (ω)
3. Acceleration (f)	Angular acceleration (α)
4. Momentum (mv)	Angular momentum ($I.\omega$)
5. Force ($P = mf$)	Torque ($G = I.\alpha$)
6. Kinetic energy ($\frac{1}{2}mv^2$)	Rotational K.E. ($\frac{1}{2}I.\omega^2$)
7. Conservation of linear momentum when no external force acts.	Conservation of angular momentum when no external torque acts.
8. Power ($P.v$)	Power ($G.\omega$)

6·18. Motion of a body in a vertical circle :

When a body of mass m attached at the end of a string is whirled in a *vertical circle*, its weight always acts vertically downwards but the centrifugal force at every point acts along the string directed away from the centre. Its circular motion is not uniform because the magnitude of its velocity decreases when it goes upward and increases when it comes downward. The tension in the string is also not constant ; it is maximum at the bottom being equal to the sum of the weight and the centrifugal force and minimum at the top being equal to the difference between the centrifugal force and the weight [Fig. 6.24].

Let its velocity at the bottom A be v_1 and that at the top B be v_2. Let the tensions at A and B be T_1 and T_2 respectively. Then,

$$T_1 = \frac{mv_1^2}{r} + mg \quad \text{and} \quad T_2 = \frac{mv_2^2}{r} - mg$$

When $T_2 = 0$, *i.e.*, when there is no tension in the string at the top, $\frac{mv_2^2}{r} = mg$ or $v_2^2 = gr$ or, $v_2 = \sqrt{gr}$. Below this *critical velocity* the string becomes slack and the body falls downward instead of going along the circular path.

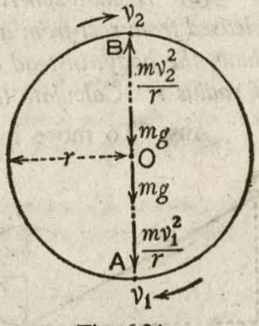


Fig. 6.24

Least velocity necessary to maintain the critical velocity :

In order to maintain the critical velocity at the top, the least velocity of the body at the bottom should be such that its magnitude gradually decreases to the above value v_2 at the top. In rising to the top from the bottom, the body gains a height $2r$ and increases its potential energy by $mg \times 2r$. This gain in potential energy is evidently equal to the loss of kinetic energy due to the decrease in velocity.

$$\text{So, } \tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_2^2 = mg \times 2r \quad \text{or, } \tfrac{1}{2}mv_1^2 = mg \times 2r + \tfrac{1}{2}mv_2^2$$

$$\text{or, } v_1^2 = 4\,gr + v_2^2$$

So, the least value of v_1 is given by $(v_1)_{min} = \sqrt{4gr + gr} = \sqrt{5\,gr}$. [$\therefore\ v_2^2 = g.r.$]

Examples : (1) *A stone of mass 10 lb is revolving in a vertical circle at the end of a string 8 ft. long, the other end of which is fixed. When the stone is at the top of the circle, its velocity is 16 ft/sec. Find the tension in the string when the stone is (i) at the top, (ii) at the bottom (iii) at a level with the centre.* $g = 32 ft/s^2$

Ans. (i) At the top, the tension $T_2 = \frac{mv_2^2}{r} - mg. = \frac{10 \times 16 \times 16}{8} - 10 \times 32 = 0.$

(ii) At the bottom $T_1 = \frac{mv_1^2}{r} + mg.$

Now, when $T_2 = 0$, the value of v_1 at the bottom is minimum.

Hence, $v_1 = \sqrt{5\,gr} = \sqrt{5 \times 32 \times 8} = \sqrt{1280}$

$$\therefore\ T_1 = \frac{10 \times 1280}{8} + 10 \times 32 = 1600 + 320 = 1920 \text{ poundals} = 60 \text{ lb wt.}$$

(iii) If at the level of the centre, the velocity of the stone $= v$, then the tension T there is given by $T = \frac{mv^2}{r}$ because the wt. of the stone is acting perpendicular to the tension and has, therefore, no component along the direction of the thread.

Now, total K.E. of the stone at the centre level
$=K.E.$ at the top$+P.E.$ due to a fall through 8 ft.
$=\frac{1}{2}mv_2{}^2+mgh=\frac{1}{2}\times10\times16\times16+10\times32\times8=3840$ ft. poundals.
Since the velocity of the stone at the centre-level is supposed to be v, we have

$$\tfrac{1}{2}mv^2=3840 \text{ or, } v^2=\frac{3840\times2}{10}=768.$$

$$\therefore \text{ Tension } T=\frac{mv^2}{r}=\frac{10\times768}{8}=960 \text{ poundals}=30 \text{ lb wt.}$$

(2) *A small spherical body slides down a frictionless inclined plane after being released from rest from a point at a height h. Arriving at the bottom of the inclined plane, the body, instead of moving in a horizontal track, describes a vertical circle of radius r. Calculate the minimum value of h for which such motion is possible.*

Ans. To move in a vertical circle, the minimum velocity of the body at the bottom of the circle A is given by [Fig. 6.25] $v=\sqrt{5.g.r}$. So, K.E. of the body at $A=\frac{1}{2}mv^2=\frac{1}{2}m\times5\ g.r.$ The body will gain this K.E. in exchange of the loss of P.E. due to a fall through h.

$$\text{So, } m.g.h=\tfrac{1}{2}\ m\times5.g.r \text{ or } h=\frac{5.g.r.m}{2\ m.g}=\frac{5}{2}r.$$

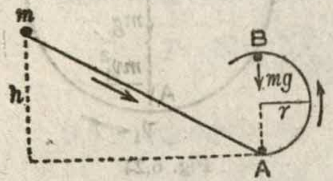


Fig. 6.25

(3) *A nail is located at a certain distance vertically below the point of suspension of a simple pendulum. The pendulum bob is released from a position where the string makes an angle of 60° with the vertical. Calculate the distance of the nail from the point of suspension such that the bob may perform revolutions with the nail as centre. Assume the length of the pendulum to be 1 metre.* [*I.I.T. 1975*]

Ans. Let N be the position of the nail. To describe a vertical circle, say of radius r around the nail, the minimum velocity of the bob at the bottom-most position *i.e.* at O_1 should be $\sqrt{5\ g.r}$ *i.e.*, K.E. of the bob at $O_1=\frac{1}{2}m\times5\ gr$.

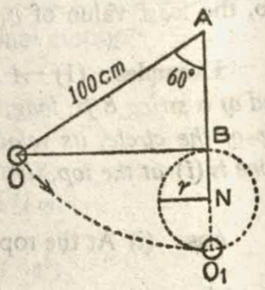


Fig. 6.26

This K.E. the bob would derive as a result of fall from O to O_1.

Now, loss of P.E. of the bob$=mg\ (AO_1-AB)$

$$=mg\ (l-l.\cos60^\circ)=l.m.g\ (1-\cos60^\circ)$$

$$=100\times m\times g\ (1-\tfrac{1}{2}) \qquad [l=100 \text{ cm}]$$

$$=50.g\times m.$$

$$\therefore \tfrac{1}{2}m\times5gr=50\times g\times m \text{ or, } r=20 \text{ cm.}$$

$\therefore$ Distance of the nail from the point of suspension $=AO_1-r=100-20$ $=80$ cm.

Exercises

Essay type :

1. Define 'angular velocity', 'angular acceleration'. and torque. Find a relation between torque and angular acceleration. *[H. S. Exam. 1979]*

Establish relations between (i) angular velocity and linear velocity and (ii) angular acceleration and linear acceleration.

2. Establish the relations between (i) angular acceleration and torque (ii) angular momentum and torque. What is angular impulse?

3. State the principle of conservation of angular momentum. . Cite an example in its support. *[H. S. Exam. 1979]*

4. Explain what you understand by centripetal and centrifugal forces. Establish an expression for centripetal force. *[H. S. Exam. 1978]*

Why is centripetal force called 'a real force' and centrifugal force 'a pseudo force?

5. Discuss briefly the forces that act upon a body which rotates uniformly in a circular path. Deduce expressions for the force. Prove the existence of the forces, taking some illustrations from everyday life.

6. Why does a cyclist lean towards the centre of the circle along which he intends to take a sharp and speedy turn? Establish a relation between the radius of the circle, the speed of the cyclist and the angle of inclination.

7. Show that the minimum horizontal velocity V with which a particle, hanging freely at the end of a string of length l must be projected so as to describe completely a vertical circle is given by $V=\sqrt{5.g.l.}$, where 'g' is the acceleration due to gravity.

8. What is a speed governor? Explain the working of a governor devised by Watt.

Short answer type :

9. (i) What is a radian ? Why is radian measure of angle satisfactory for all systems of units ?

(ii) A particle of mass m is revolving in a circular path of radius r with uniform angular velocity w. Write the magnitude of its linear velocity, linear acceleration and angular momentum. Show the directions of linear velocity and linear acceleration in a diagram. *[Jt. Entrance 1982]*

10. A wheel is rotating about an axis through its centre perpendicular to the plane of the wheel. Consider a point on the rim. When the wheel rotates with *constant angular velocity* does the point have a radial acceleration or a tangential acceleration? When the wheel rotates with *constant angular acceleration* does the point have radial acceleration or a tangential acceleration? Do the magnitudes of these accelerations change?

11. When a torque is applied on a rigid body free to rotate about an axis, the angular acceleration produced depends not only upon the size and the shape of the body but also upon the distribution of its mass with respect to the axis of rotation. Explain.

12. Mention the nature of the forces which give rise to centripetal force in the following cases :—

(i) An artificial satellite circling round the earth (ii) Banking in a curved railway line (iii) Rotation of a stone tied to a thread in a horizontal plane (iv) Revolution of electron in an atom (v) The motion of the sonometer wire when its mid-point is plucked.

13. (i) A man stands on a rotating turntable with his arms outstreched. What will happen when the man draws his hands towards his chest ? (ii) A man turns on a rotating table. He is holding two equal masses at arm's length. Without moving his arms, he drops the two masses. What change, if any, is there in his angular speed ?

14. Why does a diver, diving from a high platform, get a good number of somersaults if he curbs his body more ?

15. Will the angular velocity of earth decrease if a heavy meteor hits the earth from the sky ?

[**Hints :** Before striking the earth, the angular momentum of the meteor about the axis of the earth is zero. As action and reaction are equal, no external torque acts on the earth when a meteor falls on the earth. So, the total angular momentum of the earth and the meteor after the meteor strikes the earth=angular momentum of the earth alone before the striking. Now, the mass of the earth increases a little due to the addition of the meteor. Hence, the angular momentum of the earth decreases a little].

16. You are walking along a narrow railroad track and you are about to lose your balance. If you start falling to the left, which way do you turn your body to regain balance ?

17. To maintain a body in uniform circular motion, a force directed towards the centre is to be always applied on the body. Explain. What is that force called ?

18. When a heavy piece of metal is tied at the end of a string and revolved with a large enough speed, the string snaps. Why does it happen so ?

19. When an omnibus takes a sharp bend, the passengers inside the bus seem to be thrown outwards. How is it explained by a passenger in the bus and a person standing on the road-way ?

20. Is it possible to rotate a bucket of milk in a vertical plane without spilling the milk ?

21. Rotating machineries which keep some external parts attached to the axis of rotation have always a limit set to its angular velocity. Why ?

22. Even if the earth were perfectly spherical in shape the weight of a body would vary with latitude. Why? [*Jt. Entrance 1984*]

[**Hints :** Weight varies due to rotation of the earth about its own axis].

23. A stone tied at one end of a rope is whirled in a vertical circle. At what positions on the circle is the tension in the rope (i) greatest and (ii) least ?

24. (a) When a particle moves in a circular path with uniform speed, does its acceleration remain unaltered ?

(b) Why does a small chain set rotating at a high speed roll along a floor as though it were a rigid metal hoop ?

(c) A coin is put on a gramophone disc. The disc starts rotating but before the final speed is reached, the coin flies off. Why ?

(d) The driver of a truck travelling with a velocity v suddenly notices a wall in front of him at a distance r. Is it better for him to apply brakes or to make a circular turn without applying brakes in order to just avoid crashing into the wall? Why ? [*I.I.T. 1977*]

Objective type :

25. Pick out the correct answer in the following questions :—

(a) Which force provides the centripetal force in the case of rotation of an electron round the nucleus of an atom ? *Ans.* (i) gravitational (ii) elastic (iii) electrical (iv) mechanical.

(b) When a particle is made to rotate with uniform velocity in a circular path, in which direction is its acceleration directed ? *Ans.* (i) Towards the centre (ii) Away from the centre (iii) along the tangent to the circle.

(c) Why does cream separate out from the milk when the milk is churned ?

Ans. (i) Due to cohesive forces (ii) Due to centrifugal forces (iii) Due to frictional forces.

(d) A stone, tied at one end of a rope, is whirled along a horizontal circle with great speed. Suddenly the rope breaks. In which direction will the stone fly off ?

Ans. (i) away from the centre (ii) along a curved path (iii) along a tangential direction.

(e) Does the acceleration of a body remain constant when it revolves along a circular path with uniform angular velocity ? *Ans.* (i) yes (ii) no (iii) remains constant in magnitude but varies in direction.

(f) A mass M is moving with a constant velocity parallel to X-axis. Its angular momentum with respect to the origin is (i) zero (ii) remains constant (iii) goes on increasing (iv) goes on decreasing. [*I.I.T. 1985*]

Numerical Problems :

26. A wheel has a constant angular acceleration of 3 rad/s^2. In a 4 sec interval, it turns through an angle of 120 radians. Assuming the wheel started from rest, how long had it been in motion at the start of this 4 sec interval? [Ans. 8 sec.]

27. (i) A 150 ft. long railway line has a curve of 10° at a point. Find the radius of curvature of the curve supposed to be along the arc of a circle. [*H. S.* (*Comp*), *1963*] [*A*ns. 859 ft. (nearly)]

(ii) What angle in radians in subtended by an arc 6 ft. long on the circumference of a circle whose radius is 4 ft. [Ans. 1·5 rad]

28. (i) The minute hand of a large clock is 50 cm. long. What is its linear speed ? [Ans. ·087 cm/sec]

(ii) In an electric clock, the extremity of hour hand moves $\frac{1}{20}$th as fast as the minute hand. What is the length of the hour hand if the minute hand is 10 cm long? [Ans. 3.3 cm]

29. A stone, tied to the end of a string 200 cm. long, is whirled in a circle 50 times in every 10 seconds. Find the angular velocity and the linear speed of the stone. [Ans. 31·4 rad/sec ; 6280 cm/sec]

30. An automobile engine is revolving uniformly at 150 *r.p.m.* Being accelerated it suddenly increases its speed to 3000 *r.p.m.* What is the angular acceleration of the engine ? What will be its angular velocity 2 seconds later ? What total angle will it rotate through in 3 seconds ? [Ans. (i) 99·5 rad/sec^2. (ii) 215 rad/sec. (iii) 495 radians]

31. An atom of mass $8{\cdot}3\times10^{-22}$ gm. is rotating in a circular orbit of radius 4×10^{-8} cm. with a speed of 6×10^{10} *r.p.m.* What is its angular momentum ? [Ans. $8{\cdot}34\times10^{-27}$ units]

[Hints : Angular momentum$=m.r^2\omega$]

32. A stone, being tied to a string, is whirled along a horizontal circle with a uniform speed. If the radius of the circular path be 40 cm. and the speed of the stone 160 cm/sec., calculate the radial acceleration. [Ans. 640 cm/sec^2]

33. A body of mass 2 gm., is set to rotate in a circular path by means of a string 100 cm. long. If it makes 3 complete revolutions in 1·2 sec. find the tension of the string. [Ans. 50·35 gm. wt. (nearly)]

34. A body of mass 150 gm tied a piece of thread 20 cm long, is rotated about a pin 120 times a minute. Find the tension in the thread and the angle made by the thread with the vertical. [Ans. $4{\cdot}74\times10^5$ dynes ; 71°54′]

[Hints : If θ be the required inclination, $T\cos\theta=mg$ and $T\sin\theta=$centripetal force$=m\omega^2.r$ but $r=l\sin\theta$ $\therefore$ $T\sin\theta=m\omega^2 l\sin\theta$ or $T=m\omega^2 l$; also $\cos\theta=g/\omega^2.l.$ Here $\omega=2\pi n=2\pi\left(\frac{120}{60}\right)$].

35. A cyclist, running at 20 miles/hr speed took a turn along a curved path of radius 44 ft. What was his inclination with the vertical ? [Ans. 31°18]

36. The curvature of a railway line has a radius 150 ft and the distance between the two lines is 5 ft. If the outer line is 5 inches higher than the inner, at what maximum speed can an engine bend round the curve without exerting lateral pressure on the lines ? [Ans. 13·63 miles/hr.]

37. The two sets of wheels of a carriage are 54 inches apart and its centre of gravity is at a height of 18 inches from the ground. The carriage intends to negotiate a bend of radius 200 ft on a horizontal plane. What can be the maximum speed of the carriage ? [Ans. 98 ft/sec]

38. What is the highest speed in miles/hr. at which a 5000 lb automobile can travel around a curve of 100 ft. radius, if the road is level? Coefficient of friction between the tyres and the road=0·25. [*Jt. Entrance 1981*] [Ans. 19·7 m/hr]

39. A bucket-full of milk is being rotated in a vertical circle of 66 cm. radius. What should be the minimum speed of rotation so that the milk is not spilled even when the bucket gets inverted ? $g=980$ cm/sec^2. [Ans. 252 cm/sec (nearly)]

$\left[\textbf{Hints :}\ \frac{v^2}{r}=g\right]$

40. An automobile of mass 9600 kg. running at a speed of 100 km/hr. takes a bend of radius 150 metres. What should be the banking of the road so that the car does not skid ?

[Ans. 27·7°]

41. A cyclist, going 5 metres/s speed turns round a bend of radius 2 metre. If the weight of the cyclist be 50 kg. what is the centrifugal force acting on the cyclist ? How many time is it than his own weight ? [Ans. $6{\cdot}25\times10^7$ dynes, 1·27]

42. A cyclist is describing a circle of 20 *m* radius at a speed of 18 km/hr. What is his inclination to the vertical ? Assume the rider and the cycle in the same plane.

[*H. S. Exam 1978*] [Ans. 7°16′]

43. A cyclist, inclined at an angle θ with the ground, takes a bend of radius R at a speed V. Show that $\tan\theta=\frac{gR}{V^2}$ where g=accⁿ due to gravity. [*Tripura H. S. 1979*]

44. A car moves in a curve of radius of curvature R. The width between the wheels is x and the height of C.G. from the ground is h. With what velocity must the car move in order that the vertical force on the inside wheels shall be reduced to zero ?

[Ans. $v=\sqrt{\frac{R.g.x}{2h}}$]

45. A rail wagon has its c.g. at a height of 3 ft. above the rails which are 4 ft. apart. What would be the safe maximum speed at which it would travel an unbanked curve of radius 108 ft ? $g=32$ ft/s².

[Ans. 48 ft/s]

46. An artificial satellite, circling the earth at a very close distance takes 90 minutes to make one complete revolution. What is its acceleration ? Radius of the earth$=6{\cdot}4\times10^6$ km.

[Ans. 8·67 km/s²]

47. Calculate the increase in length of an elastic string of original length 10 ft at the end of which a stone of mass 0·5 lb is whirled at the rate of 4 rev/sec. if a load of 25 lb increases the length of the string by 2%. [Ans. 0·86 ft (nearly)]

48. A boy is sitting on the horizontal platform of a joy wheel at a distance of 15 ft from the centre. The joy wheel begins to rotate and when the angular speed exceeds 10 rev/minute, the boy just slips. What is the coefficient of friction between the boy and the platform ? $g=32$ ft/sec².

[Ans. 0·51]

49. A stone of mass 500 gm is to be rotated in a vertical circle with the help of a thread 50 cm long. The thread can withstand a tension of 20 N. When the stone is whirled with gradually increasing speed, it snapped at some instant. At what point of the circle can it happen ? What is the angular velocity of the stone then ?

[Ans. At the lowest point of the circle ; 7·7 rad/s]

50. A horizontal disc is revolving about a vertical axis at 100 r.p.m. rate. A small piece of wax falls from a height and gets stuck to the disc at a distance of 9 cm from the axis. If the mass of the piece be 10 gm and the moment of inertia of the disc $7{\cdot}3\times10^3$ gm-cm², what is the present angular velocity of the disc ?

[Ans. 90 r.p.m.]

51. A small body is kept at a distance of 7 cm from the centre of a gramophone disc. The disc started rotating with gradually increasing speed and the body is just on the point of being thrown off when the disc rotates 60 times a minute. What will be the speed of the disc when the body, kept at a distance of 12 cm from the centre, is just being thrown off ? What will be the rotational speed when a body of double the mass is kept at the previous position of 7 cm away from the centre ? [Ans. 46 *r.p.m.* ; 60 *r.p.m.*]

52. A spherical cage of radius 5 metre is erected for performing a motor cycle feat of going in a vertical circle. What should be the minimum speed of the rider at the highest point of the cage so that he may not lose contact with the caga ? [Ans. 70 cm/s]

Harder Problems :

53. An arched bridge is to be so designed that cars may pass over the bridge safely at speeds upto 180 km/hr without jumping off the road. What should be the minimum or maximum value of the radius of curvature of the arch ? [*Joint Entrance 1975*] [Ans. 255 metres/(nearly)]

54. If a body revolves 42 times in a minute along the circumference of a circle, what will be its angular velocity ? If the mass of the body be 50 gm and the radius of the circular path 20 cm. what are the normal acceleration and centripetal force of the body ?
[Ans. 4·4 rad ; 387·2 cm/s^2 ; 19360 dynes]

55. If λ be the latitude of a place on the surface of the earth, the distance of the place from the axis of rotation of the earth is $R \cos \lambda$, where R is the radius of the earth. At the equator $\lambda=0$ and at Belgrade $\lambda=45°$; what is the linear velocity with which a person rotates at (i) equator and (ii) Belgrade due to the rotation of the earth ? $R=6400$ km.
[Ans. (i) 0·46 km/s (ii) 0·32 km/s]

56. A particle of mass 1 gm is circling a path of 10 cm radius. What is the force acting on the particle, if the time for one complete revolution is π second ? [Ans. 40 dynes]

57. In a machine, a horizontal arm 2 ft long is driven by a vertical shaft and makes 600 *r.p.m.* in a horizontal plane. A 10 lb block is attached to the free end of the arm. Find out the force on the block ? [*Joint Entrance 1973*] [Ans. 78960 poundals]

58. A small ball of mass 500 gm is suspended by a light thread 200 cm long from a fixed point. The ball is revolving in a horizontal plane along a circular path of radius 100 cm and the thread is generating a cone. Find the tension in the thread and the number of revolutions executed by the ball in a minute. [Ans. 19 (nearly) ; 0·58 kg-wt]

59. A mass of 60 gm is attached to the lower end of a string 4 metres long. It revolves along a circular path at a uniform rate in a horizontal plane 3·2 metres vertically below the point of attachment of the upper end of the string. Calculate the tension in the string and the time taken by the mass to describe one complete circle.
[*Jt. Entrance 1981*] [Ans. $7{\cdot}35\times10^4$ dynes; 3·59 sec]

60. (a) A particle of mass 10 gm is suspended from the end of a weightless string of length 100 cm and is allowed to swing in a vertical plane. The speed of the mass is 200 cm/s, when the string makes an angle of 60° with the vertical. Find the speed of the particle when it is at the lowest position. $g=980$ cm/s^2. [*I.I.T. 1971*] [Ans 371·5 cm/s]

(b) A light rigid rod of length l has a mass m attached to its end, forming a simple pendulum. It is inverted and then released. What is the speed at the lowest point and what is the tension in the suspension at that instant? [Ans. $2\sqrt{gl}$; $5\,mg$]

61. A mass m on a frictionless table is attached to a hanging mass M by a thread through a hole in the table. Find the condition (v and r) with which m must spin to keep M at rest.

$$\left[\text{Ans. } \frac{v^2}{r}=\frac{Mg}{m}\right]$$

62. A string of length 1 metre is fixed at one end and carries a mass of 100 gm at the other end. The string makes $\frac{2}{\pi}$ revolutions per sec. around a vertical axis through the fixed end. Calculate (i) the angle of inclination of the string with the vertical and (ii) the linear velocity of the mass. [*I.I.T. 1976*] [Ans. (i) 52°12′ (ii) 3·16m/s]

[Hints : $\omega=\frac{2}{\pi}\text{rev/s}=\frac{2}{\pi}\times 2\pi=4\text{ rad/s}$]

63. A large mass M and a small mass m hang at the two ends of a string that passes through a smooth tube as shown in the fig 6.27. The mass m moves around a circular path which lies in a horizontal plane. The length of the string from m to the top of the tube is l and θ is the angle this length makes with the vertical. What should be the frequency of rotation of the mass m so that the mass M remains stationary ?

[*I.I.T. 1978*] [Ans. $\frac{1}{2\pi}\sqrt{Mg/ml}$]

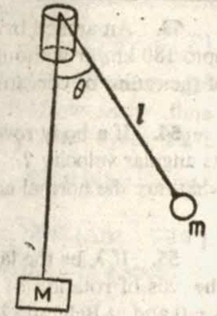


Fig. 6.27

64. A mass m on a frictionless table is connected to a hanging mass M ($M>m$) by a string which passes through a hole on the table. If the mass m rotates with a speed v along a circle of radius r on the horizontal table such that the mass M is at rest, show that $v=\sqrt{\frac{Mgr}{m}}$.

65. A particle is tied at the end of a string and is made to revolve at a constant speed in a horizontal plane The maximum tension that can be offered by the string is 50 poundal. What should be the value of the maximum velocity with which the particle can revolve in a circle of radius 8 ft ? The mass of the particle is 4 lb. [Ans. 10 ft/s]

66. The moment of inertia of a solid fly wheel about its axis is 0·1 kg-m^2. It is set in rotation by applying a tangential force of 20 N with a rope wound round the circumference, the radius of the wheel being 10 cm. Calculate the angular acceleration of the flywheel. What would be the acceleration if a mass of 2 kg were hung from the end of the rope ? $g=10$ m/s^2.

[Ans. 20 rad/s^2 ; 16·7 rad/s^2]

67. A small body of mass m rests at the top of a smooth hemisphere of radius r. It is displaced so that it slides over the hemisphere. Show that the body will lose contact with the hemisphere when it descends vertically through a height $\frac{r}{3}$ from the top.

68. The bob of a simple pendulum has mass m gm. It is pulled from its position of equilibrium through an angle θ and then released. Show that the tension of the thread when the bob passes through the position of equilibrium is given by $mg(3-2\cos\theta)$.

7

WORK, POWER AND ENERGY

7.1. Work : We frequently come across examples of work in our everyday life. When a man carries a luggage or a horse pulls a carriage or a labourer draws water from a well, each of them does some work. Work is a term commonly used by the layman to describe the expenditure of one's stored-up body energy. In science it has a more precise and exact definition.

Definition : *Work is said to be done only when a force succeeds in moving the body it acts upon and is measured by the amount of the force multiplied by the distance moved in the direction of the force.*

Suppose, a force $\vec{F}$ acting on a body at A in the direction AC moves the body from A to B [Fig. 7.1]. If the displacement of the body be $\vec{S}$ [*i.e.*, $AB=S$], then the work done $W=\vec{F}.\vec{S}$.

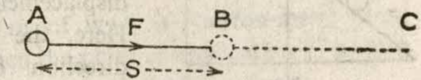


Fig. 7.1

Although the force F and the displacement S are both vector quantities, yet work which is the product of the two, is not a vector quantity ; it is a scalar quantity.

From the above definition, it is clear that however great the force may be, no work will be done unless the point of application of the force moves. For example, if you want to move a big block of stone by applying as much force as you can, you will do no work unless you move the block.

It is to be noted that the displacement of point of application of a force may be different if viewed from different frame of reference. Hence work done by a force on a body under same conditions may appear different from different frames of reference. As an illustration, suppose, a sweeper is pushing a wheel-barrow through a distance S on a railway platform. The work done by him is $F.S.$ where F is the force applied by him. If a train moves through a distance S' in the same time in the same direction, the work done by the sweeper will appear to be $F.(S-S')$ to a passenger in the train. If $S=S'$, the passenger will think that no work is done by the sweeper. Again suppose, a coolie is shifting a luggage in a running train through 50 ft while the train itself advances 200 ft in the same time. In measuring the work done by the coolie, the distance taken should be 50 ft. and not 250 ft.

Work done by or against a force :

(i) If the point of application of a force moves in the direction of the force, work is said to be done by the force [Fig. 7.1]. For example if a body be allowed to fall from a certain height, the body being attracted by the force of gravity, will fall towards the ground. Here, the body moves in the direction of the force and therefore, it may be said that the force of gravity has done the work. In all such cases, *the work done is reckoned positive.*

(ii) If, on the other hand, the point of application of the force moves in a direction opposite to that of the force, the work is said to be done against the force. In fig. 7.1 if the direction of the force be $\overrightarrow{CA}$ but the displacement of the body be from A to B, then $W=-\vec{F}.\vec{S}$; In this case, *the work done is reckoned negative.*

For example, when a body is raised to a certain height against gravity, work is done against the force of gravity.

(iii) A third case may arise, where the point of application of the force moves in a direction other than the direction of the force. For example, when a body rolls down an inclined plane, the displacement takes place along the inclined plane but the gravitational force (F) which causes the displacement, acts vertically downwards [Fig. 7.2]. Here, the effective component of the force along the direction of the displacement $= F \times \cos\ \theta$ and the work done $= F.\ \cos\ \theta.\ S$ where $\theta =$ the angle between the directions of the force and the displacement...(i)

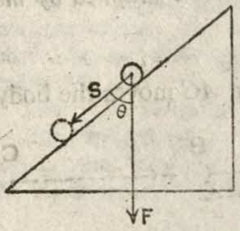


Fig. 7.2

Notice that we can write the above equation either as $(F \cos \theta).\ S$ or $F.(S \cos \theta)$. This suggests that work can be calculated in two different ways: (i) either we multiply the magnitude of displacement by the component of the force in the direction of the displacement or (ii) we multiply the magnitude of the force by the component of the displacement in the direction of the force.

Suppose, a cart on a road is pulled by a rope [Fig. 7.3]. Here the force is inclined to the horizontal direction, while the point of application of the force moves along the horizontal direction. The work done, in this case may be obtained by the above equations.

Using the concept of scalar product of two vectors (art. 2.12), we may write, $W=\vec{F}.\vec{S}$.

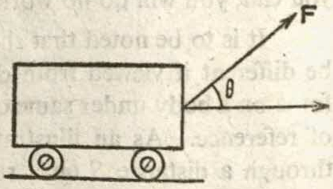


Fig. 7.3

No work forces :

If $\theta=90°$, then from eqn. (i) above, $\cos \theta=0$ and the work done$=0$; thus, when a body is moved along a horizontal plane, no work is done against the gravity (*i.e.*, against the weight of the body) because, the directions of the displacement and the gravitational force are, in this case, mutually perpendicular.

Similarly when a body is whirled along a circle, the centripetal force is directed towards the centre of the circle. At any point, this force is at right angles to the tangent drawn from the point to the circle *i.e.*, to the direction of motion of the body. Since the component of the centripetal force at right angles to itself and thus along the direction of motion is zero, no work will be done on the body. Similarly, in the case of the earth going round the sun, no work is done by the centripetal force exerted by the sun since the motion of the earth is always at right angles to the direction of the force.

Such forces which are perpendicular to the direction of motion and so do not accomplish any work, are called '*No-work forces.*'

It is to be remembered that when a force does some work, it does work against some other force. As for example, when a body is raised, work is done against the force of gravity ; when it is allowed to fall, force of gravity does work against the force of inertia of the body ; when a body is drawn along a rough surface, work is done against the force of friction etc.

7.2. Different units of work :

(i) *Absolute units* : In the *C.G.S.* system, the absolute unit of work is **erg.** It is equal to the work done when the point application of a force of 1 dyne is displaced through 1 cm. in the direction of the force.

In the *F. P. S.* system, the absolute unit of work is **foot-poundal.** It is equal to the work done when the point of application of a force of 1 poundal is displaced through 1 ft. in the direction of the force.

In the *M. K. S.* system, the absolute unit of work is Newton-metre (*n-m*) or **Joule** (*J*). It is equal to the work done when the point of application of a force of 1 newton is displaced through 1 metre in the direction of the force.

$$1\ J = 1\ \text{Newton} \times 1\ \text{metre}$$
$$= (10^5\ \text{dynes}) \times (10^2\ \text{cm}) = 10^7\ \text{ergs}.$$

(ii) *Gravitational units* : In the *C. G. S.* system, the gravitational unit of work is **gramme-centimetre.** It is equal to the work done in raising a body of mass 1 gm. vertically through 1 cm. against the force of gravity.

$$1\ gm\text{-}centimetre = g\ ergs = 980\ ergs.$$

In the *F. P. S.* system, the gravitational unit of work is **foot-pound.** It is equal to the work done in raising a body of mass 1 lb vertically through 1 ft. against the force of gravity.

$$1\ ft\text{-}lb = g\ ft\text{-}poundals = 32\ ft\text{-}poundals$$

In *M.K.S.* unit, the gravitational unit of work is **kilogram-metre.** It is equal to the work done in raising a body of mass 1 kg vertically through 1 metre against the force of gravity.

$$1\ kg\text{-}metre = 9{\cdot}8\ Newton\text{-}metre = 9{\cdot}8\ joule.$$

(iii) *Practical units* : The unit of work 'erg' is used almost everywhere. But sometimes a bigger unit becomes necessary in practical cases and it is known as the practical unit of work. It is called **Joule.**

$$1\ Joule = 10^7\ ergs.$$

Joule is also the practical unit of work in *M.K.S.* system.

7.3. Relation between ft-poundal and erg :

We know, 1 ft-poundal = 1 poundal × 1 ft.

Now, 1 poundal = 13825 dynes and 1 ft = 30·48 cm.

So, 1 ft-poundal = 13825 × 30·48 ergs.

$$= 4{\cdot}214 \times 10^5\ \text{ergs (nearly)}$$

Further, 1 ft-lb $=32$ ft-poundals.

$=32\times4{\cdot}214\times10^{5}$ ergs $=1{\cdot}35\times10^{7}$ ergs (nearly)

$=1{\cdot}35$ joules (nearly)

Examples : (1) *How much work is done in raising 1 cwt. of coal from a mine 300 ft. deep? Express your answer in gravitational and absolute units.*

Ans. (a) *Gravitational unit* : Applied force $=112$ lb. [1 cwt $=112$ lb].
The work done $=112\times300$ ft. lb $=33{,}600$ ft-lb.

(b) *Absolute unit* : Applied force $=112\times32$ poundals
The work done $=112\times32\times300$ ft-poundals
$=1{,}075{,}200$ ft-poundals.

(2) *A force of 200 dynes displaces a body through 300 cm. in the direction of the force. How much work is done? How much work will be done if the force is 10 gm-wt?*

Ans. (i) Here, $F=200$ dynes ; $S=300$ cm.
We know, $W=F\times S=200\times300$ ergs $=60{,}000$ ergs.

(ii) Here, $F=10$ gm-wt ; $S=300$ cm.
We know, $W=F\times S=10\times300$ gm-cm. $=3000$ gm-cm.
$=3000\times980$ ergs. $=2940000$ ergs.
$=0{\cdot}3$ joule (nearly)

7.4. Work done by a variable force :

The applied force may vary in magnitude and direction. In such cases, the work done is determined by dividing the whole displacement into large number of small parts so that each part may be regarded as a small straight line. Fig 7.4 shows the curved displacement of the point of application of a varying force. At every point of the path, the magnitude and direction of the force are different. To determine the work done, the whole path is divided into small straight paths like Δx_1, Δx_2 etc. As the force acts for a very small interval of time, in each of the small displacements mentioned above, the force in each case may be supposed to be constant. Thus, P_1, P_2, P_3 etc are the constant forces which act during the displacements Δx_1, Δx_2, Δx_3, etc respectively. If θ_1, θ_2, θ_3...etc are the angles made by the direction of the forces with respective displacements, then the total work done $=P_1\cos\theta_1\times\Delta x_1+P_2.\cos\theta_2\times\Delta x_2+\ldots+P_n\cos\theta_n.\times\Delta x_n=\Sigma P\cos\theta\times\Delta x.$

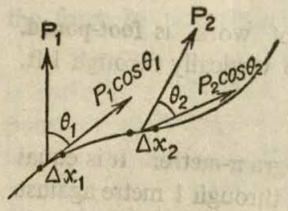


Fig. 7.4

7.5. Work done by a couple :

Consider a couple (F, F) acting on a body. The arm of the couple being AB, its moment or torque $G=F\times AB$ [Fig. 7.5].

Under the action of the couple, if the body undergoes a small angular displacement θ about the point C, the couple does some work. As a result of the angular displacement, let the point A goes to A_1 and the point B to B_1. If θ be very small, the arcs AA_1 and BB_1 may be taken to be almost straight lines, so that $AA_1 = AC \times \theta$ and $BB_1 = BC \times \theta$.

The work done by the force acting at A is given by $W_1 = F \times AA_1 = F.AC.\theta$. Similarly, the work done by the force acting at B is $W_2 = F \times BB_1 = F.BC.\theta$.

∴ Total work done by the couple

$$W = W_1 + W_2$$
$$= F.AC \times \theta + F.BC \times \theta$$
$$= F(AC + BC).\theta$$
$$= F \times AB \times \theta = G \times \theta$$

So, $W = \text{Torque} \times \text{angular displacement}$.

In the case of linear displacement, the work done $W = \text{linear force} \times \text{linear displacement}$. The comparison between the two is useful.

If the body makes 'n' complete rotations per sec., the angular displacement $= 2\pi n$; hence, the work done $W = 2\pi n \times G$

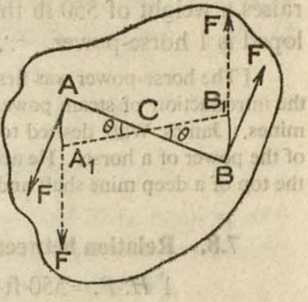


Fig. 7.5

7.6. Power :

Definition : *Power is defined as the rate of doing work.*

$$\text{or, Power } (P) = \frac{\text{Work done}}{\text{Time taken}} = \frac{W}{t} = \frac{F.S.}{t} = F.v = \text{force} \times \text{velocity}$$

If the displacement does not take place in the direction of the force, $P = F.v \cos\theta$, where θ is the angle between the force and the displacement.

Vectorially, it can be written as $P = \vec{F}.\vec{V}$.

The faster is the work done, the greater is the power. In other words, the smaller the time t in the above equation, the greater is the fraction $\frac{W}{t}$ and hence, the power P.

7.7. Different units of Power :

(i) *Absolute units* : In the *C. G. S.* system, the absolute unit of power is 1 *erg per second, i.e.*, work done at the rate of 1 erg per sec, will develop unit power in the *C. G. S.* absolute system.

In the *F. P. S.* system, the absolute unit of power is 1 *foot poundal per sec, i.e.*, work done at the rate of 1 foot-poundal per sec. will develop unit power in the *F. P. S.* absolute system.

In the *M. K. S.* system, the absolute unit of power is *watt*. Work done at the rate of 1 joule (or 1 Newton-metre) per second will develop 1 watt of power.

(ii) *Practical units* : In the *C. G. S.* system, the practical unit of power is **Watt.** When 1 joule of work is done per second, the power developed is called one watt. It is also the unit of power in *M. K. S.* system.

∴ 1 *Watt* = 1 *joule/sec* = 10^7 *ergs/sec.*

Sometimes, a bigger unit *viz*, **Kilowatt** is used.

1 *Kilo-watt* (*K. w.*) = 1000 *watts.*

Generally, power of electrical machines is expressed in watts.

In the *F. P. S.* system, the practical unit of power is called **Horse-Power** (H.P) which expresses a rate of working of 550 ft-lb/sec. Thus if a man or a machine raises a weight of 550 lb through 1 ft. in 1 second against gravity, the power developed is 1 horse-power. ∴ *1 H. P. = 550 ft-lb/sec.*

[The horse-power was first measured by James watt, the inventor of steam engine. Before the introduction of steam power, pumps worked by horses were used for removing water from mines. James Watt desired to measure the power of the newly devised steam-engine in terms of the power of a horse. He accordingly harnessed a horse to a rope passing over a pulley at the top of a deep mine shaft and found that the power of the animal was 550 ft-lb/sec.]

7.8. Relation between Horse-Power and Watt :

$$\begin{aligned} 1\ H.\ P. &= 550 \text{ ft-lb/sec.} \\ &= 550 \times 32{\cdot}2 \text{ ft poundal/sec.} \qquad [g = 32{\cdot}2 \text{ ft/sec}^2] \\ &= 550 \times 32{\cdot}2 \times 4{\cdot}21 \times 10^5 \text{ ergs/sec.} \\ &= \frac{550 \times 32{\cdot}2 \times 4{\cdot}21 \times 10^5}{10^7} \text{ joules/sec.} \\ &= 746 \text{ joules/sec (nearly)} = 746 \text{ watts (nearly)} = \tfrac{3}{4}\text{kw (nearly)} \end{aligned}$$

Alternatively, 1 watt $= \frac{1}{746}$ H. P. and 1 Kilowatt $= \frac{4}{3}$ H. P. = 1·34 H. P.

Examples : (1) *An engine can raise 10 tons of goods at a height of 30 ft. in half a minute. What is its power? Express your answer in horse-power and kilo-watt.*

Ans. 1 ton = 2240 lb. Here, the force applied = 10 × 2240 lb-wt.

∴ Work done = 10 × 2240 × 30 ft. lb : time required = ½ mnt = 30 seconds.

So, the power $= \frac{10 \times 2240 \times 30}{30}$ ft-lb/sec. $= \frac{10 \times 2240}{550}$ H. P. = 40·7 H. P.

Again, 40·7 H. P. = 40·7 × 746 watts. $= \frac{40{\cdot}7 \times 746}{1000}$ kw. = 30·4 kilo-watts.

(2) *A pump can raise 1000 gallons water to a height of 90 ft in one hour. Find the power of the engine in H. P. 1 gallon of water weighs 10 lb.*

Ans. Here, the force applied = wt. of 1000 gallons of water = 1000 × 10 lb.
Work done = 1000 × 10 × 90 ft-lb ; time required = 1 hour = 60 × 60 sec.

∴ Power $= \frac{1000 \times 10 \times 90}{60 \times 60}$ ft lb/sec. = 250 ft lb/sec $= \frac{250}{550}$ H.P. = 0·455 H.P.

(3) *A boy, weighing 100 lb. can run up a flight of 20 steps, each 9 inches high, in 5 seconds. How much horse-power does the boy develop ?* [*H. S. Exam. 1963*]

Ans. Total height of steps $= \frac{20 \times 9}{12}$ ft. Work done $= \frac{100 \times 20 \times 9}{12}$ ft-lb.

Power developed $= \frac{100 \times 20 \times 9}{12 \times 5}$ ft lb/sec. $= \frac{100 \times 20 \times 9}{12 \times 5 \times 550}$ H.P. = 0·54 H.P. (nearly)

(4) *Water is being raised from a well to a height of 30 ft by means of a 5 H.P. motor. If the efficiency of the pump be 85%, how many gallons of water will be raised per minute. 1 gallon of water weighs 10 lb.*

Ans. If the efficiency of the pump be 85%, the effective power$=\frac{85}{100}\times5=\frac{17}{4}$H.P. Suppose '*m*' gallons of water are raised per minute. So, the work done $=m\times10\times30$ ft-lb. Since, this work is done in 1 minute, the rate of doing

$$\text{work}=\frac{m\times10\times30}{60}\ \text{ft-lb/sec}=\frac{m\times10\times30}{60\times550}\ \text{H.P.}$$

$$\therefore\ \frac{17}{4}=\frac{m\times10\times30}{60\times550}\ \text{or, } m=\frac{60\times550\times17}{4\times10\times30}=467{\cdot}5\ \text{gallons}$$

(5) *A pump-set is used to lift water to a reservoir of 5000 litres capacity over an average height of 18 metres. If it takes 1 hour to fill the reservoir completely, calculate the power of pump-set if its efficiency is 75%.* $g=9{\cdot}8$ *metre*$/s^2$

Ans. Since 1 litre of water has a mass 1 kg., the wt. of 5000 litres of water$=5000\times9{\cdot}8$ kg-wt. Work done$=5000\times9{\cdot}8\times18$ joules.

Let P=power of the pump-set. Since its efficiency is 75%, work done by the pump=effective power×time$=\frac{75}{100}\times P\times60\times60$ joule.

$$\therefore\ \frac{75}{100}\times P\times60\times60=5000\times9{\cdot}8\times18\ \text{or, } P=\frac{5000\times9{\cdot}8\times18\times100}{75\times60\times60}\text{watt}=326{\cdot}6\ \text{watt.}$$

(6) *A force pump can project 11000 cu.-ft. of water with a velocity of 10 ft/sec. per minute. What is the power of the pump? Density of water=62·5 lb/cu.ft.*

Ans. Mass of water projected per minute $=11000\times62{\cdot}5$ lb.

Work done$=\frac{1}{2}mv^2=\frac{1}{2}\times11000\times62{\cdot}5\times(10)^2$ ft. poundals. Since this work is being done in 1 minute, the rate of doing work$=\frac{\frac{1}{2}\times11000\times62{\cdot}5\times(10)^2}{32\times60}$ft lb/sec.

$$\therefore\ \text{H.P.}=\frac{\frac{1}{2}\times11000\times62{\cdot}5\times(10)^2}{32\times60\times550}=32{\cdot}5\ \text{H.P.}$$

7.9. Energy :

Anything which is capable of doing work is said to possess energy.

Definition : *Energy of a body is its capacity to perform work and is measured by the amount of work done by it. Energy and work, being of same nature, are expressed by the same units.*

Human beings or animals can perform work and, therefore, they possess energy. In an automobile, the petrol vapour drives the piston of the engine and therefore, it possesses energy. When a body is raised to a certain height, it possesses energy to perform work. Numerous such examples of energy can be cited.

Most of the material comforts which we enjoy today come from utilisation of various forms of energy. Chemical energy in coal and oil, electrical energy, atomic energy, etc. are being used today in various ways for the welfare of mankind.

Energy is, in general, divided in seven different forms :—(1) *Mechanical energy*, (2) *Heat energy*, (3) *Light energy*, (4) *Sound energy*, (5) *Magnetic energy*, (6) *Electric energy* and (7) *Chemical energy*.

In this chapter, we shall confine ourselves only to mechanical energy, which is again divided into two kinds *viz* (i) kinetic and (ii) potential.

7.10. Kinetic energy :

Kinetic energy is the energy which a body possesses by virtue of its motion.

Examples of kinetic energy are moving bullets or hammer heads. They perform some work by overcoming forces when they strike something. A car moving along a road has kinetic energy of translation. In athletic sports, an athlete is found to run a distance before he takes a jump. In running some distance he acquires necessary energy for performing the work required for a jump. The rotating wheels of a moving car have kinetic energy of rotation, as well as of translation..

Measure of kinetic energy :

(i) **K.E. due to translatory motion :** When a force is applied against a moving body, the body does some work against the force before it comes to rest. Total work done by the body under such circumstances is a measure of its kinetic energy (abbreviated as K. E.).

Suppose a body of mass 'm' is moving with a velocity 'u'. A force P is now applied against the motion of the body. This would produce a retardation on the body which will finally come to rest. Let the retardation produced be 'f' and S the distance travelled by the body before it comes to rest. In this case, we can say that the body has displaced the point of application of the force P through a distance S in the direction of its displacement.

So, the work done by the body=force×displacement$=P\times S$.

Now, from Newton's second law of motion we get, $P=mf$.

Again, the final velocity of the body, $v=0$ (because the body comes to rest finally). Hence, from $v^2=u^2+2f.S.$ we have, $0=u^2+2(-f)S.$ or, $S=\frac{u^2}{2f}$

[f being retardation is $-$ve].

So, the work done by the body$=P\times S=mf\times\frac{u^2}{2f}=\frac{1}{2}mu^2$

i.e., K. E. of the body$=\frac{1}{2}mu^2=\frac{1}{2}\times$*mass*$\times$(*velocity*)^2

[**N.B.** When 'm' is expressed in gm. and u in cm/sec (*i.e.*, in the C.G.S. system), K.E.$=\frac{1}{2}mu^2$ ergs$=\frac{1}{2}\frac{mu^2}{g}$ gm-cm.

When m is expressed in lb and u in ft/sec (*i.e.*, in the F. P. S. system), K.E. $=\frac{1}{2}mu^2$ ft-poundals$=\frac{1}{2}\frac{mu^2}{g}$ ft-lb.]

Again, when a force P acts on a body of mass 'm' and changes the velocity of the body from u to v while the body moves through a distance S, then the change of K.E. of the body $=\frac{1}{2}m(v^2-u^2)=\frac{1}{2}m\times 2f.S.=m.f.S.=P.S.$ $=$work done.

i.e., the work done on a body by a force=the change of K.E. of the body

Note that when the speed (or the velocity) of the body is constant, there is no change in kinetic energy and the work done by force is zero. With uniform circular motion, for example, the speed of the particle is constant and the centripetal force does no work on the particle.

If, however, the kinetic energy of a body decreases, the work done on it by the force is *negative.* The displacement and the component of the force along the line of motion are, in this case, oppositely directed.

[**By calculus :** Let the point of application of a constant force P is displaced through a small distance ds in time dt. Work done by the force$=P.ds$.

If the total displacement is S, then the total work done $W=\int P.ds$

Now, $P=m.f=m.\frac{dv}{dt}$ and $\frac{dv}{dt}$ can be written as $\frac{dv}{ds}.\ \frac{ds}{dt}$

So, $W=\int m\left(\frac{dv}{ds}.\ \frac{ds}{dt}\right).\ ds=\int m.\ v.\ dv.=\frac{1}{2}mv^2.\left(\because\ \frac{ds}{dt}=v\right)$

So, $K.E.$ of the body$=\frac{1}{2}mv^2$]

Examples : (1) *The velocity of a body of mass 1 kg. changes from 5 cm/sec to 20 cm/sec in 5 seconds by the application of a force. What was the magnitude of the force and how much work was done in that time?* [*H. S. Exam. 1966*]

Ans. We know, the work done by the force=change of K. E. of the body

$=\frac{1}{2}\times 1000\times\{(20)^2-(5)^2\}$ ergs$=\frac{1}{2}\times 1000\times 375$ ergs$=187{\cdot}5\times 10^3$ ergs.

Again, force $P=m.f=\dfrac{m(v-u)}{t}=\dfrac{1000(20-5)}{5}=3000$ dynes.

(2) *Two blocks A and B of masses m_1 and m_2, coupled by a spring, rest on a smooth table. The blocks are pulled apart and then released. Show that the kinetic energies of the blocks at any instant are inversely proportional to their respective masses. How will the blocks move if the table is (a) perfectly frictionless and (b) rough ?*

Ans. Since no external force is acting on the system, the principle of conservation of linear momentum may be applied to the motion of the blocks. The momentum of the system before the blocks were released was zero in the $X-Y$ reference frame shown in the Fig. 7.6. So after the blocks are released, the total momentum must also be zero. This shows that the blocks A and B move in the opposite directions, A moving in the $+x$ direction while B in the $-x$ direction. From the principle of conservation of momentum, $0=m_1v_1+m_2v_2$

or, $v_1=-\frac{m_2}{m_1}.v_2$ [where v_1 and v_2 are the velocities of A and B respectively at any instant]. .. .. (i)

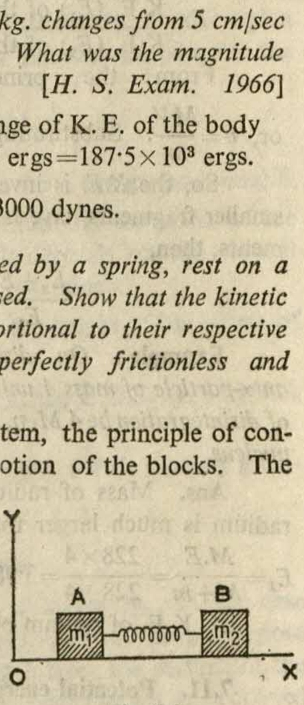


Fig. 7.6

Now, $$\frac{\text{K.E. of block } A}{\text{,, ,, ,, } B}=\frac{\frac{1}{2}m_1v_1^2}{\frac{1}{2}m_2v_2^2}=\frac{m_1}{m_2}\times\left(\frac{m_2}{m_1}\right)^2=\frac{m_2}{m_1}\quad\text{[From eqn. (i)]}$$

This shows that the K.E.'s of the blocks at any instant are inversely proportional to their respective masses.

(*a*) As mechanical energy is conserved and the table is perfectly smooth, the blocks will continue to oscillate back and forth, the energy being partly kinetic and partly potential.

(*b*) If the table is rough, frictional forces will come into play and the energy of the blocks will be dissipated. So, the motion will gradually die out.

(ii) **K. E. due to rotational motion :**

Due to rotational motion, K.E. of a body$=\frac{1}{2}.I\omega^2$, where I is the moment of inertia of the body about the axis of rotation and ω the angular velocity of the body. [ref. art. 6.7].

(iii) **K. E. due to explosive forces :**

In the case of explosion or nuclear reaction, a fragment flies off while the remaining mass recoils. If a fragment of mass m moves with a velocity v while the remaining mass M recoils with a velocity V, then,

$$\frac{\text{K.E. } (E_1) \text{ of the mass } m}{\text{K.E. } (E_2) \text{ of the mass } M}=\frac{\frac{1}{2}mv^2}{\frac{1}{2}MV^2}=\frac{mv^2}{MV^2}\dots\text{(i)}$$

From the principle of conservation of momentum, $mv=MV$

or, $v=\frac{MV}{m}$. Substituting this value in eqn. (i), $\frac{E_1}{E_2}=\frac{mM^2V^2}{m^2MV^2}=\frac{M}{m}=\frac{1/m}{1/M}$

So, the *K.E.* is inversely proportional to the masses of the particles *i.e.* the smaller fragments have larger energy. Thus, if E be the total energy of two fragments, then,

$$\frac{E_2+E_1}{E_1}=\frac{M+m}{M}\text{ or, } E_1=\frac{M.E}{M+m}$$

Example : *On disintegration, a thorium nucleus (mass=232 unit) ejects an α-particle of mass 4 units and is reduced to a radium nucleus. If the total energy of disintegration be 4 Mev, calculate the kinetic energies of α-particle and the radium nucleus.*

Ans. Mass of radium nucleus=232−4=228 unit. Undoubtedly, mass of radium is much larger than the mass of α-particle. So, the K.E. of α-particle,

$$E_1=\frac{M.E}{M+m}=\frac{228\times4}{228+4}=3{\cdot}98\,Mev$$

∴ K.E. of radium nucleus $E_2=4-3{\cdot}98=0{\cdot}02\,Mev$

7.11. Potential energy :

A body is said to have potential energy if by virtue of its position or state it is able to do work.

A car at the top of a hill or a wound up clock spring is an example of an object with potential energy. The wound up clock spring may keep a clock running for a certain length of time and a car may, by coasting downhill, travel a great distance. Generally speaking, the term "potential energy" refers to the energy given to a body by raising it above the level of the earth's surface. When something is lifted vertically through a height, work is done against its weight and this work remains stored up in the body as its potential energy.

Mechanical potential energy is of two types : (i) *Gravitational potential energy* and (ii) *Elastic potential energy.*

A body raised to certain height gives us an example of gravitational potential energy. Wound up clock spring, compressed air etc. are examples of elastic potential energy.

Another point should be borne in mind in connection with potential energy. A body or a system of bodies will always have a tendency of remaining in a position where the potential energy is minimum. *Increase in potential energy means instability.* Stable equilibrium, therefore, demands an orientation where the potential energy is minimum. For this reason, a body rolls down from higher level to lower level, water from hill-top flows down to river or sea, a coiled spring, when released, returns to its normal state etc.

7.12. Measure of potential energy of a body raised to a certain vertical height :

It has been mentioned earlier that when a body is lifted vertically through a height, work is done against its weight and this work remains stored up in the body as its potential energy.

Suppose, m=mass of the body ; then its weight$=mg$.

If it is lifted to a height 'h', the work done$=m.g.h$

So, the potential energy of the body at a height h from the ground$=m.g.h$.

Since, this potential energy arises due to the gravitational attraction of the earth, it is known as *gravitational potential energy.* It depends on the vertical height attained and not on the path in which the height is attained.

[**N. B.** (i) For a body of certain size, 'h' means the height of the centre of gravity of the body between the two positions.

(ii) If 'm' is expressed in gm. and h in cm (*i.e.,* in the C.G.S. system) P.E. $=mgh$ ergs $=mh$ gm-cm.

(iii) If m is expressed in lb and h in ft. (*i.e.,* in the F.P.S. system) P.E$=$ mgh poundals$=mh$ ft-lb.]

7.13. Gravitational potential energy may be negative :

To measure potential energy, a reference plane is necessary. In the case of gravitational potential energy, the surface of the earth usually serves the purpose of reference. If a body be raised vertically to a height h from the surface of the earth, the work done is stored in the body as its potential energy and it is reckoned as positive. If the body, on the other hand, be taken below the surface of the earth, say in a mine, it will release some energy and at the bottom, it will possess potential energy less than that on the surface. In that case, the body is said to possess negative potential energy.

If any plane, other than the surface of the earth, be regarded as the reference plane, a body situated at any point above that plane will generally have a positive gravitational potential energy while at points below the plane, a negative gravitational potential energy.

Suppose a body is kept on the floor of a lift which is moving upward. If we measure the change of potential energy of the body with reference to the floor of the lift, the change is zero because the height of the body from the floor is not changing. But if the change of potential energy is measured with reference to the ground, it has a positive value beacuse the height of the body from the ground is increasing. So, a measure of the difference of potential energy is dependent on the reference plane.

It is to be noted that the reference plane (or condition) usually chosen is the one in which the force acting on the body is zero, so that its P.E. in this position (or condition) may also be taken to be zero. The difference of P.E. in this position (or condition) and at a different position (or condition) of the body then gives the P.E. in the latter position. We cannot, measure the absolute P.E. of a body at any position much the same way as we cannot measure the absolute temperature of a body.

Another aspect of potential energy should be pointed out here. When a man lifts a weight from the floor and raises it above his head, he has, in effect, put his body between the weight and the earth. He has pushed the weight in one direction with his hand and the earth in the opposite direction with his feet. As the mass of the earth is huge, its displacement is extremely small and is not perceptible in the ordinary case. For this reason, we commonly say that the weight has gained potential energy. But it should be pointed out that the earth also gains potential energy at the same time. So, the potential energy is not a property of the weight alone but a joint property of the system consisting of the weight plus the earth. Although this aspect of potential energy should be kept in mind, we shall nevertheless, for convenience, continue to speak of "potential energy of the raised weight" as if it is a property of the weight alone.

7.14. Elastic potential energy of an extended spring :

Some work is needed to elongate or to coil up a spring. The work done remains stored in the elongated or coiled up spring as its elastic potential energy. The spring releases this energy when it uncoils or shortens to the original shape. This is the principle on which the hair-spring of a clock works. The elastic potential energy of an elongated spring can be calculated in the following way.

The elongation of a spring is proportional to the force applied on it. If a spring of length l_1 is elongated to a length l_2 then the extension (l_2-l_1) is proportional to the force applied on it. The force is, therefore, equal to $K\,(l_2-l_1)$ where K is a constant, known as the *force constant* of the spring [See art. 2.3, vide General properties of matter]. Now, when the length of the spring is l_1, the force acting on it$=0$; when the length elongates to l_2, the force becomes $K(l_2-l_1)$. During extension of the spring, the average force acting on it$=\frac{1}{2}\,K(l_2-l_1)$ and the displacement of its point of application$=l_2-l_1$

$\therefore$ Work done on the spring$=\frac{1}{2}K(l_2-l_1)\times(l_2-l_1)=\frac{1}{2}K(l_2-l_1)^2$

So, the elastic potential energy of the extended spring$=\frac{1}{2}K(l_2-l_1)^2$.

If the extension of the spring be called x, the elastic potential energy of the spring$=\frac{1}{2}\,k.x^2$. [$\because\ l_2-l_2=x$].

Example : *A block weighing 16 lb slides on a horizontal frictionless table with a speed of 8 ft/sec. It is brought to rest in compressing a spring in its path. By how much is the spring compressed if its force constant is 1·5 lb/ft ? $g=32$ ft/s^2.*

Ans. The kinetic energy of the block$=\frac{1}{2}mv^2=\frac{1}{2}(\frac{16}{32}).\ (8)^2=16$ ft-lb. This kinetic energy is equal to the potential energy of the spring when it is compressed by the block. If x be the compression, the potential energy of the spring$=\frac{1}{2}\,k.x^2$ $=\frac{1}{2}\times1{\cdot}5\times x^2=0{\cdot}75\times x^2$ ft. lb

$$\therefore\ 0{\cdot}75\times x^2=16 \quad \text{or,}\ x^2=\frac{16}{0{\cdot}75} \quad \therefore\ x=4{\cdot}6 \text{ ft. (nearly)}$$

7.15. Distinction between energy and power :

Apparently energy and power may mean the same thing but actually there are differences between them. The following are the differences :

Energy	*Power*
1. The capacity of a body in performing work is called its energy.	1. The rate of doing work is called the power of the body.
2. Energy of a body is measured by the total work done by the body under the circumstances in which it is placed. There is no question of time in measuring the energy.	2. Power is measured by the work done in unit time. Total work done divided by the time taken gives the power. Hence, in measuring power, time comes into consideration.
3. Energy is a scalar.	3. Power is a scalar.
4. In C.G.S. system, the absolute and gravitational units of energy are respectively erg and gm-cm and in F.P.S. system, ft-poundal and ft-pound.	4. In C.G.S. system, the absolute and gravitational units of power are erg/*s* and gm-cm/*s* respectively and in F.P.S. system, ft-poundal/*s* and ft-lb/*s*.

Examples : (1) *A body of mass 50 gm is allowed to fall freely from a certain height. What will be its K.E. after 5 seconds?*

Ans. We shall have to find the velocity acquired by the body 5 sec. after its fall. Now, $u=0$, $f=g=980$ cm/sec^2, $t=5$ sec. ; $v=?$

We know, $v=u+gt=0+980\times5=4900$ cm/sec.

So, the *K.E.* of the body$=\frac{1}{2}mv^2=\frac{1}{2}\times50\times(4900)^2$ ergs

$=25\times(4900)^2$ ergs$=6\times10^8$ ergs.$=60$ joule

(2) *A bullet of mass 25 gm. travelling at a speed of 500 metres/sec, penetrated a target and its velocity was reduced to 100 metres/sec after penetration How much energy was expended due to penetration?*

Ans. The initial K. E. of the bullet$=\frac{1}{2}\times25\times(500\times100)^2$ ergs

$=\frac{1}{2}\times25\times25\times10^8$ ergs.

The final K.E. of the bullet$=\frac{1}{2}\times 25\times(100\times 100)^2$ ergs$=\frac{1}{2}\times 25\times 10^8$ erg.

$\therefore$ Expended energy$=\frac{1}{2}\times 10^8\times(25\times 25-25)$ ergs

$=\frac{1}{2}\times 25\times 10^8\times(25-1)=25\times 12\times 10^8=3\times 10^{10}$ ergs

(3) *A ball flying at a velocity $v_1=15$ m/sec is thrown back by a racket in the opposite direction with the velocity $v_2=20$ m/sec. Find the change in momentum of the ball if the kinetic energy changes by $\Delta E=8{\cdot}75$ joules.* [*Jt. Entrance 1984*]

Ans. Change in kinetic energy $\Delta E=\frac{1}{2}\,m\,(v_2^2-v_1^2)=\frac{1}{2}\,m\,(400-225)=\frac{1}{2}m\times$ 175 joules. According to the question, $\Delta E=8{\cdot}75$ joules.

$\therefore\quad 8{\cdot}75=\frac{1}{2}\,m\times 175$ or, $m=\frac{1}{10}$ kg

Now, the change of momentum$=mv_1-m\,(-v_2)=m\,(v_1+v_2)=\frac{1}{10}\times(15+20)=$ $3{\cdot}5$ kg-m/*s*.

[**N.B.** Ene gy is a scalar quantity but momentum is a vector quantity]

(4) *A hammer of mass 10 kg is allowed to fall from a height of 5 metre on a peg. The pointed end of the peg is in contact with the ground. If the average resistive force of the ground be 49490 Newton, calculate the depth to which the peg would penetrate into the ground. $g=9{\cdot}8$ m/s².*

Ans. Here, the *P.E* of the hammer = work done against the resistive force of the ground. If the peg enters *x* metre into the ground, the total fall of the hammer $=(5+x)$ metre. Hence, *P.E.* of the hammer$=m.g.h.=10\times 9{\cdot}8\times(5+x)$ joule.

Again, the work done against the resistive force$=49490\times x$ joule.

$$\therefore\quad 10\times 9{\cdot}8\times(5+x)=49490\times x\quad\therefore\quad x=\frac{490}{49392}\text{ metre}=0{\cdot}9\text{ cm (nearly)}$$

(5) *A bullet of mass m, travelling horizontally with a velocity v struck a heavy wooden block of mass M and got stuck into it. The combined mass then began to move in the same direction. What fraction of the kinetic energy of the bullet will be retained as mechanical energy after the collision ? What will happen to the remaining part ?*

Ans. Before collision, *K.E.* of the bullet$=\frac{1}{2}mv^2$; let the velocity of the combined mass after the collision be *V*. From the principle of conservation of momentum, $(M+m).V=m.v$ or, $V=\dfrac{m}{M+m}\cdot v$. Total *K.E.* of the bullet and the block after collision$=\frac{1}{2}(M+m)V^2=\frac{1}{2}(M+m)\,\dfrac{m^2.v^2}{(M+m)^2}=\frac{1}{2}\cdot\,\dfrac{m^2v^2}{M+m}$.

$\therefore$ Fraction of the initial *K.E.* of the bullet retained by the combined mass$=\frac{1}{2}\dfrac{m^2v^2}{M+m}\div\frac{1}{2}mv^2=\dfrac{m}{M+m}$.

The remaining part will be converted into sound and heat energy.

(6) *An ideal massless spring S can be compressed 1 metre by a force of 100 Newtons. The same spring is placed at the bottom of a frictionless inclined plane inclined at an angle 30° to the horizontal as shown in the fig. 7.7. A 10 kg mass is released from rest at the top of the incline and is brought to rest momentarily after*

compressing the spring 2 metres. Calculate (i) the distance through which the mass slides before it comes to rest (ii) the velocity of the mass just before it reaches the spring. $g=10\ m/s^2$.

Ans. (i) Let the mass slides a *total distance d* along the inclined plane before coming to rest after compressing the spring. Here, P.E. lost by the sliding mass $=mg\times d\times \sin 30°$. This must obviously be equal to the work done in compressing the spring. Now, force constant of the spring $k=100\ n/m$.

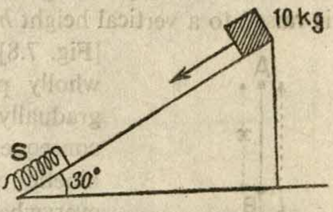


Fig. 7.7

$\therefore$ Work done in compressing the spring through a distance $x=\frac{1}{2}k.x^2=\frac{1}{2}\times 100\times (2)^2 =200$ joule

$\therefore$ $mg\times d.\sin 30°=200$ or, $10\times 10\times d\times \frac{1}{2} =200$ $\therefore$ $d=\dfrac{200\times 2}{100}=4$ metre.

(ii) Since the total distance 4 metres also includes the 2 metre distance through which the spring is compressed, the distance through which the mass slides before reaching the spring $=4-2=2$ metres.

$\therefore$ From the equation, $v^2=u^2+2f.S$, we have,
$v^2=0+2.g.\sin 30°\times 2=0+2\times 10\times \frac{1}{2}\times 2=20$ $\therefore$ $v=\sqrt{20}=4{\cdot}47$ m/s.

7.16. Transformation and conservation of energy :

Different forms of energy are inter-related, *i.e.*, one form of energy can be transformed into another. As a matter of fact, all natural phenomena are the manifestations of such transformations.

Water, we know, flows from higher level to lower level. While at higher level, a mass of water possesses potential energy but while flowing down to lower level, the potential energy is transformed into kinetic energy, which again, may be transformed into electric energy by means of turbine and generator.

When electric energy is supplied to the thin filament of an electric bulb, we get light and heat. It illustrates how electric energy is converted into light and heat energies.

Super-heated steam drives a steam-engine. Here, the heat energy of the steam is transformed into mechanical energy of the steam engine.

A swinging pendulum bob is a good example of a body whose energy can be either kinetic or potential or a mixture of both. Its energy is all potential at the extreme ends of the swing and all kinetic when passing through the rest position. At any other point, the energy is partly kinetic and partly potential.

Numerous such examples of transformations can be cited. Now when any form of energy is transformed into another, a general proposition says that no energy is lost during transformation. Scientists believe that the total quantity of energy in the universe is constant. We can neither create any new energy nor can destroy any. This principle is known as the *principle of conservation of energy*.

7.17. Principle of conservation of mechanical energy :

In the following cases, we shall prove that the sum of kinetic and potential energy is always constant, *i.e.*, the total mechanical energy is conserved.

(A) **A body falling freely under gravity** : Suppose, a body of mass '*m*' is raised to a vertical height *h* from the ground against gravity and is at rest at *A*. [Fig. 7.8]. At the point *A*, the energy of the body is wholly potential. Now, when the body falls, its velocity gradually increases as it approaches the ground and consequently its potential energy changes gradually into kinetic energy. But, it can be proved that *P.E.*+*K.E.* is everywhere constant.

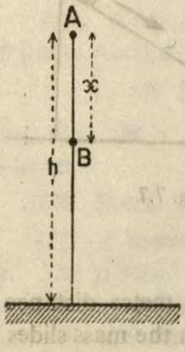


Fig. 7.8

Now, *P.E.* of the body at *A* $=mgh$.

and *K.E.* ,, ,, ,, ,, ,, $=0$

$\therefore$ Total energy of the body at $A=mgh+0=mgh$.

Suppose, now, the body reaches the point *B*, after falling through a height *x*. At *B*, the energy of body will be partly kinetic and partly potential because it is yet to reach the ground and has gained some motion.

P.E. of the body at $B=mg(h-x)=mgh-mgx$.

If 'v' be the velocity acquired by the body at *B*, then its *K.E.* at $B=\frac{1}{2}mv^2$.

Now, during fall the initial velocity $u=0, f=g$, and $S=x$.

We have, $v^2=u^2+2f.S=0+2.g.x=2gx$. So, *K.E.* at $B=\frac{1}{2}m x 2gx=mgx$

So, *K.E*+*P.E* or the total energy of the body at *B*,

$$=mgh-mgx+mgx=mgh.$$

=total energy of the body at *A*.

This proves that the sum of kinetic energy and potential energy of a falling body is everywhere constant during its fall or that the principle of conservation of energy is valid for a freely falling body.

(B) **A body freely falling along a smooth inclined plane** :

Consider a body of mass '*m*' at the point *A* on a smooth inclined plane, inclined at an angle θ to the horizon. The height of the body from the horizontal line is *h* [Fig. 7.9]. At *A*, the potential energy of the body$=m.g.h$ and kinetic energy$=0$. So, the total energy of the body at $A=m.g.h.+0=m.g.h.$

Now, suppose, the body slides along the inclined plane to *B*, travelling a distance *S*, and acquiring a velocity *v* at *B*. The acceleration of the body down the inclined plane$=g.\sin\theta$.

From the relation, $v^2=u^2+2.f.S.$ we have $v^2=0+2.g.\ \sin\theta.\ S.$ $=2g.\sin\theta\times S.$

So, the kinetic energy of the body at *B*

$$=\frac{1}{2}mv^2=\frac{1}{2}m\times 2.g\sin\theta\times S=mg.\sin\theta\times S.$$

If the height of the body at *B* measured from horizontal be x, the potential energy of the body at $B. = m.g.x$

$\therefore$ The total energy at $B = mg \sin\theta . S + mg.x$

Now, from the fig. 7·9, we have, $S \sin\theta = (h - x)$

So, the total energy at $B = mg(h-x) + mgx$ $= m.g.h. =$ Total energy at *A*.

So, it is seen that the total mechanical energy is conserved while the body slides down a smooth inclined plane.

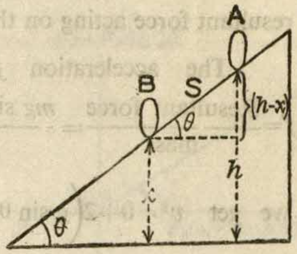


Fig. 7.9

7.18. Dissipation of energy : When transformation of energy takes place, no energy is lost but some energy is dissipated. The input energy in a machine is never equal to output—the output being always somewhat less than the input. Some energy is dissipated in the form of heat in overcoming the frictional forces existing between the different parts of the machine. It is not practically possible to have a mechanical system absolutely free from friction or other resistive forces. These forces always act in such a way as to oppose the motion. When a body is thrown up, the frictional and viscous forces of air act downward and oppose the upward motion of the body. When the body falls, they act upward and again oppose the downward motion of the body. When a body is dragged along a rough surface, friction opposes the motion. A swinging pendulum dissipates energy due to friction and viscosity of the air and gradually slows down its motion and finally comes to rest. In all these cases, the dissipated energy appears in the form of heat or some other irrecoverable form of energy. When this dissipated heat energy is taken into account, the total energy will be found to be constant but no useful work will be available in exchange of dissipated heat energy. In this respect Lord Kelvin realised a general proposition wherein he said that whenever energy is used or transformed, a part of it will always be dissipated and will never be available for doing useful work. This is known as the '*Principle dissipation of energy*'.

7.19. Work done against Friction : Dissipation of energy :

Let us now discuss the energy dissipation due to friction in the case of a body sliding down a rough inclined plane.

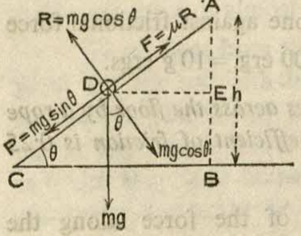


Fig. 7.10

At the point *A*, where the particle is at rest, the energy is wholly potential and it is equal to $m.g.h$, where m = mass of the particle and $AB = h$. Let the particle slide down to *D* through a distance $AD = S$. At this instant, the vertical distance fallen $= AE = S. \sin\theta$. So, its *P.E* at *D* $= mg\,(h - S \sin\theta)$. To find the *K.E* at *D*, we are to find the velocity attained by the particle at *D*. we can do it in the following way.

The force on the particle down the plane, $P = mg \sin\theta$. As the plane is rough frictional force F will act up the plane. Now, $F=\mu R=\mu mg\cos\theta$. So, the resultant force acting on the particle$=P-F=mg\sin\theta-F$

The acceleration f of the particle down the plane is given by $f=\dfrac{\text{Resultant force}}{\text{mass}}=\dfrac{mg\sin\theta-F}{m}=g\sin\theta-\dfrac{F}{m}$. From the equation, $v^2=u^2+2.f.s$,

we get $v^2=0+2\left(g\sin\theta-\dfrac{F}{m}\right).S=2\,g.S\sin\theta-\dfrac{2\,F.S.}{m}$

$\therefore$ $K.E$ of the particle at D,

$$=\tfrac{1}{2}\,mv^2=\tfrac{1}{2}\,m\left(2gS.\sin\theta-\frac{2F.S}{m}\right)=mgS.\sin\theta-F.S.$$

$\therefore$ Total energy at $D=mg(h-S\sin\theta)+mgS\sin\theta-F.S=mgh-F.S$

This shows that the total energy at D is less than the total energy at A by a quantity equal to $F.S$, which is the work done against frictional force. This amount of work is dissipated as heat and no useful work is available in exchange of this dissipated energy.

So, the amount of energy dissipated in sliding down a distance $S=F.S=\mu\,mg\cos\theta\times S$.

When the particle slides down the whole length of the incline and arrives at C, the total energy dissipated $=\mu mg\cos\theta\times AC$.

Examples : (1) *A particle is falling down a rough inclined plane. Assuming the frictional resistance to be 0·2 times the normal reaction and the angle of inclined plane to be 60°, calculate the acceleration of the particle. Also calculate the change of the sum of K.E. and P.E. as the particle falls through a length of 1 metre along the inclined plane. The mass of the particle is 1 gm.* [*Jt. Entrance 1982*]

Ans. Referring to the art 7.19 we have seen,

$$f=g\sin\theta-\frac{F}{m}.=g.\sin\theta-\frac{0{\cdot}2R}{m}=g.\sin\theta-\frac{0{\cdot}2\,mg\cos\theta}{m}$$

$$=g.\frac{\sqrt{3}}{2}-0{\cdot}2g\times\tfrac{1}{2}\quad(\theta=60^\circ).$$

$$=\tfrac{1}{2}g.\,(\sqrt{3}-0{\cdot}2)=\tfrac{1}{2}\times g\times(1{\cdot}73-0{\cdot}2)=0{\cdot}76\times g\ \text{cm/sec}^2.$$

Again, change of the sum of $K.E+P.E=$Work done against frictional force $=\mu.m.g\cos\theta\times S=0{\cdot}2\times1\times g\times\cos60^\circ\times100$ erg. $=10\,g$ ergs.

(2) *A body with a mass of 10 kg is pulled 10 metres across the floor by a rope making an angle of 30° with the horizontal. If the coefficient of friction is 0·25 find the work done.* $g=9{\cdot}8m/s^2$.

Ans. Let F be the force applied. Component of the force along the floor $F_x=F.\cos30^\circ=F\times{\cdot}866$ and the component perpendicular to the floor, $F_y=$ $=F.\sin30^\circ=0{\cdot}5\,F$.

Now, the normal reaction $R = mg - F_y = mg - 0·5\,F$

$\therefore$ the frictional force, $f = \mu R$

$= 0·25(mg - 0·5F)$.

To overcome the friction and slide the body, the component F_x must at least be equal to f. So, $0·866\,F = 0·25\,(mg - 0·5F)$

$= 0·25\,(10 \times 9·8 - 0·5\,F)$

$= 24·5 - 0·125\,F$

$\therefore\quad F = 24·5\,N$ (nearly)

Hence, the work done $= F\cos 30° \times x$

$= 24·5 \times 0·866 \times 10$

$= 221·17J$ (nearly)

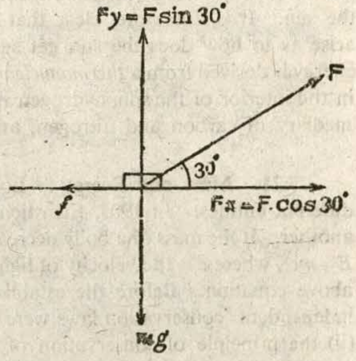


Fig. 7.11

(3) *A body of mass 50 gm is projected vertically upwards with a velocity of 100 m/s. It rises to a height of 400 metre. Find the resistance of the air. With what velocity will it strike the ground after it comes down?*

Ans. At the instant of projection, *K. E.* of the body$= \frac{1}{2}\,mv^2 = \frac{1}{2} \times 50 \times (10^4)^2 = 25 \times 10^8$ ergs. and $P.E. = 0$.

At the greatest height, the *P.E.* of the body$= m.g.h. = 50 \times 980 \times 400 \times 100 = 19·6 \times 10^8$ ergs and $K.E. = 0$.

$\therefore$ Work done against resistance of the air$=$Difference of the total energies at the two positions$= 25 \times 10^8 - 19·6 \times 10^8 = 5·4 \times 10^8$ ergs.

If P be the resistance force of the air, then,

$$P \times 400 \times 100 = 5·4 \times 10^8 \text{ or } P = 1·35 \times 10^4 \text{ dynes.}$$

Now, when the body falls from the greatest height, the same amount of energy will be lost due to friction. Hence if v be the velocity on reaching the ground, then $\frac{1}{2}mv^2 = P.\,E.$ at the greatest height$-$loss of energy due to friction.

$$= 19·6 \times 10^8 - 5·4 \times 10^8 = 14·2 \times 10^8$$

$$\therefore\quad v^2 = \frac{14·2 \times 10^8 \times 2}{50} = 56·8 \times 10^6 \text{ or } v = 7·53 \times 10^3\text{cm/s} = 75·3 \text{ m/s.}$$

7.20. Solar energy is the ultimate source of energy :

The sun is the store-house of an enormous quantity of energy and it is the ultimate source of almost all terrestrial energy.

We get heat from the sun. Trees and plants get their food from the atmosphere and the soil with the help of sun-rays. Most of the food that we take for our growth get their energy from sun-rays. Coal is a good source of energy but the plants from which coal is formed derived their growth from the action of sun light in which they were bathed. Water from rivers, tanks and ponds get evaporated constantly due to the heat of the sun and produce clouds which ultimately pour rain. Accumulated rain waters in the mountain come down as torrential river, from which we produce electric energy. In this way if we try to trace the origin of any terrestrial source of energy we will ultimately arrive at the sun.

[Although solar energy is the ultimate source of all terrestrial energy, yet it is very surprising that we, on the surface of the earth, receive only a small fraction of the total energy radiated by the sun. It is, therefore, clear that the energy radiated by the sun is colossal. Question may arise as to how does the sun get such colossal energy ? Scientists have shown that the solar energy is derived from a *thermonuclear process*. Under the extermely high-temperature conditions in the interior of the sun, hydrogen nuclei fuse together to form helium nuclei through the intermediary of carbon and nitrogen, and the resulting loss in mass is converted into energy.]

7.21. Mass and Energy : Long ago, mass and energy were considered to be entirely different entities. In 1905, Eienstien proved they are equivalent *i.e.* one can be converted into another. If the mass of a body decreases by m, the energy E released by it is given by the equation, $E=mc^2$, where c is the velocity of light. Experiments on nuclear reactions proved the truth of the above equation. Before the establishment of the above-mentioned mass-energy relation, two independent conservation laws were existent *viz.* (i) the principle of conservation of mass and (ii) the principle of conservation of energy. Eienstien's equivalence principle has helped the modern physics to do away with first law and to retain the second one.

As the value of c, the velocity of light is enormous, the above equation shows that at the cost of a very small amount of mass, we can get a very large amount of energy. The huge energy that the sun is continuously emitting is derived as a result of conversion of a small quantity of mass into energy during the fusion process taking place continuously in the sun.

7.22. Conservative and non-conservative forces :

The system in which mechanical energy remains conserved is called a *conservative system* and the forces acting in the system are called *conservative forces.* Force of gravitation, for example, is a conservative force. When a body is taken vertically to a certain height, work is done against the force of gravity and the work done remains stored in the body as its potential energy. But when the body falls from the height it does some work by virtue of its potential energy and the work done by the body is equal to the work done previously on it, *i.e.*, the energy remains conserved. So in a conservative system, work once done can be restored and the net work done in the whole cycle is zero.

But when a body is dragged along a rough ground, some work is done against the friction. In this process the body does not gain any potential energy. If the body be taken back to its initial position, friction will again act against the motion and some work will have to be done again instead of getting back the previous work. So, in a complete cycle, the net work is not zero. For this reason, friction and other resistive forces are regarded as *non-conservative forces.*

So it may be said that forces where work is restorable are called conservative forces and the forces where work is not restorable are called non-conservative forces.

Exercises

Essay type :

1. What do you understand by 'work' and power' ? What are their practical units in the C.G.S. and F.P.S. systems ? What is the relation between the two units ? [*Tripura H. S. 1981*]

2. What do you mean by 'work done by a couple' ? If a body makes 'n' complete rotations, show that work done$=2\pi nG$ when G is the torque applied.

3. What do you mean by 'horse-power' ? Obtain a relation between horse power and watt.

4. How would you find out the work done by a variable force, varying both in magnitude and direction.

5. Describe the difference between kinetic energy and potential energy, mentioning the ways in which they are measured.

6. What do you mean by kinetic energy and potential energy ? "A carriage of 5 H. P"—What does it mean ? [*Tripura H. S. 1978*]

7. Explain what you understand by 'kinetic energy'. Show that the work done by a force is equal to the change of kinetic energy of the body. [*Tripura H. S. 1981*]

8. Prove that if a piston moves in a cylinder due to a constant pressure, the work done in each stroke is equal to the product of the pressure and the volume swept out by the piston.

9. Define kinetic energy. Calculate the kinetic energy of a particle of mass M moving with a velocity v. [*H. S. Exam 1978* ; *Tripura H. S. 1981*]

10. Prove that sum total of K. E. and P. E. of a body falling freely under gravity is everywhere constant. [*H. S. Exam 1979, Tripura H. S. 1980*]

11. What do you mean by transformation and conservation of energy ? Give a few illustrations.

12. What are conservative and non-conservative forces ? Show that the force of gravity is a conservative force but frictional force is a non-conservative force.

13. Differentiate between gravitational potential energy and elastic potential energy. Obtain an expression for the elastic potential energy of a stretched spring of unstretched length l_o.

Short answer type :

14. What is the difference between 'work done by a force' and 'work done against a force' ? Explain with illustrations.

15. A single force acts on a body in rectilinear motion. Velocity-time graph of the body is shown in Fig. 7.12. Find the sign (+*ve* or −*ve*) of the work done by the force on the body in each of the intervals *OA*, *AB*, *BC* and *CD*.

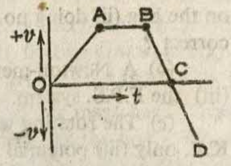


Fig. 7.12

16. Explain the following with reasons :

(*a*) A man is swimming up-stream in such a manner that he is stationary with respect to the shore. Is he doing any work ?

(*b*) A car is moving along a horizontal roadway with a constant velocity and no net force is acting on it. Is any work being done on the car ?

(*c*) In a tug-of-war, the weaker team is gradually yielding to the stronger team. Which team is doing work ?

(*d*) The earth is moving round the sun in a circular orbit and is acted upon by a force. Hence work must be done on the earth by this force. Do you agree ?

[**Hints :** No ; the force is a centripetal force which is at right angles to the motion ; hence no work is done.]

17. When two boys play catch on a train, does the kinetic energy of the ball depend on the speed of the train ?

[**Hints :** No ; the ball being in the train, its relative velocity with respect to the train is zero. Hence the K. E. of the ball depends only on the speed given to it by the boys.]

18. What happens to the P.E. a lift loses in coming down from the top of a building to a stop at the ground floor ?

19. Can a body have momentum without energy and energy without momentum ?

[**Hints :** A body can have P.E., without momentum but whenever a body has a momentum it has motion and therefore, energy.]

20. Does the work done in raising a load on to a platform depend on how fast it is raised ? Does power depend ?

21. A man, carrying a bucket of water, stands in a lift which is ascending with uniform velocity. Explain (i) whether the man is doing any work on the bucket and (ii) whether the energy of the bucket remains unchanged. [*H. S. Exam 1978, '81*]

[**Hints** : (i) No work is done because the point of application of the fore applied by the man is not changing with respect to the lift (ii) The P.E. of the bucket increases as it ascends higher.]

22. A heavy body and a light body have equal momentum. Will their kinetic energies be equal ?

[**Hints** : $MV = mv$; as $M > m$; $V < v$.

$$\text{Now, } \frac{\text{K.E. of the heavy body}}{\text{,, ,, ,, light ,,}} = \frac{\frac{1}{2}MV^2}{\frac{1}{2}mv^2} = \frac{MV.V}{mv.v} = \frac{V}{v}$$

As $V < v$, K.E. of the heavy body < K. E. of the light body.]

23. When compressed air is allowed to do some work, what type of energy is stored in it ?

24. Two springs A and B are identical, except that A is stiffer than B (*i.e.* $K_A > K_B$), On which spring is more work done if (i) they are stretched by same amount (ii) they are stretched by the same force ?

25. When a gas-filled balloon rises up, it gains both K.E. and P.E. How does the principle of conservation apply in this case ? [*Jt. Entrance 1985*]

[**Hints** : The bouyant force of the displaced air is greater than the weight of the balloon. The resultant force does work in lifting the balloon. The work done on the balloon by the resultant force equals the gain of K.E.+P.E. So the principle is valid in this case.]

26. Do you consider the resistive force of air conservative ?

Objective type :

27. (*a*) A man is walking along a road with a heavy bag in one hand. He is (i) doing work on the bag (ii) doing no work on the bag (iii) doing work only if the bag is lighter. Which one is correct ?

(*b*) A Newton-metre is the unit of work in (i) the C.G.S. system (ii) the M.K.S. system (iii) the F.P.S. system. Which one is correct ?

(*c*) The rotating wheels of a moving bi-cycle have (i) translational K.E. only, (ii) rotational K.E. only (iii) potential energy (iv) translational and rotational K.E. Which one is correct ?

(*d*) A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to (i) $\sqrt{t}$ (ii) $\sqrt[4]{t^3}$ (iii) $\sqrt{t^3}$ (iv) t^2.

[*I.I.T. 1984*]

(*e*) A body is rotating with uniform speed in a circular path. Everything of the body remains constant except (i) velocity (ii) acceleration (iii) kinetic energy (iv) none of it.

(*f*) When a watch-spring is wound, it has (i) gravitational potential energy (ii) elastic potential energy (iii) kinetic energy. Which one is correct ?

(*g*) A system of bodies has always a tendency of remaining in a position where (i) the potential energy is maximum (ii) the potential energy is minimum (iii) the kinetic energy is minimum. Which one is correct ?

(*h*) A body of mass m has $K.E. = E$. Its momentum is (i) $2mE$ (ii) $\sqrt{2mE}$ (iii) $\sqrt{mE}$

Numerical Problems :

28. How much work is done in raising a mass of 1 kg to a height of 10 metres ? The kinetic energy of a body is 1 joule ; a force of 1 mega-dyne is applied against its motion. How much distance will the body go before it comes to rest ? [Ans. 98·1 joules ; 10 cm]

29. If a person, weighing 11 stones, climbs a mountain 6000 ft. high how much work does he perform ? (1 stone = 14 lb). [Ans. 924,000 ft. lb]

30. 10 stone cubes each having 12 cm long arm, are scattered on the ground. What work is done in arranging the cubes one over the other ? Density of stone = 2·5 gm/cc.

[**Hints** : Total height = 120 cm, the height of the C.G. = 60 cm. The C.G. of the first cube is at a height of 6 cm. ∴ $W = 10 \times (12)^3 \times 2{\cdot}5 \times 980\ (60-6)$ erg $= 22{\cdot}86 \times 10^8$ ergs.]

31. An engine can project 13200 lb of water per minute with a velocity of 48 ft/sec. Calculate the H.P. of the engine. [Ans. $14\frac{2}{5}$]

32. A man, weighing 180 lb, can climb up the top of a tower 200 ft. high in 5 minutes. Calculate the power of the man in H.P. [Ans. 0·218]

33. How fast can a crane raise a piece of iron weighing 1650 lb. if it exerts 4 H.P. power ? [*H. S.* (*Comp.*) *1966*] [Ans. 1·38 ft./sec.]

34. A pump can raise 5,000 gallons of water from a well to an average height of 20 ft. per minute. If the efficiency of the pump is 70%, find its power. [1 gallon = 10 lb] [Ans. 43·3 H.P.]

35. If clouds were 1 mile above the earth and rain fell sufficiently to cover 1 sq. mile at sea-level, $\frac{1}{2}$ inch deep, how much work was done in raising the water to the clouds ? [Ans. $3{\cdot}83\times10^{11}$ ft. lb (nearly)]

36. Find the work done by a man weighing 144 lb in carrying 56 lb of bricks up a ladder 20 ft long making an angle of 30° with the horizontal. [Ans. 2000 ft-lb]

37. An engine of 10 H.P. power is used to raise water to a reservior 300 ft. high. If the efficiency of the engine be 80%, find the amount of water delivered into the reservior per minute. [*H. S.* (*Comp*) *1962*] [Ans. 880 lb]

38. A well 60 ft. deep and 8 ft. in diameter has water to a height of 30 ft. A pump empties it in 1 hour. Calculate the horse-power of the engine. [Ans. 2·14]

39. A bullet of mass 100 gm is fired from a gun with a speed of 400 metres/sec. What is the kinetic energy of the bullet ? [Ans. 8×10^{10} ergs]

40. What is the H.P. of an engine which can raise 250 gallons of water per minute to a height of 40 yds ? 1 gallon = 10 lb. [Ans. 9·09]

41. A body of mass 1 kg is thrown upward with a velocity of 200 cm/s from the top of a tower 10 metres high. What is its K.E. just before touching the ground ? [*Tripura H. S. 1980*] [Ans. 10^9 erg]

42. A neutron, one of the constituents of a nucleus, was found to pass two points 12 metres apart in a time-interval of 3.6×10^{-4} sec. Assuming its speed was constant, find its kinetic energy. The mass of a neutron is 1.7×10^{-27} kg. [Ans. $9{\cdot}3\times10^{-19}$ joule]

43. A running man has half the kinetic energy that a boy of half his mass has. The man increases his speed by 100 cm/sec and then has the same kinetic energy as the boy. What were the original speeds of man and boy ? [Ans. Man = 2·4 m/s Boy = 4·8 m/s]

44. A bomb, weighing 40 kg was dropped from a plane at a height of 1 km from the ground. What will be its K.E. (i) after 10 sec. (ii) just before touching the ground ? [Ans. (i) 192080 joule (ii) 392000 joule]

45. A body of mass 5 kg fell from a height of 5 metres on to a peg. If the peg penetrates 10 cm into the ground, calculate the resistive force of the ground in newtons. [Ans. 2499*N*]

46. What power is needed to move a 500 kg car up a 30° incline with constant speed of 36 km/hr against frictional force of 50 kg ? $g = 9{\cdot}8$ m/s^2. [Ans 29·4 K.W.]

47. An electric motor of power 300 watts is used to drive the stirrer in a water bath. 60% of the energy supplied to the motor is expended in stirring the water. Calculate the work done on the water in 1 minute. [*I.S.C. 1979*] [Ans. 10,800 joules]

48. A trunk with a mass 100 kg is pulled 20 metres across the floor by a rope making an angle of 35° with the horizontal. If $\mu = 0{\cdot}25$ find the work done. [Ans. 4117 joules (nearly)]

49. A body of mass 1 kg is let fall from a tower 100 metres high. Calculate the K.E. of the body (i) 1 sec. after its release and (ii) when at the bottom of the tower. [*H. S. Exam 1981*] [Ans. (i) $48{\cdot}02\times10^7$ erg. (ii) 98×10^8 erg]

50. A body of mass 100 gm has a momentum of 2000 gm-cm/s. What is its kinetic energy ? If the body attains the momentum in 19 sec from rest, find the force acting on the body. [*Tripura H. S. 1981*] [Ans. 2×10^4 eng ; 200 dyne]

51. A force of 5 lb is found to stretch a screen door spring 6 inches. What is the P.E. of the spring when opening the door stretches it 18 inches ? [Ans. 11.3 ft-lb]

52. A body of mass 0·5 gm slides along a rough inclined plane having inclination of 30° to the horizontal. If the frictional force is 0·4 times the normal reaction, find the change of the sum of K.E.+P.E. as the body falls through a distance of 200 cm along the incline [Ans. 34·6 *g* ergs]

53. A particle of mass 5 gm is initially moving along a straight line with a velocity of 10 cm/sec and then a force of $10\sqrt{2}$ dynes is applied to it at an angle of 45° to the initial direction of motion. Calculate the change in its kinetic energy during the first second.

[*I.I.T. 1965*] [Ans. 120 ergs]

Harder Problems :

54. A horse is drawing a carriage with a force of 50 lb inclined to 30° with the horizontal. If the carriage moves at the rate of 60 miles/hr, what work will be done by the horse in 10 minutes ? Express the power of the horse in practical unit ? [Ans. $2{\cdot}275\times10^5$ ft-lb ; 0·7 H.P.]

55. Eight stone cubes, each having 10 cm long arm are lying scattered on the ground. What work is done in arranging the cubes one over the other. Density of stone=2·5 gm/cc.

[Ans. $5{\cdot}86\times10^5$ erg]

56. Water is to be pumped to a tank 27·8 ft above a reservoir. The bottom of the tank measures 5 ft×5 ft. What work is to be done in pumping water to a depth of 4·4 ft in the tank ? If the power of the pump is ½kW, what time will be required for the work ?

[*Jt. Entrance 1976*] [Ans. $2{\cdot}06\times10^5$ ft-lb ; 17 mnt 18 sec]

57. A person, weighing 60 kg is going up a flight of steps with a load of mass 20 kg in his hand. There are 20 steps, each of height 20 cm. If the person ascends the steps in 20 seconds, what is his power ? [*I.I.T. 1974*] [Ans. 313·6 watt]

58. A bullet of mass 20 gm travelling horizontally at 100 metre/s, embeds itself in the centre of a block of wood of mass 1 kg which is suspended by a light vertical string 1 metre in length. Calculate the maximum inclination of the string to the vertical. Cos 37°=0·8038. [Ans. 37°]

[Hints : $20\times100=1020\times v \therefore v=\frac{100}{51}$ m/s. Now $\frac{1}{2}Mv^2=Mgh$

$$\therefore h=\frac{v^2}{2g}=\frac{10^4}{(51)^2}\times\frac{1}{2\times9{\cdot}8}=0{\cdot}1962. \text{ But } h=l(1-\cos\theta)$$

$$\therefore 1-\cos\theta=\frac{0{\cdot}1962}{l}=\frac{0{\cdot}1962}{1} \text{ or } \cos\theta=0{\cdot}8038 \therefore \theta=37^\circ]$$

59. An upward force $F=196$ Newton is applied upon a body of mass 10 kg till it is raised vertically upwards by a distance of 10 metres. Calculate the work done by F and the work done against gravity. Here the work done by F is much greater than the gain in gravitational P.E. Show using clear calculations that the law of conservation of energy is quantitatively satisfied here.

[*Joint Entrance 1978*] [Ans. 1960 joules, 980 joules]

60. A lorry and a motor car are travelling with equal kinetic energies. By applying brakes, both are brought to rest with equal retarding force. Which one will come to rest within a shorter distance ? [Both will take equal distance]

[Hints : Here, $\frac{1}{2}m_1u_1^2=\frac{1}{2}m_2u_2^2$ and $m_1f_1=m_2f_2 \quad \therefore \frac{u_1^2}{f_1}=\frac{u_2^2}{f_2}$

Now, $S_1=\frac{u_1^2}{2f_1}$ and $S_2=\frac{u_2^2}{2f_2} \quad \therefore S_1=S_2$]

61. A block weighing 2 lb is forced against a horizontal spring of negligible mass, compressing the spring an amount $x_1=6$ inches. Upon releasing the block, it moves on a horizontal table a distance $x_2=2$ ft before coming to rest. The force constant of the spring is 8 lb/ft. What is the coefficient of friction between the block and the table ? [Ans. 0·25]

62. A rectangular block of mass 10 gm rests on a rough plane which is inclined to the horizontal at an angle of $\sin^{-1}(0{\cdot}05)$. A force of 0·03 Newton, acting in a direction parallel to the line of greatest slope, is applied to the block so that it moves up the plane. When the block has travelled a distance of 1·1 metre from its initial position, the applied force is removed. The block moves on and comes to rest again after travelling a further 0·25 metre. Calculate

(i) the work done by the applied force (ii) the gain in P.E. of the block (iii) the coefficient of sliding friction between the block and the plane. $g=9{\cdot}8\ m/s^2$.

[Ans. (i) 0·33 *J* (ii) 0·0066 *J* (iii) 0·2 (approx.)]

63. A body of mass 2 kg is being dragged with a uniform velocity of 2 metre/s on rough horizontal plane. The coefficient of friction between the body and the surface is 0·2. Calculate the amount of work done in 5 sec. $g=9{\cdot}8\ m/s^2$.

[*I.I.T. 1980*] [Ans. 39·2 joule]

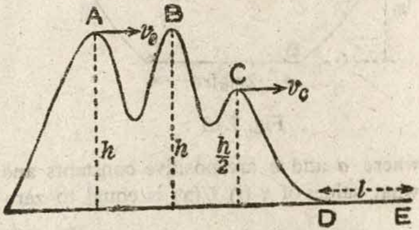


Fig. 7.13

64. A point mass m starts from the point A of a curved track of the form shown in fig 7.13 with a horizontal velocity v_0. The track is frictionless except for the small horizontal portion DE of length l. Height of $C=h/2$. Calculate (i) the velocity of the mass at points B and C. (ii) What constant deceleration is required to stop the mass at E if the retarding frictional force starts operating at point D ? [Ans. (i) $v_B=v_0$; $v_C=\sqrt{v_0^2+gh}$ (ii) $(v_0^2+2gh)/2l$]

65. A body of mass m accelerates uniformly from rest to a speed V in time T. Show that the work done on the body as a function of time t in terms of V and T is $\frac{1}{2}\frac{mV^2}{T^2}.t^2$

[**Hints** : Acc^n $f=V/T$; If s is the displacement in time t, then $s=\frac{1}{2}\frac{Vt^2}{T}$. Now force$=mf=mV/T$

$\therefore$ Work done$=\frac{mV}{T}\times\frac{1}{2}\frac{V.t^2}{T}=\frac{1}{2}.\frac{mV^2.t}{T^2}$]

66. A block of mass M with a semicircular track of radius R rests on a horizontal frictionless surface. A uniform cylinder of radius r and mass m is released from rest at the top point A [Fig. 7.14]. The cylinder slips on the semicircular frictionless track. How far has the block moved when the cylinder reaches the bottom (point B) of the track ? How fast is the block moving when the cylinder reaches the bottom of the track ? [*I.I.T. 1983*]

$$\left[\text{Ans. } \frac{m(R-r)}{m+M}\ ;\ \frac{m}{M}\sqrt{2g(R-r)}\right]$$

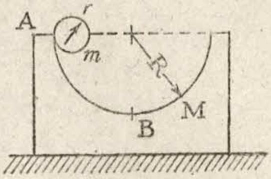


Fig. 7.14

67. A car, moving at a speed of 72 km/hr reaches the foot of a smooth incline when the engine is switched off. If the inclination of the plane to the horizontal be 30° how far does the car go up the incline before coming to rest ? $g=9{\cdot}8\ m/s^2$. [Ans. 40·8 metre]

68. A bullet of mass 10 gm is fired horizontally into a 4 kg wooden block at rest on a horizontal surface. The coefficient of friction between the block and the surface is 0·25. The bullet gets embeded in the block and the combination moves 20 metres before coming to rest. With what speed did the bullet strike the block ? [Ans. 198·3 *m/s*]

[**Hints** : K.E. of the bullet$=\frac{1}{2}mv^2=\frac{1}{2}\times{\cdot}01\times v^2$; Work done by the combination against friction$=\mu.(4+{\cdot}01)\times 9{\cdot}8\times 20$;.$\therefore$ $\frac{1}{2}\times{\cdot}01\times v^2=0{\cdot}25\times 4{\cdot}01\times 9{\cdot}8\times 20$]

69. A test tube of mass 15 gm, closed with a cork of mass 1 gm contains some volatile liquid. The test tube is suspended by a string of length 8 cm. What is the minimum speed with which the cork must fly off on heating the test tube so that the tube may describe a full vertical circle about the point of suspension ? Assume that the string always remains taut.

[Ans. 26·56 metre/*s*]

70. A boy of mass M standing on a smooth horizontal plane of ice throws a ball of mass m horizontally on the surface of the ice. If x be their distance apart after a time t, show that the amount of work done by the boy in throwing the ball is $\frac{1}{2}\left(\frac{x}{t}\right)^2\cdot\frac{mM}{M+m}$.

71. A particle of mass M slides down a track with elevated ends and a flat central part as shown in fig 7.15. The flat part is 2 metre long and the curved portions are frictionless. For the flat part, the coefficient of friction, is 0·2. The particle is released from A which is 1 metre above the BC line. Where does the particle finally come to rest ?

[Ans. Mid point of BC]

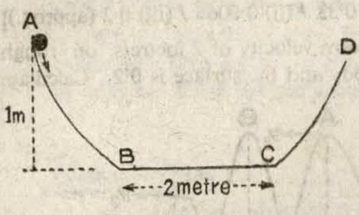


Fig. 7.15

72. Potential energy function for the force between two atoms in a diatomic molecule can be expressed approximately as : $U(x)=\frac{a}{x^{12}}-\frac{b}{x^{6}}$, where a and b are positive constants and x is the distance between the atoms. Find for what values of x (i) $U(x)$ is equal to zero and (ii) $U(x)$ is minimum.

[**Hints :** $U(x)$ is minimum when $\frac{d}{dx}[U(x)]=0$] [Ans, (i) $x=\left(\frac{a}{b}\right)^{\frac{1}{6}}$ (ii) $x=\left(\frac{2a}{b}\right)^{\frac{1}{6}}$]

73. If the magnitude of the force of attraction between a particle of mass m_1 and one of mass m_2 is given by $F=Km_1m_2/x^2$, find the work done to increase the separation of the masses from $x=x_1$ to $x=x_1+d$ [Ans. $(Km_1m_2d)/x_1(x_1+d)$

[*Hints* : Work done for small displaced $dx=F.dx$. So, total work done$=\int_{x_1}^{x_1+d} F.dx$.]

GENERAL PROPERTIES OF MATTER

1

GRAVITATION

1.1. Introduction : From the study of the motion of planets round the sun, ancient astronomers, like Tycho Brahe and John Kepler, formulated certain laws governing the planetary motion but could not offer any explanation as to why they are moving. It was Sir Isaac Newton, who in an effort to interpret those laws discovered the Law of Gravitation which deals with the forces that hold the planets in their orbits round the sun.

1.2. Newton's law of gravitation:

Any two particles in this universe attract each other with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between them. This is Newton's law of gravitation.

If m_1 and m_2 be the masses of two particles and d the distance between them, the force of attraction F between them is given by, $F \propto \frac{m_1 m_2}{d^2}$ or, $F = G. \frac{m_1 m_2}{d^2}$ where G is a constant.

This constant 'G' is known as the '*Gravitational constant*'. In the C.G.S., system, its value is $6{\cdot}6576 \times 10^{-8}$; it means that two particles, each of mass 1 gm, when separated by a distance of 1 cm. will exert upon each other, a force of attraction equal to $6{\cdot}6576 \times 10^{-8}$ dynes. This value was first determined experimentally by Henry Cavendish in 1798. It is, indeed, clear that gravitational forces among terrestrial bodies are extremely small. In the M. K. S. unit, the value of G is $6{\cdot}66 \times 10^{-11}$.

It is to be noted that the gravitational forces between two particles (say, m_1 and m_2) are an action-reaction pair. The particle m_1 exerts a force on the particle m_2 that is directed towards the particle m_1 along the line joining the two. Likewise the particle m_2 exerts a force on the particle m_1 that is directed towards the particle m_2 along the same line. These forces are equal in magnitude but oppositely directed.

Definition and units of G :

From Newton's law we get $F = G. \frac{m_1 m_2}{d^2}$. If $m_1 = m_2 = 1$ and $d = 1$, then, $G = F$ *i.e. the gravitational constant may be defined to be numercally equal to the force of attraction between two unit masses kept at unit distance apart.*

From the above equation, we get $G = \frac{F.\, d^2}{m_1 m_2}$

The dimension of G is as follows :

$$[G] = [F]\,[d^2]/\,[m_1]\,[m_2]$$
$$= [MLT^{-2}]\,[L^2]/\,[M^2]$$
$$= [M^{-1}L^3T^{-2}]$$

So, in the C. G. S. system, the unit of G may be written as $cm^3\ sec^{-2}\ gm^{-1}$ or dyne $cm.^2/gm^2$. In the M.K.S. system, the unit of G is $metre^3\ sec^{-2}\ kg^{-1}$ or Nm^2/Kg^2. It is a scalar quantity.

1.3. Universality of Newton's law of gravitation :

Newton's law of gravitation is found to be valid over a wide range from the small terrestrial distance to the vast astronomical distance of the space as well as from minute atomic particles to huge heavenly bodies. It cannot be verified directly, but its best proof is that assuming it to be true, the astronomical calculations made are found to be true. It admirably explains the motion of a planet round the sun. Further the law is independent of the medium, the nature of the attracting masses, temperature and the chemical composition of the masses. For all these reasons, the law is regarded as a universal law.

But there are certain small divergences. In the vicinity of the sun where the sun's attraction is very great, the law does not give satisfactory result. For example, it is found that the perihelion of the planet Mercury undergoes slight rotation in every hundred year which is not explained by the law of gravitation. Further, the law of gravitation cannot be given a complete universal character because of the fact that according to the theory of relativity, the mass of a body depends upon its motion and the measurement of distance upon the reference frame of the observer. The law is also found to be invalid in a very small distance of the order of 10^{-7} cm or still smaller intermolecular distances.

Example : *What will be the force of attraction between two masses of 5 kg and 8 kg when they are 10 cm apart.* $G=6{\cdot}7\times10^{-11}\ Nm^2/kg^2$.

Ans. We know, $F=G.\dfrac{m_1m_2}{d^2}$; Here $m_1=5$ kg ; $m_2=8$ kg ; $G=6{\cdot}7\times10^{-11}$ Nm^2/kg^2 ; $d=10$ cm$=0{\cdot}1$ metre

$$\therefore\ F=\frac{6{\cdot}7\times10^{-11}\times5\times8}{(0{\cdot}1)^2}=26{\cdot}8\times10^{-8}\ \text{newton.}$$

1.4. Gravitational attraction for extended bodies :

Newton's law of gravitation, as enunciated in art 1.2, applies to two particles. But in reality, a body, however small, must have some dimension. How can we find the gravitational attraction between such extended bodies?

If the bodies are situated far apart, their dimensions may be neglected in comparison with the large distance between them and in that case, considering the bodies as point masses, Newton's law may be applied without much error. But if the distance apart be small, the dimension cannot be neglected and complications will arise in the calculations. In all such cases, an extended body may be regarded as being made up of a large number of particles and the attractive force between each two particles to be found out. Then all these forces are to be added up vectorially because the magnitudes and directions of the attractive forces are all different. Needless to say, that it is almost impossible to find out, in this way, the attractive force between two extended bodies.

This difficulty, however, does not exist if the bodies under consideration, are spherical either solid or hollow, for it may be proved that whatever be the distance, so far as the attraction at an external point is concerned, a solid sphere or a shell behave as if its whole mass is concentrated at its centre. So, to calculate the attraction between two spheres of extended size, we shall consider as if there are two point masses situated at their centres, each of value same as the whole mass of the sphere, and their distance apart same as the distance between the centres of the spheres.

Heavenly bodies like sun, stars, planets, satellites etc. are taken as spherical. Hence, in order to calculate the gravitational attraction between them, we apply Newton's law considering their whole masses concentrated at their respective centres.

1.5. Gravitational field and potential of the earth :

We know that an electric field exists surrounding an electric charge. In the same way, a magnetic field exists surrounding a magnetic pole. If a charge or a magnetic pole be brought in the electric field or in the magnetic field, the charge or the pole experiences force. Likewise, there exists a gravitational field surrounding the earth because a mass brought near the earth experiences a force towards the centre of the earth.

If a point mass m be kept at a distance d from the centre of the earth ($d >$ the radius of the earth) the force experienced by the body is given by,

$F = G.\ \frac{M.m}{d^2}$, where M=mass of the earth. Here the mass of the earth is supposed to be concentrated at its centre. If $m=1$, the force is given by $F = \frac{GM}{d^2}$. This force on a unit mass placed at a distance d from the centre of the earth is called *the intensity of earth's gravitational field at that point.* From the above relation, it is clear that the gravitational intensity is different at different points, being smaller further the point is from the centre of the earth.

In the same way, gravitational potential at a point, due to the earth is defined as the amount of work done in moving a unit mass from infinity to the point. As the gravitational force between the earth and the unit mass is a force of attraction, the work done is negative and the gravitational potential is, also, negative. Consider a particle of mass m placed at a distance r from the centre of the earth. Gravitational intensity on the mass $F = \frac{GM}{r^2}$, where M is the mass of the earth, supposed concentrated at its centre. If the mass m be now moved through a distance dr towards the earth, the work done $= -F.dr. = \left(-\frac{GM}{r^2}\right)dr$. So, the total work done in transferring the mass from infinity to a point distant r from the centre of the earth

$$= \int_0^{\propto} -\frac{GM}{r^2}.\ dr = \left[\frac{GM}{r}\right]_0^{\propto} = -\frac{GM}{r}$$

According to definition, the potential V at the point is $= -\frac{GM}{r}$.

This shows that the gravitational potential due to the earth or any other mass M, is maximum at infinity and is equal to zero. At all other points, the potential is less than zero *i.e.* has a negative value.

Example : *At a point on the line joining the centres of the earth and the moon, the gravitational intensities of the aforesaid two bodies are equal and opposite. Find the distance of the point from the centre of the earth. The mass of the earth is 81 times the mass of the moon and their centres are* $3{\cdot}84\times10^5$ *km apart.*

Ans. Let the distance of the point from the centre of the earth be d. Then the gravitational intensity due to the earth at the point $=G.\frac{M_e}{d^2}$ and that due to the moon $=\frac{G.M_m}{(3{\cdot}84\times10^5-d)^2}$. According to the question,

$$G.\frac{M_e}{d^2}=G.\frac{M_m}{(3{\cdot}84\times10^5-d)^2} \quad \text{or} \quad \frac{M_e}{M_m}=\frac{d^2}{(3{\cdot}84\times10^5-d)^2}$$

$$\text{But } \frac{M_e}{M_m}=81 \quad \therefore \quad 81=\frac{d^2}{(3{\cdot}84\times10^5-d)^2} \quad \text{or} \quad 9=\frac{d}{3{\cdot}84\times10^5-d}$$

$$\text{or, } d=3{\cdot}46\times10^5 \text{ km.}$$

1.6. Gravity and acceleration due to gravity : *The attraction of the earth on any terrestrial body or on a body situated very near the earth, is called gravity.* The force of which we are constantly aware in our daily life, is the force of gravity which endows everybody with its 'weight'. Gravity is, however, a special case of gravitation.

We know, from Newton's second law of motion that when a force acts on a body, the body gets an acceleration. Hence when a body falls downward, being acted on by the force of gravity, it falls with an acceleration which is known as the *acceleration due to gravity.* It is usually expressed by the letter 'g'.

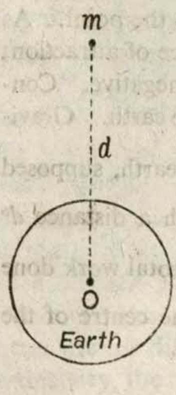


Fig. 1.1

Suppose a body of mass m is at a distance 'd' from the centre of the earth [Fig. 1.1]. If the body be released, it falls towards the earth with an acceleration, known as the acceleration due to gravity. If the mass of the earth be M and F the force of attraction, then according to Newton's law of gravitation, $F=G.\frac{M.m.}{d^2}$

Here, the whole mass of the earth is supposed concentrated at its centre O.

If the body, now, falls with an acceleration 'g', then according to Newton's second law of motion, $F=m.g.$

$$\therefore\ mg=G.\frac{M.m}{d^2} \text{ or, } g=\frac{G.M}{d^2}$$

Since G and M are constants, $g\propto\frac{1}{d^2}$, *i.e.*, the acceleration due to gravity at a place varies inversely as the square of its distance from the centre of the earth.

The above relation further shows that acceleration due to gravity at a place does not depend on the mass of the body but depends on the mass of the earth.

It is to be noted that g is equal to the gravitational field intensity at a point distant d from the centre of the earth [ref art 1·5]. Further if $d=R$ (radius of the earth) the value of g is given by $g=\frac{GM}{R^2}=\frac{G\times\frac{4}{3}\pi R^3.\rho}{R^2}=\frac{4\pi}{3}.G.R.\rho.$ where $\rho=$ mean density of the earth.

Why does a body fall towards the earth and not the earth towards the body? It has been said that the body of mass 'm' when let fall, falls towards the earth due to the force of gravity. But according to Newton's law of gravitation, the force of gravity is mutual. Why, then, do we find the body moving towards the earth and not the earth towards the body? The answer is very simple.

We have seen that the force of attraction F between the body and the earth is given by $F=G.\frac{Mm}{d^2}$

Now, the acceleration with which the body moves towards the earth

$$=\frac{\text{force on the body}}{\text{mass of the body}}=\frac{F}{m}=\frac{G.Mm}{d^2 m}=\frac{GM}{d^2}$$

Again, the acceleration with which the earth moves towards the body

$$=\frac{\text{force on the earth}}{\text{mass of the earth}}=\frac{F}{M}=\frac{G.M.m}{d^2 M}=\frac{Gm}{d^2}$$

$$\therefore\ \frac{\text{acceleration of the body}}{\text{,, \quad ,, \quad ,, earth}}=\frac{M}{m}$$

Since the mass M of the earth is very great comapred to the mass m of the body, the acceleration of the body will also be very great compared to that of the earth. This is why we see the body moving towards the earth and not the earth towards the body.

1.7. Variation of the value of acceleration due to gravity :

The value of 'g' at a place may change due to the following reasons :—(i) non-spherical shape of the earth (ii) altitude of the place (iii) depth inside the earth (iv) diurnal motion of the earth. We shall, now, discuss these factors one by one.

(i) **For the non-spherical shape of the earth** : The shape of the earth is not exactly spherical. Its polar regions are a bit flattened while the equatorial regions a bit bulged out. Consequently, the equatotial radius and the polar radius are not equal, equatorial radius being about 13 miles more than the polar radius. We have seen that the value of 'g' at a place is inversely proportional to the square of the distance of the place from the centre of the earth. Hence, the value of 'g' at the poles will be greater than the value at the equator.

(ii) **Due to the altitude of the place :** Suppose, the acceleration due to gravity at a place on the surface of the earth is 'g', and that at a height 'h' is 'g_1'. Taking the earth as a complete sphere, the acceleration due to gravity, we know, varies inversely as the square of the distance of the place from the centre of the earth. So, if R be the radius of the earth.

$$\frac{g}{g_1}=\frac{(R+h)^2}{R^2}=\frac{R^2+2.R.h+h^2}{R^2}=1+\frac{2h}{R}+\frac{h^2}{R^2}$$

'h' being very small compared to R, $\frac{h^2}{R^2}$ may be neglected.

So, $\frac{g}{g_1}=1+\frac{2h}{R}$ $\therefore$ $g_1=\frac{g}{1+\frac{2h}{R}}=g\left(1+\frac{2h}{R}\right)^{-1}=g\left(1-\frac{2h}{R}\right)$

This equation shows that with the increase of altitude 'h' the value of 'g' decreases.

Further, $g-g_1$ =decrease in the value of acceleration due to gravity due to altitude$=\frac{2.h.g}{R}$ .. .. .. (i)

[Alternative calculus method :

The small variation of g due to a small variation of distance from the centre of the earth is best handled by calculus. We know, $g=\frac{GM}{R^2}$ where R is not necessarily the earth's radius but the distance from the earth's centre. By differentiating we get, $dg=-\frac{2\,GM}{R^3}.\ dR$. Dividing the second equation by the first, we get $\frac{dg}{g}=-2\frac{dR}{R}$. The above relation is true if dg and dR are small compared with g and R.]

Examples : (1) *How far away from the earth does the acceleration due to gravity become 1% of its value at the earth's surface? Assume that the earth is a sphere of radius* $6{\cdot}38\times10^8$ *cm.*

Ans. Suppose, the required height is h cm. If the acceleration due to gravity there be g_1 and that on the surface of the earth be g, then $\frac{g_1}{g}=\frac{R^2}{(R+h)^2}$

But according to the problem, $\frac{g_1}{g}=\frac{1}{100}$

$$\therefore \frac{1}{100}=\frac{R^2}{(R+h)^2} \text{ or, } \frac{1}{10}=\frac{R}{R+h}$$

$$\therefore h=9.R=9\times6{\cdot}38\times10^8 \text{ cm.} =57{\cdot}42\times10^8 \text{ km.}$$

(2) *At what height above the earth's surface has g diminished by* $\frac{1}{10}$*th of 1%? Radius of the earth=6380 km.*

Ans. Here $\frac{dg}{g}=\frac{1}{10}$ th of $\frac{1}{100}=\frac{1}{1000}=\cdot001$

$$\therefore \quad -\frac{dg}{g}=-\frac{2dR}{R} \text{ [minus because } g \text{ decreases]}$$

$$\text{or } dR=\frac{R}{2}\cdot\frac{dg}{g}=\frac{6380}{2}\times\cdot001=3\cdot2 \text{ km. (nearly)}$$

(iii) **Due to depth inside the earth :** Let O be the centre of the earth, R the radius and x the distance of a body of mass 'm' from the surface of the earth inside it [Fig. 1.2]. If, now, an imaginary sphere be drawn with O as centre and $(R-x)$ as radius, it will divide the earth into two parts—one, a solid internal sphere of radius $(R-x)$ and the other, a thick shell of thickness x. The body of mass 'm' is just outside the internal sphere and inside the thick shell. Under this circumstances, it may be proved that the body will feel gravitational attraction due to the internal sphere only and the shell will exert no force on it. If F be the force of attraction experienced by the body, then,

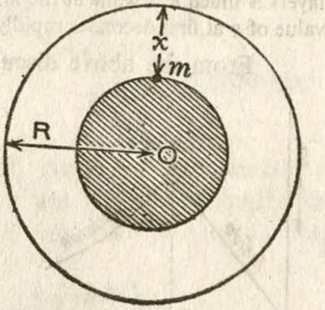


Fig. 1.2

$$F=G\times\frac{\text{mass of the internal sphere}\times m}{(R-x)^2}$$

If, again, the average density of the earth be ρ, then, the mass of the internal sphere$=\frac{4}{3}\pi(R-x)^3\,\rho$.

$$\therefore \quad F=\frac{G\times\frac{4}{3}\pi(R-x)^3.\rho.m.}{(R-x)^2}=G.\times\tfrac{4}{3}\pi(R-x).\rho.m.$$

$$\text{So, } g_x=\frac{\text{force on the body}}{\text{mass of the body}}=\frac{F}{m}=G\times\tfrac{4}{3}\pi(R-x).\rho$$

Here, G and ρ are constants ; so if x be increased, *i.e.*, if the body be taken to a greater depth inside the earth, $(R-x)$ will decrease and hence g_x will decrease. If it were possible to take the body at the centre of the earth, then g_x would have been equal to zero because $x=R$ and $R-x=0$; So, it can be said that *the acceleration due to gravity at the centre of the earth is zero.*

If g be the acceleration due to gravity on the surface of the earth,

$$\text{then } g=\frac{4\pi}{3}\times G.R.\rho. \quad ..(i) \qquad \text{So,} \frac{g_x}{g}=\frac{\frac{4\pi}{3}\times G\,(R-x).\rho}{\frac{4\pi}{3}\times G.R.\rho.}=1-\frac{x}{R}$$

$$\therefore \quad g_x=g\left(1-\frac{x}{R}\right)=\frac{g}{R}(R-x)$$

So, *the acceleration due to gravity at a point inside the earth, is proportional to the distance of the point from the centre of the earth.*

Also, $g-g_x=\frac{x}{R}.g$ ..(ii)

Comparing equations (i) and (ii), we can say that the acceleration due to gravity at a point above the surface of the earth is less than the acceleration due to gravity at the corresponding point inside the earth.

[**Note :** The earth is not a homogeneous sphere. The density of the earth at upper layers is much less while at the lower layers, the earth material is very dense. Consequently, the value of g at first decreases rapidly with the depth but later on, the rate of decrease becomes less.]

From the above discussion, it is clear that the value of g decreases as we go higher up as well as inside the earth. Above the earth, g decreases inversely as the square of the distance from the centre of the earth while inside the earth, it decreases proportionately with the distance from the centre of the earth. This change of the value of g with the distance from the centre of the earth has been shown in fig. 1.3.

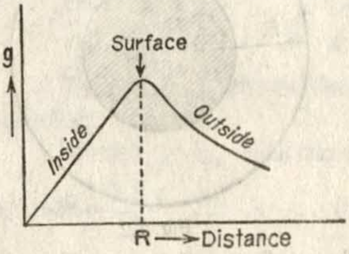


Fig. 1.3

Example : *The ratio of the acceleration due to gravity in a deep mine and on the surface of the earth is* $\frac{790}{800}$. *Assuming the earth to be of uniform density throughout and the radius of the earth is 4000 miles, calculate the depth of the mine.*

Ans. If the depth of the mine be x miles, then, we know, the acceleration due to gravity there is $g_x=G.\frac{4}{3}\pi(R-x)\,\rho$.

The force of gravity on the body at the surface of the earth

$$=G.\frac{\frac{4}{3}\pi R^3\rho.m.}{R^2}=G.\frac{4}{3}R.\rho.m.$$

So, acceleration due to gravity, $g=\frac{\text{force}}{\text{mass}}=\frac{G.\frac{4}{3}\pi R.\rho.m.}{m}=\frac{4}{3}G\pi R.\rho.$

$$\therefore \frac{g_x}{g}=\frac{G.\frac{4}{3}\pi(R-x)\rho}{G.\frac{4}{3}\pi R.\rho.}=\frac{R-x}{R}=1-\frac{x}{R}$$

$$\therefore \frac{790}{800}=1-\frac{x}{4000}\,;\ \therefore\ x=\frac{4000\times10}{800}=50 \text{ miles.}$$

(iv) **Due to the earth's diurnal motion :** The earth rotates about its own axis from the west to the east. Had the earth been at rest and a sphere of uniform density, the acceleration due to gravity of a body would have been same everywhere on the surface and would have been directed to the centre of the earth. But due to the diurnal motion of the earth, a part of the gravitational force on the body is expended in counteracting the centrifugal force of the body which partakes of the diurnal motion along with the earth. Consequently, its acceleration due to gravity becomes less than what it would have been if the earth were at rest.

Let a body of mass *m* be placed at *A*, whose latitude is ϕ. If ω be the angular velocity of the earth about its own axis, the body also executes a rotational motion with same angular velocity along a circle of radius $r=R.\cos\phi$. Due to this rotational motion, the body experiences a centrifugal force $m\omega^2 r$ acting outward as shown in fig. 1.4. The force of gravity *i.e.* the weight *mg* which would act on the body, *if the earth were at rest* (*g* being the acceleration due to gravity with the earth at rest) would obviously act towards the centre of the earth. A part of this force balances the outward centrifugal force and keeps the body on the surface of the earth. If the centrifugal force $m\omega^2 r$ be resolved, then for the motion of the body, the weight balances the component $m\omega^2 r\cos\phi$. Consequently, the apparent weight of the body at $A=mg-m\omega^2 r.\cos\phi=mg-m\omega^2 R\cos^2\phi$. If g_ϕ be the acceleration due to gravity at A, then,

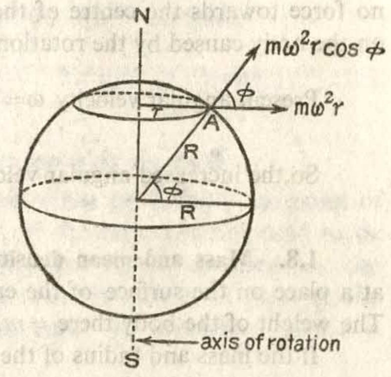


Fig. 1.4

$$m.g_\phi=mg-m\omega^2 R\cos^2\phi \quad \therefore\ g_\phi=g\left(1-\frac{R\omega^2\cos^2\phi}{g}\right)$$

Now, $\dfrac{R.\omega^2}{g}=\dfrac{1}{289}$, taking $g=978{\cdot}03$ cm/s^2 at the equator. So, $g_\phi=g\left(1-\dfrac{\cos^2\phi}{289}\right)$

Now, at the equator $\phi=0$, so, $\cos\phi=1$, g_ϕ is minimum. (978/cms^2)

Again at the poles $\phi=90°$, so, $\cos\phi=0$ and g_ϕ is maximum. (983·2 cm/s^2)

So, with the increase of latitude, the value of *g* increases.

[N.B. Since weight of a body is proportional to *g* (art 1.10), the weight will vary for all the reasons stated above in accordance with the variation of *g*].

Mean value of the acceleration due to gravity : The mean values of '*g*' on the surface of the earth in different systems are as follows : In the *C.G.S.* system, $g=980$ cm/sec^2 ; in the *F.P.S.* system, $g=32$ ft/sec^2, and in the *M.K.S.* system, $g=9{\cdot}8$ metre/ sec^2.

Example : *Show that the objects lying at the equator will start flying off the surface of the earth, if the speed of rotation of the earth increases 17 times its present value.*

Ans. The apparent acceleration due to gravity of an object at equator ($\phi=0$) is given by, $g_\phi=g\left(1-\dfrac{R.\omega^2}{g}\right)$ where ω is the present speed of rotation.

If the speed of rotation increases 17 times, the apparent acceleration due to gravity at the equator, $g_{\phi'}=g\left\{1-\dfrac{R(17\omega)^2}{g}\right\}=g\left\{1-\dfrac{289.R.\omega^2}{g}\right\}$

But $\dfrac{R.\omega^2}{g}=\dfrac{1}{289}$; hence, $g_{\phi'}=g\left\{1-\dfrac{289}{289}\right\}=0$

Therefore the body lying at the equator will start flying off because there is no force towards the centre of the earth to counter balance the centrifugal force on the body caused by the rotation of the earth.

Present angular velocity $\omega = \frac{2\pi}{T} = \frac{2 \times 3{\cdot}14}{24 \times 60 \times 60}$ rad/s.

So, the increased angular velocity $= \frac{2 \times 3{\cdot}14 \times 17}{24 \times 60 \times 60}$ rad/s $= 0{\cdot}00123$ rad/s.

1.8. Mass and mean density of the earth : Consider a body of mass 'm' at a place on the surface of the earth where the acceleration due to gravity is 'g'. The weight of the body there $= m.g.$

If the mass and radius of the earth be M and R respectively, the gravitational attraction of the earth on the body $= \frac{G.\ Mm}{R^2}$

Since weight of a body is the total attraction of the earth on the body, we have, $mg = G.\frac{M.m}{R^2} \quad \therefore \quad M = \frac{g.R^2}{G}$

Taking $g = 980$ cm/sec^2 ; $R = 6{\cdot}36 \times 10^8$ cm. and $G = 6{\cdot}67 \times 10^{-8}$ *C.G.S.*, we have, $M = 5{\cdot}94 \times 10^{27}$ gm.

Again, assuming the earth to be a sphere of uniform density ρ,

$M = \frac{4}{3}\pi\ R^3.\rho \quad \therefore \quad \frac{4}{3}\pi R^3 \rho = \frac{gR^2}{G}$ or $\rho = \frac{3g}{4\pi RG}$

Substituting the values of g, R and G, we get : $\rho = 5{\cdot}52$ gm/c.c.

But the earth is not made of homogeneous material of uniform density. The density of the terrestrial material in the upper strata is only 2·7 gm./c.c. So, the density at the lower strata is much greater than 5·52 gm/c.c.

Example : *If the mean density of the earth be 5500 kg/m³, Gravitational constant be 6·7×10⁻¹¹ Nm²/Kg² and the radius of the earth 6400 km, calculate the value of the acceleration due to gravity on the surface of the earth.*

Ans. We know, $\rho = \frac{3g}{4\pi R.G.}$ or $g = \frac{4\pi}{3}.\ R.G.\rho$. Putting the values, we get,

$$g = \frac{4\pi}{3} \times 6400 \times 1000 \times 6{\cdot}7 \times 10^{-11} \times 5500 \text{ m/s}^2 = 9{\cdot}9 \text{ m/s}^2 \text{ (approx).}$$

Mass of the Sun :

The mass of the sun can be computed from the period of revolution of the earth round it. If ω be the angular velocity and T the time-period of revolution of the earth round the sun, then, for the circular motion of the earth,

$G.\ \frac{M_s.M_e}{r^2} = M_e \omega.^2 r$ or, $M_s = \frac{r^3.\omega^2}{G}$, where r is the distance between the centre of the sun and the centre of the earth. But $\omega = \frac{2\pi}{T}$; $\therefore \quad M_s = \frac{4\pi^2 r^3.}{T^2.\ G.}$

Taking $T=365$ days, $r=1\cdot5\times10^{13}$ cm, $G=6\cdot70\times10^{-8}$ we have,

$$M_s=\frac{4\pi^2(1\cdot5\times10^{13})^3}{(365\times24\times60\times60)^2\times6\cdot70\times10^{-8}}=2\times10^{33}\text{ gm. (nearly)}$$

1.9. Motion of a body through a frictionless tunnel in the earth :

Consider a tunnel bored through the centre of the earth from one point of the surface to the diametrically opposite point. If the tunnel is supposed to be frictionless, it may be proved in the following way that a body dropped into the tunnel will move simple harmonically from one point of the surface to the opposite point along the diameter. For this reason *the earth is sometimes called a bottomless well.*

Suppose the body dropped into the tunnel is at P at any instant at a distance x from the centre (O) of the earth [Fig. 1.5]. If a sphere be imagined to be drawn with O as centre and x as radius then we know that this sphere will exert gravitational attraction on the body at P, the shell of thickness AP having no effect on the body. The shell exerts no attraction at any point inside it.

If ρ be the mean density of the earth and m the mass of the body dropped, then the force of attraction F on the body is,

$$F=\frac{G\times\text{mass of the internal sphere}\times m}{x^2}$$

$$=\frac{G\times\frac{4}{3}\pi x^3\rho\times m}{x^2}=\frac{4}{3}\,\pi\,G.\,\rho.\,m.\,x.$$

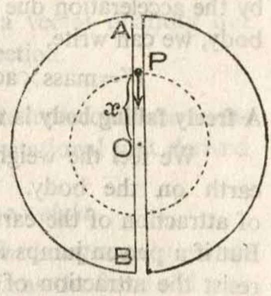


Fig. 1.5

The acceleration of the body $f=\frac{F}{m}=\frac{4}{3}\cdot\frac{\pi\,G.\,\rho.\,m.\,x.}{m}=\frac{4}{3}\,\pi\,G.\rho.x.$

We see that $f\propto x$ and the acceleration is directed towards the centre of the earth. This shows that the motion of the body satisfies the condition of *S. H. M.*. So it may be said that the body will execute simple harmonic oscillations along the tunnel AB.

Now, the time-period of the oscillation, T is given by

$$T=2\pi\sqrt{\frac{\text{Displacement}}{\text{acceleration}}}=2\pi\sqrt{\frac{x}{\frac{4}{3}\pi G\rho x}}=2\pi\sqrt{\frac{3}{4\pi G\rho}}$$

If 'g' be the acceleration due to gravity on the surface of the earth then, we have seen in art 1.8, $\rho=\frac{3g}{4\pi RG}$ $\qquad\therefore\quad T=2\pi\sqrt{\frac{3}{4\pi G}\times\frac{4\pi RG}{3g}}=2\pi\sqrt{R/g}$

Taking $R=6400$ kilometres and $g=980$ cm./sec^2, we get

$$T=2\pi\sqrt{\frac{6400\times10^5}{980}}\text{ sec}=84\text{ min. }35\text{ sec. (nearly).}$$

It shows that the body will travel the whole diameter of the earth once in 42 min. 21 sec. Note that this time does not depend on the mass of the body dropped in the tunnel.

[**N. B.** The same result holds good even if the tunnel is imagined to be bored in aninclined way *i.e.* not through the centre of the earth.]

1.10. Weight of a body : When we place a body on our palm, we feel a downward force on it. If the body be very heavy, we may feel it difficult to hold it on our palm or to lift it from the ground. What is the reason of it? The reason is that the body is attracted by the earth or that the force is a force of gravity.

Definition : *The weight of a body is the force with which it is attracted towards the earth.* It should be borne in mind that *weight is a force.*

From Newton's second law of motion, we know, force=mass×acceleration.

So, to measure the force of gravity on a body, its mass is to be multiplied by the acceleration due to gravity. Since this force gives the weight (W) of the body, we can write,

$$W = \text{mass} \times \text{acceleration due to gravity} = m \times g$$

A freely falling body is weightless :

We feel the weight of a body when we resist the force of attraction of the earth on the body. Thus when we put a load on our back, we resist the force of attraction of the earth on the load by our muscles and then we feel its weight. But if a person jumps from some height with some load on his back, he need not resist the attraction of the earth as long as he is falling and hence he does not feel any weight on his back during free fall. But as soon as his feet touch the ground he again feels that some load has been put on his back. For this reason Galileo first said that '*a falling body has no weight*'. A simple experiment with a spring balance illustrates it nicely.

Let us take a spring balance. Suppose it reads zero when no load is put on the hook of the balance. Now put a load on the hook and suppose the pointer reads 5 lb. This shows that the load weighs 5 lb. Now, allow the spring balance to fall down vertically along with the load. During free fall, it will be seen that the pointer reads zero, as if there is no load on the hook. This conclusively proves that during free fall, the load becomes weightless.

Difference between mass and weight : In everyday conversation, the distinction between mass and weight is not very important but in science we must be careful to distinguish between the two. They have the following differences :

(i) The mass of a body means the matter contained in the body but the weight is a force *viz*, the force with which a body is attracted towards the earth.

(ii) The mass of a body multiplied by 'g' gives the weight of the body ; hence the two cannot be identical.

(iii) The mass of a body remains unchanged wherever the body may be taken to, but the weight of a body changes due to the change in the value of 'g'. Thus, the weight of a body is less at the top of a mountain than at the surface of the earth. Since at the centre of the earth $g=0$, a body will be weightless there.

Moreover, the weight of a body will differ if it is taken to a different planet or to the moon, but its mass will remain the same. The mass of other planets or of the moon being different from the mass of the earth, the gravitational attraction at those places will be different. For example, the weight of a body at the surface of the moon will be one sixth of the weight at the surface of the earth, because the mass of the moon is about one sixth of the mass of the earth. If, for some reason or other, the earth were to lose its gravitational attraction, all bodies would be rendered weightless although the masses would remain unaffected.

Since the mass of a body is not affected by the motion, temperature, electrification, magnetisation etc. of the body, it is considered as an *intrinsic property* of the body while weight, which is variable, is not so.

Professor Albert Einstein, however, proved with the help of the famous theory of Relativity that the mass of a body increases with the speed of the body when the speed is comparable with the velocity of light. But for ordinary velocity, the change is not appreciable.

(iv) Weight has magnitude and direction, *i.e.*, it is a vector quantity but mass is a scalar quantity having only magnitude and no direction.

1.11. Gravitational units of force : Besides absolute unit of force mentioned earlier, there is another unit of force, known as gravitational unit derived from the law of gravitation.

In the *C.G.S.* system this unit of force is called *gramme-weight.*

Definition : *The force with which a body of mass 1 gm is attracted towards the earth, is called a gramme-weight.* Sometimes, it is known as the force of 1 gm.

Now, 1 gm-wt $=1$ gm $\times g$ dynes $=980$ dynes (nearly)

In the *F.P.S.* system, the name of the gravitational unit of force is *pound-weight* (lb wt).

Definition : *The force with which a body of mass 1 lb is attracted towards the earth is called a lb-wt.* Sometimes it is known as the force of 1 lb.

Now, 1 lb-wt $=1$ lb $\times g=g$ poundals $=32$ poundals (nearly).

In the *M.K.S.* system, the gravitational unit of force is *kilogram weight.*

Definition : *The force with which a body of mass 1 kg is attracted towards the earth is called a kilogram weight.* It is sometimes known as the force of 1 kg.

1 kilogram weight $=1$ kilogram $\times g=9{\cdot}8$ Newtons (N).

1.12. Motion under gravity : The ancient Greek philosopher Aristotle described the motion of a freely falling body by saying that the heavier the body, the faster it would fall. This, at first thought, seems to be true but actually it is not so. In the latter part of the sixteenth century, the great Italian scientist Galileo tried some experiments which convinced him that it was merely the disturbing effect of air resistance causing a light object fall more slowly than a heavy one. In vacuum, all bodies fall at the same rate.

Galileo also established certain laws in connection with a body falling under gravity. These laws are known as the laws of falling bodies.

1.13. Laws of falling bodies : The following are the laws of bodies falling freely under gravity :

(i) Falling freely from rest, all bodies fall with equal rapidity.

(ii) Falling freely from rest, the velocity acquired by a body is proportional to the time of fall.

(iii) Falling freely from rest, the space travelled by a body is proportional to the square of the time of fall.

Explanation of the laws : 1st law—It has been mentioned before that the law was established by Galileo in 1589. Newton verified it experimentally by his famous 'guinea and feather' experiment, which is illustrated in fig. 1.6.

A long glass cylinder containing a feather (light substance) and a guinea (heavy coin) is connected by a flexible tube to a vacuum pump. If after evacuation, the tube is turned upside down, the feather and the coin will be found to fall together. When the air is admitted and the tube again turned upside down, the feather will flutter slowly to the bottom while the coin will reach the bottom quickly. It shows that in the absence of air resistance, all bodies fall with equal rapidity which again means that the acceleration due to gravity at a place is same for all bodies.

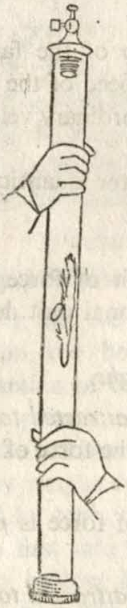


Fig. 1.6

2nd law—If the body falls for t seconds and the velocity acquired by it at the end of the time be v, then according to the law, $v \propto t$.

Thus, if the velocity of the body at the end of first second be 32 ft/sec, the velocity at the end of 2nd or 3rd. second will be 32×2 ft/sec or 32×3 ft/sec etc.

3rd law—If 'h' be the height dropped through by the body in time t, then according to the law, $h \propto t^2$.

Thus, if the body descends 16 ft at the end of first second, then it will descend $16 \times (2)^2$ ft. and $16 \times (3)^2$ ft. at the end of second and third second respectively and so on.

1.14. Equations of motion under gravity :

(*a*) *For bodies falling downward* : When a body falls downward under gravity, its motion is accelerated and the acceleration is the acceleration due to gravity 'g'. In ordinary equations of rectilinear motion (ref. Mechanics art 1.5), if the acceleration 'f' is replaced by 'g', we shall get the equations of motion of falling bodies. The quantity 'S' in those equations should, however, be replaced by the height 'h'. So, for falling bodies, we can write,

$$v = u + g.t$$

$$h = u.t + \tfrac{1}{2}g.t^2$$

$$v^2 = u^2 + 2g.h.$$

(*b*) *For bodies moving upward* : When a body is projected straight upward, its speed will rapidly diminish until at some point it comes momentarily to rest and then falls back towards the earth, acquiring again at the ground the same speed it had upon projection. Experiment shows that the time taken to rise to the highest point of its trajectory is equal to the time taken to fall from there to the ground. This implies that the upward motions are just the same as the downward motions but in reverse.

To treat the motion mathematically, it is convenient to use the aforesaid equations taking the point of projection as the *origin* and adopting the following conventions of signs, for projection upward :

(i) Distances above the origin are positive.
(ii) Distances below the origin are negative.
(iii) Velocities upward are positive.
(iv) Velocities downward are negative.
(v) Acceleration downward (gravity) is negative.

Whether a body is moving up or down, the acceleration g is always downward. So, for projection upward, the aforesaid equations are to be used with a minus sign put before g.

(*c*) **Greatest height attained by a body :**

Let a body be projected vertically upward with a velocity u. At the greatest height, the body will be momentarily at rest. If the greatest height attained be H, then from the equation $v^2=u^2+2f.S$, we may write $0=u^2-2.g.H$ [g is $-ve$].

$$\therefore \quad H=\frac{u^2}{2g} \quad ..(\text{i})$$

(*d*) **Time to attain the greatest height :**

If T be the time required by the body to attain the greatest height, then from **the equation, $v=u+ft$,** we have, $0=u-gT$ or $T=\frac{u}{g}$. ..(ii)

(*e*) **Time to fall from the greatest height :**

If 'T'' be the time to fall from the greatest height, then from the equation, $S=ut+\frac{1}{2}gt^2$ we have, $H=\frac{1}{2}gT'^2$ or $T'^2=\frac{2H}{g}=\frac{2}{g}\times\frac{u^2}{2g}$ or, $T'=\frac{u}{g}$ ..(iii)

From eqn (ii) and (iii) we see that $T=T'$ *i.e. time to attain the greatest height is equal to the time to fall from the greatest height.*

$$\therefore \quad \text{Total time of flight}=\frac{2u}{g}$$

The examples below will illustrate the applications of the above rules and equations.

Examples : (1) ***With what velocity should a body be projected so as to attain just a height of 144 ft? When will the body be at a height of 80 ft from the ground? Explain the double answer.***

Ans. Just at the height of 144 ft. the body will be momentarily at rest. Here $v=0$; $h=144$ ft ; $g=-32$ ft/sec^2 ; $u=?$

We know, for a body projected upward, $v^2=u^2-2.g.h$,

$\therefore\ 0=u^2-2.32.144$ or, $u^2=2.32.144 \quad \therefore \quad u=96$ ft/sec.

In the second case, $u=96$ ft/sec ; $h=80$ ft ; $g=-32$ ft/sec^2; $t=$? We know $h=ut+\frac{1}{2}gt^2$ or, $80=96.t-\frac{1}{2}.32.t^2$

or, $5=6t-t^2$ or, $t^2-6t+5=0 \quad \therefore \quad t=5$ sec ; 1 sec.

Double answer shows that the body attains a height of 80 ft while ascending and will again come to a height of 80 ft. from the ground while descending after attaining the greatest height.

(2) *A stone is let fall from a balloon ascending vertically with a velocity of 32 ft/sec. If at that time, the height of the balloon from the ground is 3200 ft. find* (*a*) *the greatest height the stone will attain* (*b*) *the time taken by the stone to come back to the same point again* (*c*) *the velocity with which it will touch the ground.*

Ans. (*a*) When the stone is dropped, its velocity is the same as that of the balloon and at the greatest height, this velocity of the stone will be reduced to zero. So, $u=32$ft/sec ; $v=0$; $g=-32$ ft/sec^2 ; $h=?$

$$\text{We know, } v^2=u^2+2\,g.h \text{ or, } 0=(32)^2-2.32.h \ \therefore\ h=\frac{32\times 32}{2\times 32}=16 \text{ ft.}$$

So, the greatest height attained by the stone from the ground $=3200+16=$ 3216 ft.

(*b*) When the stone comes back to the same point from where it is dropped, the vertical height$=0$, *i.e.*, $h=0$. So, here $h=0$, $u=32$ ft/sec^2; $g=-32$ft/s^2; $t=?$

We know, $h=u.t+\frac{1}{2}.g.t^2$ or, $0=32t-\frac{1}{2}.32.t^2$ or, $16t=32 \ \therefore\ t=2$ seconds.

(*c*) When the stone touches the ground it goes below the point of projection. According to the conventions of sign, $h=-3200$ ft. ; $g=-32$ ft/sec^2, $u=32$ ft/sec ; $v=?$

We have, $v^2=u^2+2.g.h.=(32)^2+2(-32)(-3200)=32\times 32\times 201$

$\therefore\ v=454{\cdot}4$ ft/sec.

(3) *A stone is dropped from a tower 300 ft. high and at the same time, another stone is projected upward from the foot of the tower with a velocity of 100 ft/sec. When and where will the two stones meet?*

Ans. Suppose after t seconds, they meet each other. Now, for the *downward* motion of the first stone, we have, $u=0$, $g=32$ ft/sec^2, $t=t$, $h=?$

From, $h=ut+\frac{1}{2}gt^2=0+\frac{1}{2}.32t^2=16t^2$

The second stone ascends $(300-h)$ ft. So, for the *upward* motion of the second stone, we have, $u=100$ ft/sec ; $t=t$, $h_1=300-h$; $g=-32$ ft/sec^2

From, $h_1=ut+\frac{1}{2}gt^2$

we have, $300-h=100t-\frac{1}{2}.32.t^2=100.t-16t^2$

or, $300-16t^2=100t-16t^2 \quad [\because\ h=16t^2] \quad \therefore\ t=3$ sec.

So, the stones meet each other $16\times(3)^2=144$ ft. below the top of the tower.

(4) *A ball falls from a height of 10 metres with an initial downward velocity v_0. It collides with the ground, loses 50% of its energy in collision and then rises back to the same height. Find (i) the initial velocity v_0 and (ii) the height to which the ball would rise after collision if the initial velocity v_0 was directed upward instead of downward.* *[I. I. T. 1979]*

Ans. (i) Let v be the velocity of the ball just before it touches the ground. Then, $v^2=v_0^2+2g.h.$....(i)

Let v_1 be the velocity with which the ball started to move upward after colliding against the ground. Since it ascended the same height, we have $v_1^2=2g.h.$ Further since, the energy is reduced by 50%, due to collision, we have, $\frac{1}{2} mv_1^2=\frac{1}{2}\times\frac{1}{2} mv^2$ or $v_1^2=\frac{1}{2}v^2$ $\therefore$ $\frac{1}{2}v^2=2.g.h.$

So, from eqn. (i) $v_0^2=v^2-2g.h.=4g.h.-2gh=2g.h.=2\times 980\times 10\times 100$

$\therefore$ $v_0=1400$ cm/s$=14$ metre/s.

(ii) When the initial velocity v_0 is directed upwards, let the greatest height attained be h_1. Since the velocity at the greatest height is zero, we have, $v_0^2=2gh_1$..(i)

Let the velocity of the ball just before it touches the ground be v_2. Then $v_2^2=2g(h+h_1)$. Suppose, the velocity with which the ball moves upward after collision is v_3. As the loss of energy is 50%, we have, $\frac{1}{2}mv_3^2=\frac{1}{2}\times\frac{1}{2}mv_2^2$ or $v_3^2=\frac{1}{2}.v_2^2$ Let the height attained be h'. Then,

$$2gh'=v_3^2=\frac{1}{2}.v_2^2=\frac{1}{2}\,[2g(h+h_1)]=g(h+h_1)$$

$$\therefore\ h'=\frac{1}{2}(h+h_1)=\frac{1}{2}\left(10+\frac{14\times 14}{2\times 9{\cdot}8}\right)=10 \text{ metres [From eqn. (i)]}$$

(5) *An obstacle 14·7 metres high is at a horizontal distance of 19·6 metres from an observer. At what angle should the observer project a body with an initial velocity of 19·6 metres/s, so that it can just pass over the obstacle? Given $g=9{\cdot}81$ m/s². Air resistance may be neglected.* *[Joint Entrance, 1980]*

Ans. Let the body be projected with a velocity u inclined at an angle θ with the horizontal [Fig. 1.7]. The horizontal component of the velocity of projection$=u\cos\theta=19{\cdot}6\cos\theta$ and vertical component$=u\sin\theta=19{\cdot}6\sin\theta$.

While passing over the obstacle, the body has travelled a horizontal distance of 19·6 metres with horizontal velocity 19·6 cos θ metres/s. If t be the time required then $19{\cdot}6=19{\cdot}6\cos\theta\times t$

$$\text{or, } t=\frac{1}{\cos\theta}=\sec.\ \theta.$$

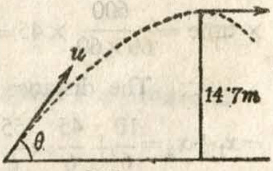


Fig. 1.7

Again, while passing over the obstacle, the body has travelled a vertical distance of 14·7 metre with vertical component 19·6 sin θ in time t. From the equation $S=ut+\frac{1}{2}ft^2$, we get,

$$\begin{aligned}14{\cdot}7&=19{\cdot}6\sin\theta\times t-\tfrac{1}{2}gt^2\\&=19{\cdot}6\sin\theta\times\sec\theta-\tfrac{1}{2}\times 9{\cdot}8\times\sec^2\theta\\&=19{\cdot}6\tan\theta-\tfrac{1}{2}\times 9{\cdot}8\times\sec^2\theta.\end{aligned}$$

or, $3 = 4\tan\theta - \sec^2\theta = 4\tan\theta - 1 - \tan^2\theta$.
or, $\tan^2\theta - 4\tan\theta + 4 = 0$ or $\tan\theta = 2$
$\therefore\ \theta = \tan^{-1} 2$.

(6) *A pilot jumps out of an aeroplane with a parachute when the plane is just above the point A on the surface of the earth and moving horizontally with a velocity of 600 km/hr. After 10 seconds, the parachute unfolds when the man descends with a retardation of 2 m/s². When he is about to touch the ground his velocity is 8 m/sec. Find the height of the plane from the surface of the earth and the distance of the spot where the man drops from the point A.* ($g = 9{\cdot}8$ *m/sec²*)

[*Jt. Entrance 1984*]

Ans. If S_1 be the height descended by the pilot during first 10 sec, then from the equation $S_1 = \frac{1}{2}gt^2$, we get $S_1 = \frac{1}{2} \times 9{\cdot}8 \times 100 = 490$ metres. At the end of first 10 sec., if his downward velocity be u, then $u = gt = 9{\cdot}8 \times 10 = 98$ m/s. When he is about to touch the ground his downward velocity is 8 m/s. After the unfolding of parachute, his downward motion is decelerated. If S_2 be the height descended during this decelerated motion, then from $v^2 = u^2 + 2f.S$, we can write, $(8)^2 = (98)^2 + 2(-2) \times S_2 = (98)^2 - 4 \times S_2 \quad \therefore \quad S_2 = 2385$ metre.

So, the height of the plane from the surface of the earth $= S_1 + S_2 = 490 + 2385 = 2875$ metres. Again, the horizontal displacement x_1 of the pilot during the first 10 sec interval is given by,

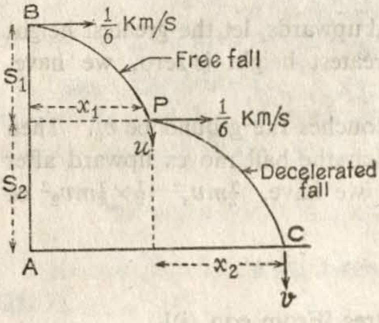


Fig. 1.8

$$x_1 = \text{his horizontal velocity} \times \text{time} = \frac{600}{60 \times 60} \times 10 = \frac{1}{6} \times 10 = \frac{10}{6} \text{ km.}$$

Time during which the pilot descends with deceleration is, suppose, t. Since $v = u + gt$. we have, $8 = 98 + (-2) \times t$ or $t = 45$ sec.

The horizontal displacement during this interval (x_2) = horizontal velocity $\times$ time $= \frac{600}{60 \times 60} \times 45 = \frac{45}{6}$ km.

$\therefore$ The distance of the spot C where the man drops from the point A $= x_1 + x_2 = \frac{10}{6} + \frac{45}{6} = \frac{55}{6}$ km. $= 9\frac{1}{6}$ km.

1.15. Motion of planets and satellites : From early civilisation stars, planets etc. aroused curiosity in the minds of the people. In the middle of the sixteenth century, Tycho Brahe, a Danish astronomer, carefully observed the motion of the planet 'Mars' and collected some important data. Later in the year 1609, another astronomer of Denmark, John Kepler, realised after further investigation and observation of planetary motions that planets move round the sun under some force. Still later, in the middle part of seventeenth century, the celebrated physicist

Sir Isaac Newton established the laws of gravitation and told that the planets move round the sun being attracted by the sun's gravitational force and the satellites round their respective planets due to the planet's gravitational attraction.

Kepler's laws of planetary motion :

1. The path of a planet is an elliptical orbit with the sun at one of its focil.
2. The radius vector, drawn from the sun to a planet sweeps out equal areas in equal time *i.e.*, the area swept out per unit time is constant.
3. The square of the period of revolution of a planet round the sun is proportional to the cube of its mean distance from the sun.

If T be the period of revolution of a planet and r the mean distance of the planet from the sun, then according to the third law, $T^2 \propto r^3$ or $\frac{T^2}{r^3}=$constant.

1.16. Orbital velocity and time of revolution of a planet :

If the orbits of the planets round the sun be supposed circular (in reality, the orbits are elliptical), we can find the speed and the time period of revolution of the planets and also establish the third law of Keplar by the application of the law of gravitation. Suppose, a planet of mass 'm' is revolving round the sun of mass M in a circular orbit of radius r (Fig. 1.9). We know that a body moving in a circular path requires centripetal force. In the case of planets, the centripetal force arises out of the gravitational force exerted by the sun on them. If linear speed of the planet be v, the centripetal force required by it is given by $F=\frac{mv^2}{r}$. Again, the gravitational force exerted by the sun on the planet$=G.\frac{M.m}{r^2}$ $\therefore$ $\frac{GMm}{r^2}=\frac{mv^2}{r}$

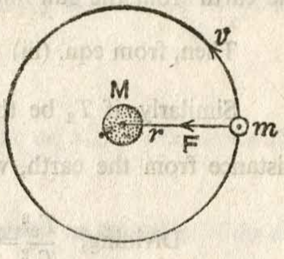


Fig. 1.9

$$\text{or, } v=\sqrt{\frac{GM}{r}} \quad \ldots \quad \text{(i)}$$

This is the orbital velocity of the planet.

If T be the time taken by the planet to make one complete trip around its orbit, then, $v=\frac{2\pi r}{T} \ldots \quad \ldots$(ii) Squaring (i) and (ii) we get,

$$\frac{4\pi^2 r^2}{T^2}=\frac{GM}{r} \text{ or, } T^2=\frac{4\pi.^2 r^3}{G.M.} \quad \ldots \quad \ldots \text{(iii)}$$

Since, G and M are constants, $T^2 \propto r^3$ *i.e.*, the square of the orbital period of the planet is proportional to the cube of its distance from the sun. This law was formulated by John Kepler long before the law of gravitation was established. Relation (iii) shows that the smaller the value of r *i.e.* smaller the distance of the planet from the sun, the smaller the value of T or the length of the year for it. *A planet will, therefore, have a shorter year if nearer to the sun than when at a distance from it.*

The satellites move round their respective planets. They derive the centripetal force from the gravitational attraction of the planets. Following the same procedure, it may be shown that the squares of the orbital periods of the satellites are proportional to the cubes of their distances from the planets.

Examples : (1) *The radius of the earth's circular orbit round the sun is 93×10^6 miles. The earth goes round the orbit once in 365 days. Calculate the orbital velocity of the earth in miles per hour.*

Ans. We know, $v = \frac{2\pi r}{T} = \frac{2 \times 3{\cdot}14 \times 93 \times 10^6}{365 \times 24} = 66{\cdot}7 \times 10^3$ miles/hr.

(2) *The moon describes a circular orbit of radius $3{\cdot}8 \times 10^5$ km about the earth in 27 days and the earth describes a circular orbit of radius $1{\cdot}5 \times 10^8$ km round the sun in 365 days. Find the ratio of the mass of sun to that of the earth.*

Ans. Let M_s=mass of the sun ; M_e=mass of the earth ; d_1=the distance of the earth from the sun and T_1=time period of revolution of the earth.

Then, from eqn. (iii) of art 1·16 $T_1^2 = \frac{4\pi^2 . d_1^3}{G . M_s}$.

Similarly, if T_2 be the time-period of revolution of the moon and d_2=its distance from the earth, we have, $T_2^2 = \frac{4\pi^2 . d_2^3}{G . M_e}$

$$\text{Dividing, } \frac{T_2^2}{T_1^2} = \frac{M_s}{M_e} . \frac{d_2^3}{d_1^3} \quad \therefore \quad \frac{M_s}{M_e} = \frac{T_2^2}{T_1^2} \times \frac{d_1^3}{d_2^3}$$

Here, $T_2 = 27$ days ; $T_1 = 365$ days ; $d_1 = 1{\cdot}5 \times 10^8$ km and $d_2 = 3{\cdot}8 \times 10^5$ km.

$$\therefore \quad \frac{M_s}{M_e} = \left(\frac{27}{365}\right)^2 \times \left(\frac{1{\cdot}5 \times 10^8}{3{\cdot}8 \times 10^5}\right)^3 = 3{\cdot}37 \times 10^5$$

(3) *How many days will there be in one year if suddenly the earth were one half of its present distance from the sun ?*

Ans. If r and T be the present distance and the present time of revolution of the earth, then according to eqn. (iii) of art 1·16, $T = 2\pi\sqrt{\frac{r^3}{GM}}$

If the distance becomes half of the present value, *i.e.* $\frac{r}{2}$, let the time-period of revolution be T_1. Then, $T_1 = 2\pi\sqrt{\frac{(r/2)^3}{GM}} = 2\pi\sqrt{\frac{r^3}{8GM}}$

$$\text{Dividing, } \frac{T_1}{T} = \sqrt{\frac{1}{8}} = \frac{1}{2\sqrt{2}} \quad \therefore \quad T_1 = \frac{T}{2\sqrt{2}}$$

But $T = 365$ days ; So $T_1 = \frac{365}{2\sqrt{2}} = 129$ days. Hence the number of days in one year, when the earth is half its present distance from the sun, is 129.

1.17. Escape velocity : When a body is projected upward, it returns to the ground after sometime due to the earth's gravitational attraction. Can a body be projected with such a velocity so as not to return to the earth's surface again ? It goes without saying that for a body to escape from the earth and never return, it must be launched with a velocity which will take the body beyond the gravitational field of the earth. Such velocity is known as the *escape velocity.* If the radius of the earth be R, and the acceleration due to gravity at a point on the surface of the earth be 'g' then, it can be proved that the escape velocity of a body v_e at that position is

$$v_e=\sqrt{2.g.R} \quad \text{Also } g=\frac{GM}{R^2}\ ; \quad \therefore\ v_e=\sqrt{\frac{2GM}{R}}.$$

[**Proof :** Let the mass of the body$=m$; the mass of the earth$=M$; the distance of the body from the earth's centre$=x$; radius of the earth$=R$. The attraction on the body$=\frac{GMm}{x^2}.$

If the body is displaced through dx against this attractive force, the work done$=\frac{GMm}{x^2}.\ dx.$

So, total work done in taking the body out of the earth's attraction$=\int_R^{\alpha}\frac{GMm}{x^2}dx=$

$$GMm.\int_R^{\alpha}\frac{1}{x^2}dx=GMm\left[-\frac{1}{x}\right]_R^{\alpha}=\frac{GMm}{R}$$

If v_e be the velocity of escape, its initial $K.E.=\frac{1}{2}mv_e^2$. Evidently, the K.E. is equal to the above work done. Hence, $\frac{1}{2}mv_e^2=\frac{GM.m}{R}$ or, $v_e=\sqrt{\frac{2GM}{R}}$

Again from art 1.7 ,we know, g, the acceleration due to gravity on the surface of the earth, is given by $g=\frac{GM}{R^2}$ $\therefore\ v_e=\sqrt{2gR}.$]

It is to be noted that in the above expression for the escape velocity, the mass m of the projected body does not enter. This shows, therefore, that the escape velocity at a place is same for all bodies, big or small.

Now taking $R=6400$ km ; $g=980$ cm/sec^2, we see that

$$v_e=11{\cdot}20\times10^5 \text{ cm/sec}=11{\cdot}2 \text{ km/sec}$$
$$=7 \text{ miles/sec (nearly)}$$

So, a terrestrial body when projected upward with a velocity 7 miles/sec, will never return to the earth again. The acceleration due to gravity on the surface of the moon is about $\frac{1}{6}$th of that on the surface of the earth. Consequently the escape velocity on the moon will be much smaller compared with that on the earth.

Example : *With what velocity must a body be projected vertically upward from the earth's surface to rise to a height equal to the earth's radius.* $g=9{\cdot}8\ m/s^2$ *and radius of the earth*$=6{\cdot}38\times10^6 m.$

Ans. The gravitational potential on the surface of the earth$=-\frac{GM}{R}$ where M and R are the mass and the radius of the earth. Similarly, at a height $2R$ from the centre, the potential$=-\frac{GM}{2R}$

$$\therefore \text{ Change of gravitational potential} = GM\left(\frac{1}{R} - \frac{1}{2R}\right)$$

If m be the mass of the projected body, the potential energy of the body at height R from the surface = mass × potential change

$$= GMm\left(\frac{1}{R} - \frac{1}{2R}\right) = \frac{GMm}{2R}.$$

Let v be the velocity of projection. Its $K.E. = \frac{1}{2}mv^2$

$$\therefore \frac{1}{2}mv^2 = \frac{GMm}{2R} \quad \text{or} \quad v^2 = \frac{GM}{R}.$$

But we know, $g = \frac{GM}{R^2}$ [see art 1·7]. So $v^2 = \frac{GM}{R} = \frac{g.R^2}{R} = g.R$

$$\therefore v = \sqrt{g.R} = \sqrt{6{\cdot}38 \times 10^6 \times 9{\cdot}8} = 7{\cdot}9 \times 10^3 \text{ m/s} = 7{\cdot}9 \text{ km/s}.$$

Rarity of light gases in earth's atmosphere :

We know that light gaseous elements like hydrogen, helium etc. are rare in the earth's atmosphere. The explanation can be obtained from the consideration of escape velocity. It has been found by calculation that the root mean square (R.M.S.) velocity of hydrogen molecules at N.T.P. is about 1 mile/sec. But during the early stage of formation of the earth, the temperature was much higher and the R.M.S. velocity of hydrogen molecules was about 2 or 3 miles per second. So it is very likely that an appreciable fraction of the molecules had velocities above the escape velocity. Such fast molecules would soon be lost by escape from the earth's atmosphere. This explains the rarity of hydrogen, helium etc. in the earth's atmosphere.

The moon has no atmosphere around it and similar is the planet mercury. They have small masses and consequently small escape velocity. They could not have held any gaseous atmosphere around them because the gas molecules had initial velocity greater than the escape velocity. The same thing applies to satellites of all planets. It is worth while to mention, however, that the gaseous atmosphere round the earth produces a blanketing effect and has made the temperature conducive to the growth of life on earth.

1.18. Artificial satellite : The scientists of the Soviet Union launched the first artificial satellite on 4th October, 1957. It moved round the earth as the moon does. There after, many other countries, including India, launched a number of such satellites and artificial planets. The first artificial satellite that India launched was named "*Arya Bhatta.*"

When an artificial satellite takes off from the ground to orbit the earth, its initial take off direction is vertically upward. As the rocket gains height, control fins or jets are set to make it turn slowly toward horizontal trajectory. To find the velocity which an artificial satellite must acquire to circle the earth just as the moon does, we shall proceed as follows :

Imagine a tower several hundred miles high, from the top of which projectiles are launched in a horizontal direction (Fig. 1.10) With a low initial velocity, the projectile will follow a nearly parabolic path and will meet the earth's surface at *A*. At a somewhat higher velocity, the trajectory will be a bigger parabola and the projectile may reach the earth's surface at *B*. At a still higher velocity, the projectile in falling towards the earth, will follow a circular path of radius *r*. This particular velocity is called the *orbiting velocity*. At still higher velocities, such as shown at *D*, the projectile will follow an elliptical path or escape completely from the earth. We have seen in the art 1.15 that by simply equating the centripetal force for a satellite moving in a circular orbit to the gravitational force, we may obtain directly the orbiting velocity. According to that article, the orbiting velocity $v=\sqrt{\frac{GM}{r}}$ .. (i)

[r=radius of the circular orbit]

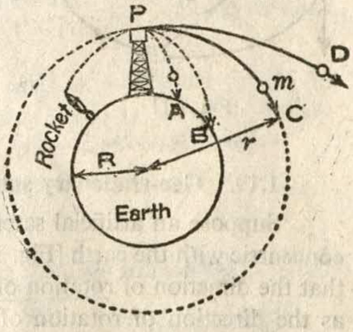


Fig.1.10

This equation holds for any satellite moving in a circular orbit around any astronomical body in the universe.

If h be the elevation of the satellite from the surface of the earth, $r=R+h$, where R= radius of the earth. Hence $v=\sqrt{\frac{GM}{R+h}}$

Also, if T be the time-period of revolution of the satellite, then, $v=\frac{2\pi(R+h)}{T}$

$\therefore \quad \frac{2\pi(R+h)}{T}=\sqrt{\frac{GM}{R+h}}$ or $T=2\pi\sqrt{\frac{(R+h)^3}{GM}}$. ..(ii)

The following table shows how the path of a satellite of the earth depends upon its velocity of projection v_0 in relation to orbital velocity v and escape velocity v_e.

(a) $v_0=v\rightarrow$ circular path around the earth.

(b) $v_0<v\rightarrow$ elliptic path—returns to the earth.

(c) $v_0>v$ but $<v_e\rightarrow$ elliptic path around the earth.

(d) $v_0=v_e\longrightarrow$ parabolic path—never returns to the earth.

(e) $v_0>v_e\rightarrow$ hyperbolic path—escapes from the earth.

Satellite circling very near the earth :

In the special case of a satellite circling the earth close to the surface, the orbital radius is approximately the radius R of the earth. Under these conditions, the centripetal force is just mg, the weight of the satellite and we have,

$mg=G\frac{Mm}{R^2}$ or, $GM=gR^2$

Substituting this value in the expression for the orbital velocity, in equation (i) we get $v=\sqrt{\frac{gR^2}{r}}=\sqrt{gR}$ [$\because R=r$] Also, $T=2\pi\sqrt{\frac{R^3}{GM}}=2\pi\sqrt{\frac{R^3}{gR^2}}=2\pi\sqrt{\frac{R}{g}}$.

If the orbit of a satellite is not circular, but elliptical, the magnitude of the orbital velocity changes from one point to another. As shown in Fig. 1.11, the velocity at the extreme left end (A) of the major axis is maximum while that at the right end (B) is minimum. The first point (A) is called the *perigee* and the other point (B) the *apogee* of the orbit. As the satellite 'm' recedes from its perigee position the backward gravitational pull of the planet M slows it down, until at the apogee the orbital speed is minimum. As the satellite moves along the opposite side of its orbit, the gravitational pull speeds it up again until at the perigee its speed again becomes maximum.

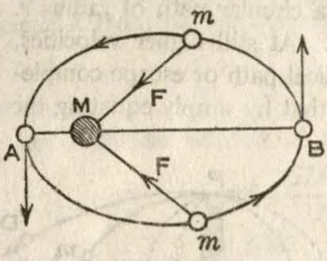


Fig. 1.11

1.19. Geo-stationary satellite and parking orbit :

Suppose an artificial satellite is circling the earth in the plane of the equator concentric with the earth [Fig. 1.12.] Also suppose that the direction of rotation of the satellite is same as the direction of rotation of the earth about its own axis. If r be the radius of the satellite's orbit and v be the orbital velocity, then $\frac{mv^2}{r}=\frac{GMm}{r^2}$. But $GM=g.R^2$ where R=radius of the earth. So, $\frac{mv^2}{r}=\frac{mgR^2}{r^2}$ or $v^2=\frac{g.R^2}{r}$; If T be the time-period of revolution of the satellite, $v=\frac{2\pi r}{T}$

$$\therefore \quad \frac{4\pi^2r^2}{T^2}=\frac{gR^2}{r} \quad \text{or} \quad T^2=\frac{4\pi^2.r^3}{g.R^2}$$

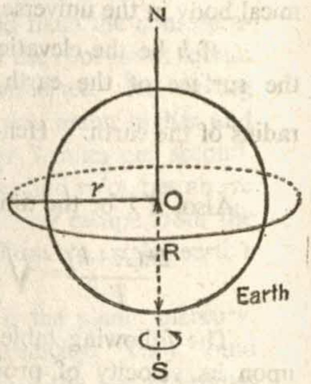


Fig. 1.12

If T be equal to the earth's period of revolution round its axis (*i.e.* 24 hours), the satellite will remain stationary at the same place over the earth while the earth rotates. Such artificial satellites are called *geo-stationary satellites* and their orbits are known as *parking orbits*. Putting several geo-stationary satellites in parking orbits, television programmes of music, sports etc. may be relayed continuously from one part of the world to another.

Putting T=24 hours in the above equation, the radius r of the parking orbit may be calculated as follow : $r=\sqrt[3]{\frac{T^2g.R^2}{4\pi^2}}=\sqrt[3]{\frac{(24\times 3600)^2\times 980\times(6{\cdot}4\times 10^8)^2}{4\pi^2}}$ cm

$=42400$ km. (nearly).

So, to put a geo-stationary satellite in its parking orbit, its height above the surface of the earth should be $(42400-6400)=36000$ km. (nearly).

[**Note :** The artificial satellite INST-1, launched hy Indian scientists in April 1982, was a geo-stationary satellite.]

Examples : (1) *If the radius of the earth be 6400 km, mass 6×10^{27} gm and $G=6\cdot6\times10^{-8}$ c.g.s., what will be the escape velocity for a terrestrial body?*

Ans. If g be the acceleration due to gravity on the surface of the earth, the escape velocity $v_e=\sqrt{2gR}$. If m be the mass of the body, $mg=\frac{G.Mm}{R^2}$ $\therefore$ $g=\frac{GM}{R^2}$

$$\therefore\ v_e=\sqrt{\frac{2GM\times R}{R^2}}=\sqrt{\frac{2\,GM}{R}}$$

Substituting the values, we get $v_e=\sqrt{\frac{2\times6\cdot6\times10^{-8}\times6\times10^{27}}{6400\times10^5}}=11\cdot2\times10^5$ cm/s.

(2) *An artificial satellite is circling round the earth at an altitude of 700 km from the surface of the earth. Calculate the orbiting velocity of the satellite. Radius of the earth $=6300$ km and 'g' on the surface of the earth$=980$ cm/sec^2*

Ans. We know, the orbiting velocity, $v=\sqrt{\frac{gR^2}{r}}$

Here, $r=6300+700=7000$ km$=7000\times10^5$ cm; $R=6300$ km$=6300\times10^5$ cm.

$$\therefore\ v=\sqrt{\frac{980\times(6300\times10^5)^2}{7000\times10^5}}=6300\times10^5\sqrt{\frac{980}{7000\times10^5}}=63000\times\sqrt{\frac{980}{7}}$$

$$=63000\times11\cdot88=7\cdot5\times10^5\text{ cm/sec.}=7\cdot5\text{ km/sec}$$

(3) *A small satellite revolves round the earth, the radius of its orbit being slightly greater than the radius of the earth. Calculate the time of revolution and the orbital velocity of the satellite. $g=980$ cm/sec^2 and the radius of the earth $R=6300$ km.*

Ans. We know that in the case of a satellite circling the earth close to its surface, the orbital velocity is given by,

$$v=\sqrt{gR}=\sqrt{980\times6300\times10^5}=78\cdot4\times10^4\text{ cm/sec.}=7\cdot84\text{ km/sec.}$$

Again, time of revolution, $T=\frac{2\pi R}{v}=\frac{2\times3\cdot14\times6300}{7\cdot84}$ sec.

$$=\frac{2\times3\cdot14\times6300}{7\cdot84\times3600}\text{hr.}=1\text{ hr. }24\text{mnt. (nearly)}$$

(4) *At what height above the surface of the earth should a satellite be so positioned that it appears stationary with respect to an observer on the earth? Given radius of the earth, $R=6400$ km. mass of the earth$=6\cdot03\times10^{27}$ gm ; $G=6\cdot67\times10^{-8}$ C.G.S.*

Ans. The satellite will appear stationary to an observer on the earth if the time period of revolution of the satellite in its orbit equals 24 hours. Let the height of the satellite from the surface of the earth$=h$; then its height from the centre of the earth $r=h+R$. Its orbital velocity $v=\sqrt{\frac{GM}{r}}$ [art. 1.15].

The circumference of the orbit $=2\pi r$. Hence, the time-period of revolution,

$$T=\frac{2\pi r}{v}=\frac{2\pi(r)^{\frac{3}{2}}}{(GM)^{\frac{1}{2}}} \quad \text{So, } 24\times 60\times 60=\frac{2\times 3{\cdot}14\times (r)^{\frac{3}{2}}}{\sqrt{6{\cdot}03\times 10^{27}\times 6{\cdot}67\times 10^{-8}}}$$

$$\text{or, } r=\left[\frac{(24\times 60\times 60)\times(6{\cdot}03\times 10^{27}\times 6{\cdot}67\times 10^{-8})^{\frac{1}{2}}}{2\times 3{\cdot}14}\right]^{\frac{2}{3}}$$

$$=42{\cdot}40\times 10^{8}\text{cm}=42400 \text{ km}.$$

∴ The height of the satellite above the surface of the earth $=(42400-6400)$ $=36000$ km.

(5) *An artificial satellite is circling round the earth with the same centre as the centre of the earth. Prove that its escape velocity in that condition is just 1·414 times its speed.*

Ans. Let m = mass of the satellite and v_0 = orbital velocity and r_0 = radius of its orbit. During motion, the centrifugal force on the satellite is balanced by the gravitational attraction of the earth. Hence, $\frac{GMm}{r_0^2}=\frac{mv_0^2}{r_0}$ [M = mass of the earth]. or, $v_0^2=\frac{GM}{r_0}$ ......(i)

From art 1.16, we know, that at a distance r_0 from the centre of the earth, the escape velocity $v_e=\sqrt{\frac{2GM}{r_0}}$ or $v_e^2=\frac{2\,GM}{r_0}$ ...(ii)

From eqns (i) and (ii), we get $v_e^2=2.v_0^2$ or, $v_e=\sqrt{2}.v_0=1{\cdot}414\ v_0$.

*1.20. Kinetic energy and potential energy of a satellite :

An artificial satellite circling round the earth has both kinetic and potential energies. If the mass of the satellite be m and v its orbital velocity, its $K.E=\frac{1}{2}mv^2$. Let M be the mass of the earth.

For a satellite moving in a circular orbit of radius r, we know, $\frac{mv^2}{r}=\frac{G.Mm}{r^2}$

∴ Kinetic energy, $E_k=\frac{1}{2}\,mv^2=\frac{GMm}{2r}$...(i)

Taking the gravitational potential due to the earth at infinity to be zero, the potential at a distance r from the centre $=-\frac{GM}{r}$ [negative because the gravitational force is a force of attraction]

So, $P.E.$ of the satellite at that position, E_p = potential × mass $=-\frac{GMm}{r}$ ..(ii)

From the above equations it is seen that the *potential energy* (E_p) *of an orbiting satellite is double the kinetic energy* (E_k) but is negative.

Now, the total energy of the satellite at the orbit,

$$E=E_k+E_p=-\frac{GMm}{r}+\frac{GMm}{2r}=-\frac{GMm}{2r}\ldots\text{(iii)}$$

The energy of the satellite diminishes gradually due to the resistance offered by the atmosphere of the earth. Hence, the radius of its orbit also diminishes. Suppose, the orbital radius diminishes from r to r_1. The total energy of the satellite in the new orbit $=-\frac{GMm}{2r_1}$. Clearly this energy is less than the previous energy.

$$\text{So,}\ -\frac{GMm}{2r_1}<-\frac{GMm}{2r}\quad\text{or}\quad\frac{GMm}{2r_1}>\frac{GMm}{2r}$$

From eqn. (i) it can be said that above two expressions give the kinetic energies of the satellite in orbits of radius r_1 and r respectively.

Thus, we may conclude that when the satellite moves from an orbit of higher radius to an orbit of lower radius, its kinetic energy and hence its orbital velocity increase. It may appear confusing because it has earlier been said that the satellite falls from higher energy orbit to a lower energy orbit. This apparent anomaly may, however be explained if we remember that the *P.E.* decreases by twice as much as the *K.E.* increases. Thus, there is, on the whole, a loss of energy.

It may be mentioned here that the motion of an electron round the nucleus of an atom is in many ways, similar to the motion of the satellite round the earth. An electron, revolving round the nucleus, possesses both kinetic and potential energies and when transition takes place from outer to inner orbit, the kinetic energy of the electron increases but its potential energy decreases.

Example : *Two artificial satellites A and B of same mass are circling the earth at distances R and 3R respectively from the surface of the earth, R being the radius of the earth. Assuming the orbits of the satellite circular, compare their kinetic and potential energies.*

Ans. Height of the satellite A from the centre of the earth $=R+R=2R$ and that of $B=R+3R=4R$.

Taking mass of each satellite as m and that of the earth as M,

$$P.E.\ \text{of}\ A=-\frac{GMm}{2R}\ \text{and of}\ B=-\frac{GMm}{4R}\quad\therefore\quad\text{ratio}=2:1$$

If the orbital velocities of the satellites be v_A and v_B respectively *K.E.* of $A=\frac{1}{2}mv_A^2$ and of $B=\frac{1}{2}mv_B^2$. As $v_A=\sqrt{\frac{GM}{2R}}$ and $v_B=\sqrt{\frac{GM}{4R}}$

$$K.E.\ \text{of}\ A=\frac{1}{2}\frac{GMm}{2R}\ \text{and of}\ B=\frac{1}{2}\frac{GMm}{4R}\quad\therefore\quad\text{ratio}=2:1$$

1.21. Weightless condition : The weight of a body means the force with which it is attracted towards the earth. So, a body will be weightless, if, somehow, this gravitational force of the earth be counter-balanced. In an artificial satellite or in a space-craft which is circling the earth, the gravitational force becomes counterbalanced and the astronaut inside the space-craft remains in a weightless condition. We have seen earlier that to launch a satellite or a space-craft in its orbit, it should be raised to a particular altitude and then be given a horizontal velocity. In launching the first artificial satellite, the Russian scientists calculated that if the satellite be given a horizontal velocity 18000 miles/hour (*i.e.* about 25,000 km/hr) at an altitude of 900 km. from the surface of the earth, the gravitational pull of the earth would be counterbalanced by the centrifugal force of the satellite and the satellite would maintain its orbit. As long as the two forces are not equal, the satellite will have a tendency of either coming towards the earth or escaping from it into the space. So long as it remains in its orbit, the two forces are counterbalanced and the astronaut is in a weightless condition.

A state of weightlessness may occur in any system complying with the following conditions: (i) no other external forces, except those of the gravitational field are acting on the system; (ii) the size of the system is not too large, so that the intensity of the gravitational field is the same at all points of the system at each instant of time, (iii) the system is in translatory motion. These conditions are realised in a freely falling lift, artificial satellites and spaceships in free flight.

It goes without saying that weightless condition is an unusual condition for a normal man. It produces various kinds of uneasiness and unpleasant reactions, including permanent health damage. For this reason, an astronaut, before being sent to the space, is acclimatized for a long time in a simulated weightless condition on the surface of the earth. Now-a-days, two separated space vehicles are bound together by cables and while rotating, they simulate normal weight conditions in each of them.

Simple pendulum

1.22. Some terms in connection with a simple pendulum : A simple pendulum, in general, consists of a small heavy mass in the form of a bob suspended by means of a light inelastic string. But for theoretical discussion, an ideal simple pendulum is imagined in which the bob is a mere particle, the string is weightless and perfectly non-elastic and the point of suspension absolutely free from friction. All these conditions are difficult to realise in practice. For experimental purpose, therefore, a simple pendulum is made by attaching a length of light thread to a small lead or brass sphere. The thread is fixed firmly between two small pieces of wood held by a clamp and stand.

Definition : *A simple pendulum is defined as a small heavy body suspended by a light inextensible string.*

When such a simple pendulum is pulled aside a little and then released, it will oscillate to and fro. It may be proved that the motion of a simple pendulum is simple harmonic (See art 1.5 Vibrations and Waves).

Suppose AO is a simple pendulum, oscillating to and fro along the path POQ [Fig. 1.13]. The point A is called the *point of suspension* and the oscillating mass the *bob*. *The effective length* or simply the length of the pendulum is defined as the distance from the point of suspension to the centre of gravity of the bob. Since the bob is a sphere, the distance from the point A to the centre of the sphere gives the effective length of the pendulum. If l be the length of the thread and r the radius of the bob, the effective length $L=l+r$.

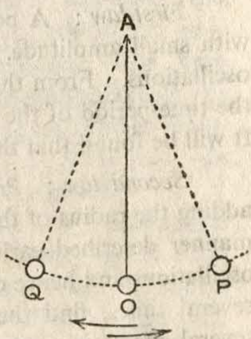


Fig. 1.13

One complete to and fro movement of the pendulum from the point P (say) to Q and back to P is called an *oscillation* or *vibration*. The time taken for one complete oscillation is called the *period of oscillation*.

As the pendulum swings to and fro, the maximum displacement of the bob, from its rest position is called *amplitude*. In fig. 1.13 OP or OQ denotes the amplitude. The angle between the extreme position and the rest position of the string is called the *angular amplitude* ($\angle OAP$, say) of the pendulum.

The number of complete oscillations done by a pendulum in one second is called its *frequency*.

Provided the amplitude is small, *i.e.*, not more than 4°, the periodic time T is related to the effective length L by the following relation. $T=2\pi\sqrt{\frac{L}{g}}$ where 'g' is the acceleration due to gravity at the place.

1.23. Laws of pendulum : The motion of a pendulum is guided by four laws, known as the laws of pendulum. The following are the laws :—

(1) **Law of isochronism :** Time-period of a given simple pendulum is a constant quantity and is independent of the amplitude of oscillation, provided the amplitude is small (*i.e.*, within 4°).

(2) **Law of length :** Time-period of a simple pendulum at a particular place is proportional to the square root of its effective length, *i.e.*, $T \propto \sqrt{L}$ when g is constant.

(3) **Law of acceleration :** For a pendulum of given length, the time-period is inversely proportional to the square root of the acceleration due to gravity at the place, *i.e.*, $T \propto \frac{1}{\sqrt{g}}$ when L is constant.

(4) **Law of mass :** Time-period of a pendulum at a particular place is independent of the mass or material of the bob provided the effective length of the pendulum remains the same.

1.24. Experimental verification of the laws of pendulum :

First law : A pendulum with suitable effective length is allowed to oscillate, with small amplitude. Note the time taken by the pendulum to execute, say 25 oscillations. From this, find the time for one oscillation. In the same way, find the time-period of the pendulum allowing to it oscillate with different amplitudes. It will be found that the time-period remains constant in every case.

Second law : Prepare a simple pendulum and find its effective length by adding the radius of the spherical bob with the length of the thread. In the manner described earlier, note the time taken by the pendulum to complete 25 oscillations and hence calculate its time-period. Changing the length of the thread several times, find the various time-periods. From these observations, we get several effective lengths and the corresponding time-periods. If the square root of the effective length is divided by its corresponding time-period, the result will be found to be equal in each case, *i.e.*, $\frac{\sqrt{L}}{T}$=constant. In other words, $T \propto \sqrt{L}$.

Third law : Strictly speaking there is hardly any laboratory method for direct verification of this law. To verify it, the same pendulum with its effective length unchanged is to be carried to different places where the value of '*g*' is different. In each place, the time-period is to be determined by the method already described. It will be found that $T \times \sqrt{g}$=constant *i.e.*, $T \propto \frac{1}{\sqrt{g}}$.

Fourth law : Several pendulums with bobs of different materials and of different masses are taken but the lengths of the threads are so adjusted that the effective lengths of all the pendulums are equal. When these pendulums are allowed to oscillate, it will be seen that the period of oscillation of each of these pendulums is the same.

1.25. An interesting example of variation of time-period of a simple pendulum :

A hollow sphere is filled with water and is hung by a long thread. A small hole is made at the bottom of the sphere and water flows out slowly through the hole. It is observed that the period of oscillation of the sphere first increases and then decreases—Explain.

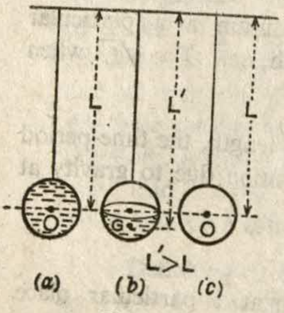


Fig. 1.14

Explanation : A sphere suspended by a long thread is nothing but a simple pendulum. Its *effective length* is the distance between the point of suspension to the *C.G.* of the sphere. From the law of length of the pendulum we know $T \propto \sqrt{L}$, where L is the effective length *i.e.*, T increases when L increases and vice-versa. Now,

(i) When the sphere is full of water, the *C.G.* of the system consisting of the sphere plus the water is at the centre of the sphere *O*. Hence, time-period $T \propto \sqrt{L}$ [Fig. 1.14(a)]

(ii) When water flows out through the hole at the bottom of the sphere, the upper part of the sphere becomes gradually empty and the *C.G.* of the system goes below the centre of the sphere, thereby increasing the effective length. The time period of oscillation consequently increases. When the upper hemisphere of the bob becomes completely empty, the *C.G.* of the whole system goes further below the centre O to G [Fig. 1.14(b)], making the effective length L' maximum. In that circumstances, the time-period of the pendulum becomes the greatest.

(iii) With further leakage of water through the hole, the lower hemisphere starts evacuating but this makes the *C.G.* of the system go up, shortening the effective length. So, the time-period of oscillation now diminishes. When the sphere becomes completely empty, the *C.G.* returns to the centre O of the sphere [Fig. 1.14 (c)] and the original effective length L and hence the original time-period are restored.

1.26. Second's pendulum and Pendulum clock : A simple pendulum whose length is so adjusted that its time-period is 2 seconds is called a *second's pendulum.* Its length can be calculated in the following way :

We know, $T=2\pi\sqrt{\frac{L}{g}}$; Here, $T=2$ seconds, $\pi=\frac{22}{7}$ and $g=980$ cm/sec^2

Hence, $2=2\times\frac{22}{7}\sqrt{\frac{L}{980}}$ or, $L=\frac{980\times4}{4\times(\frac{22}{7})^2}=\frac{980}{9\cdot86}=99\cdot391$ cm.

In the F.P.S. system, $g=32$ ft/sec^2 ; so, $L=\frac{32\times4}{4\times(\frac{22}{7})^2}=\frac{32}{9\cdot86}=3\cdot245$ ft.

Reason for a pendulum clock going 'fast' or 'slow' :

A pendulum clock keeps time by the oscillation of its pendulum ; but for various reasons the time-period of oscillation of a pendulum may change and hence the clock may fail to keep correct time.

Thus, at the top of a hill where the value of 'g' is less than that at the surface of the earth, the time-period of a pendulum will be longer and the pendulum clock will go slower. For the same reason, the clock will go slower inside a mine than on the surface of the earth.

The value of 'g' is less at the equator than at the poles. Consequently, pendulum clock will gain more or go faster at the poles than at the equator.

Due to temperature variation during winter and summer, the effective length of a pendulum changes for which a pendulum clock cannot keep correct time. Arrangements are made to compensate the change of length, enabling the clock to keep correct time. Such pendulums are known as *compensated pendulums.*

1.27. Determination of the value of 'g' by simple pendulum :

In art 1.22 it has been mentioned that within small amplitude (less than 4°) the time-period T of a simple pendulum at a place is related to its effective length L by the following equation : $T=2\pi\sqrt{\frac{L}{g}}$ or, $g=4\pi^2\frac{L}{T^2}$

So, knowing the effective length L and the time-period T, the value of 'g' at a place can be found out.

Experiment : Prepare a simple pendulum, having long effective length. Oscillate it and find its time-period with the help of a stopwatch. Care should be taken so that the amplitude does not exceed the limit stated above. Find the value of $\frac{L}{T^2}$. Next change the length of the thread several times and determine the time-period of the pendulum in each case. From all these observations, calculate the value of $\frac{L}{T^2}$. Substituting this value in the equation mentioned above, 'g' can be found out.

1.28. Drawbacks of simple pendulum :

Though simple in theory and very convenient to perform, the method of finding the value of g by a simple pendulum is not quite an accurate one due to various drawbacks. The following are the important drawbacks of a simple pendulum :

(a) A simple pendulum is just an ideal conception, not realisable in actual practice for we can neither have a point mass nor a weightless string; so that the string too has a moment of inertia about the suspension axis.

(b) The resistance and the buoyancy of air can not be fully eliminated. It has appreciable effect on the motion of the bob.

(c) The relation giving the time-period of the pendulum is true only for oscillations, having an infinitely small amplitude.

(d) The motion of the bob is not strictly linear ; it has also a rotatory motion about the axis of suspension passing through the point of suspension.

(e) At the extremities of the amplitude on either side, the bob has a relative motion with respect to the suspension thread.

1.29. (i) Determination of the height of a hill by a simple pendulum :

Suppose the time-period of a simple pendulum on the surface of the earth is T sec. and that at the top of a hill 'h' cm. high, is T' sec.

Now, if the acceleration due to gravity at the surface and at the top of the hill be g and g' respectively, then we can write,

$$T=2\pi\sqrt{\frac{L}{g}} \quad \text{and} \quad T'=2\pi\sqrt{\frac{L}{g'}} \quad \text{Dividing we get, } \frac{T'}{T}=\sqrt{\frac{g}{g'}} \quad \ldots \quad \ldots\text{(i)}$$

Now, suppose, M gm = mass of the earth ; R cm = radius of the earth and G = Gravitational constant. Applying Newton's law of gravitation,

$$g=\frac{GM}{R^2} \quad \text{and} \quad g'=\frac{GM}{(R+h)^2}. \quad \text{Dividing, we get, } \frac{g}{g'}=\frac{(R+h)^2}{R^2}=\left(1+\frac{h}{R}\right)^2$$

$$\text{or, } \sqrt{\frac{g}{g'}}=1+\frac{h}{R} \quad \ldots \quad \ldots\text{(ii)}$$

From equations (i) and (ii) we have, $\frac{T'}{T}=1+\frac{h}{R}$ or $h=R\left(\frac{T'}{T}-1\right)$

So, knowing T and T', the value of 'h' can be found out.

(ii) Determination of the depth of a mine by a simple pendulum :

Let the time-period of a simple pendulum on the surface of the earth where the acceleration due to gravity is g, be T. Then $T=2\pi\sqrt{\frac{L}{g}}$ where L=length of the pendulum. If the pendulum be now taken to the bottom of a mine whose depth is h, its time-period will be different. If T' be the time-period and g' the acceleration due to gravity, then $T'=2\pi\sqrt{\frac{L}{g'}}$ Dividing, we get $\frac{T'}{T}=\sqrt{\frac{g}{g'}}$ .. (i)

But from art 1.7 (iii), we know, g' at a depth h is $g'=\frac{g}{R}(R-h)$ where R= radius of the earth. Putting this value in eqn. (i),

$$\frac{T'}{T}=\sqrt{\frac{g.R}{g(R-h)}}=\sqrt{\frac{R}{R-h}} \text{ or } \frac{T'^2}{T^2}=\frac{R}{R-h} \quad \therefore h=R\left(1-\frac{T^2}{T'^2}\right)$$

So knowing T and T', the value h can be found out.

Examples : (1) *The length of a second's pendulum is 100 cm. If another simple pendulum makes 25 complete oscillations in a minute at that place, what is the length of that pendulum ?*

Ans. Time-period of the second's pendulum $=2$ seconds.

,, ,, ,, other pendulum $=\frac{60}{25}$ seconds

In the case of a second's pendulum, $2=2\pi\sqrt{\frac{100}{g}}$

,, ,, ,, ,, the other pendulum, $\frac{60}{25}=2\pi\sqrt{\frac{l}{g}}$ [l=its length]

Dividing we get, $\frac{25\times2}{60}=\sqrt{\frac{100}{l}}$ or $\frac{10}{12}=\sqrt{\frac{100}{l}}$

Squaring, $\frac{100}{144}=\frac{100}{l}$ $\therefore$ $l=144$ cm.

(2) *The length of a simple pendulum is double that of the other. If the time-period of the second pendulum is 3 sec., what is the time-period of the first ?*

Ans. From the laws of pendulum, we get $\frac{T_1^2}{T_2^2}=\frac{l_1}{l_2}$

Here, $l_1=2l_2$ and $T_2=3$ sec ; So, $\frac{T_1^2}{9}=\frac{2l_2}{l_2}=2$ or $T_1^2=18$

$\therefore$ $T_1=4{\cdot}24$ sec. (nearly)

(3) *Two pendulums of time-periods 1·6 sec and 1·7 sec are started at the same time. How many seconds later will one gain one complete oscillation over the other? How many oscillations does the faster one execute in the mean time?*

Ans. Suppose, after x seconds, one pendulum gains one complete oscillation over the other. Now, the number of oscillations made by the first pendulum in x sec $=\frac{x}{1{\cdot}6}$. Similarly the number of oscillations executed by the second pendulum in the same time $=\frac{x}{1{\cdot}7}$. Now, according to the problem, $\frac{x}{1{\cdot}6}-\frac{x}{1{\cdot}7}=1$

or, $x\times 0{\cdot}1=1{\cdot}6\times 1{\cdot}7 \quad \therefore \quad x=16\times 1{\cdot}7=27{\cdot}2$ seconds.

The number of oscillations executed by the faster pendulum (*i.e.*, whose period is 1·6 sec) during this time $=\frac{27{\cdot}2}{1.6}=17$.

(4) *A faulty pendulum (second's) loses 20 seconds per day. Find the required alteration in length so that it may keep correct time.*

Ans. In a day, there are $24\times 60\times 60=86400$ seconds. Since the pendulum loses 20 seconds a day, it swings $(86400-20)=86380$ times in a day. So, the time for half oscillation $t=\frac{86400}{86380}$ sec.

Since it is a second's pendulum, $t=\pi\sqrt{\frac{l_1}{g}}$ (l_1=length of the defective pendulum).

$$\text{or, } \frac{86400}{86380}=\pi\sqrt{\frac{l_1}{g}} \quad \text{or, } \left(\frac{86400}{86380}\right)^2=\frac{\pi^2.l_1}{g}$$

$$\text{or, } l_1=\frac{g}{\pi^2}\left(1+\frac{20}{86380}\right)^2=\frac{g}{\pi^2}\left(1+\frac{2\times 20}{86380}\right)=\frac{g}{\pi^2}\left(1+\frac{2}{4319}\right)$$

If l be the length of a correct second's pendulum, then $t=1=\pi\sqrt{\frac{l}{g}}$ or $l=\frac{g}{\pi^2}$

$\therefore$ Alteration in the length $=l_1-l=\frac{g}{\pi^2}\times\frac{2}{4319}=\frac{980}{(3{\cdot}14)^2}\times\frac{2}{4319}={\cdot}046$ cm.

(5) *A second's pendulum keeps correct time at a certain place on the surface of the earth, but loses 10 seconds at the top of a hill. Determine the height of the hill, given the radius of the earth=6400 kilometres.*

Ans. If 'g' be the acceleration due to gravity on the surface of the earth, then for a second's pendulum, $2=2\pi.\sqrt{\frac{l}{g}}$

If T' be the time period and g' the acceleration due to gravity at the top of the hill, then, $T'=2\pi\sqrt{\frac{l}{g'}}$.

Dividing, $\frac{T'}{2}=\sqrt{\frac{g}{g'}}$; But we know, $\sqrt{\frac{g}{g'}}=1+\frac{h}{R}$ [See art 1.28]

$$\therefore \quad \frac{T'}{2}=1+\frac{h}{R}$$

Since the pendulum loses 10 seconds at the top of the hill, it executes $(86400-10)=86390$ swings in a day. Hence the time of half oscillation, *i.e.*, $T'/2$ $=\frac{86400}{86390}$ sec.

$$\therefore \quad 1+\frac{h}{R}=\frac{86400}{86390} \text{ or, } \frac{h}{R}=\frac{10}{86390} \text{ or } h=\frac{10\times 6400}{86390}\text{ km}=0{\cdot}74 \text{ km}=740 \text{ metres.}$$

(6) *The length of a simple pendulum is 2 metres. Assuming small amplitude, what would its time-period be in a lift accelerating upward at a rate of* $2{\cdot}2$ *metres/*s^2. $g=9{\cdot}8\ m/s^2$.

Ans. For a simple pendulum, $T=2\pi\sqrt{\frac{l}{g}}$. When the lift moves upward with an acceleration f, the resultant acceleration on the bob of the pendulum $=g+f=9{\cdot}8+2{\cdot}2=12\ m/s^2$.

$$\therefore \quad T=2\pi\sqrt{\frac{2}{12}}=\frac{2\times 3{\cdot}14\times 1}{\sqrt{6}}=\frac{2\times 3{\cdot}14}{2{\cdot}45}=2{\cdot}56 \text{ sec.}$$

[*N.B.* : (i) When a pendulum is taken in a lift which is moving *upward* with a uniform acceleration f or moving *downward* with a uniform deceleration f, the time-period is $T=2\pi\sqrt{\frac{l}{g+f}}$.

(ii) When the lift is moving upward with a uniform deceleration f or downward with a uniform acceleration f, the time-period is $T=2\pi\sqrt{\frac{l}{g-f}}$.

(iii) When the lift is moving downward with acceleration g, the time-period $T=\propto$ i.e. the pendulum will not oscillate.]

(7) *A simple pendulum of length l and mass m is suspended in a car which is travelling with a constant speed v around a circle of radius r. Assuming small amplitude, calculate the time-period of the pendulum.*

Ans. When the pendulum is at rest, its time-period is given by $T=2\pi\sqrt{\frac{l}{g}}$.

While travelling in a circular path, the pendulum bob will be subjected to centrifugal acceleration, v^2/r in addition to g. The direction of centrifugal acceleration being horizontal outward, the resultant acceleration of the bob

$$=\sqrt{g^2+\left(\frac{v^2}{r}\right)^2}=\sqrt{g^2+\frac{v^4}{r^2}}=\frac{1}{r}\sqrt{g^2r^2+v^4} \quad \therefore \quad T=2\pi\sqrt{\frac{l.r.}{(g^2r^2+v^4)^{\frac{1}{2}}}}$$

Exercises

Essay type :

1. What is Newton's law of gravitation ? [*H. S. Exam 1981*]. How far is the law universal ? What is acceleration due to gravity ? How does it depend upon distance ? [*H. S. Exam 1982*]

2. State the difference between mass and weight of a body. What do you mean by 'a force of 1 lb' and 'a force of 1 gm' ?

3. How does the acceleration due to gravity inside the earth decrease with depth ? What is its value at the centre of the earth ?

4. What are the different causes for the variation of the value of acceleration due to gravity ?

5. Show that the acceleration due to gravity at a certain distance below the earth's surface is greater than at the same distance above the earth's surface.

6. A frictionless tunnel is bored along one of the diameters of the earth. Show that a body would travel, when dropped, the whole length of the tunnel in about 42 min. 21 sec.

7. A frictionless tunnel is bored along one of the diameters of the moon . Show that a body would travel, when dropped, the whole length of the tunnel in about 54 minute 44 second. Mass of the moon$=7{\cdot}35\times10^{25}$ gm and its radius$=1{\cdot}75\times10^{8}$ cm.

8. State the law of falling bodies. How would you verify experimentally the first law ? A body is released from rest and falls freely. Compute the velocity and position after 1, 2, 3 and 4 seconds.

9. State Kepler's laws of planetary motion. How can you establish Kepler's third law from the law of gravitation ?

10. Find expressions for the orbital velocity and the time of revolution of a planet revolving in a circle around the sun.

11. A satellite is circling the earth close to the surface. Show that its orbital velocity is given by $v=\sqrt{g\,R}$, where R is the radius of the earth and g the acceleration due to gravity on the surface of the earth.

12. Show that the smallest time-period of revolution of an artificial satellite in a circular orbit round a planet (assumed spherical) is determined only by the average density of matter of the planet. [*Jt. Entrance 1975*]

13. When an artificial satellite moves from an orbit of greater radius to one of smaller radius, its kinetic energy increases but its total energy decreases. Explain the statement with proper mathematics.

14. What is a simple pendulum ? Write the equation connecting the time-period and effective length of a simple pendulum. State the laws of simple pendulum. [*H. S. Exam 1981*]

15. What is a second's pendulum ? Calculate its length at a place where $g=980$ cm/s^2. How can you find the value of 'g' with a pendulum ? [*H. S. Exam 1980*]

Short answer type :

16. (i) Which one is greater—the force of attraction of the earth on 1 lb of lead or the force of attraction of lb of lead on the earth ?

(ii) If the force of gravity acts on all bodies in proportion to their masses, why does not a heavy body fall faster than a light body ?

17. A body is weighed on the surface of the earth, on the sea-level and on the top of a mountain. Will you notice any change in the weight of the body ? Explain your answer carefully.

18. Where will a body weigh more—at the poles or at the equator ? Would we have more sugar to the kilogram at the pole or the equator ?

19. How does the acceleration due to gravity at a place alter due to the altitude of the place ?

20. Two identical trains are moving on rails along the equator on the earth in opposite directions. Will they exert the same pressure on the rails ? [*I.I.T. 1985*]

[Hints : No ; the train, travelling due east, has its angular velocity about the axis of rotation of the earth increased because the earth itself rotates about the axis from west to east. The centrifugal force on the train increases and hence its apparent weight decreases. Reverse is the case for the other train which is moving from east to west.]

21. (i) What is escape velocity? Is it possible to keep an artificial satellite stationary at a particular place of the sky in relation to an observer on the earth? What is that satellite called ?

(ii) Objects at rest on the earth's surface move in circular paths with a period of 24 hours. Are they "in orbit" in the sense that an earth satellite is in orbit ?

22. For a body projected vertically upward, prove that (i) the time of ascent to the greatest height is equal to the time of descent from the greatest height and (ii) the velocity at the time of projection is equal to the velocity attained at the time of fall at the position of projection.

23. A man standing on a platform at some height above the ground throws one ball straight up with initial speed v_0 and then throws another ball straight down with the same initial speed. Which ball, if either, has the larger speed when it hits the ground ? Neglect air resistance.

24. Will the time-period of a simple pendulum change in the following cases :—(i) if the temperature is raised (ii) if a hollow bob is taken instead of a solid bob. (iii) If the hollow bob is completely filled with water. (iv) if half of the hollow bob is filled with water. (v) if the pendulum is taken up in an aeroplane with acceleration g ? (vi) if it is taken down a lift with acceleration g ?

[**Hints** : (i) due to rise of temperature, the volume and hence the radius of the hollow bob increase and thereby, the effective length also increases. Hence, time-period increases (ii) If the bob is hollow, its mass decreases but the radius remains unaltered. Hence time-period remains the same (iii) when the hollow bob is completely filled with water, its mass increases but not its radius. So, time-period remains unaltered (iv) when the hollow bob is half-filled with water the C.G. of the bob goes below the centre, thereby increasing the effective length. Hence, time-period increases (v) when raised upward with an acceleration g, the effective acceleration on the bob increases and hence time-period decreases. (vi) if it is taken down with acceleration g, it will not swing.]

25. Explain, giving reasons, whether, the time-period of a simple pendulum increases, decreases, or remains same in the following cases : (i) it is taken from the surface of the earth to the top of a mountain (ii) it is taken inside a mine (iii) the radius of its bob is increased (iv) the mass of its bob is increased, keeping the diameter unchanged.

26. Will there be any change in time-period of a pendulum if it is taken in an aeroplane high up with (i) uniform velocity (ii) uniform acceleration?

27. One clock is based on an oscillating spring, the other on a pendulum. Both are taken to Mars. Will they keep the same time as they kept on the earth ? Will they agree with each other ?

28. How is the period of a pendulum affected when its point of suspension is (i) moved horizontally with acceleration f (ii) moved vertically upward with acceleration f (iii) moved vertically downward with acceleration $f<g$?

29. An astronaut on the surface of the moon finds that the period of a simple pendulum there is much larger than on the earth and that the pendulum continues to oscillate for a much longer time than on the earth. What information regarding the moon could be obtained from these observations? [*I. I. T. 1977*].

[**Hints** : (i) g is smaller (ii) No atmosphere].

30. The bob of a pendulum is a sphere of iron. A powerful electromagnet is placed below the bob. On passing current through the electromagnet, the time-period of the pendulum is found to decrease. Why?

[**Hints** : Electromagnet exerts a vertical downward force on the bob. This increases the value of 'g' artificially. Hence the time-period decreases].

31. From the top of a building, a ball is dropped while another is thrown horizontally at the same time. Which ball will strike the ground first?

[**Hints** : Both the balls will strike the ground at the same time. The ball thrown horizontally moves vertically downward with same uniform acceleration as the other. Its horizontal motion will not disturb its vertical motion.]

32. Elucidate why a space vehicle in orbit is a freely falling body with an acceleration towards the earth equal to the value of 'g' at that point.

33. A body is simply released in space from an orbiting artificial satellite. Will the body drop down or move like the satellite?

Objective type :

34. A few questions are given below. Give the answer in 'Yes' or 'No', form :—

(a) Is gravitational attraction between two masses in air same as that in water?

(b) Is gravitational attraction dependent on temperature?

(c) Does the weight of a body depend on the motion of the body?

(d) Will a pendulum clock gain when it is taken to the bottom of a mine?

(e) A body is thrown vertically upwards. It returns to the ground after some time. Are the two times of flight equal?

(f) A projectile fired from the ground follows a parabolic path. Is the speed of the projectile minimum at the top of the path ?

(g) Is the escape velocity on the surface of the moon greater than at the surface of the earth?

(h) A simple pendulum of period T has a metal bob which is negatively charged. It is allowed to oscillate above a positively charged metallic plate. Will the time-period decrease?

(i) The numerical value of the angular velocity of the rotation of the earth should be.... rad/s in order to make the effective acceleration due to gravity equal to zero at the equator. Fill in the gap. [*I.I.T. 1984*]

(j) If two satellites are revolving such that r_1 is more than r_2, then the outer satellite will have (i) more orbital velocity (ii) equal orbital velocity (iii) less orbital velocity. Which is correct?

(k) If the radius of the earth were to shrink by 1% its mass remaining same, the value of g on the earth's surface would (i) decrease (ii) remain unchanged (iii) increase. Which is correct ? [*I.I.T. 1981*]

(l) The sun attracts all bodies on the earth. At midnight when the sun is directly below, it attracts an object in the same direction as the attraction of the earth on that body ; at noon when the sun is directly above, it pulls on the object in a direction opposite to the pull of the earth. Hence all objects should weigh more at midnight that at noon. Is it correct ?

Numerical problems :

35. The force with which a sphere of mass 40 kg attracts another sphere of mass 15 kg. placed 20 cm away from its centre is $\frac{1}{10}$th of 1 mg-wt. Find, from these, the value of the gravitational constant. [Ans. $6{\cdot}53 \times 10^{-8}$ c.g.s.].

36. (i) Two smooth small spheres each of mass 10 gm. are kept at 1 metre apart on a smooth horizontal surface. After what time will they unite being attracted by the force of gravitation? [Ans. 14·2 days (approx)]

(ii) How far from the earth must a body be along a line towards the sun so that the sun's gravitational pull balances the earth's ? The sun is 9.3×10^7 miles away and its mass is 3.24×10^5 Me. [Me=mass of the earth] [Ans. 1.6×10^5 miles (nearly)]

37. At what altitude from the centre of the earth will the acceleration due to gravity be $\frac{1}{16}$*th* of its value on the surface of the earth? Assume the earth to be a sphere of 6300 km. radius. [Ans. 252,00 km.]

38. At what altitude and at what depth from the surface of the earth will the acceleration due to gravity be half that on the surface? Radius of the earth=6300 km. [Ans. 2612 km ; 3150 km]

39. Calculate the mass of the earth from the following data : radius of the earth = 6400 km ; $g=9{\cdot}8$ metre/s^2 ; $G=6{\cdot}7\times10^{-8}$ c.g.s. [*H. S. Exam. 1982*] [Ans. 6×10^{24} kg]

40. Calculate the acceleration due to gravity on the surface of the earth if the mean density of the earth = 5500 kg/m^3 ; Gravitational constant = $6{\cdot}7\times10^{11}$ N.m.^{2}/kg^2 and radius of the earth = 6400 km. [Ans. 9·9 metre/s^2]

41. With the help of the given data, calculate the mass of the sun : Distance of the sun from the earth = $1{\cdot}49\times10^{13}$ cm ; $G=6{\cdot}66\times10^{-8}$ c.g.s. ; 1 year = 365 days.

[*Tripura H.S. 1981*] [Ans. $1{\cdot}98\times10^{33}$ gm]

42. A stone is projected upward with a velocity 40 ft/sec. What is the greatest height attained? What time will it take to return to the ground? ($g=32$ ft/sec^2)

[Ans. (i) 25 ft. (ii) 2·5 sec.].

43. Two stones are projected upward simultaneously. One ascended 112 ft. more than the other and reached the ground 2 sec later. Find the velocities of projection of the stones. ($g=$ 32 ft/sec^2) [Ans. 128 ft/sec ; 96 ft/sec]

44. A stone is thrown horizontally with a velocity 400 ft/sec from the top of a tower 400ft. high. When and where will it strike the ground? [Ans. 5 sec ; 2000 ft away]

45. A body is thrown vertically upward with a velocity 100 ft/sec. When will it be at a height of 80 ft. from the ground? Explain the double answer. $g=32$ ft/sec^2.

[Ans. 0·94 sec ; 5·3 sec]

46. From the top of a tower, a stone is thrown vertically upward with a velocity 30 meter/s and 4 sec. later another stone is dropped from the same point. Both the stones reach the ground at the same time. Find the height of the tower and time of fall of the second stone.

[80 metre; 4 sec.]

47. A stone falling from the top of a vertical tower has descended x ft. when another is let fall from a point y ft below the top. If they fall from rest, and reach the ground together, show that the height of the tower is $(x+y)^2/4x$ ft. [*H.S. Exam. 1971*]

48. From a vertically rising balloon, a stone is dropped when it was at a height of 200 ft from the ground. The stone reached the ground in 6 seconds. What was the velocity of the balloon?

[Ans. 62·66 ft/sec]

49. A body is dropped from a balloon at rest. It covers a tower 81 ft high during last quarter second of its journey. Find the height of the balloon and the velocity of the body when it touches the ground. [Ans. 1681 ft ; 328 ft/sec]

50. A body is projected upward with a velocity of 80 ft/sec. How long will it ascend? What is the greatest height attained? What will be its velocity half the way up?

[Ans. 2·5 sec ; 100 ft ; 56·57 ft/sec.]

51. A parachutist after bailing out falls 50 metres without friction. When the parachute opens, he decelerates downward 2 metres/s^2. He reaches the ground with a speed of 3 metres/sec. How long is the parachutist in the air ? At what height did he bail out ?

[Ans. 17 sec ; 290 metres]

52. The world's first satellite was reported to be circling the globe at a distance of 560 miles. Calculate its minimum orbital velocity and the period of revolution. Radius of the earth = 4000 miles, g = 32 ft/sec^2. [Ans. 24340 ft/sec ; 1 hr, 43 mnt 27 sec]

53. What speed should a particle have in order that it may become a satellite of the earth and go round in a circular orbit 300 miles above the surface. Radius of the earth = 3960 miles g = 32 ft/sec^2. [Ans. 4·7 miles/sec.]

54. A small satellite revolves round a planet of mean density 10 gm/cc, the radius of its orbit being slightly greater than the radius of the planet. Calculate the time of the revolution of the satellite. $G=6{\cdot}66\times10^{-8}$ C.G.S. units. [Ans. 1 hr. 26 mnt.]

55. A second's pendulum loses 5 seconds a day. By how much must it be shortened in order to keep correct time? [Ans. 0·0115 cm.]

56. The length of a second's pendulum is increased by 1%. How many seconds will it lose in a day? [Ans. 430]

57. A second's pendulum keeps correct time at a place where $g=981$ cm/s². It is taken to another place where $g=978$ cm/s². How many seconds will the pendulum go slow in a day at the second place? [Ans. 132]

58. A second's pendulum keeps correct time at the foot of a hill 1482 metre high. How many seconds will the pendulum lose per day when taken to the top of the hill? Radius of the earth $=6400$ km. [Ans. 20]

59. The mass and diameter of a planet are twice those of the earth. What will be the period of oscillation of a pendulum on this planet if it is a second's pendulum on the earth? [Ans. 2·8 sec]

[**Hints** : $g=G\times\dfrac{Mm}{r^2}$ and $g_p=G.\dfrac{m\,2M}{(2r)^2}$ $\therefore$ $\dfrac{g_e}{g_p}=2$

Again, $2=2\pi\sqrt{\dfrac{l}{g_e}}$ and $T=2\pi\sqrt{\dfrac{l}{g_p}}$ $\therefore$ $\dfrac{T}{2}=\sqrt{\dfrac{g_e}{g_p}}=\sqrt{2}$ or $T^2=8$ or, $T=2{\cdot}8$ sec].

60. Show that the time-period of a simple pendulum at a depth x below the surface of the earth is inversely proportional to $\sqrt{R-x}$, where R = radius of the earth. What will be the time-period when $x=R/2$? The pendulum is a second's pendulum on the surface of the earth. [Ans. 2·8 sec]

Harder problems :

61. Find the difference in the values of g at pole and equator due to diurnal motion of the earth. Radius of the earth $=6{\cdot}37\times10^8$ cm. [Ans. 3·4 cm/s^2]

62. The diameters of two planets A and B are respectively 8000 miles and 4000 miles. The mass of the planet B is 0·12 times the mass of A. If a body weighs 100 lb on the planet A, what will it weigh on B ? [Ans. 48 lb]

63. Suppose, a man is measuring force by the gravitational attraction exerted by two unit masses separated by unit distance. What will be the value of G, the constant of gravitation according to this new unit ? [*Jt. Entrance 1976*] [Ans. $1{\cdot}5\times10^7$]

64. If the earth were a solid sphere of iron of radius $6{\cdot}37\times10^6$ metre and of density 7·86 gm/cc. what would be the value of the acceleration due to gravity at its surface, taking the gravitational constant to be $6{\cdot}58\times10^{-8}$ *c.g.s.* unit ? [Ans. 1380 cm/sec²]

65. If the earth were to cease rotating about its axis, what will be the change in the value of g at a place of latitude 45°, assuming the earth to be a sphere of radius $6{\cdot}38\times10^6$ metres ? [Ans. 1·6895 cm/s²]

66. A lift is ascending with an acceleration of 2 metre/s^2. When its velocity is 8 metre/s, a piece of iron from the ceiling of the lift falls. If the height of the lift-box be 3 metre, find the time after which the piece will reach the floor of the lift. What height will it rise or fall ? $g=9{\cdot}8$ metre/s^2. [Ans. 0·7 sec (approx) ; 3·2 m(approx) rise]

[**Hints** : When the piece of iron got loose, both the piece and floor were ascending with the same speed. Afterwards, the acceleration of the floor was 2 m/s² upward but that of the piece was 9·8 m/s² downward.]

67. Show that the height fallen through by a body in time t is $\dfrac{1}{n^2}$ times the height fallen through by the same body in time $\dfrac{t}{n}$.

68. A block of ice starts sliding down from the top of an inclined roof of a house (inclination being 30° with the horizontal) along a line of maximum slope. The heighest and the lowest points of the roof are at heights of 8·1 metres and 5·6 metres respectively from the ground. At what horizontal distance from the starting point will the block hit the ground ? [*I.I.T. 1972*] [Ans. 9 metres]

69. A wooden block of mass 10 gm is dropped from the top of a cliff 100 metres high. Simultaneously a bullet of mass 10 gm is fired from the foot of the cliff vertically upwards with a velocity of 100 metres/s. Where and after what time will they meet ? If the bullet after striking the block gets embeded in it, how high will it rise above the cliff before it starts falling ? $g=9{\cdot}8$ m/s^2 [*I.I.T. 1973*] [Ans. 1 sec after ; 95·1 metre from the foot ; 77·55 metres]

70. A rocket is fired vertically and ascends with a constant vertical acceleration of 64 ft/sec^2 for 1 minute. Its fuel is then completely exhausted and it continues as a free particle. What is the maximum altitude reached ? What is the total time elapsed from take-off until the rocket strikes the earth ? [Ans. $3{\cdot}5\times10^5$ ft ; 5·5 mnt]

71. The accelerations due to gravity at two places are g_1 and g_2. From same height at two places if a body be dropped, it takes t less to reach the ground in the second place than the first but the velocity on reaching the ground in the second place is greater by v than in the first place. Prove that $g_1.g_2=v^2/t^2$.

72. From an elevated platform A, a stone is thrown vertically upwards. When the stone reaches a point h below the platform, its velocity is double of what it had at a height h above the platform. Show that the greatest height attained by the stone above the platform is $\frac{5}{3}.h$.

73. A ball of mass 100 gm is projected vertically upward from the ground with a velocity of 49 m/sec. At the same time another identical ball is dropped from a height of 98 metre to fall freely along the same path as that followed by the first ball. After some time, the two balls collide and stick together and finally fall to the ground. Find the time of flight of the masses. [*I.I.T. 1985*] [Ans. 6·5 sec (nearly)]

74. Particles P and Q of mass 20 gm and 40 gm respectively are simultaneously projected from the points A and B on the ground. The initial velocities of P and Q make 45° and 135° angles respectively with the horizontal AB as shown in Fig. 1.15. Each particle has an initial speed of 49 m/s. The separation of AB is 245 metre. Both particles travel in the same vertical plane and undergo a collision. After the collision, P retraces its path. Determine the position of Q when it hits the ground. How much time after the collision does the particle Q take to reach the ground? Take $g=9{\cdot}8$ m/s^2. [*I.I.T. 1982*] [Ans. 122·5 m from A ; 3·53 sec]

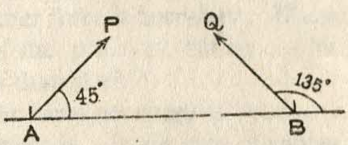


Fig. 1.15

75. A satellite is to be put into orbit 500 km above the earth's surface. If its vertical velocity after launching is 2000 metre/s at this height, calculate the magnitude and direction of the impulse required to put the satellite directly into the orbit, if its mass is 50 kg. Assume $g=10$ m/s^2 and radius of the earth $R=6400$ km. [Ans. 4×10^{10} gm-cm/s ; 14·6° to the orbit tangent]

76. An artificial satellite is circling the earth at an altitude of $R/2$ where R=radius of the earth ($6{\cdot}38\times10^6$ metre). Find the period of revolution of the satellite. $g=9{\cdot}8$ metre/s^2. [Ans. 2 hr. 35 mnt]

77. An artificial satellite is circling the earth at an altitude of 300 km. If the radius of the earth be 6400 km, what is the time-period of revolution of the satellite ? What is its orbital velocity ? $g=9{\cdot}8$ metre/s^2. [Ans. 1 hr. 34m. 8 sec; 7·7 km/s]

78. Assuming the earth to be a uniform sphere of mass-density σ, show that the time-period of revolution of an artificial satellite circling the earth just outside the equatorial plane depends on σ.

[**Hints** : $T=2\pi\sqrt{\dfrac{(R+h)^3}{GM}}$; here $h=0$; $M=\dfrac{4\pi}{3}.R^3.\sigma$ $\therefore$ $T=2\pi\sqrt{\dfrac{R^3}{GM}}=2\pi\sqrt{\dfrac{3}{4\pi G.\sigma}}$]

79. A satellite of mass m is orbiting the earth in a circular orbit of radius r. It starts losing energy slowly at a constant rate Q due to friction. If M and R denote the mass and radius of the earth respectively, show that the satellite falls on the earth in a time $t=\dfrac{GMm}{2Q}\left(\dfrac{1}{R}-\dfrac{1}{r}\right)$.

[**Hints** : Total energy of the satellite when in the orbit of radius $r = -\frac{GMm}{2r}$.

" " " " " " on the surface on the earth $= -\frac{GMm}{2R}$

$\therefore$ Loss of energy in falling to the earth $= \frac{GMm}{2R} - \frac{GMm}{2r} = \frac{GMm}{2}\left(\frac{1}{R} - \frac{1}{r}\right)$

If the satellite takes time t to fall to the earth, energy lost $= Q.t$.

$$\therefore Q.t = \frac{GMm}{2}\left(\frac{1}{R} - \frac{1}{r}\right) \quad \text{or} \quad t = \frac{GMm}{2Q}\left(\frac{1}{R} - \frac{1}{r}\right)\Big]$$

80. The length of a second's pendulum at Calcutta is 100 cm and at Darjeeling it is 95 cm. If a body be transferred from Darjeeling to Calcutta, how will its weight increase ? [Ans. $\frac{1}{19}$]

81. A simple pendulum of length l is suspended from the roof of a room. Its time-period of oscillation is T_1. If its length be diminished by a, its time-period becomes T_2. Show that

$$g = \frac{4\pi^2 a}{T_1^2 - T_2^2}$$

82. A spherical bob is so suspended from a point by a thread that it is describing a circle of radius r on a horizontal plane with angular velocity ω and the thread is generating a cone. Show that the tension in the thread $T = m\sqrt{g^2 + r^2\omega^4}$, where m = the mass of the bob.

2

ELASTICITY

2.1. Elasticity : You know that a rubber string or a steel spring when stretched, elongates in length but shrinks back to the original length when the stretching force is withdrawn. The string or the spring, in the above case, suffers strain due to the stretching force. A force of reaction develops inside the material of the body, as soon as the body is strained and this force of reaction helps the body to regain its original shape and size when the external force is withdrawn.

Definition : *When a body is acted on by a system of forces in equilibrium, the body may undergo a change either in shape or in size or in both and it returns to its original shape or size when the system of deforming forces is withdrawn. This property of the substance is known as elasticity.*

Whenever we think of an elastic substance, we commonly cite a piece of rubber as a good example of it in the sense that if we apply a stretching force, the piece of rubber gets elongated readily and as soon as we remove the stretching force, the piece quickly returns to its original length. But to produce a small deformation in a piece of steel wire, a much greater force is necessary. Elastic property of the steel is not so noticeable as that of the piece of rubber. This, however, does not mean that rubber is more elastic than steel.

From the view point of Physics, the greater the force necessary to produce a deformation in the body, the more elastic the substance is. In the case of rubber, a deformation may easily be produced but in the case of steel, a greater force is necessary to produce a small deformation. In this sense, a piece of steel is more elastic than a piece of rubber.

A knowledge of the elastic properties of different materials is essential for many works—specially in engineering science and technology. In the construction of bridges, buildings etc. large beams of iron or other metal are very often used and their suitability is judged by ascertaining to what extent they can bear load or what is their elastic limit etc.

2.2. Some important definitions in connection with elasticity :

(i) **Strain** : Whenever a system of forces, in equilibrium, acts on a body, the forces do not produce any bodily motion but produce a relative displacement of the various particles of the body causing a change of shape, or size or both. This change is generally called the deformation of the body. This deformation of the body is known as *strain* and is measured by the ratio of the change in configuration to the original configuration. The idea of this measure of strain will be clear when it is taken up in connection with different kinds of strain.

(ii) **Stress** : When a relative displacement of various parts of a body takes place on the application of a system of forces, a resisting force is called into play inside the body tending to bring it back to its unstrained condition. This restoring force is called a *stress*.

The stress, being a reaction to the external force, is according to Newton's third law of motion, equal to the external force, and is measured by the force applied externally per unit area of the body. Hence,

$$\text{Stress} = \frac{\text{Applied force on the body}}{\text{Area of cross-section of the body}}$$

If the applied force be inclined to the surface, the component of the force normal to the surface, per unit area, is called the **normal stress** while the component along the surface, per unit area, is known as the **tangential** or **shearing stress.**

In elongating a wire or to change the volume of a body, the stress is always normal but in changing the shape of a body, the stress is always a shearing stress. Further, the normal stress may be *compressive* or *tensile* according as it produces a decrease or an increase of the length respectively or the volume of a body.

(iii) **Perfectly elastic** : If a body, after being strained, regains fully its original shape or size when the external forces are withdrawn, it is called a perfectly elastic body. The maximum stress that a substance will bear and yet return to its initial shape or size when the deforming force is removed is called its *elastic limit.* Different materials have different elastic limit. Steel, for example, has high elastic limit but lead has low.

(iv) **Perfectly rigid** : A body will be called perfectly rigid if it does not suffer any strain under any amount of stress. No substance is perfectly rigid, though glass is a very close approach to it.

(v) **Breaking weight** : It has been said before that a substance behaves as a perfectly elastic substance up to its elastic limit. If the externally applied force exceeds this limit, the body will not return fully to its initial condition when the force is removed. The body will retain certain strain. If the external force be gradually increased, a stage may come when the substance breaks or snaps. The corresponding weight or load is called the breaking weight.

2.3. Force constant of a spring :

Robert Hook experimentally found that the deformation of an elastic body is directly proportional to the force applied on the body so long as the elastic limit of the body is not exceeded. For a spring, for example, if the tensile force applied on it be F and the increase in length be x, then $F \propto x$ or $F = k.x$. The spring exerts an equal but opposite force $-F$. For the spring then, $F = -k.x$. The minus sign indicates that x and F are in opposite directions.

The constant of proportionality, k is known as the *force constant* of the spring. If $x = 1$, $F = k$. *The force constant of a spring is, therefore, defined as the force per unit elongation.* It denotes the *stiffness* of the spring. The larger the value of k, the stiffer is the spring.

Examples : (1) *A horizontal spring is found to be stretched 4 inches from its equilibrium position when a force of 1·5 lb acts on it. Then a 4 lb body is attached to the end of the spring and is pulled 6 inches along a horizontal smooth table from the equilibrium position.* (*i*) *Find the force constant of the spring.* (*ii*) *Also find the force exerted by the spring on 4 lb body just before it is released.*

Ans. (i) A force of 1·5 lb *on* the spring produces a displacement of 4 inches or $\frac{1}{3}$ ft. Hence $k=\frac{F}{x}=\frac{1\cdot5}{\frac{1}{3}}=4\cdot5$ lb/ft.

(ii) The spring is streched 6 inches or $\frac{1}{2}$ ft. Hence the fore exerted *by* the spring is $F=-k.x=-(4\cdot5\times\frac{1}{2})=-2\cdot25$ lb.

The minus sign indicates that the force is directed opposite to the displacement.

(2) *The force constant of a spring of length l is k. The spring is cut into two parts of length l_2 and l_1 such that $l_1=nl_2$, where n is an integer. If the force constants of the two parts are k_1 and k_2, find their magnitudes.*

Ans. Here, $l=l_1+l_2=l_1+\frac{l_1}{n}=\frac{l_1(n+1)}{n}$ .. ..(i)

also, $l=nl_2+l_2=l_2(n+1)$ .. (ii)

Now, for a given spring, its force constant is inversely proportional to its length *i.e.* $k\propto\frac{1}{l}$ or, $kl=$constant. or, $k_1l_1=kl=kl_1\frac{(n+1)}{n}$ $\therefore$ $k_1=\frac{k}{n}(n+1)$

Similarly, $k_2l_2=kl=kl_2(n+1)$ $\therefore$ $k_2=k(n+1)$

(3) *Two massless springs of force constants K_1 and K_2 are joined end to end. Show that the resultant force constant K of the system is given by* $K=\frac{K_1K_2}{K_1+K_2}$.

Ans. Suppose the spring A has force constant K_1 and the spring B a force constant K_2. They are joined end to end. Let a force P be applied at the end of the spring B [Fig. 2.1]. If x_1 and x_2 be the extensions of the springs A and B respectively, then

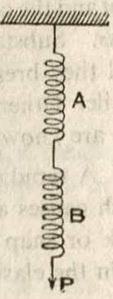


Fig. 2.1

$$K_1x_1=P=K_2x_2. \quad \therefore \ P=\frac{x_1}{\frac{1}{K_1}}=\frac{x_2}{\frac{1}{K_1}}; \quad \text{Then } P=\frac{x_1+x_2}{1/K_1+1/K_2}$$

If K be the resultant force constant of the system, then

$$K.(x_1+x_2)=P \quad \text{or} \quad P=\frac{x_1+x_2}{\frac{1}{K}}$$

Comparing these relations, we get $\frac{1}{K}=\frac{1}{K_1}+\frac{1}{K_2}$ or $K=\frac{K_1K_2}{K_1+K_2}$

2.4. Elastic behaviour of a solid : load-extension graph :

To study the elastic behaviour of a solid material, generally a wire of the material is taken and gradually increasing loads are put on one end of the wire, the other end being rigidly fixed to a stout support. Due to the loads, extensions in the length of the wire will take place and a graph drawn between the load and extension so produced provides us an easy means of studying the elastic properties of thmate erial.

Fig. 22. shows such a load-extension graph for a solid material. From the graph, it is seen that the portion of the curve from O to A is straight, *i.e.*, the extension is proportional to the load. If at any point between O and A, the load is removed, the wire returns to its original state. So, the substance behaves as a perfectly elastic substance up to the point A which is the elastic limit for the substance. Along OA, the wire is said to undergo *elastic deformation*.

Loading beyond A results in an increasing deviation from a straight line curve and if the load be now removed, say at B, a permanent set occurs, the wire returning to C instead of O. OC measures the permanent extension. Under this circumstances, the material becomes *plastic*. If the load be still increased (say up to D), the extension becomes greater than the load demands and the material no longer obeys the elastic law. The wire may become thinner at different places. The point D is known as the *yield point* of the substance. Brass and bronze and many other alloys, however, appear to have no yield point. These materials increase in length beyond the elastic limit as the load is increased without the appearance of the plastic stage.

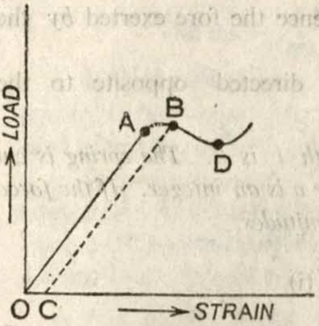


Fig. 2.2

If the load be still further increased, the wire snaps at one of the places where it gets thinner. The point of the graph where this thing happens is called the *breaking point* and the corresponding load or stress is called the *breaking weight* or the *breaking stress*. Substances which elongate considerably and undergo plastic deformation until they break, are called *ductile* substances. Lead, copper and wrought iron are ductile. Other substances however, break just after the elastic limit is reached ; they are known as *brittle*. Glass and high carbon steels are brittle.

A rapid change of stress on an elastic material produces elastic fatigue in it, which causes a degradation of its elastic property. The material may then break or snap at a stress less than the normal breaking stress or even at a load within the elastic limit.

2.5. Hooke's Law :

Hooke's law states that *within elastic limit, the stress applied is proportional to the strain produced.* This law was established by Robert Hooke in 1678. In words, it may be said, stress $\propto$ strain or, $\frac{\text{stress}}{\text{strain}}$ =constant.

The constant is known as the *modulus of elasticity.*

2.6. Different kinds of strain and modulii of elasticity :

Due to the application of external forces on an elastic body, there may be a change in length, or volume or shape of the body, giving different kinds of modulii too.

(*a*) **Longitudinal or tensile strain and Young's modulus :**

Due to the application of tensional forces, if an elastic body changes in its *length* only, the strain is called the *longitudinal* or *tensile strain* and is measured by the ratio of the change in length to the original length. The corresponding stress is called the longitudinal or tensile stress.

The ratio of the longitudinal stress to the longitudinal strain is called the Young's modulus.

$$\text{Hence, Young's modulus } (Y) = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

As liquids and gases have no definite length, Young's modulus is a special property of solid substances only.

Suppose a spring of length L cm. is suspended from a rigid support (Fig. 2.3). Weights can be placed on the scale-pan attached to its lower end. Suppose m gm. of load is placed on the pan and the spring increases in length by l cm. Here, the strain of the spring is longitudinal strain which is measured by the increase in length per unit length of the spring. Hence,

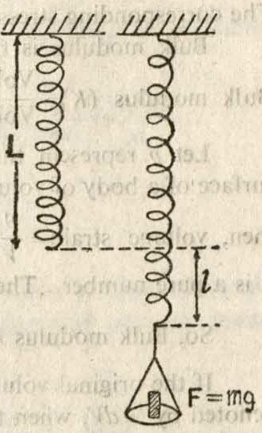


Fig. 2.3

$$\text{Longitudinal strain} = \frac{\text{Increase in length}}{\text{Original length}} = \frac{l}{L}$$

If the area of cross-section of the spring be 'a', then,

$$\text{Longitudinal stress} = \frac{\text{Applied force}}{\text{cross-sectional area}} = \frac{mg}{a}$$

$$\therefore \; Y = \frac{\text{longitudinal stress}}{\text{,, \quad strain}} = \frac{mg}{a} \div \frac{l}{L} = \frac{mgL}{a.l}$$

'Young's modulus of steel is 2×10^{12} dynes/sq. cm.'—this statement means that to produce unit strain in a steel wire, a force of 2×10^{12} dynes will have to be applied per sq. cm. of the cross-section of the wire.

Units of Young's modulus :

It is important to note that a strain, being a ratio of two lengths, has no units ; It is simply a number. A stress being a ratio of force and cross-sectional area is expressed in dynes/sq. cm. or poundals/sq. ft. So, the unit of Young's modulus will be dynes/cm.^{2} in C.G.S. system and poundals/sq. ft. in F.P.S. system. In M.K.S. system, the unit of Young's modulus is Newton/metre^2. Y for steel is about 2×10^{12} dynes/cm^2 ; for copper and brass, the values are $1{\cdot}2 \times 10^{12}$ and 2×10^{12} dynes/cm^2 respectively.

Dimension of Young's modulus :

Strain, being a ratio, has no dimension. Now, dimension of stress

$$= \frac{\text{dimension of force}}{\text{,, \quad ,, area}} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

$$\therefore \; [Y] = \frac{\text{Dimension of stress}}{\text{,, \quad ,, strain}} = [ML^{-1}T^{-2}]$$

(*b*) **Volume strain and Bulk modulus** : Due to the application of external forces, if an elastic body changes its *volume*, without changing its shape, the strain is called the volume strain. For example, if pressure be applied uniformly from all sides normally on a sphere, the volume of the sphere will diminish but the shape will remain the same. Here, the strain undergone by the sphere is a volume strain. The corresponding stress is called the volume stress.

Bulk modulus is the ratio of the volume stress to volume strain. Hence,

$$\text{Bulk modulus } (K)=\frac{\text{Volume stress}}{\text{Volume strain}}$$

Let p represent the magnitude of the *pressure* applied uniformly over the surface of a body of volume V to produce a change in the volume by v. We have, then, volume strain $=\frac{v}{V}$. Like longitudinal strain, volume strain has no unit ; it is a pure number. The volume stress $=p$.

$$\text{So, Bulk modulus } K=\frac{\text{volume stress}}{\text{volume strain}}=p\div\frac{v}{V}=\frac{p.V}{v}.$$

If the original volume of the substance be V, the change in volume may be denoted by $-dV$, when the pressure increases by a small amount dP ; the minus sign shows that the volume decreases. Thus, $K=-\frac{dP}{dV/V}=-\frac{V.dP}{dV}$

Since solid, liquid and gas—all possess some volume, bulk modulus is a general property of matter. Its unit and dimension are the same as those of Young's modulus. Bulk modulus of water (between 1 and 30 atmospheric pressure) is about 2×10^{10} dyne/cm^2 and that of mercury $=27\times10^{10}$ dyne/cm^2.

The reciprocal of bulk modulus is known as *compressibility* $(1/K)$.

(*c*) **Shearing strain and Rigidity modulus** : Due to the application of external forces, if an elastic body changes its *shape* without changing its volume, the strain is called the shearing strain and the corresponding stress the shearing stress.

$$\text{Now, Rigidity modulus}=\frac{\text{shearing stress}}{\text{shearing strain}}.$$

As liquids and gases have no definite shape, rigidity modulus is a special property of solid substances.

Suppose, we have a cube of an elastic material so that its lower surface *DIHC* is rigidly fixed while a tangential force *F* acts on its upper surface *AEGB* [Fig. 2.4]. Under the circumstances, every layer of the cube parallel to the face *AEGB* will slide relatively to one another in the same direction and finally the cube will be deformed into a parallelopiped. By such straining, it is only the shape and not the volume of the body that changes.

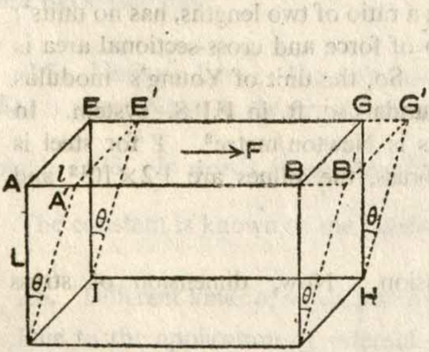


Fig. 2.4

If θ be the angle through which the face *AEID* or the face *BGHC* turns, then it is said that the cube has been sheared through an angle θ and this angle measures the shearing strain. Now, from the fig. 2.4, it is seen that

$$\theta=\frac{AA'}{AD}=\frac{l}{L}=\frac{\text{Displacement of the face } ABGE}{\text{Distance of the face from the bottom}}$$

If $L=1$, then $\theta=l$, *i.e.*, the relative displacement between two surfaces at unit distance apart measures the shearing strain.

$$\text{In this case, the shearing stress}=\frac{F}{\text{areas of the face } ABGE}=\frac{F}{a}.$$

if a is the area of the face *ABGE*.

$$\text{Hence, modulus of rigidity } (n)=\frac{F}{a}\div\frac{l}{L}=\frac{F.l}{a.l}.$$

The unit and dimension of rigidity modulus are same as those of Young's modulus. Rigidity modulus of copper$=4{\cdot}8\times10^{11}$ dynes/cm^2 and of quartz$=3\times10^{11}$ dyne/cm^2.

Torsion of a cylindrical wire :

A body may undergo shearing strain when twisted. Thus, if a wire of circular cross-section be rigidly fixed at one end and a couple be applied at the other, the wire will be twisted and the material will experience a shearing strain. In this case, however, the difference between the angle of shear and the angle of twist is to be noted.

Consider a wire of radius r and length l, fixed at the upper end and twisted by a force applied at the lower end [Fig. 2.5]. If we take a section of the cylindrical wire between radii r and $r+dr$, then a slice of the material *PQAO* has been sheared through an angle α to a position PQA_1O, where O is the centre of the lower end of the wire. Due to twisting, the radii OA, however, moves to OA_1 such that $\angle AOA_1=\theta$. Here, the angle θ is known as the angle of twist. Clearly, α and θ are not equal *i.e.* angle of shear and the angle of twist are not equal. If we take different sections of the cylindrical wire between different radii, it is clear that the angle of shear will be different for different sections but the angle of twist will remain same. It may be proved that the modulus of rigidity of the material is given by $n=\frac{2l\tau}{\pi r^4}$, where τ is called the torsional couple for unit twist $(\theta=1)$.

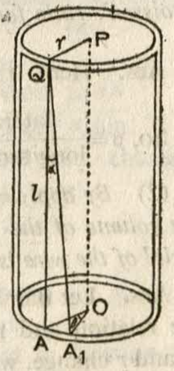


Fig. 2.5

Example : *The lower surface of an aluminium cube of side 10 cm. is rigidly fixed and a tangential force of $7{\cdot}5\times10^{10}$ dyne is applied on the upper surface. As a result, the upper surface is displaced through $0{\cdot}03$ cm. relative to the lower one. Find the modulus of rigidity of aluminium.*

Ans. We know, $n=\frac{F}{\alpha}\cdot\frac{L}{l}$. Here, $F=7{\cdot}5\times10^{10}$ dynes ; $\alpha=10\times10=100$ sq. cm. ; $L=10$ cm. and $l=0{\cdot}03$ cm.

$$\therefore\ n=\frac{7{\cdot}5\times10^{10}}{100}\times\frac{10}{0{\cdot}03}=2{\cdot}5\times10^{11}\text{ dyne/sq. cm.}$$

(*d*) **Poisson's ratio :** When a wire is stretched along its length it elongates but at the same time, it gets thinner, *i.e.*, its diameter decreases. Generally it may be said that a longitudinal strain is always attended with a lateral strain. If the strain be small, the lateral strain is found to be proportional to the longitudinal strain.

Poisson's ratio is defined as the ratio between the lateral strain and the longitudinal strain. In other words,

$$\text{Poisson's ratio }(\sigma)=\frac{\text{lateral strain}}{\text{longitudinal strain}}$$

It is needless to mention that Poisson's ratio is a property of solids only. Being a ratio of two strains, it has no dimension or unit ; it is a pure number.

Suppose, L=the length of a wire ; D=diameter of the wire ; l=increase in length of the wire and d=corresponding decrease in its diameter.

Here, lateral strain$=\frac{d}{D}$ and longitudinal strain$=\frac{l}{L}$.

$$\text{Hence } \sigma=\frac{d/D}{l/L}=\frac{d.L}{l.D}.$$

Examples : (1) *The diameter of a metal rod is 25 cm. When stretched by a given load, its elongation is 4 in 10^5 but its width decreases by 3×10^{-4} cm. Calculate the Poisson's ratio for the metal.*

Ans. Here, the longitudinal strain$=\frac{4}{10^5}$ and lateral strain$=\frac{3\times10^{-4}}{25}$

$$\text{So, }\sigma=\frac{\text{lateral strain}}{\text{longitudinal strain}}=\frac{3\times10^{-4}}{25}\div\frac{4}{10^5}=0{\cdot}3.$$

(2) *By applying force to a wire of length l and radius r, the wire is elongated. If the volume of the wire remains unchanged, show that the Poisson's ratio for the material of the wire is ½.*

Ans. Let the volume of the wire be V. Then $V=\pi r^2.l$. Differentiating the above relation and remembering that the volume of the wire remains constant but l and r change, we have, $0=\pi.r^2\times\delta l+l\times2\pi r.\delta r$.

[δl=change in length and δr=change in radius]

$$\therefore\ r.\delta l=-2l.\delta r \text{ or } \frac{-\delta r/r}{\delta l/l}=\tfrac{1}{2}$$

But $\delta r/r$ denotes the lateral strain and $\delta l/l$ denotes the longitudinal strain. Negative sign shows that as l increases, r decreases and vice versa.

$$\therefore\ \text{Poisson's ratio }\sigma=\tfrac{1}{2}.$$

[**Note :** σ for rubber is about 0·48. This shows that the volume of rubber remains almost unchanged when stretched. Metals, in general, increase in volume when stretched.]

2.7. Relations among the elastic constants :

Young's modulus (y), Bulk modulus (k), rigidity modulus (n) and Poisson's ratio (σ)—these four quantities are known as elastic constants. They are inter-related. The relations are as follows :—

$$\frac{3}{n}+\frac{1}{k}=\frac{9}{y} \quad \ldots\ \ldots(\text{i})$$

$$1+\sigma=\frac{y}{2n} \quad \ldots\ \ldots(\text{ii})$$

$$1-2\sigma=\frac{y}{3k} \quad \ldots\ \ldots(\text{iii})$$

If any two constants are known or found out by experiment, the other two can be calculated from the above relations.

Further, from the above relations, it may be proved mathematically that Poisson's ratio for different materials lies between $+0{\cdot}5$ and -1. Now negative Poisson's ratio for a material means that the material undergoes longitudinal extension and lateral extension simultaneously. But no such material is known to have such property. So, in reality, the value of Poisson's ratio for all materials varies from $+0{\cdot}5$ to 0.

2.8. Comparison between three modulii of elasticity :

Young's modulus (Y)	Bulk modulus (k)	Rigidity modulus (n)
1. $Y=\frac{\text{tensile stress}}{\text{tensile strain}}$	$k=\frac{\text{Volume stress}}{\text{Volume strain}}$	$n=\frac{\text{shear stress}}{\text{shear strain}}$
2. Relates to change in *length* (tensile)	Relates to change in volume ('bulk')	Relates to change in *shape* ('shear')
3. $Y=\frac{F/A}{l/L}$	$k=\frac{V.dP}{-dV}$	$n=\frac{F/A}{\alpha}$
4. Applies only to solids.	Applies to all materials (low value for gases).	Applies to solids and liquids.
5. Used in stretching wires, bending beams, linear thermal changes.	Used in velocity of sound formula for all materials.	Used in twisting wires, helical spring etc.

2.9. Work done in stretching a wire :

In order to stretch a wire, work must be done by the applied force. The energy so spent is stored up in the wire and is called the *energy strain*. When the external force is removed, the wire regains its original length and the energy strain appears as heat.

Suppose a wire of length L is fixed at one end and a tensile force F is applied at the free end. If the increase in length be l, then Young's modulus $Y=\frac{F/\alpha}{l/L}$, where α is the cross-section of the wire. Hence $F=\frac{Y.\alpha.l}{L}$.

Suppose, a small increment of length dl takes place after this. Evidently, the work done for this increment $=F\times dl$. If the total increment l of length be imagined to be made up of a large number of such small increments dl, then the total work done in stretching the wire through l, is given by,

$$W=\int_0^l F.dl=\int_0^l \frac{Y.\alpha.l}{L}\,dl=\frac{Y.\alpha.}{L}\int_0^l l\,dl=\frac{Y.\alpha.}{L}.\ \frac{l^2}{2} \quad .. \quad \text{(i)}$$

Now, the volume of the wire $=L.\alpha$. Hence, work done or energy strain per unit volume $=\frac{W}{L.\alpha}=\frac{1}{2}.\ \frac{Y.\alpha}{L}.\ \frac{l^2}{L.\alpha}=\frac{1}{2}\frac{Y.\alpha.l}{L.\alpha}\times\frac{l}{L}=\frac{1}{2}.\ \frac{F}{\alpha}.\ \frac{l}{L}=\frac{1}{2}\times$ stress $\times$ strain.

The same relation can be established for volume strain and shearing strain. In general, therefore, it may be said that *work done per unit volume in deforming a body*$=\frac{1}{2}\times$ *stress* $\times$ *strain.*

It is to be noted that when stress and strain are expressed in c.g.s. system, the work done will be expressed in erg.

Examples : (1) *A wire 50 cm. long and 1 sq. mm. in cross-section has Young's modulus* $1{\cdot}24\times10^{12}$ *dynes/sq. cm. How much work is done in stretching it through 1 mm.* ?

Ans. Work done $W=\frac{1}{2}.\frac{Y.\alpha.l^2}{L}$ [eqn. (i)]

Here, $Y=1{\cdot}24\times10^{12}$ dynes/sq. cm. ; $\alpha=1$ sq. mm. $=\cdot01$ sq. cm. ; $l=1$ mm $=0{\cdot}1$ cm. and $L=50$ cm.

$$\therefore\quad W=\frac{1}{2}.\ \frac{1{\cdot}24\times10^{12}\times\cdot01\times(0{\cdot}1)^2}{50}=1{\cdot}24\times10^6 \text{ ergs}=0{\cdot}124 \text{ joule.}$$

(2) *A wire of length 5 metre and of uniform circular cross-section of radius 1 mm. is extended by 1·5 mm. when subjected to a uniform tension of 100 newton. Calculate the energy strain per unit volume of the wire.*

Ans. Energy strain/unit volume $=\frac{1}{2}\times\frac{F}{\alpha}.\ \frac{l}{L}$

Here, $F=100$ newton ; $\alpha=\pi r^2=\pi(10^{-3})^2=\pi\times10^{-6}$ sq. m.

$l=1{\cdot}5$ mm. $=1{\cdot}5\times10^{-3}$ metre ; $L=5$ metre.

$$\therefore\quad \text{Energy strain/unit volume}=\tfrac{1}{2}\times\frac{100}{\pi\times10^{-6}}\times\frac{1{\cdot}5\times10^{-3}}{5}$$

$$=4{\cdot}7\times10^3 \text{ joule.}$$

[Note the units of different quantities.]

2.10. Determination of Young's modulus in a laboratory :

Two wires *AB* and *CD* of the same material and sectional area are suspended from a rigid support and they carry two hooks at the free ends [Fig. 2.6]. With one of them (*CD*) is provided a scale graduated in millimetres. A weight is suspended from its hook to keep the wire free from kinks. This wire is called the reference wire. The other wire *AB*, called the experimental wire, is provided with a vernier (*V*) which can easily slide along the millimetre scale attached with the reference wire.

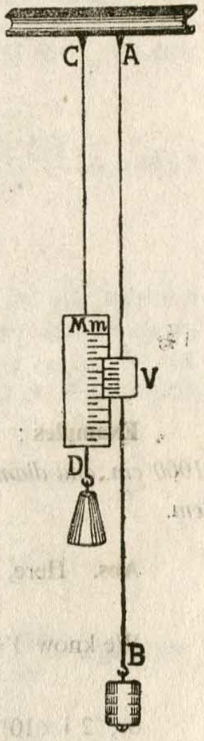


Fig. 2.6

To start with the experiment, a certain load is put on the hook attached with experimental wire to make it free from kinks. The position of the vernier against the main scale is noted, and from this the initial reading is obtained. Then successively equal weights are put on this hook in regular increasing order and the readings at each step are noted from which extensions are calculated. It is to be noted that the maximum load to be put on the hook should not exceed the elastic limit of the material of the experimental wire.

Extensions are also calculated from the readings recorded when loads are withdrawn in the same order in which they were put in. A graph is then plotted with load against extension. A straight line is obtained which passes through the origin *O* [Fig. 2.6(i)]. This shows that stress and strain are proportional to each other, *i.e.*, it experimentally verifies Hooke's law.

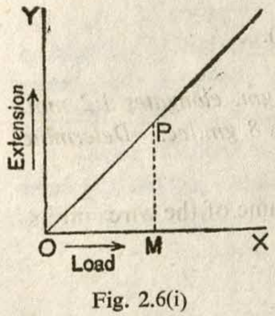


Fig. 2.6(i)

Now, on the straight line a point *P* is taken and a perpendicular *PM* is drawn. *OM* represents a load for which the extension is represented by *PM*. Then, the radius (r) of the wire is measured carefully with the help of a screw gauge at different points of the wire and at every point two readings at right angles to each other are taken. The length of the experimental wire from the point of support to the point of attachment of the vernier is determined. Suppose, the length is *L*.

If *OM* represents m gm. load and *PM* the corresponding extension l, then,

longitudinal stress $=\frac{mg}{\pi r^2}$ and longitudinal strain $=\frac{l}{L}$.

$$\therefore \quad Y=\frac{\text{longitudinal stress}}{\text{longitudinal strain}}=\frac{mg}{\pi r^2}\div\frac{l}{L}=\frac{mgL}{\pi r^2 l}$$

All the quantities on the right hand side of the above equation being known, Y can be easily found out.

Y for some materials

Materials	Y (in dynes/sq. cm.)
Steel	2×10^{12}
Copper	$0{\cdot}95-1\times 10^{12}$
Aluminium	$0{\cdot}62-7\times 10^{12}$
Brass	$0{\cdot}3\times 10^{12}$

Examples : (1) *How much load is required to elongate a steel wire of length 1000 cm. and diameter 0·2 cm. through 2·5 cm. ?* Y *for steel* $=2{\cdot}1\times 10^{12}$ *dynes/sq. cm.*

Ans. Here, $Y=2{\cdot}1\times 10^{12}$; $L=1000$ cm. ; $l=2{\cdot}5$ cm. ; $r=\frac{0{\cdot}2}{2}=0{\cdot}1$ cm.

We know $Y=\frac{mg.L}{\pi r^2.l}$ or, $2{\cdot}1\times 10^{12}=\frac{m\times 980\times 1000}{\pi(0{\cdot}1)^2\times 2{\cdot}5}$

or, $2{\cdot}1\times 10^{12}=\frac{m\times 98}{3{\cdot}14\times 25}\times 10^7$ or, $2{\cdot}1\times 10^5=\frac{m\times 98}{3{\cdot}14\times 25}$

$$\therefore\quad m=\frac{3{\cdot}14\times 2{\cdot}1\times 25\times 10^5}{98}\text{ gm}=168\text{ kg. (nearly).}$$

(2) *A uniform steel wire 250 cm. long weighing 16 gm. elongates 1·2 mm. when stretched by a force of 8 kilo wt. The density of steel is 8 gm./cc. Determine the value of Young's modulus for steel.*

Ans. Let the radius of the wire be r cm. The volume of the wire $=\pi r^2\times 250$ cc. The mass of the wire $=\pi r^2\times 250\times 8$ gm.

$$\therefore\quad \pi r^2\times 250\times 8=16,\quad\text{or, } \pi r^2=\frac{16}{250\times 8}=\frac{1}{125}$$

Now, we know, $Y=\frac{m.g.L}{\pi r^2.l}$ Here, $m=8\times 1000$ gm. ; $L=250$ cm. ; $\pi r^2=\frac{1}{125}$; $l=0{\cdot}12$ cm.

$$\therefore\quad Y=\frac{8\times 1000\times 980\times 250}{\frac{1}{125}\times 0{\cdot}12}=\frac{8\times 1000\times 980\times 250\times 125}{0{\cdot}12}$$

$$=2\times 10^{12}\text{ dynes/sq. cm.}$$

(3) *How much load, in kilograms, will be required to elongate a steel wire 628 cm. long and 2 mm. in diameter through 1 mm. ? Y for steel*$=2\times10^{12}$ *dynes/sq. cm. and* $g=980$ *cm./sec*^{2}. [*H. S. Exam. 1964*]

Ans. We know, $Y=\frac{m.g.L}{\pi r^2.l}$ Here, $g=980$ C.G.S. ; $L=628$ cm. ; $l=1$ mm. $=0\cdot1$ cm. ; $r=1$ mm. $=0\cdot1$ cm. ; $Y=2\times10^{12}$ dynes/sq. cm.

$$\therefore\quad 2\times10^{12}=\frac{m\times980\times628}{3\cdot14\times(0\cdot1)^2\times0\cdot1}\quad\text{or,}\quad 2\times10^6=\frac{m\times98\times628}{314}=m\times98\times2$$

$$\text{or,}\quad m=\frac{10^6}{98}\text{ gm.}=\frac{10^3}{98}\text{ kg.}=10\cdot21\text{ kg.}$$

(4) *The Poisson's ratio and the Young's modulus of a light rod vertically suspended from one end, having length 300 cm. and radius 3 cm. are respectively 0·3 and* 2×10^{12} *dynes/cm*^{2}. *A load of 1200 kg. is placed at the other end of the rod. Find the lateral strain of the rod.* [*H. S. Exam. 1980*]

Ans. We know, $Y=\frac{\text{longitudinal stress}}{\text{,, strain}}$

$$\text{or,}\quad 2\times10^{12}=\frac{1200\times10^3\times980}{\pi(3)^2}\times\frac{1}{\text{longitudinal strain}}$$

$$\therefore\quad \text{longitudinal strain}=\frac{2\times98}{3\pi\times10^6}$$

$$\text{Now, Poisson's ratio}=\frac{\text{lateral strain}}{\text{longitudinal strain}}\quad\text{or,}\quad 0\cdot3=\frac{\text{lateral strain}}{2\times98}\times3\pi\times10^6$$

$$\therefore\quad \text{lateral strain}=\frac{0\cdot3\times2\times98}{3\pi\times10^6}=0\cdot624\times10^{-5}$$

(5) *A body of mass 2 kg. and density 2·7 gm./c.c. is suspended by a steel wire of length 1 metre and diameter 1 mm. If the body is now completely immersed in water, find the change in length of the wire. Y for steel*$=2\times10^{12}$ *dynes/cm*^{2} *and* $g=1000$ *cm./s*^{2}.

Ans. Change in length is equal to the difference of elongations produced when the body is in air and completely immersed in water. When the body is in air, let the elongation be l_1. Then,

$$Y=\frac{F}{\alpha}\times\frac{L}{l_1}\quad\text{or}\quad 2\times10^{12}=\frac{2\times10^3\times1000}{\pi(\cdot05)^2}\times\frac{100}{l_1}\quad\text{or}\quad l_1=\frac{1}{25\pi}.$$

When the body is immersed in water, let the elongation be l_2. In this case, the apparent wt. of the body=Real wt.−wt. of displaced water

$$=2\times10^3\times1000-\frac{2\times10^3}{2\cdot7}\times1000=\frac{1\cdot7}{2\cdot7}\times2\times10^6\text{ gm.-wt.}$$

$$\therefore\ \text{From } Y=\frac{F}{\alpha}\times\frac{L}{l},\ \text{we have,}\ 2\times10^{12}=\frac{1\cdot7\times2\times10^6}{2\cdot7\times\pi(0\cdot05)^2}\times\frac{100}{l_2}\quad\text{or}\quad l_2=\frac{1\cdot7}{2\cdot7\times25\pi}$$

$$\therefore\ l_1-l_2=\frac{1}{25\pi}-\frac{1\cdot7}{2\cdot7\times25\pi}=\frac{1}{2\cdot7\times25\times\pi}=4\cdot72\times10^{-3}\text{ cm.}$$

(6) *A steel wire, 2 mm. in diameter is just stretched between two points at 20°C. What will be the tension in the wire if the temperature falls to 10°C. Co-efficient of linear expansion of steel*$=1{\cdot}1\times10^{-5}$.

Ans. Due to the fall in temperature, the wire will try to shrink in length. Therefore a tension will be applied on the wire.

Here, decrease in length $=l_{20}-l_{10}=l_{10}\times\alpha\times(20-10)=\alpha\times l_{10}\times10$

$$\therefore \text{ longitudinal strain}=\frac{l_{20}-l_{10}}{l_{10}}=\frac{\alpha\times l_{10}\times10}{l_{10}}=\alpha\times10$$
$$=1{\cdot}1\times10^{-5}\times10=1{\cdot}1\times10^{-4}.$$

Now, if the tension be W dynes, then longitudinal stress $=\dfrac{W}{\pi(0{\cdot}1)^2}$ dynes/sq. cm.

$$\therefore\ 2{\cdot}1\times10^{12}=\frac{W}{\pi(0{\cdot}1)^2}\times\frac{1}{1{\cdot}1\times10^{-4}}$$

$$\text{or, } W=2{\cdot}1\times10^{12}\times3{\cdot}14\times(0{\cdot}1)^2\times1{\cdot}1\times10^{-4}\text{ dynes}$$
$$=2{\cdot}1\times3{\cdot}14\times1{\cdot}1\times10^{6}\text{ dynes}=7{\cdot}253\times10^{6}\text{ dynes.}$$

(7) *A 20 kg. weight is suspended from a length of copper wire 1 mm. in radius. If the wire breaks suddenly what will be the change in temperature of the wire? Young's modulus for copper*$=12\times10^{11}$ *dynes/cm*^{2}; *density of copper*$=9$ *gm./c.c.; sp. heat*$=0{\cdot}1$ *and* $J=4{\cdot}2\times10^{7}$ *ergs/cal.*

Ans. We know, $Y=\dfrac{F}{\alpha}\times\dfrac{L}{l}$. or $l=\dfrac{F}{\alpha}.\dfrac{L}{Y}=\dfrac{20\times10^3\times980\times L}{\pi(0{\cdot}1)^2\times12\times10^{11}}=\dfrac{2\times98\times L}{12\times\pi\times10^4}$

$$\text{Now, work done}=\tfrac{1}{2}\times F\times l=\tfrac{1}{2}\times\frac{20\times10^3\times980\times2\times98\times L}{12\times\pi\times10^4}$$
$$=\frac{980\times98\times L}{6\pi}\text{ erg.}$$

This energy will remain stored in the wire as its potential energy. When the wire breaks, this energy will appear as heat energy which will increase the temperature of the wire.

Now, Heat produced $=$ mass $\times$ sp. heat $\times$ rise of temperature
$$=\pi(0{\cdot}1)^2\times L\times9\times0{\cdot}1\times\theta\qquad[\theta=\text{rise in temp.}]$$
$$=9\pi(0{\cdot}1)^3\times L\times\theta\text{ cal.}$$

We know, $W=J.H$ or, $\dfrac{980\times98\times L}{6\pi}=4{\cdot}2\times10^7\times9\pi\times(0{\cdot}1)^3\times L\times\theta$

$$\therefore\ \theta=\frac{980\times98}{6\times4{\cdot}2\times10^7\times9\times(\pi)^2\times(0{\cdot}1)^3}=0{\cdot}0043^\circ\text{C}$$

(8) *A piece of copper wire has twice the radius of a piece of steel wire. One end of the copper wire is joined to one end of the steel wire so that both can be subjected to the same longitudinal force. By what fraction of its length will the steel wire have stretched when the length of the copper wire has increased by 1%. Y for steel is twice that for copper.*

Ans. We know, $Y=\frac{F}{\pi r^2}\cdot\frac{L}{l}$. Let us use subscripts 1 and 2 to refer copper and steel respectively. Then, $Y_1=\frac{F}{\pi r_1^2}\cdot\frac{L_1}{l_1}$ and $Y_2=\frac{F}{\pi r_2^2}\cdot\frac{L_2}{l_2}$

Now, $\frac{Y_2}{Y_1}=\left(\frac{r_1}{r_2}\right)^2\left(\frac{l_1}{l_2}\right)\left(\frac{L_2}{L_1}\right)$; But $\frac{Y_2}{Y_1}=2$; $\frac{r_1}{r_2}=2$; $\frac{l_1}{L_1}=\frac{1}{100}$

$\therefore\ 2=4\cdot\frac{1}{100}\cdot\frac{L_2}{l_2}\quad\therefore\ \frac{l_2}{L_2}=\frac{2}{100}$

Thus the steel wire will increase by 2% of its original length.

(9) *A metallic wire of length 3 metre and diameter 1 mm. is stretched by a load of 10 kg. If Young's modulus of the material of the wire be $Y=12\cdot5\times10^{11}$ dynes/cm² and Poisson's ratio$=0\cdot26$, find the contraction in diameter of the wire.*

Ans. Suppose L=length of the wire, D=diameter ; δL=longitudinal extension ; δD=contraction in diameter ; here, longitudinal strain$=\frac{\delta L}{L}$ and lateral strain$=\frac{\delta D}{D}$. If F be the tension and α the cross-section of the wire,

$Y=\frac{F}{\alpha}\Big/\frac{\delta L}{L}\qquad\therefore$ longitudinal strain $\frac{\delta L}{L}=\frac{F}{Y.\alpha}$

Now, $\sigma=\frac{\text{lateral strain}}{\text{longitudinal ,,}}=\frac{\delta D/D}{\delta L/L}\quad\therefore\ \frac{\delta D}{D}=\sigma\times\frac{\delta L}{L}=\frac{F.\sigma}{Y.\alpha}$

$\therefore\ \delta D=\frac{\sigma.F.D}{Y.\alpha}=\frac{0\cdot26\times10\times10^3\times980\times0\cdot1}{\pi(0\cdot05)^2\times12\cdot5\times10^{11}}=2\cdot6\times10^{-5}$ cm.

(10) *A steel wire of diameter 0·8 mm and length 1 metre is clamped firmly at two points A and B which are 1 metre apart and in the same horizontal plane. A body is hung from the middle point of the wire such that the middle point sags 1 cm lower from the original position. Calculate the mass of the body. Given $Y=2\times10^{12}$ dynes/cm².* [*I.I.T. 1970*]

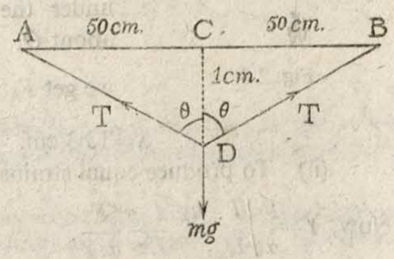


Fig. 2.7

Ans. Suppose a mass m is hung from the middle point C of the wire and the point C after sag of 1 cm, comes to the point D [Fig. 2.7]. Due to the load, both halves of the wire AC and CD will have some longitudinal extension.

Now, $(AD)^2=(AC)^2+(CD)^2=(50)^2+(1)^2=2501\quad\therefore\ AD=50\cdot01$ cm.

So, the increase in length of the portion $AC=50\cdot01-50=0\cdot01$ cm.

The longitudinal strain in the wire$=\frac{0\cdot01}{50}=2\times10^{-4}$

The cross-sectional area of the wire$=\frac{\pi d^2}{4}=\frac{\pi(0\cdot08)^2}{4}$ cm²

Now, Young's modulus $Y=\frac{\text{long. stress}}{\text{,, strain}}$ or $2\times10^{12}=\frac{\text{long stress}}{2\times10^{-4}}$

$\therefore$ Long stress $=2\times10^{-4}\times2\times10^{12}=4\times10^{8}$ dynes/cm^2

So, the tension on each half of the wire, $T=$ stress $\times$ cross-sectional area

$$=4\times10^{8}\times\pi\times\frac{(0{\cdot}08)^2}{4}$$

$$=\pi\times64\times10^{4}\text{ dynes.}$$

From fig, it is clear that $mg=2T.\cos\theta=2\times\pi\times64\times10^{4}\times\frac{1}{50{\cdot}01}$

$$\therefore\quad m=\frac{2\times\pi\times64\times10^{4}}{50{\cdot}01\times980}=82\text{ gm.}$$

(11) *A light rod of length 20 cm. is suspended from the ceiling horizontally by means of two vertical wires of equal length tied to its ends. One of the wires is made of steel and is of cross-section 0·1 sq. cm. and the other is of brass of cross-section 0·2 sq. cm. Find out the position along the rod at which a weight may be hung to produce (i) equal stresses in both wires (ii) equal strains in both wires.*

Y for brass$=10\times10^{11}$ dynes/cm^2 and for steel$=20\times10^{11}$ dynes/cm^2.

[*I.I.T. 1974*]

Ans. Let A and B be steel and brass wires [Fig. 2.8], PQ the rod, F_A and F_B are the tensional forces in the wires A and B respectively.

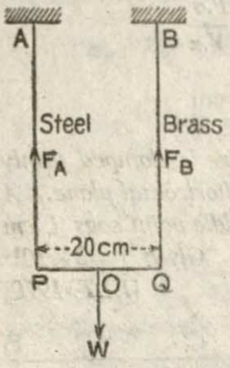


Fig. 2.8

(i) To produce equal stresses, we have, $\frac{F_A}{\alpha_1}=\frac{F_B}{\alpha_2}$

or $\frac{F_A}{F_B}=\frac{\alpha_1}{\alpha_2}$ where α_1 and α_2 are the cross-sectional areas of the wires A and B respectively.

But $\frac{\alpha_1}{\alpha_2}=\frac{0{\cdot}1}{0{\cdot}2}=\frac{1}{2}$ $\therefore$ $\frac{F_A}{F_B}=\frac{1}{2}$ or $2F_A=F_B$.

Let the weight W be suspended at O, say at a distance x from the end P. Since the system is in equilibrium under the forces F_A, F_B and W, taking moments about O,

we get $F_A\times x=F_B\,(20-x)$ or $x=\frac{F_B}{F_A}(20-x)=2(20-x)$

$\therefore$ $x=13{\cdot}3$ cm.

(ii) To produce equal strains, l/L ratio should be same for both the wires.

Now, $Y=\frac{F}{\alpha}\Big/\frac{l}{L}$ or $\frac{l}{L}=\frac{F}{\alpha.Y}$

$$\therefore\ \frac{F_A}{\alpha_1.Y_A}=\frac{F_B}{\alpha_2.Y_B}\quad\text{or}\quad\frac{F_A}{F_B}=\frac{\alpha_1}{\alpha_2}\times\frac{Y_A}{Y_B}=\frac{0{\cdot}1}{0{\cdot}2}\times\frac{20\times10^{11}}{10\times10^{11}}=1.$$

$\therefore$ $F_A=F_B$.

Taking moments about O, we have, $F_A.x=F_B\,(20-x)$

or $F_A.x=(20-x).F_A$. or, $x=10$ cm.

Exercises

Essay type :

1. What is Young's modulus ? How would you determine Young's modulus of the material of a wire in the laboratory ? Can you verify Hooke's law from this experiment ? [*Tripura H. S. 1980*]

2. How can you demonstrate with the help of a simple experiment that a body elongates under a stretching force and that the body regains its initial state when the force is withdrawn. For which property of the matter such thing happens ? Mention the law in this connection.

3. How is shearing strain measured ? Clearly explain Young's modulus, Bulk modulus, Rigidity modulus and Poisson's ratio. [*Tripura H. S. 1981*]

4. Find the work done in stretching a wire. What happens to the energy when the wire is allowed to return to its original length ?

Short answer type :

5. What do you understand by elasticity of matter ? Why is steel considered more elastic than rubber ? [*H. S. 1981, '82*]

6. Write the exact definitions of :—(i) strain, (ii) stress, (iii) perfectly elastic, (iv) elastic limit. What are the units of stress and strain ? [*H. S. Exam. 1982*]

7. What is Hooke's law ? What are Young's modulus and volume strain ? "The Young's modulus of steel is 2×10^{12} dynes/sq. cm."—Explain the statement.

8. Define : (a) longitudinal stress, (b) longitudinal strain, (c) Young's modulus. Determine the dimension and the unit of Young's modulus in the C.G.S. and M.K.S. systems.

9. Define Poisson's ratio. What are its maximum and minimum values ? Write the relations it has with the modulii of elasticity.

10. Explain : (i) Bulk modulus, (ii) Rigidity modulus and (iii) Poisson's ratio.

11. A stretched wire suddenly snapped. Would there be any change in its temperature ? Draw the load-extension graph for a solid.

12. In determining the Young's modulus of a steel wire, the wire is hung vertically and load is put on the other end. Why a long and thin wire is taken in the experiment ? Why is a second similar wire fixed by its side ?

13. What is force constant of a spring ? What property of the spring does it denote ?

14. Show that if a wire of length L is elongated by an amount l, the energy-strain per unit volume$=\frac{1}{2}Yl^2/L^2$, where Y=Young's modulus.

Objective type :

15. Pick out the correct answer :—

(a) Steel is considered to be more elastic than rubber because (i) steel is harder than rubber (ii) steel has greater density than rubber (iii) greater stress is required for a particular strain in the case of steel.

(b) Due to the application of external forces, if an elastic body changes its volume without changing its shape, the strain is called (i) longitudinal strain (iii) shearing strain (iii) volume strain.

(c) A certain force F is required to break a piece of cord. A force (i) $2F$ (ii) F (iii) $F/2$ will be required to break a piece of cord of the same material but twice as long as the first.

(d) If a body does not suffer any strain under any amount of stress, the body is called (i) a perfectly elastic body (ii) a perfectly rigid body (iii) a plastic body.

(e) A wire is stretched by a load. Suddenly the wire snaps. As a result, the temperature of the wire will (i) decrease (ii) increase (iii) remain the same.

(f) The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied : (i) length = 50 cm ; diameter = 0·5 mm (ii) length = 100 cm ; diameter = 1 mm (iii) length = 200 cm ; diameter = 2 mm (iv) length = 300 cm ; diameter = 3 mm. [*I.I.T. 1981*]

[**Hints** : Extension $= \frac{4T.L}{\pi d^2 Y}$; T and Y are constants. So extension $\propto \frac{L}{d^2}$]

Numerical problems :

16. A load of 25 kg. is attached to a wire of diameter 0·4 cm. and a length of 100 cm. of the wire is found to elongate to 102 cm. Find the Young's modulus of the material of the wire. [Ans. $0{\cdot}975 \times 10^{10}$ dynes/sq. cm.]

17. A wire of length 2 metres and diameter 1 mm. supports a weight 8 kilo at one end. If the material of the wire has Young's modulus of $1{\cdot}995 \times 10^{12}$ dynes/sq. cm., find the increment in length of the wire. [Ans. 1 mm.]

18. A load of 20 kg. is suspended from the end of a wire of length 600·5 cm., and of cross-section 1 sq. mm. When the load is taken off, the length decreases by 0·5 cm. Find the Young's modulus for the material of the wire. [Ans. $2{\cdot}35 \times 10^{12}$ dynes/sq. cm.]

19. A uniform steel wire, of length 250 cm. and mass 16 gm. has density 8 gm./c.c. If mass of 12 kg. is needed to elongate it through 1·8 mm., what is its Young's modulus ? [*Tripura H. S. 1978*] [Ans. 2×10^{12} dynes/cm^2]

20. Two wires of different material are, each 2 metres long ; the diameter of one is 2 mm. while that of the other is 3 mm. Each of them is stretched by a load of 10 kg. The elongation of the first wire is found to be double than that of the second. Compare their Young's modulii. [Ans. 9 : 8]

21. A weight of 3 kg. is suspended from one end of a copper wire, whose length is 2 metres and diameter is 0·5 mm. The elongation of the wire is 2·38 mm. What is the Young's modulus of copper ? [Ans. $1{\cdot}26 \times 10^{12}$ dynes/sq. cm.]

22. A copper wire of length 300 cm. and radius 0·5 mm. increases in length by 3 mm. when stretched by a load of 10 kg. Calculate the Young's modulus of copper. [Ans. $1{\cdot}2 \times 10^{12}$ dynes/cm^2]

23. A mass of 8 kg is hung from one end of an iron wire, 2 metre long and 1 mm. in diameter. If the Young's modulus of iron be 2×10^{12} dynes/cm^2, find the elongation of the wire. [*Tripura H. S. 1980*] [Ans. 0·099 cm.]

24. A mass of 10 kg. is hung from one end of a copper wire 3 metre long and 1 mm. in diameter. Y for copper is $12{\cdot}5 \times 10^{11}$ dynes/cm^2. What is the elongation of the wire ? If Poisson's ratio of copper be 0·26, what is the lateral compression ? [Ans. 0·3 cm. (nearly) ; $2{\cdot}6 \times 10^{-4}$ mm.]

25. A certain type of wire is found to cross its elastic limit when the strain exceeds $\frac{1}{100}$. It has a diameter of ·04 inch and Young's modulus for its substance is 12×10^6 poundals/sq. inch. What is the greatest weight that can be hung from a length of it without causing permanent stretching ? [Ans. 4·71 lb.]

26. If the strain in a wire, 10 metres long, is 0·001%, what will be its elongation ? If the wire has a cross-sectional area 2 sq. mm. and is stretched by 1 kg., what will be its stress, in dynes/sq. cm. ? [Ans. 0·1 mm. ; 49×10^6 dynes/sq. cm.]

27. When a mass of 50 lb. is placed at the middle of a plank supported at its ends, the depression of the middle point is 2 inches. What will be the depression when the load is 75 lb. ? How much load will be required to produce a depression of 3·5 inches ? [Ans. 3 inches ; 87·5 lb.]

28. A steel rod, 1 sq. cm. in cross-section is rigidly clamped at two ends and there is no room for expansion at the ends. If the initial temperature of the rod be 0°C and the final temperature be 30°C, what will be the force at the two ends? [Y for steel $= 2{\cdot}1 \times 10^{12}$ dynes/sq. cm. ; $\alpha = 1{\cdot}1 \times 10^{-5}$ per °C]. [Ans. $6{\cdot}93 \times 10^8$ dynes]

29. By applying equal and opposite tangential forces on the two opposite sides of an aluminium cube of side 3 cm., an angle of shear of 0·01° is produced. If the modulus of rigidity of aluminium be 7×10^{11} dynes/cm², what is the force applied ? [Ans. 11×10^{8} dynes]

[**Hints** : Shearing strain$=\dfrac{0{\cdot}01\times\pi}{180}$ rad.]

30. What stress is required to produce elongation of $\frac{1}{10}$th of 1% of the length of a wire ? The Young's modulus of wire is 12×10^{11} dynes/cm² and diameter is 0·56 mm. What force is needed to produce the above stress ? [Ans. $1{\cdot}2\times10^{9}$ dynes/cm² ; $29{\cdot}6\times10^{5}$ dynes]

31. Find the work done in stretching a wire, 1 sq. mm. in cross-section and 2 metres in length through 0·1 mm. Y for the material of the wire$=2\times10^{12}$ dynes/sq. cm. What is the energy stored per unit volume ? [Ans. 5000 erg.; $2{\cdot}5\times10^{3}$ ergs/c.c.]

32. A flexible wire of length 2 metres and cross-section 1 sq. mm. is stretched tight between two points A and B in the same horizontal plane. A mass of 10 gm. hung from the mid-point of the wire depresses the point through 1 cm. below the line AB. Find Young's modulus for the material of the wire. [Ans. $9{\cdot}8\times10^{11}$ dynes/sq. cm.]

33. A cylindrical copper wire of length 1·5 metre and diameter 2 mm. is joined face to face with an indentical steel wire and a composite wire of length 3 metre is thereby made. A load is hung from one end of the composite wire so that it increases in length by 3·003 metre. Calculate the strain in copper and iron wires. Find also the force applied. Y for copper$=1{\cdot}2\times10^{12}$ and for steel$=2\times10^{12}$ dynes/cm². [Ans. Cu$=1{\cdot}25\times10^{-3}$; Steel$=0{\cdot}75\times10^{-3}$; $4{\cdot}71\times10^{7}$ dynes]

34. A force of 10 newton is required to elongate (within elastic limit) a rope of radius 3·5 mm. by $\frac{1}{20}$th of its length. What is Young's modulus for the material of the rope ? [*H. S. Exam. 1982*] [Ans. $5{\cdot}1\times10^{7}$ dynes/cm²]

35. A steel rod 25 cm. long has a cross-sectional area of 0·8 sq. cm. What force would be required to stretch this rod by the same amount as the expansion produced by heating it through 10°C ? Co-eff^{n} of linear expansion of steel$=10^{-5}/°C$, Y for steel$=20\times10^{11}$ dynes/sq. cm. [*I.I.T. 1971*] [Ans. 16×10^{7} dynes]

36. A wire AB of radius 0·3 mm and length 300 cm is attached to a fixed support and hangs vertically. The wire carries a load 4 kg-wt attached to its mid-point and also another weight of 4 kg-wt at its lower end B. If Y for the material of the wire be 2×10^{12} dynes/cm², find the depression of the end B. [Ans. 0·312 cm]

[**Hints** : The tension at C, the mid-point$=8$ kg-wt and that at $B=4$ kg-wt. If the elongation of the lower half$=x$ cm, that of the upper half$=2x$. So, the depression$=3x$]

37. A steel wire of cross-sectional area 0·5 mm² is held between two fixed supports. If the tension in the wire is negligible and it is just taut at a temperature of 20°C, find the tension when the temperature falls to 0°C. [*I.I.T. 1973*] [Ans. $2{\cdot}52\times10^{6}$ dynes]

[α for steel$=12\times10^{-6}/°C$; $Y=2{\cdot}1\times10^{12}$ dynes/cm²]

Harder problems :

38. A sphere of mass 25 kg. and radius 10 cm. is fastened at one end of a steel wire, the other end being fixed at the ceiling of a room. The point of suspension is 521 cm. above the floor. When the sphere oscillates like a pendulum, it just touches the floor at its lowest point of oscillation. What is the velocity of the sphere at the lowest point ? Young's modulus of steel $=20\times10^{11}$ dynes/cm² ; Initial length of the wire$=500$ cm., radius of the wire$=0{\cdot}05$ cm. [*I.I.T. 1971*] [Ans. 375 cm./s]

[**Hints** : If F the tension in the wire, $F=Y.\alpha.\dfrac{l}{L}=20\times10^{11}\times\pi(0{\cdot}5)^{2}\times\dfrac{1}{500}$
$=31{\cdot}4\times10^{6}$ dynes

Elongation of the wire $(l)=521-520=1$ cm. Now, tension is the sum of weight of the sphere and its centripetal force, *i.e.* $F=mg+\dfrac{mv^{2}}{r}$.]

39. Two springs of force constants K_1 and K_2 are joined end to end. Show that the force constant K of the composite spring so formed is given by $K=\dfrac{K_1K_2}{K_1+K_2}$.

40. A lead sphere of radius 2 cm. is suspended by a rubber thread. If the extension of the thread is 0·1%, calculate the Young's modulus of rubber. The cross-sectional area of the thread is 0·04 sq. cm. and density of lead=5·5 gm./c.c. [Ans. $0{\cdot}45\times10^{10}$ dyne/cm^2]

41. When a mass of 5 kg. is hung from one end of a vertical wire of length 220 cm. and radius 1 mm., it elongates through 0·07 cm. What is the Young's modulus of the material of the wire ? $g=1000$ cm/s^2 and $\pi=\frac{22}{7}$. [*Jt. Entrance 1976*] [Ans. 5×10^{11} dyne/cm^2]

42. A wire of length 200 cm. and diameter 1·22 mm. are stretched between two rigid supports. When a load is suspended from the middle point of the wire, the point sags through 2 cm. What is the value of the load ? Y for the material of the wire=$12{\cdot}3\times10^{11}$ dynes/cm^2.
[Ans. 115 gm.]

43. A glass tube of uniform cross-section and of length 120 cm. is closed at one end. It is filled with water. When the tube is stretched vertically downward, it elongates through 1 cm. but the column of water only increases through 0·7 cm. in length. What is the Poisson's ratio of glass ? [Ans. 0·336]

44. A wire of length L and cross-section α is stretched through l $(l<<L)$. Show that if the elastic limit is not exceeded, the potential energy of the wire increases by $\dfrac{\alpha Yl^2}{2L}$.

45. The Poisson's ratio of a material is σ. Show that if α be the longitudinal strain, the volume strain=$(1-2\sigma)\alpha$.

46. A wire of length 1 metre when stretched by a load of 10 kg. increases in length by 0·1 cm. The cross section of the wire is 0·1 cm^2. Calculate the Young's modulus of the material. If the Poisson's ratio be $\frac{1}{3}$, calculate the change in volume of the wire.
[*Jt. Entrance 1982*] [Ans. $9{\cdot}8\times10^{10}$ dynes/cm^2 ; 0·0033 c.c.]

[**Hints :** $V=\pi r^2l$; $dV=\pi r^2dl+2\pi r.ldr$. Now $\sigma=-\dfrac{dr/r}{dl/l}$

$\therefore\ dV=\pi r^2.dl-2\pi r^2.\sigma dl=\pi r^2.dl(1-2\sigma)=\alpha.dl(1-2\sigma)$]

47. A disc of mass M is placed on a table. A stiff spring is attached to it and is vertical. To the other end of the spring is attached a disc of negligible mass. What minimum force should be applied to the upper disc to press the spring such that the lower disc is lifted off the table when the external force is suddenly removed ? [*I.I.T. 1976*] [Ans. $M.\ g$]

48. A gold wire 0·32 mm. in diameter, elongates by 1 mm. when stretched by a force of 330 gm. wt. and twists through 1 radian when equal and opposite torques of 145 dyne-cm. are applied at its ends. Find the value of the Poisson's ratio for gold. $g=981$ cm/s^2.
[Ans. 0·429]

[**Hints :** torque=$\dfrac{\pi nr^4}{2l}$ and $n=\dfrac{Y}{2(1+\sigma)}$]

3

HYDROSTATICS

3.1. Density of a substance : Everybody realises that a given volume of one material has, in general, a different weight than the same volume of some other material. For instance, we ordinarily say that iron is 'heavier' than wood. More exactly, we should say that any given volume of iron is heavier than the same volume of wood, *i.e.* the density of iron is greater than the density of wood.

Definition : *The density of a substance is defined as the mass per unit volume of the substance.* Stated as a formula, $D=\frac{M}{V}$ where D stands for density, M for mass and V for volume. In other words, $M=D\times V$.

Sometimes, the letter 'ρ' is used to indicate the density of a substance.

Units of density :

C.G.S. system : If 1 c.c. of a substance contains 1 gm. of mass, its density is taken as the unit in the C.G.S. system and is called 1 gm/c.c. The density of pure water at 4°C is taken as 1 gm/c.c.

F.P.S. system : If 1 cu. ft. of a substance contains 1 lb. of mass, its density is taken as the unit in the F.P.S. system and is called 1 lb/cu. ft. 1 cu. ft. of water has mass 62·5 lb. So, the density of water in the F.P.S. system is 62·5 lb/cu. ft.

M.K.S. system : If 1 cubic metre of a substance contains 1 kg of mass, its density is taken as the unit in the M.K.S. system and is called 1 kg/metre^3.

It is to be borne in mind that the density of a substance in the C.G.S. system is different from its density in the F.P.S. or M.K.S. system. So, whenever the density of a substance is mentioned, its unit should also be mentioned along with it. For example, it is incorrect to say that the density of mercury is 10·5 ; it should be 10·5 gm./c.c. The density of mercury in the F.P.S. system is, however, $10{\cdot}5\times 62{\cdot}5$ lb/cu. ft.

Importance of density measurement : Architects and engineers must have the knowledge about the densities of various building materials when engaged in the design of bridges, flyovers and other massive structures. Chemists often make a density determination as a test of the purity of a substance.

To measure the density of a substance, we are to measure the mass and the volume of a piece of the substance. Mass, we can determine by a balance while the method of measurement of volume of a regularly shaped body had been discussed earlier.

Examples : (1) *A piece of iron weighs 740 gm. and has a volume 100 c.c. What is the density of iron ?*

Ans. Here, $M=740$ gm., $V=100$ cc.; We know $D=\frac{M}{V}=\frac{740}{100}=7{\cdot}4$ gm./c.c.

(2) *The radius of a sphere made of steel is 1 cm. and its mass is 32·7 gm. ; what is the density of steel ?*

Ans. Volume of the sphere $=\frac{4}{3}\pi\times(\text{radius})^3=\frac{4}{3}\times\frac{22}{7}\times(1)^3=\frac{88}{21}$ c.c.

So, the density of steel $=\dfrac{\text{mass of the sphere}}{\text{volume of the sphere}}=\dfrac{32{\cdot}7}{88/21}=7{\cdot}8$ gm./c.c.

3.2. Specific gravity :

We have seen that equal volumes of different substances weigh differently. One c.c. of gold, for example, is heavier than one c.c. of copper or 1 c.c. of iron is heavier than 1 c.c. of wood. Sp. gravity of a substance (solid or liquid) means how many times a piece of the substance is heavier than an equal volume of a standard substance. Water is accepted as the standard in the cases of solids and liquids. For example, sp. gr. of gold is 19·32—this means that a piece of gold is 19·32 times heavier than an equal volume of water.

Definition : *The specific gravity (sp. gr.) of a substance is defined as the ratio of the weight of any volume of it to the weight of an equal volume of water.* In other words,

$$S=\frac{\text{Weight of any volume of the substance}}{\text{,, \quad ,, an equal volume of water}}$$

[Note : Density of water changes with temperature. At 4°C, water has maximum density and for a correct definition of sp. gr. equal volume of water at 4°C is always taken. For ordinary purposes, temperature need not be mentioned.]

In the above definition, any volume of the substance may be taken. Suppose, unit volume of the substance is taken. Then

$$S=\frac{\text{Weight of unit volume of the substance}}{\text{,, \quad ,, \quad ,, \quad ,, \quad ,, water}}$$

But, weight of unit volume of any substance is known as its **density**.

$$\text{Hence } S=\frac{\text{Density of the substance}}{\text{,, \quad ,, water}}$$

Specific gravity being a ratio of two densities, is a pure number, having no units. Sometimes, it is known as *relative density*.

In the C.G.S. system, the density of water, (at 4°C) is 1 gm./c.c. ; so, in this system, $S=\dfrac{\text{Density of the substance}}{1}$

In other words, the specific gravity and the density of a substance are numerically equal in the C.G.S. system. In the F.P.S. system, however, the density of water is 62·5 lb/cu. ft.

$$\text{So, } S=\frac{\text{Density of the substance}}{62{\cdot}5}$$

or, $S\times62{\cdot}5=$ Density of the substance (in the F.P.S.)

Temperature correction of sp. gravity : It has been mentioned earlier that for a correct determination of sp. gravity of a substance an equal volume of water at 4°C should be taken. But the temperature of water during the experiment may not remain at 4°C. How, then, can we find the correct value of the specific gravity? Evidently a correction is necessary to the result obtained experimentally. The correction can be done in the following way. We know,

$$S=\frac{\text{Weight of a piece of substance}}{\text{,, \quad ,, equal volume of water at } 4^\circ\text{C}}$$

If 't' be the temperature of water during the experiment, the above equation can be written as :

$$S=\frac{\text{weight of the substance}}{\text{wt. of equal volume of water at } t^\circ C}\times\frac{\text{wt. of equal volume of water at } t^\circ C}{\text{,, \quad ,, \quad ,, \quad ,, \quad ,, \quad ,, ,, } 4^\circ C}$$

The first part of the R.H.S. of the above equation represents the sp. gr. of the substance at $t^\circ C$ and the second part the sp. gr. of water at $t^\circ C$. Hence,

S=sp. gr. as experimentally determined×sp. gr. of water at $t^\circ C$.

There is a table showing specific gravities of water at different temperatures. So, correction should be made by multiplying the observed sp. gr. of the substance by the sp. gr. of water at the room-temperature.

3.3. Difference between specific gravity and density :

(1) Sp. gr. is a pure number, having no unit. Density on the other hand, has definite units.

(2) In the C.G.S. system, the specific gravity and the density are numerically equal. Sp. gr. of gold, for example, is 19 and its density is 19 gm/c.c.

(3) In the F.P.S. system, the specific gravity and the density are numerically different. Sp. gr. of gold is 19 but its density in the F.P.S. system is 19×62·5 lb/cu.ft.

3.4. Pressure :

If a body be placed on a surface, it exerts a force equal to its weight on the surface. The pressure that the surface feels does not only depend on the aforesaid force but also on the area of contact between the body and the surface. When a man stands on a level ground, his weight acts over the area of his feet in contact with the ground. But when he stands on some pieces of stones, particularly if the stones are pointed, the area of contact between his feet and the stones becomes less and the pressure increases. That is why a person feels pain in walking over the pointed stones or gravels.

Definition : *Pressure is defined as the force per unit area of the surface on which it acts.* Hence, Pressure$=\dfrac{\text{force}}{\text{area}}$.

Liquid also exerts pressure, in the same way, on a surface with which it comes in contact.

Experiments : (1) Fill a tall jar with water. Insert a test-tube with its closed end downwards into a certain depth of water and suddenly release it. The test-tube will at once jump out of water. This is due to the pressure exerted by the water on the bottom of the test-tube.

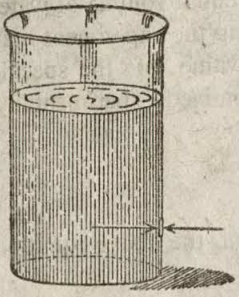


Fig. 3.1

(2) Pour water into a vessel which has a small opening on its wall (Fig. 3.1). Water will come out of the hole, which can be stopped by placing an obstacle of the same size as the hole. But to keep the obstacle on the hole, an external force opposite to the flow of water is to be constantly applied on the hole. This shows that water exerts pressure on the walls of the vessel.

3.5. Pressure of a liquid at a point and thrust :

Imagine a small area A round the point in the liquid where the pressure is to be determined, parallel to the free surface of the liquid. If it be supposed that the liquid exerts a force F on the area A, then the pressure P at the point is given by $P=\frac{F}{A}$.

Definition : *The force exerted normally on unit area by the liquid is defined as the pressure of the liquid.*

The thrust on an area means the total force exerted by the liquid on the area. Hence, *thrust=pressure × area.*

In the C.G.S. system, the absolute unit of thrust is dyne and the absolute unit of pressure is dyne/sq. cm. The gravitational unit of thrust is gramme-weight while that of pressure is gramme-weight/sq. cm.

In M.K.S. unit pressure is expressed by Newton/metre^2 and thrust by Newton.

In the F.P.S. system, the absolute unit of thrust is poundal and the absolute unit of pressure is poundal/sq. ft. The gravitational units are pound-weight (lb-wt) and lb-wt/sq. ft. respectively.

3.6. Calculation of pressure at a point in a liquid :

Suppose we consider a horizontal area A round the point where pressure is to be calculated, at a depth h below the surface of a liquid of density d (Fig. 3.2). Standing on the area is a vertical column of liquid of volume $h.A.$, the mass of which is given by $h.A.d.$ Hence, the thrust on the area is $h.A.d.g.$

Therefore, pressure

$$=\frac{\text{Thrust}}{\text{Area}}=\frac{h.A.d.g}{A}=h.d.g.$$

i.e., *Pressure=depth × density × acceleration due to gravity*

or, Pressure ∝ depth × density

[∵ 'g' is constant at a place]

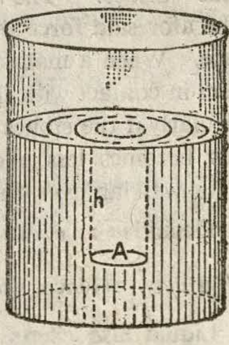


Fig. 3.2

For this reason, pressure is alternatively expressed in terms of depth. For example, 'a pressure of 100 metres of water' means the pressure exerted by a column of water of height 100 metres.

[**Note** : From the relation $P=h.d.g.$ it is evident that the pressures at all points on a horizontal line in a given liquid are equal, because d and g for a given liquid are constants and h for all points in a horizontal line is also constant.]

Example : (1) *Calculate the pressure at a depth of 200 cm. in a liquid whose density is 1·03 gm/c.c.*

Ans. Here, $h=200$ cm. ; $d=1{\cdot}03$ gm/c.c. ; $g=980$ cm/sec^2.

The required pressure, $P=h.d.g.=200\times1{\cdot}03\times980$

$=201880$ dynes/sq. cm. (absolute unit)

or, $P=200\times1{\cdot}03=206$ gm-wt/sq. cm. (gravitational unit)

(2) *The diameter of a cylinder is 14 cm. and its height is 40 cm. What will be the thrust on the bottom of the cylinder if it is filled with mercury of density 13·6 gm/c.c. ?*

Ans. Pressure at any point on the bottom

$P=h.d.g.=40\times13{\cdot}6\times980$ dynes/sq. cm.

The area of the bottom $=\pi r^2=\frac{22}{7}\times7\times7=154$ sq. cm.

∴ Thrust on the bottom $=$ pressure $\times$ area

$=40\times13{\cdot}6\times980\times154$ dynes

$=82100480$ dynes (absolute)

Alternatively, thrust on the bottom $=40\times13{\cdot}6\times154$

$=83776$ gm-wt (gravitational)

3.7. Liquid, at rest, exerts pressure at a point within it in all directions with equal magnitude :

Take a thistle funnel B and close its mouth with a thin rubber membrane in a water-tight manner. Attach a rubber tubing at the other end of the funnel and connect this rubber tubing with a horizontal glass tube A fitted in a frame-work as shown in fig. 3.3. There is a scale fixed with the frame-work which runs alongside the glass tube. Take a bit of coloured liquid C in the glass tube for serving the purpose of an index.

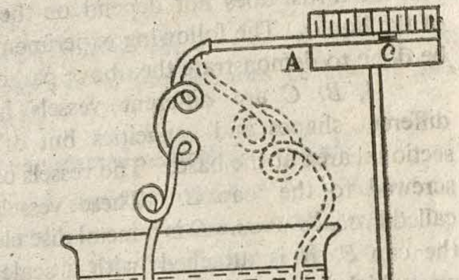


Fig. 3.3

Experiment : Insert the thistle funnel within the liquid keeping its face downward at a certain depth. It will be found that the index C has changed its position proving thereby that a vertical upward pressure is exerted by water on the membrane closing the mouth of the funnel. This pressure compresses the air inside the funnel and the rubber tubing, and the compressed air in its turn, has pushed the index to a different position.

If the face of the thistle funnel be next turned upward and then sideways as shown in the figure, keeping its depth from the free surface constant, it will be seen that the position of the index remains unaltered. We can, therefore, conclude that *liquid exerts equal pressure at all points on the same horizontal line.*

Further, keeping the face of the funnel downward if it be dipped more and more, the index will be found to move slowly further away, showing that *pressure of a liquid increases with the increase of depth.*

The above fact can also be demonstrated by the following experiment.

A wide glass jar, having three or four openings on its wall is taken. Close the openings by means of wax. Fill the jar with water. Now quickly make perforations through the wax by a needle. Jets of water will be found coming out of the perforations. It will be seen that the jet at the lowest level goes the farthest distance while that at the highest level goes the least [Fig. 3.4]. This shows that the pressure exerted by water at the lowest point is the greatest and that at the highest point is the least. In other words, *pressure of a liquid increases with the depth of the liquid.*

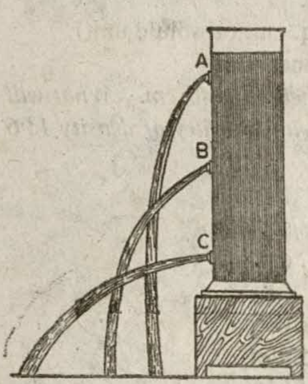


Fig. 3.4

3.8. Thrust exerted by a liquid on the base of a vessel depends on the area of the base and the height of the liquid :

When a vessel is filled with a liquid the thrust on the base of the vessel does not depend on the mass of the liquid but depends on the base area and the height of the liquid. This is referred to as an *hydrostatic paradox* because it seems queer that the thrust does not depend on the mass of the liquid. The following experiment may be done to demonstrate the above paradox.

A, *B*, *C* are different vessels having different shapes and capacities but of same sectional area at the base. The vessels can be screwed to the cap *E*. These vessels are called *Pascal's vases*. *D* is a metal disc closing the cap *E*. It is attached with a scale pan on which counterpoising weights may be placed. A pointer *P* can slide along the rod *R* which records the level of water in a vessel screwed to the cap *E*. [Fig. 3.5]

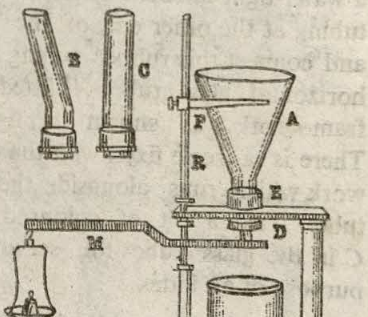


Fig. 3.5

Now place some counterpoising weights on the scale pan. The lever arm *M* will keep the disc *D* pressed to the cap *E* with some force. Put the vessel *A* in the cap *E* and pour water in the vessel. Record the level of water by the pointer *P* when the water column in the vessel just forces open the disc *D* and water spills out. Evidently, the thrust of water, in this case, equals the weight of the counterpoising weights placed on the scale pan. Strangely enough, it will be seen that when the vessel *A* is replaced by *B* or *C*, the disc *D* will be forced open by equal height of water columns in them although they contain different quantities of water. This shows, therefore, the thrust exerted by a liquid depends not on the quantity but on the height and the base area of the liquid column.

3.9. Equilibrium of two liquids in a U-tube :

If two liquids which do not mix with each other and have different densities be poured in a U-tube, it will be found that the levels of the two liquid columns are horizontal (*C* and *D*) but not at the same height [Fig. 3.6]. It may be proved in the following way that the heights of liquid columns h_1 and h_2 above the surface of contact *AB* are inversely proportional to their densities.

Suppose, we pour mercury first. Mercury will enter into both the limbs of the U-tube and will attain the same height. If, through left hand limb, water be poured over mercury, it will exert pressure on the mercury column in that limb and the level of mercury in that limb will go down while the level in the other limb will go up. When the system comes to rest, suppose the height (*CA*) of the water column $=h_2$ and that of the mercury column (*DB*) from the common surface of contact $AB=h_1$.

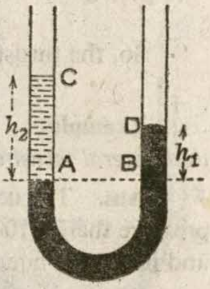


Fig. 3.6

If ρ_1 and ρ_2 be the densities of mercury and water respectively then pressure at *B* due to h_1 column of mercury $=h_1\rho_1.g$. Similarly pressure at *A* due to h_2 column of water $=h_2.\rho_2 g$. As the system is in equilibrium,

$$h_1.\rho_1.g=h_2.\rho_2.g$$

$$\text{or, } \frac{h_1}{h_2}=\frac{\rho_2}{\rho_1}$$

This shows that the heights of liquid columns from the surface of contact are inversely proportional to their densities. If one of the densities be known, the other may be found out from this experiment.

3.10. Total lateral thrust on a surface immersed vertically in a liquid :

In order to determine the thrust on a surface immersed in a liquid, we ordinarily multiply the area of the surface by the pressure of the liquid because, we know, thrust = pressure × area. The relation is obviously applicable when the pressure at every point of the surface is equal as in the case of a surface held horizontally in a liquid. But if the surface be held vertically, the depth and hence the pressure at every point of the surface is not equal and the above relation becomes inapplicable. In such cases, the average pressure on the surface multiplied by the surface area gives the total thrust on the surface.

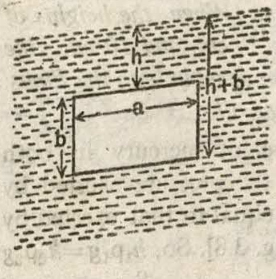


Fig. 3.7

Consider a rectangular surface of length 'a' and breadth 'b' immersed vertically at a depth 'h' in a liquid [fig. 3.7]. Finding the average lateral pressure on the surface and then by multiplying it with the surface area we get the total thrust on the surface.

Since the upper edge of the surface is at a depth 'h' below the free surface of the liquid, the lateral pressure there $=h.\rho.g.$ [ρ=density of the liquid]

Again, since the lower edge is at a depth $(h+b)$, the lateral pressure there $=(h+b)\rho.g.$ Hence the average lateral pressure on the surface

$$=\tfrac{1}{2}[h.\rho.g+(h+b)\rho.g]=\rho.g(h+\tfrac{b}{2})$$

So, the thrust on the surface=Average pressure × surface area

$$=\rho.g.(h+\tfrac{b}{2})\times a\times b.$$

Examples : (1) *A 1500 ft. long dam holds 100 ft. deep water. What is the total lateral thrust on the dam ?*

Ans. The depth of water at the lowest point of the dam=100 ft. So, lateral pressure there=100×62·5 lb/sq. ft. The depth of water at the highest point=0 and hence the lateral pressure=0.

$$\therefore \text{ the average lateral thrust}=\frac{100\times 62{\cdot}5+0}{2}=50\times 62{\cdot}5 \text{ lb/sq. ft.}$$

Hence, the total lateral thrust=area × average pressure

$$=(1500\times 100)\times 50\times 62{\cdot}5$$
$$=4{\cdot}7\times 10^{8} \text{ lb (nearly)}$$

(2) *To what height should a cylindrical vessel be filled with a homogeneous liquid to make the force with which the liquid presses on the sides of the vessel equal to the force exerted by the liquid on the bottom of the vessel ?*

Ans. Average pressure acting on the sides of the vessel corresponds to that of half the depth of the liquid. Let h be the required depth of the liquid.

Force on the wall=area × pressure$=2\pi r\ h\times\frac{h}{2}.\ \rho.g.$ [r=radius of the cylindrical vessel] Force on the bottom$=\pi r^2\times h.\ \rho.\ g.$

According to the problem, $2\pi rh\times\frac{h}{2}.\ \rho.\ g=\pi r^2\times h.\ \rho.\ g.\quad\therefore\quad h=r.$

So, the height to which the vessel is to be filled is equal to the radius of the cylinder.

(3) *A U-tube contains some mercury at the bottom. Water is poured into one limb and kerosene oil in the other. When the heights of the column are respectively 8 cm. and 10 cm., the mercury stands at the same level in both the limbs. Find the density of kerosene oil.*

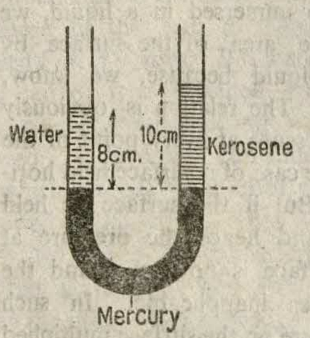


Fig. 3.8

Ans. Since the level of mercury in both the limbs is the same, the pressure exerted by 8 cm. of water column is equal to that exerted by 10 cm. of kerosene oil [Fig. 3.8]. So, $h_1\rho_1 g=h_2\rho_2 g$

or, $8\times 1\times g=10\times\rho_2\times g\quad\therefore\quad \rho_2=\frac{8}{10}=0{\cdot}8$

So, density of kerosene oil=0·8 gm/c.c.

(4) *One limb of a U-tube has cross-sectional area 3 sq. cm. and other limb 1 sq. cm. Keeping the tube vertical, some mercury is poured in the tube. Thereafter, 60 c.c. of water is poured through the wider limb over the mercury column. Find through what distance the mercury level in the wider limb goes down. Density of mercury=13·6 gm/c.c.*

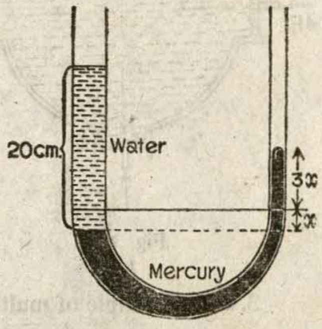


Fig. 3.9

Ans. Before water is poured, mercury will attain same height in both the limbs. After the water is poured, mercury columns in the limbs will attain different heights. The height of water column standing over mercury in the wider limb $=\dfrac{\text{volume}}{\text{cross-section}}=\dfrac{60}{3}=20$ cm. Suppose, mercury level in the wider limb goes down through x cm. [Fig. 3.9]. The height of mercury column in the other tube, therefore, goes up by $3x$ cm. Hence, height of mercury column in the narrower tube from the line of contact between mercury and water in the wider tube $=x+3x$ $=4x$ cm.

In other words, the pressure of 20 cm. of water column in the wider tube = the pressure of $4x$ cm. of mercury column in the narrower tube.

$$i.e.\ 4x\times 13{\cdot}6\times g=20\times 1\times g \quad \text{or,} \quad x=\frac{20}{4\times 13{\cdot}6}=0{\cdot}368 \text{ cm.}$$

3.11. Pascal's law for the transmission of liquid pressure :

The law states that a *pressure exerted anywhere on a mass of confined liquid is transmitted with undiminished magnitude in all directions and the transmitted pressure acts normally to the surface in contact with the liquid.*

The law was first established by the French scientist and philosopher, Blaise Pascal, in 1650.

Experiments : (*a*) Pierce a hole in a rubber ball and fill the ball with water. Make several perforations in the ball with a pin. If the ball be now pressed with fingers, water will force out through the perforations radially [Fig. 3.10]. This shows that the pressure exerted by fingers is transmitted equally in all directions by water.

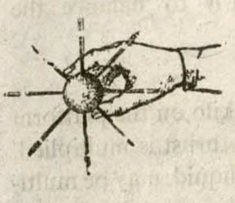


Fig. 3.10

(*b*) There are four openings of different cross-sections in a vessel, which are kept closed by water-tight pistons. If the vessel be now filled with water and the piston *A* pushed inward, the other pistons will be found to move outward. This evidently proves that the pressure applied by the piston *A* has been transmitted in all directions by water.

Further, if the pistons have area of cross-sections 1, 2, 3 & 4 units respectively, starting from A, and a force F applied on the piston A then, to keep the other pistons in position, inward forces of $2F$, $3F$ and $4F$ are to be applied on B, C and D pistons respectively [Fig. 3.11]. This shows that the force applied per unit area of the piston A is transmitted undiminished to all other pistons.

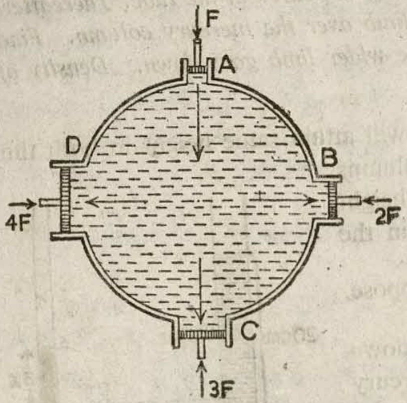


Fig. 3.11

Moreover, if the direction of outward motion of other pistons be carefully observed when the piston A is pushed inward, it will be clear that the transmitted pressures are applied normally on the pistons.

3.12. Principle of multiplication of thrust from Pascal's law :

Two cylinders—one wide and the other narrow are joined by a tube below [Fig. 3.12]. The cylinders are provided with two movable pistons which carry two platforms on their top. The cylinders are filled up with water and a weight W_2 is placed on the platform M. If A_2 be the area of the platform M, then the downward pressure on the platform $M = \frac{W_2}{A_2}$. This pressure is evidently carried to the water in contact with the piston. According to Pascal's law, water will transmit this pressure with undiminished magnitude in all directions and hence, the transmitted upward *pressure* on the piston $N = \frac{W_2}{A_2}$.

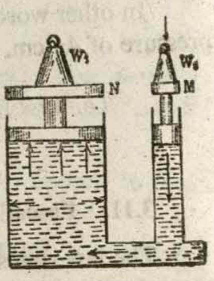


Fig. 3.12

If the area of the piston N be A_1, then the upward *thrust* on this piston $= \frac{W_2}{A_2} \times A_1 = W_2 \cdot \frac{A_1}{A_2}$.

Suppose, a weight W_1 is placed on the platform of piston N to balance the above thrust. Then, $W_1 = W_2 \cdot \frac{A_1}{A_2}$.

If, now, the area A_1 is 100 times the area A_2, then a load of 5 kilo on the platform M will develop a thrust on the piston N equal to 500 kilo, *i.e.*, thrust is multiplied. In this way, a small thrust applied at one point of a confined liquid, may be multiplied to a large thrust at some other point. This is known as *the principle of multiplication of thrust.*

3.13. Hydraulic Press : The above principle finds an important industrial application in the hydraulic press. This press is sometimes known as *Bramah's* *press* because a British Engineer named Bramah made some improvements on it. This type of machine has numerous uses, from the compression of soft materials such as waste paper, cotton, oil seeds into compact bales and oil to the shaping of motor-car bodies, forging of steel armour plate and garage lift.

Description : A section of the press has been shown in the fig. 3.13. In its simplest form, the hydraulic press consists of a cylinder *Q* and a piston of large diameter *B*, connected by a pipe *K* to a force pump consisting of a cylinder *P* of much smaller diameter and a piston *A*. The piston *B* carries a platform on which bales of cotton, paper, jute etc. can be kept for compression. *R* is a stout iron plate resting on four pillars. V_1 and V_2 are two valves which allow upward passage to water. Whenever water tries to come down, the valves press against the cylinder tightly and close the passage. *S* is a reservoir of water, connected with the cylinders *P* and *Q*. There is a tap *T* which normally remains closed.

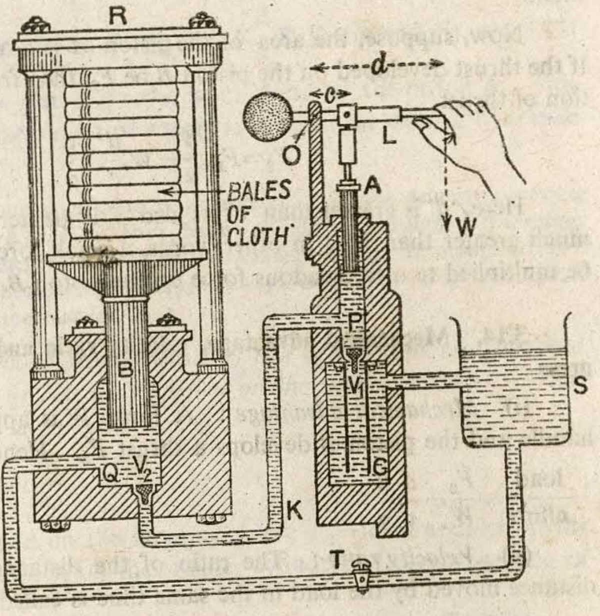


Fig. 3.13

Action : When the force pump *P* is worked by the lever *L*, water forces the valve V_1 open and fills up the cylinder *P* and the tube *K*. If now the piston *A* is pushed down, the valve V_1 closes but due to the increased pressure of water, the valve V_2 opens and water rushes into the cylinder *Q* and exerts pressure on the piston *B*. According to Pascal's law, the pressure applied on the piston *A* is transmitted undiminished to the piston *B*. The thrust developed on the piston *B* is as much greater than the thrust applied on the piston *A* as the area of *B* greater than the area of *A*. In the other words, the piston *B* tries to move up with a large thrust and the bales of cotton etc. are compressed very much against the fixed iron plate *R*.

When one compression is done, water from the cylinder *Q* is removed to the reservoir *S* by opening the tap *T* and the machine becomes ready for the second compression.

Total thrust developed in a hydraulic press : Besides the multiplication of thrust according to Pascal's principle, the law of lever also adds to some extent to the total thrust in a hydraulic press.

In fig. 3.13, *L* is a lever of second class. The piston *A* is fixed between the fulcrum *O* and the effort *W*. If the distance of the piston from the fulcrum be ***c***, and that between the effort W and the fulcrum be d, then according to the principle of lever, the force F_1 developed on the piston is,

$$F_1 \times c = W \times d \quad \text{or} \quad F_1 = W \times \frac{d}{c}.$$

In the lever of second class, the distance 'd' is generally greater than the distance 'c' and hence F_1 is greater than W. Here, the force is multiplied to some extent.

Now, suppose, the area of the piston A is α and that of the position B is β. If the thrust developed on the piston B be F_2, then from the principle of multiplication of thrust,

$$F_2 = F_1 \cdot \frac{\beta}{\alpha} = W \cdot \frac{d}{c} \cdot \frac{\beta}{\alpha} \quad \ldots \quad \text{(i)}$$

Here, 'd' is greater than 'c', as also 'β' is greater than 'α'. So, F_2 will be very much greater than W. In other words, a small force applied on the lever L, will be multiplied to a tremendous force on the piston B.

3.14. Mechanical advantage, velocity ratio and the efficiency of a hydraulic press :

(i) *Mechanical advantage* : A force W is applied at the end of the lever handle and the piston B develops a thrust F_2. Hence, the mechanical advantage

$$= \frac{\text{load}}{\text{effort}} = \frac{F_2}{W} = \frac{d}{c} \cdot \frac{\beta}{\alpha}.$$

(ii) *Velocity ratio* : The ratio of the distance moved by the effort to the distance moved by the load in the same time is called velocity ratio of a machine.

If the piston A moves through a distance S_1 in the barrel and the piston B through a distance S_2 then, $S_1 \times$ area of the piston $A = S_2 \times$ area of the piston B

$$\therefore \quad \text{Velocity ratio} = \frac{S_1}{S_2} = \frac{\text{area of the piston } B}{\text{,, \ ,, \ ,, } A} = \frac{\beta}{\alpha}.$$

The above expression gives the velocity ratio between the two pistons only. If the *total* velocity ratio of the whole press is required we must multiply $\frac{\beta}{\alpha}$ by the velocity ratio of the pump handle treated as a lever. So,

$$\text{total velocity ratio} = \frac{\beta}{\alpha} \cdot \frac{d}{c} = \text{mechanical advantage.}$$

(iii) *Efficiency* : The ratio of useful work done by the machine to the total work put into the machine is called the efficiency of a machine. Usually, this ratio is expressed as a percentage ; so we may write,

$$\text{Efficiency} = \frac{\text{Work output}}{\text{Work input}} \times 100\%$$

It may be shown that efficiency $= \dfrac{\text{Mechanical advantage}}{\text{Velocity ratio}} \times 100\%$

3.15. Conservation of energy in the hydraulic press :

If the piston A moves through a distance S_1 in the barrel, the volume of water that enters the cylinder $Q=S_1.\alpha$. If the entry of this water forces the piston B to move through a distance S_2, then clearly $S_1.\alpha=S_2.\beta$ or, $S_1=\frac{\beta}{\alpha}.S_2$.

Now, the work done on the piston $A=F_1.S_1=F_1.\frac{\beta}{\alpha}.S_2$

And the work done by the piston $B=F_2.S_2=F_1\frac{\beta}{\alpha}.S_2$ [From eqn. (i), art. 3.13]

$\therefore$ the work done on the piston A=the work done by the piston B. This shows that the principle of conservation of energy is valid in the case of hydraulic press.

In this connection, an important point is to be considered. We can develop a very big force in exchange of a small force in a hydraulic press, but we do not gain in energy. The energy put in the machine is equal to the energy delivered by the machine—rather, the output energy is somewhat less than the input energy because some energy is always lost in overcoming the frictional and other mechanical resistances of the machine.

Examples : (1) *A hydraulic press consists of two cylinders of cross-sections 1sq. ft. and 20 sq. ft. A force of 200 lb is applied on the smaller cylinder. What force will be developed on the larger piston?*

Ans. We know, $F_1=F_2\times\frac{A_1}{A_2}$

where, F_1=force developed on the large cylinder ; F_2=force applied on the smaller cylinder. A_2=cross-section on the smaller cylinder ; A_1=cross-section of the larger cylinder.

Here, $F_2=200$ lb, $A_1=20$ sq. ft.; $A_2=1$ sq. ft. ; $F_1=?$

We have, $F_1=200\times\frac{20}{1}=4000$ lb.

(2) *The two arms of a lever of a hydraulic press are 4 inches and 2 feet long. The diameter of the smaller piston is 2 inches while that of the larger one is 20 inches. If a force of 25 lb be applied at the end of the lever, find the thrust exerted on the larger piston.*

Ans. Suppose, a force F_1 is developed in the smaller piston. According to the principle of lever, $25\times2=F_1\times\frac{4}{12}$ $\therefore$ $F_1=150$ lb. [4 inches $=\frac{4}{12}$ ft.]

Now, suppose the thrust exerted on the larger piston be F_2. Then, according to the principle of multiplication of thrust,

$$F_2=F_1\times\frac{\text{area of the larger piston}}{\text{,, ,, ,, smaller piston}}=F_1\times\frac{\pi(10)^2}{\pi(1)^2}=F_1\times100=150\times100=15{,}000\text{ lb.}$$

(3) *The two cylinders of a hydraulic press have a diameter of 80 cm and 16 cm respectively, the small piston plunger is fitted with a lever 80 cm. long and the fulcrum of the piston is 8 cm. from the piston. When an effort of 20 kg-wt. is applied at the end of the lever, find (i) the velocity ratio (ii) the mechanical advantage and (iii) the*

efficiency of the press when a load of 4 metric ton can be raised in this way. [*1 metric ton=1000 kg*]

Ans. (i) Here ratio of the areas $=\frac{\beta}{\alpha}=\frac{\pi r_2{}^2}{\pi r_1{}^2}=\left(\frac{40}{8}\right)^2=25$

The ratio of the lever arms $=\frac{d}{c}=\frac{80}{8}=10$

$\therefore$ Velocity ratio $=\frac{\beta}{\alpha}\cdot\frac{d}{c}=25\times10=250.$

(ii) Mechanical advantage $=\frac{\text{load}}{\text{effort}}=\frac{4\times1000\times g}{20\times g}=200$

(iii) Efficiency $=\frac{\text{Mechanical adv.}}{\text{Velocity ratio}}\times100\%=\frac{200}{250}\times100\%=80\%$

(4) *A bottle is corked after filling it completely with oil. The diameters of the neck and the bottom of the bottle are ½ inch and 3 inches respectively. If a force of 5 lb be applied on the cork, what force will be developed at the bottom?*

Ans. Area of the neck $=\pi r_1{}^2=\pi(\frac{1}{4})^2$ sq. inches $[r_1=\frac{1}{4}$ inch$]$

" " " bottom $=\pi r_2{}^2=\pi(\frac{3}{2})^2$ sq. inches $[r_2=\frac{3}{2}$ inches$]$

Now, the pressure on the neck $=\frac{5}{\pi(\frac{1}{4})^2}=\frac{80}{\pi}$ lb/sq. inch.

So, the force developed at the bottom $=\frac{80}{\pi}\times\pi(\frac{3}{2})^2=180$ lb.

Exercises

Essay type :

1. Explain what you mean by sp. gravity. Show that in the C.G.S. system the specific gravity and the density are numerically equal to each other.

2. State Pascal's law and explain it fully. How can you get the principle of multiplication of thrust from this law? [*H. S. Exam. 1978* ; *Tripura H. S. 1979, '81*]

3. What is a hydraulic press? Describe it and explain its action. What are its uses? [*Tripura H. S. 1979, '81*]

4. Find the total thrust developed in a hydraulic press. What is its mechanical advantage? Is the principle of conservation of energy applicable in the case of hydraulic press?

5. Obtain an expression for the velocity ratio and the efficiency of a hydraulic press.

6. (a) Calculate the total lateral thrust on a plane immersed vertically in a liquid.

(b) Describe an experiment to show that liquids exert pressure in all directions. [*H. S. Exam. 1981*]

(c) Water is poured to the same level in each of the three vessels, all having same base area but different shapes. If the pressure is the same at the bottom of each vessel, the force experienced by the base of each vessel is the same. Why then do the three vessels have different weights when put on a scale?

Short answer type :

7. What is pressure ? On what factors does it depend ? Things can be cut by the sharp edge of a knife easily but cannot be cut by the blunt edge. Explain the reason. [*H. S. Exam. 1982*]

8. Distinguish between 'thrust' and 'pressure' in a liquid. What is the pressure at a point inside a liquid ?

9. What is density ? What is its unit ? What is the relation between mass, volume and density ?

10. What are the differences between sp. gravity and density ? Why in the definition of sp. gravity, water is taken at 4°C ?

11. Why is it necessary to make the dam of a water reservoir thicker at the bottom than at the top ? [*H. S. Exam. 1982*]

12. Answer the following questions:—(a) What is meant by a pressure of 100 ft. of water ? (b) Can a hydraulic press multiply energy ? (c) Why does a pointed nail penetrate into the wall when struck lightly ?

Objective type :

13. Some of the statements below are true and some are false. Write *T* against those which are true and *F* against those which are false.

(a) In the C.G.S. system, the specific gravity and the density of a substance are numerically equal.—

(b) Sp. gravity of gold is 19·32 ; its density in the F.P.S. system is 19·32 lb/c.ft.—.

(c) The pressure of a liquid does not depend on its density but depends on its depth. —

(d) We can develop a very big force in exchange of a small force in a hydraulic press but we do not gain in energy. —

(e) In a hydraulic press, a given force is multiplied many times. Hence the principle of conservation of energy does not apply in this case. —

Numerical problems :

14. A coil of copper wire, the diameter of which is 1·2 mm., weighs 150 gm. If the density of copper be 8·9 gm/c.c. what is the total length of the wire ? [Ans. 1490 cm.]

15. The weight of a box made of teak wood is 100 lb. What is the weight of a similar box made of oak, if the ratios of densities of teak wood to oak is 11 : 17. [Ans. 154·5 lb.]

16. The sp. gravity of iron is 7·2. Find its density in the C.G.S. and in the F.P.S. systems. [Ans. 7·2 gm/c.c. ; 7·2 × 62·5 lb/cu. ft.]

17. Sp. gravity of sea-water is 1·025. If 1 cu. ft. of fresh water weighs 62·5 lb. what will be the pressure at a depth of 10 ft. in sea-water ? [Ans. 640·6 lb/cu. ft.]

18. What is the pressure at the bottom of a clear lake of water 10 metres deep ? Atmospheric pressure =76 cm. of mercury ; density of mercury=13·6 gm/c.c. [*H. S. Exam. 1982*] [Ans. 2033·6 gm-wt/cm^2]

19. A person, weighing 66 kg. holds himself vertically on the heel of his right leg. If the diameter of the heel is 2 cm., what pressure is exerted on the ground? $\pi=\frac{22}{7}$. [Ans. 21 kg/cm^2]

20. A diver dives 50 ft. deep in sea-water. If the area of his costume is 20 sq. ft. and if sea-water is 1·025 times heavier than fresh water, find the total thrust on his costume. Density of fresh water=62·5 lb/cu. ft. [Ans. 64062·5 lb]

21. The area of cross-section of flat-bottomed tube is 4 sq. cm. It dips 8 cm. and 10 cm. in water and alcohol respectively. What is the weight of the tube ? What is the density of alcohol ? [Ans. 32 gm. ; 0·8 gm/c.c.]

22. The length, breadth and height of a rectangular box are 10 ft., 8 ft. and 6 ft. respectively. If the box is completely filled up with water, calculate the total thrust on the bottom of the box. [Ans. 30,000 lb]

23. A rectangular reservoir of water is 4 ft. long, 2 ft. wide and 2 ft. high. Find the thrust on the sides of the reservoir when it is full of water. Density of water 62·5 lb/cu. ft. [Ans. 1000 lb ; 500 lb ; 250 lb]

24. A cubical box is full of water. Show that the thrust on any vertical wall is half the weight of the water contained in the box.

25. A lock-gate of a canal is 12 ft. wide. The depth of water on one side of the gate is 16 ft. and that on the other is 10 ft. What is the thrust on the gate ? 1 cu. ft. water weighs 62·5 lb. [Ans. 585,00 lb.]

26. The smaller piston of a hydraulic press has a diameter 1 inch, and the larger one has a diameter 1 ft. If a force of 56 lb be applied on the smaller piston, how much force will the larger one develop ? (Cross-sections of the pistons are circular) [*P. U. 1962*] [Ans. 8064 lb]

27. A book binder uses a hydraulic press. The diameter of the smaller piston of the press is 1 inch and the diameter of the bigger piston is 6 inches. The ratio of the lever arms of the press is 1 : 4. Find the mechanical advantage of the machine. [Ans. 144]

28. The diameters of two pistons of a hydraulic press are respectively 3 inches and 30 inches. The smaller piston is attached at distance of 2 ft. from the fulcrum of a lever of length 12 ft. What force must be applied at the free end of the lever in order to produce a force of 5000 lb. wt. at the bigger piston ? [*Tripura H. S. 1981*] [Ans. 8·3 lb. wt.]

29. The cross-sectional area of the bottom of a water-filled bottle is 30 sq. cm. It has a cork in its neck. The area of the cork is 1 sq. cm. If a force of 40 gm-wt is applied on the cork, what force will be developed on the bottom ? [Ans. 1200 gm-wt]

30. (a) The diameter of the smaller piston of a hydraulic press is 1 inch and that of the larger one is 6 inches. It is operated by a hand lever, the ratio of its arms being 1 : 8. If a force of 30 lb be applied at the end of the lever arm, calculate the force developed on the goods placed on the press. [Ans. 8640 lb]

(b) A piston of cross-sectional area 100 cm^2 is used in a hydraulic press to exert a force of 10^7 dynes on water. What is the cross-sectional area of the other piston which supports a truck having a mass of 2000 kg ? [*I.I.T. 1972*] [Ans. $1{\cdot}96\times10^4$ sq. cm.]

[**Hints :** $\dfrac{2000\times10^3\times980}{\alpha}=\dfrac{10^7}{100}$]

Harder problems :

31. The two arms of a U-tube have cross-sections of 1 sq. cm. and 0·1 sq. cm. respectively. Some water is poured into the tube which enters into both the limbs of the U-tube. What volume of a liquid of density 0·85 gm/c.c. should now be poured into the wider limb so that water level in the narrower tube may rise 15 cm ? [*Jt. Entrance 1976*] [Ans. 17·91 c.c.]

32. At what depth in a lake, the pressure of water will be double than the atmospheric pressure ? [Ans. 10·2 metres]

33. A vertical U-tube of uniform inner cross-section contains mercury in both its arms. A glycerine (density=1·3 gm/c.c.) column of length 10 cm. is introduced into one of its arms. Oil of density 0·8 gm/c.c. is poured in the other arm until the upper surfaces of oil and glycerine are in the same horizontal level. Find the length of oil column. Density of mercury=13·6 gm/c.c. [*I.I.T. 1972*] [Ans. 9·61 cm.]

[**Hints :** If a horizontal line be drawn through the surface of contact between mercury and glycerine, then, Pressure of 10 cm. of glycerine=Pressure of h cm. of oil+Pressure of $(10-h)$ cm. of mercury.]

34. A U-tube, with both ends open, has a cross-section of 1 sq. inch. The vertical branches of the tube rise to a height of 33 inches and contain mercury upto a height of 6·8 inches in each limb. Find the greatest amount of water that may be poured into one of the branches. Density of mercury=13·6 gm/c.c. [Ans. 27·2 cu. inches]

35. A ship, when unloaded, dips 10 metres in sea-water. The average cross-sectional area along the line of contact with water is 2000 sq. metre. How much load may be put in the ship so that it may dip 10·5 metres instead of 10 metres ? Density of sea-water$=1{\cdot}025$ gm/c.c. [Ans. $1{\cdot}025\times10^6$ Kg]

36. A hollow right circular cone of height h and semi-vertical angle α rests with its base on a horizontal table. If the cone is filled with a liquid of density ρ, the weight of the empty cone becomes equal to the weight of the liquid it contains. Find the thrust on the base of the cone and the pressure exerted on the table when the cone is filled with the liquid. [Ans. $\pi\rho g h^3 \tan^3\alpha$; $\frac{2}{3}h\rho g$]

[**Hints :** Thrust$=\pi r^2\times h\times\rho\times g$ but $r=h.\tan\alpha$
Weight of liquid$=\frac{1}{3}\pi r^2 h.\ \rho g=\frac{1}{3}\pi\ (h\tan\alpha)^2.h\ \rho g$]

37. A U-tube is partly filled with water. Another liquid which does not mix with water, is poured into one limb until it stands a distance h above the water level on the other limb, which has meanwhile risen a distance d. Find the density of the liquid. [Ans. $2d/(2d+h)$]

[**Hints :** As water level has risen a distance d in one limb, it has fallen through the same distance in the other limb. So, the height of the liquid column above the water-level$=2d+h$. Now pressure due to liquid column$=(2d+h)\rho.g$ and that due to water column$=2d.g$. So $(2d+h).\rho.g=2d.g$.]

38. A pump, installed on the ground floor of a garage works a hydraulic jack on the upper floor. The cross-sections of the pistons are 1 sq. inch and 60 sq. inches respectively. The bigger piston is at a height of 10 ft. from the other one. If a force of 24·33 lb be applied at the lower piston, find the greatest weight that the jack will lift. [Ans. 1199·4 lb]

39. In a form of hydraulic press, the bore of the pump is 1 inch and that of the main cylinder is 1 ft. The pump is operated by a lever of velocity ratio 5. What is the velocity ratio of the whole press ? If the efficiency is 85%, what is the greatest force the press can exert when a force of 5 lb-wt is applied at the end of the lever ? [Ans. 720 ; 3060 lb-wt.]

40. The radius of the small piston of a hydraulic press is 8 cm. and that of the large piston is 96 cm. It is worked by a lever of which the ratio of the arms is 3 : 28. If a force of 27 kg-wt. is applied on the handle, what is the force developed by the large piston ? What is the velocity ratio of the whole machine ? [Ans. 36288 Kg-wt ; 1344]

4

FLOATING BODIES AND ARCHIMEDES' PRINCIPLE

4.1. Calculation of resultant thrust on a body immersed in a liquid :

ABCDEFGH is a cube whose edges are of length l. The cube is completely immersed in a liquid kept in a vessel. The upper surface of the cube *ABCD* is at a depth h_1, and the lower surface *EFGH* at a depth h_2 from the free-surface of the liquid (Fig. 4.1). We are to calculate the total thrust applied by the liquid on the cube.

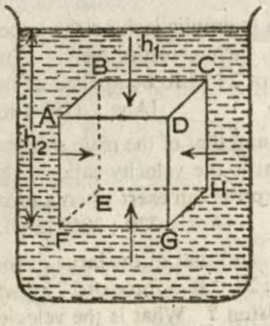


Fig. 4.1

The thrusts experienced by the vertical surfaces of the cube, such as *ABEF* and *CDGH*, are horizontal and opposite, because the pressure is always normal to the surface in contact with the liquid. Since the surfaces like *ABEF* or *CDGH* are vertical, the pressures on every point on them are horizontal. As a result, each vertical surface experiences a thrust equal and opposite to the thrust experienced by the opposite vertical surface. But on every point of the upper surface *ABCD*, the liquid exerts pressure and it is equal to $h_1 d.g$, where d=density of the liquid.

So, the total downward thrust on this surface =Pressure × area of *ABCD* surface$=h_1 d.g.\times l^2$ $=l^2.h_1 d.g.$

On the surface *EFGH*, however, an upward pressure is exerted by the liquid. We know that at a point in a liquid, the downward and upward pressures are equal in magnitude. As the surface *EFGH* is at a depth of h_2 the upward pressure at every point on it$=h_2 d.g.$

So, the total upward thrust on this surface$=h_2.d.g.\times l^2=l^2 h_2.d.g.$

Since $h_2>h_1$, the upward thrust on the surface *EFGH* is greater than the downward thrust on the surface *ABCD*.

Hence, the resultant upward thrust on the cube

$$=l^2 h_2 dg-l^2 h_1 dg$$
$$=l^2.d.g(h_2-h_1)$$
$$=l^3.d.g \quad [\because\ h_2-h_1=l]$$

But l^3 is the volume of the cube and $l^3\times d$ is the mass of an equal volume of liquid. So, $l^3.d.g$=weight of an equal volume of liquid.

Thus we see that a cube completely immersed in a liquid experiences an upward thrust equal to the weight of a mass of liquid having same volume as the cube.

The above statement is true not only in the case of a regularly shaped body like a cube but also of a body of any shape either partly or fully immersed in a liquid. In general we can say that *when a body is wholly or partly immersed in a liquid, it experiences an upthrust equal to the weight of the liquid displaced by the body.*

The upthrust is referred to as *buoyancy* of the liquid and it acts through the centre of gravity of the displaced liquid, which is known as the *centre of buoyancy.*

4.2. Apparent loss of weight of a body immersed in a liquid:

You must have noticed that a body weighs less in water than in air. A heavy bucket which is difficult to move in air, may easily be moved in water. Stone boulders, which are very heavy, are easily moved by flood water. A bucketful of water can easily be lifted from the bottom of a well but becomes very heavy the moment it clears the surface of the water in the well. All these examples show that a body loses a part of its weight when immersed in a liquid. This is evidently due to the upthrust exerted by the liquid. If W_1 be the weight of the body in air and W_2 the weight of the liquid displaced by it, then the apparent weight of the body inside the liquid $= W_1 - W_2$.

4.3. Experiment to demonstrate the apparent loss of weight of a body :

Suspend a solid cylinder *A* from the hook of a spring balance. The reading of the spring balance will give the weight of the cylinder in air. Now, immerse the cylinder slowly in water kept in a tall jar *B* [Fig. 4.2]. You will see that the readings of the spring balance are decreasing, showing that the cylinder is losing its weight. The loss of weight will be the greatest when the cylinder is completely immersed in water. The cylinder will, however, regain its initial weight when it is taken out of water. This shows that the loss of weight is an *apparent loss.*

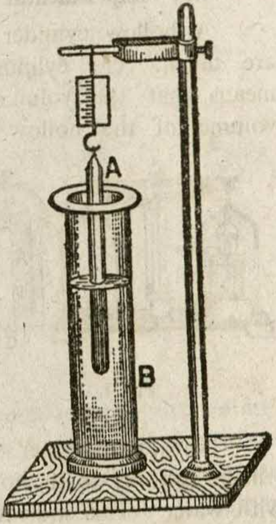


Fig. 4.2

A typical problem :

A bucket containing some water is suspended from the hook of a spring balance. A piece of stone tied to a thread is immersed in water of the bucket, the other end of the thread being held by the hand. Will there be any change in the reading of the spring balance?

Explanation : When the piece of stone is immersed in water it displaces an equal volume of water. The level of water in the bucket, as a result, is raised. As the thrust on the bottom of the bucket depends on the height of the water column and not on the mass of water, the raised level exerts a greater thrust and the spring balance gives a greater reading. The weight of the stone does not affect the reading of the spring balance as it is not directly in contact with the balance.

4.4. Archimedes' Principle :

Experiments to measure the upthrust of liquids were first carried out by the Greek philosopher Archimedes in the third century B. C. From the results of his work he established a principle which is now known as *Archimedes' Principle.* The principle states :

When a body is immersed, partly or wholly in a liquid, it appears to lose a part of its weight and the loss of weight is equal to the weight of the liquid displaced by the body.

[**N. B.** Archimedes' principle also applies to gases and therefore the term 'liquid' in the above statement should be replaced by 'fluid'. The application of the principle to gases will be taken up later.]

4.5. Experimental verification of Archimedes' principle :

A hollow cylinder *B*, commonly called the bucket and a solid cylinder *A*, are taken. The cylinder *A* just fits inside the hollow cylinder *B*, which means that the volume of the solid cylinder is just equal to the internal volume of the hollow cylinder. Now, suspend the cylinder *B* from the end of a balance beam. From the hook below the cylinder *B*, suspend the cylinder *A*. Counterpoise the balance by placing suitable weights on the right hand scale pan. Now, immerse the cylinder *A* completely in water kept in a beaker [Fig. 4.3]. Take care that the beaker does not touch the scale pan. You will now see that the balance beam is tilted towards the right showing that the cylinder *A*, while immersed in water, loses some weight.

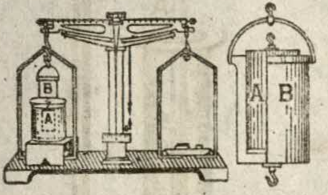


Fig. 4.3

Now, pour water slowly into the hollow cylinder *B*. The balance beam will be restored to its horizontal position when the bucket *B* is completely filled with water. This indicates that the apparent loss of weight of the cylinder *A* is equal to the weight of water whose volume is the same as the volume of *A*.

This, therefore, verifies Archimedes' principle experimentally.

4.6. Applications of Archimedes' principle :

We can determine the following quantities by the application of Archimedes' principle :

(a) the volume of an irregularly shaped body;

(b) the density of a material.

(*a*) **Determination of the volume of a body :** Let the weight of the body in air $=W_1$. Now suspend the body from one end of a balance beam and keep it immersed in water taken in a vessel [Fig. 4.4]. Determine its weight in this condition and let it be W_2.

From Archimedes' principle, we have,

$W_1-W_2=$apparent loss of wt. of the body$=$wt. of an equal volume of water.

If the measurements are done according to the C.G.S. system, the wt. of an equal volume of water$=(W_1-W_2)$ gm.

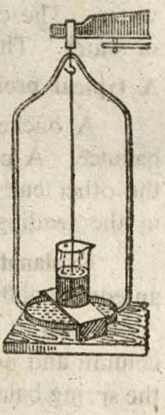


Fig. 4.4

Since, the density of water = 1 gm/c.c., the volume of the water under consideration $=(W_1-W_2)$ c.c.

As the body displaces its own volume of water, the volume of the body $=(W_1-W_2)$ c.c.

In the *F.P.S.* system, the wt. of equal volume of water $=(W_1-W_2)$ lb.

The density of water in the F. P. S. system, being 62·5 lb/cu. ft., the volume of the water under consideration $=\frac{W_1-W_2}{62\cdot5}$ cu. ft.

As the body displaces its own volume of water, the volume of the body $=\frac{W_1-W_2}{62\cdot5}$ cu. ft.

(*b*) **Determination of the density of a material :**

$$\text{Density of a material}=\frac{\text{Wt. of a piece of the substance}}{\text{volume of the piece.}}$$

The volume of the body can be ascertained in the previous way. Hence, its density in the C.G.S. system $=\frac{W_1}{W_1-W_2}$ gm/c.c.

Similarly, in the F.P.S. system, the density

$$=\frac{W_1}{\frac{W_1-W_2}{62\cdot5}}\text{ lb/cu. ft.}=\frac{W_1\times62\cdot5}{W_1-W_2}\text{ lb/cu. ft.}$$

We can also judge the purity of a metal by this method. Suppose we are asked to ascertain whether a piece of silver is genuine or not. If we determine the density of the material of the piece according to the above method and compare it with the density of pure silver which is 10·5 gm/c.c. we can easily determine whether the piece is all silver or not.

Examples : (1) *The weights of a body in air and water are respectively 50 gm and 30 gm. What would be its weight in a liquid of sp. gr. 0·8 ? Calculate the sp. gr. of the body in relation to the liquid.* [*H. S. Exam. 1984*]

Ans. Loss of weight of the body in water = 50 − 30 = 20 gm = weight of an equal volume of water.

So, the volume of the body = 20 c.c. The body, when immersed in the liquid, displaces 20 c.c. of the liquid. The weight of displaced liquid $=20\times0\cdot8=$ 16 gm.

So, the weight of the body in the liquid = 50 − 16 = 34 gm.

Sp. gr. of the body in relation to the liquid

$$=\frac{\text{Wt. of the body}}{\text{Wt. of an equal vol. of liquid}}=\frac{50}{16}=3\cdot125$$

(2) *A piece of alloy of iron and aluminium has a mass 588 gm and volume 100 c.c. If the specific gravities of iron and aluminium be 8 and 2·7 respectively calculate the ratio of* (*i*) *the volumes and* (*ii*) *the masses of the constituent metals of the alloy.*

Ans. Let the volume of iron in the alloy be V c.c. Then the volume of aluminium $=(100-V)$ c.c

Now, the mass of iron $=8V$ gm and that of aluminium $=(100-V)\times 2{\cdot}7$ gm.

$\therefore\ 8V+(100-V)\times 2{\cdot}7=588$ or, $8V-2{\cdot}7V=588-270$ or, $5{\cdot}3V=318$

$\therefore\ V=60$ c.c. and ratio of the volumes $=\frac{60}{40}=\frac{3}{2}$

Again, mass of iron $=8V=8\times 60$ gm and that of aluminium $=40\times 2{\cdot}7$ gm.

$$\therefore \text{ ratio of the masses}=\frac{8\times 60}{40\times 2{\cdot}7}=\frac{40}{9}$$

4.7. Floatation and immersion of a body :

We know that a body, immersed in a liquid, experiences an upthrust, known as buoyancy which acts vertically upward through the centre of buoyancy. The weight of the body, on the other hand, acts vertically downward through the centre of gravity. As no other forces act on the body, the following three cases may arise in respect of floatation and immersion of the body :

(i) the weight of the body $W_1>$ the buoyancy W_2. In this case, the weight of the body is greater than the weight of the displaced liquid and hence the body sinks in the liquid. Generally, if the density of the material of a body is greater than that of a liquid, the body sinks in the liquid. For example, a piece of iron or stone sinks in water;

(ii) the weight of the body, $W_1=$ the buoyancy W_2. In this case, the weight of the body is equal to the weight of the displaced liquid and hence the body floats anywhere in the liquid. For example, if a drop of olive oil be placed anywhere in a mixture of equal volumes of water and alcohol, the drop will remain at rest there because the density of olive oil is same as the density of the above mixture;

(iii) the weight of the body $W_1<$ the buoyancy W_2. In this case, the weight of the body is less than the weight of the displaced liquid and hence the body feels an upward thrust and tries to float up. When the density of the material of a body is less than that of a liquid, the body when placed on the liquid floats. A piece of wood, for example, floats on water.

4.8. Condition of equilibrium of floating bodies :

When a piece of wood or other material of density less than that of water is placed in water, it sinks until the weight of water displaced is just equal to its own weight. It then floats at rest. Hence for a body floating at rest in a liquid, the following two conditions must be fulfilled. :

(*i*) *The floating body must displace its own weight of the liquid in which it floats.*

(*ii*) *The centre of gravity of the floating body and the centre of buoyancy must be in the same vertical line.*

A little more detail about the second condition is necessary. Let us suppose that G is the $C.G.$ of a body. The weight W_1 of the body is acting vertically downward through G. G' is the centre of buoyancy, *i.e.*, the weight W_2 of displaced liquid is acting vertically upward through G' [Fig. 4.5]. According to the first condition of floatation $W_1 = W_2$. But from the figure, it is clear that two equal and opposite forces cannot keep a body in equilibrium unless they act in the same line. Hence, for the equilibrium of the floating body, G and G' must be in the same vertical line.

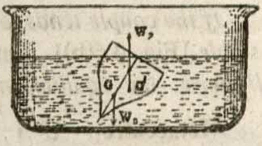


Fig. 4.5

A floating body is apparently weightless :

A body lighter than a liquid when kept fully immersed in the liquid and then released, floats up. When the floating body comes to rest, its weight becomes equal to the weight of the liquid displaced by it. The weight of the body acts vertically downward but the weight of the displaced liquid acts vertically upwards. These two equal but opposite forces neutralise each other and the floating body apparently loses all its weight. For this reason, a body floating in a liquid, is said to be apparently weightless.

4.9. Stability of a floating body :

A floating body may remain in three different states of equilibrium *viz.* (i) *neutral equilibrium*, (ii) *stable equilibrium* and (iii) *unstable equilibrium.*

It has already been mentioned that when a body floats freely, the *C.G.* of the body and the centre of buoyancy must be in the same vertical line.

A body is said to float in a neutral equilibrium when a slight tilting of the body does not produce any change of position of the centre of buoyancy. A floating sphere, for example, is in neutral equilibrium because, whatever may be its position, its centre of buoyancy remains unchanged. *The C. G. and the centre of buoyancy, in this case, remain always in the same vertical line.*

But if a floating body, when slightly tilted, makes its centre of buoyancy change its position, a couple acts on the body tending to bring the body back to its previous position or to displace it more. In the former case, the body is said to float in stable equilibrium and in the latter case, it is said to float in unstable equilibrium.

Let G be the *C. G.* of a floating body and H the centre of buoyancy. The two points are in the same vertical line *ab*, called the *central line* [Fig. 4.6(*a*)]. When the body is slightly tilted, the centre of buoyancy moves to G'. The weight (W) of the body acts vertically downwards through G while the buoyancy (W') acts vertically upwards through

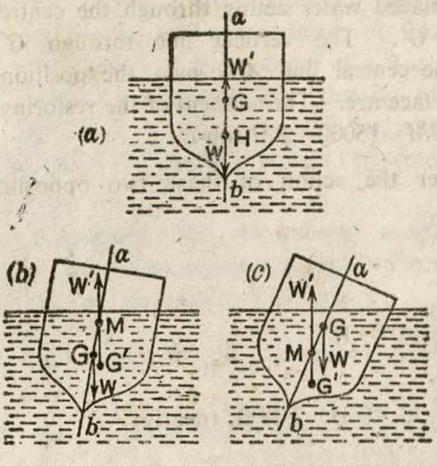


Fig. 4.6

G'. Since the liquid displaced in the tilted position, is practically the same as before, $W' = W$. Hence, they constitute a couple.

If the couple tends to restore the body to its undisturbed position, the equilibrium is stable [Fig. 4.6(b)]. On the other hand, *if the couple tends to displace the body still more, the equilibrium is unstable* [Fig. 4.6(*c*)].

Metacentre : If a vertical line be drawn through G', the new centre of buoyancy in the tilted position of the body, to cut the central line *ab* at M, the point M is called the **metacentre.** The distance GM is called the *metacentric height.*

The equilibrium of the body will be stable if the C.G. is below the metacentre and unstable if the C. G. is above the metacentre [Fig. 4.6(*b*) & (*c*)].

The position of the metacentre is of great importance in ship-designing because the stability of the vessel depends on its position. The vessel will be in stable equilibrium as long as the metacentre is above the *C.G.* of the ship. To ensure this, the *C.G.* of the vessel is kept as low as possible. This is why a ship often carries a ballast.

Example : *If a load of 30 ton, moved 20 ft across the deck of a ship of 15,000 ton displacement, causes the ship to tilt through 1°, what is its metacentric height ? sin* $1° = 0{\cdot}0175$.

Ans. The load (30 ton wt) is shifted from X to Y along the deck through a distance of 20 ft. This is equivalent to the application of a couple of moment 30×20 ton-ft tending to turn the ship clockwise [Fig. 4.7]. As the ship tilts, a restoring couple acts on the ship which is formed by the weight of the ship (W) acting through the centre of gravity G of the ship and the equal weight of displaced water acting through the centre of buoyancy G'. The vertical line through G' intersecting the central line AG, gives the position (M) of the metacentre. The moment of the restoring couple $= W \times BM = 15000 \times BM$ ton-ft.

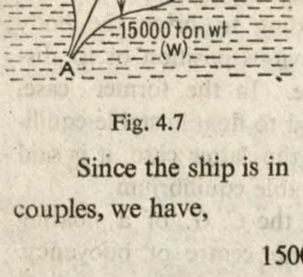


Fig. 4.7

Since the ship is in equilibrium under the action of these two opposite couples, we have,

$$15000 \times BM = 30 \times 20$$

$$\text{or} \quad BM = \frac{30 \times 20}{15000} = 0{\cdot}04 \text{ ft.}$$

$$\text{Now, } \sin 1° = \frac{BM}{GM} \text{ or } 0{\cdot}0175 = \frac{0{\cdot}04}{GM} \quad \therefore \; GM = 2{\cdot}28 \text{ ft. (nearly)}$$

So, the metacentric height of the ship $= 2{\cdot}28$ ft. (nearly).

4.10. A floating body displaces liquid whose weight is same as the weight of the body :

Take a block of wood and ascertain its weight by means of a balance. Take a displacement can with a beaker under its spout [Fig. 4.8]. Water is poured in until it runs out from the spout. When the water has ceased dripping, the beaker is removed and replaced by another beaker which has been previously dried and weighed.

Now, float the piece of wood carefully on the water. Some water will come out of the spout and will be collected in the beaker. The beaker with the water collected is weighed. From this, the weight of water displaced by the piece of wood can be found. It will be seen that the weight of the piece of wood is equal to the weight of displaced water.

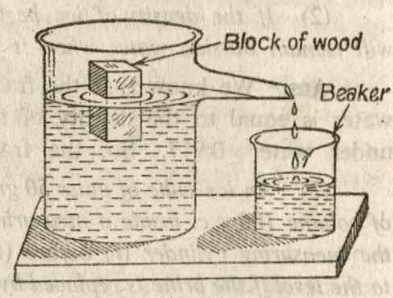


Fig. 4.8

4.11. Two important facts in connection with a floating body :

(*a*) Suppose, a body of volume V c.c. and density ρ gm/c.c. is floating on a liquid of density d gm/c.c. If v c.c. be the volume of the body immersed in the liquid then the volume of the displaced liquid is also v c.c. According to the condition of floatation,

Wt. of the body = Wt. of the displaced liquid

$$\text{or,}\quad V.\rho.=v.d \quad \text{or,}\quad \frac{v}{V}=\frac{\rho}{d}$$

$$\text{i.e.}\quad \frac{\text{Volume of the immersed portion of the body}}{\text{Total volume of the body}}=\frac{\text{density of the body}}{\text{,, ,, ,, liquid}}$$

(*b*) Suppose, the fraction of the total volume of the body immersed $=n$; then according to the condition of floatation, $V\rho=n.V.d.$ or, $\rho=n.d.$ *i.e.*, if 'n'th part of the volume of a floating body be immersed in a liquid, then the density of the body $=n\times$ density of the liquid.

If the body floats in water for which $d=1$ gm/c.c., $\rho=n.$, *i.e.*, the density of the body is numerically equal to the fraction of the volume of the body immersed in water.

Examples : (1) *An iceberg, weighing 1000 kg floats on sea-water. Find the portion of its volume immersed in sea-water. Density of ice = 0·917 gm/c.c. and that of sea water = 1·03 gm/c.c.*

Ans. Volume of the iceberg $=\frac{1000\times1000}{0\cdot917}$ c.c.

$$\text{Now, we have,}\quad \frac{\text{volume of the immersed portion}}{\dfrac{1000\times1000}{0\cdot917}}=\frac{0\cdot917}{1\cdot03}$$

$$\therefore \text{ volume of the immersed portion} = \frac{0{\cdot}917}{1{\cdot}03} \times \frac{1000 \times 1000}{0{\cdot}917} \text{ c.c.}$$

$$= 970874 \text{ c.c.} = 9{\cdot}7 \times 10^5 \text{ c.c.}$$

(2) *If the density of ice be 0·917 gm/c.c., what fraction of a piece of ice will remain outside water when it floats?*

Ans. We know that the fraction of the volume of the body immersed in water is equal to the density of the solid. Hence, the fraction of the volume under water $= 0{\cdot}917$. So, the fraction outside water $= (1 - 0{\cdot}917) = 0{\cdot}083$.

(3) *An ice cube of mass 50 gm floats on the surface of a strong brine solution of volume 100 c.c. inside a measuring cylinder. Calculate the level of the solution in the measuring cylinder (i) before (ii) after all the ice is melted. What happens to the level if the brine is replaced by water? Density of brine solution $= 1{\cdot}1$ gm/c.c.*

Ans. (i) Before melting, the cube displaces 50 gms of solution whose volume $= \frac{50}{1{\cdot}1} = 45{\cdot}4$ c.c. So, the level of the solution before melting $= (100 + 45{\cdot}5) =$ 145·5 c.c. mark.

After all the ice is melted, 50 gm of water is produced whose volume is 50 c.c. So after complete melting, the level of the solution $= (100 + 50) = 150$ c.c. mark.

(ii) If pure water is taken instead of brine, the displaced water before melting has volume $= 50$ c.c. So the level will touch $(100 + 50) = 150$ c.c. mark.

As ice on complete melting, produces a volume of water equal to the volume displaced by it when it was floating, there will be no change in the level of the solution when ice completely melts into water.

4.12. Some cases of floatation :

(1) **Floatation of ice on water :** Bodies which are less dense than water float, those more dense sink. Ice floats on water, because the density of ice (0·917 gm/c.c.) is less than that of water (1 gm/c.c.). A piece of ice floats on water with $\frac{11}{12}$*th* of its volume inside and $\frac{1}{12}$*th* of its volume outside water because 11 c.c. of water at 0°C freeze into 12 c.c. of ice at 0°C.

(2) **Floatation of ship :** A lump of iron sinks in water but a huge ship made of iron floats. How does it happen?

If the lump of iron be given such a shape that it can displace a volume of water whose weight is greater than its own weight, the lump will float. Although ships are made of iron, yet they are hollow and largely empty and can displace a large quantity of water whose weight is greater than the weight of the ship together with its cargo.

The density of fresh water of rivers is less than the density of saline sea-water. Consequently, when a ship sails from saline sea-water to fresh river water, a greater volume of the ship sinks.

If you look at a ship anchored at any port, you will find some white lines on its body. They are known as *Plimsoll lines*. The ships are to be loaded in such a way as not to dip in water more than what the Plimsoll lines indicate. Of the Plimsoll lines, one marked with '*FW*' indicates the maximum extent to which the ship may sink and the one marked with '*W*' indicates the minimum extent of sinking. Before this system came into practice, there was no law regulating the loading of ships. Consequently accidents occurred frequently due to over-loading, specially in stormy weathers. Captain Samuel Plimsoll made a strong movement against such over-loading and was successful in getting a law enacted in British Parliament in 1876 which ultimately formulated the use of Plimsoll lines.

A body heavier than water can be made to float if a lighter body of suitable size be attached with it. The lighter body will displace larger quantity of water, producing greater upthrust without appreciably increasing the weight of the body. Life-belts or buoys work on this principle. Life-belts are made with light bags filled with air and help swimmers to float.

(3) **How does a man swim** ?

When the lungs are filled with air, the human body has a slightly smaller overall density than water, and so can float. But, as every swimmer knows, the body must be almost completely immersed in order to displace a large enough weight of water. For human beings, swimming is not natural as it is for animals because overall density of animal body is smaller than that of water. Sea-water being saline produces grater upthrust than fresh-water. *This is why it is easier to swim in sea-water than in fresh water.*

(4) **Cartesian diver** : It is a hydrostatic toy which illustrates the conditions of floatations and immersion discussed earlier.

The toy is a small hollow figure made of thin glass and having a hollow, open-ended tail. Normally, being full of air, it floats on water kept in a tall glass jar whose mouth is closed tightly by a thin rubber sheet (Fig. 4.9). This toy is called the *cartesian diver*. If, now, the rubber sheet is pressed, the diver sinks. Pressure on the sheet increases the pressure inside the jar, with the result that water is forced into the toy through the hollow tail. This makes the toy heavier than an equal volume of water and the toy dives into the water. If the pressure on the rubber sheet be released, the compressed air inside the toy expands and the excess water is forced out. The toy then, floats up. By regulating the pressure on the rubber sheet, the toy can be made to sink, float up or remain stationary anywhere inside water according as its total weight is greater than, equal to or less than the weight of the water it displaces.

Hence the toy illustrates the conditions of floatation and immersion of a body in a liquid.

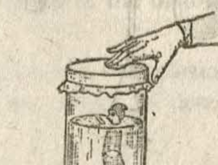


Fig 4.9

(5) **Submarines** : Submarines can move on the surface of water as well as inside water. It works in the same way as the cartesian diver.

Submarines have several chambers known as '*the ballast tanks*'. The buoyancy of a submarine depends on the quantity of water in its ballast tanks. When it is required to dive, water is admitted to these tanks. When water is ejected from the tanks by means of compressed air, the submarine rises to the surface and floats just like ordinary ships.

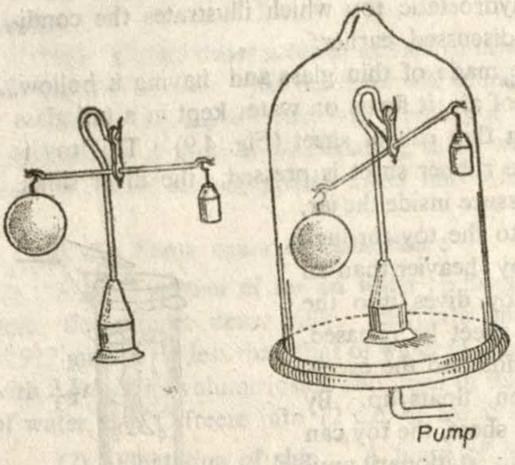
4.13. Buoyancy of air and application of Archimedes' principle to gases :

While discussing Archimedes' principle, we have seen that a body immersed in a liquid, experiences buoyancy due to which the body appears to lose some of its weight. We have also seen that the apparent loss of weight of the body is equal to the weight of the liquid displaced by the body. In this respect, the behaviours of liquids and gases are alike. A body immersed in a gas or in air (which is a gas) likewise experiences buoyancy and an apparent loss of weight. So, it may be said that when we weigh a body in air, we do not get its real weight. We get a weight slightly less than its real weight. Since air is very light, we do not ordinarily feel any difference but by making suitable arrangement we can demonstrate such difference in weight of a body.

Generally, we can say : *When a body is immersed partly or wholly in a fluid, it appears to lose a part of its weight and the loss of weight is equal to the weight of the fluid displaced by the body.*

The following experiment illustrates the buoyancy of air.

Fig. 4.10 shows a special type of balance. It is called a baroscope. It has no scale-pan; instead there is a hollow glass sphere at one end of the balance beam and a small lead weight or brass weight at the other end. Since the volume of the sphere is bigger than the volume of the lead weight, the sphere will displace greater volume of air and will consequently, feel greater buoyancy. Hence, apparent loss of weight will be greater in the case of the sphere than in the case of the lead weight. But their sizes are so that they balance each other in air, *i.e.*, their apparent weights in air are equal [Fig. 4.10(i)]. They are now placed on the receiver of an air-pump and are covered by a bell-jar. The joint between the bell-jar and receiver is made airtight by vaseline or wax. If air is now pumped out, you will see that the balance beam is no longer horizontal; the end carrying the sphere has tilted down showing that it has become heavier than the lead weight [Fig. 4.10(ii)]. Why does it happen so?



(i) Fig. 4.10 (ii)

In absence of air, the sphere and the weight will feel no buoyancy and they will get back their *real weight*. But as the loss of weight due to buoyancy was greater for the sphere, its real weight (which is the apparent weight plus the loss of weight due to buoyancy) will be greater than that of the lead weight. This is why, the end of the beam carrying the sphere went down when the air was pumped out. So it may be said that a body will weigh more in vacuo than in air.

(a) **Archimedes' principle is not applicable in the case of an artificial satellite moving round the earth in a circular orbit**: Everything inside an orbiting satellite is in weightless condition (Ref. art. 1.21—Gravitation). When a body is immersed in a liquid inside the satellite, the body, being weightless, will not experince any buoyancy due to the displaced liquid. As a result, there will not be any apparent loss of weight of the body. So, the question of applicability of Archimedes' principle in such cases does not arise.

For the same reason, Archimedes' principle is not also applicable in the case of a freely falling body because a freely falling body is weightless.

(b) **A pound of cotton is heavier than a pound of iron** : You may have heard something like the above. It may seem confusing to you because if both of them are one pound in weight how can one be heavier than the other? But a little thinking will make the matter clear. The weight mentioned above is generally the weight in air which is not the real weight but the apparent weight of cotton or iron. Since the volume of 1 lb of cotton is much greater than the volume of 1 lb of iron, cotton feels greater buoyancy and hence suffers greater loss of weight than iron. Hence in vacuum, cotton will be heavier than iron.

(c) **Weight of air-filled bladder** : Get a thin rubber bladder or a balloon, and weigh it in a balance in a deflated condition. Now inflate it with air and weigh it again. The two weights will be found equal. Does it mean that air has no weight?

When the balloon is inflated, it displaces some air and since the sheet is very thin, the volume of the air inside the balloon and the volume displaced by it are equal. Hence, gain in weight due to the air inside is compensated by the loss due to buoyancy of the displaced air. As a result, the weight of the balloon before and after inflation remains unchanged.

From such an experiment, long ago Voltaire came to an erroneous conclusion that 'air has no weight'. As a matter of fact, the weight of air cannot be determined by an experiment of the above nature.

(d) **The lifting of balloons** : A hydrogen-filled balloon rises in air. The density of air is about 14 times that of hydrogen. The total weight of the balloon consisting of fabric and hydrogen is thus much less than the weight of air it displaces. *The difference between the two represents the lifting power of the balloon.* For a balloon of big size, the lifting power is so high that the balloon, together with its passengers, can rise high into the air. In such a balloon flight, Glashier and Coxwell once ascended to a height of about 29,000 ft. It is to be remembered that one cannot go as high as one pleases in a balloon because at higher altitude, the density and pressure of air become less. The lifting power of a balloon consequently diminishes until the weight of the balloon becomes equal to the lifting power when the balloon remains stationary.

Besides hydrogen, helium is also used as a filling agent. Hydrogen is combustible but helium is not. The advantage of hydrogen lies in the fact that it is lighter than helium and therefore, produces greater lifting power.

*4.14. Buoyancy correction of weighing in a common balance :

When we weigh a substance in a common balance we do not get the actual weight of it because both the substance and the standard counterpoising weights will have some apparent loss of weight due to buoyancy of air.

Suppose m=mass of the body and ρ=density of the body. m_1=mass of the standard weights required for counterpoising the body and ρ_1=its density.

The volumes of the body and the counterpoising weights are $\frac{m}{\rho}$ and $\frac{m_1}{\rho_1}$. The buoyancy of air on them is equal to the weights of these volumes of air respectively. If d be the density of air, the apparent weight of the body $W=mg-\frac{m}{\rho}.d.g$ and the apparent weight of the counterpoising weights $W_1=m_1g-\frac{m_1}{\rho_1}d.g$. Since $W=W_1$, we have, $m\left(1-\frac{d}{\rho}\right)=m_1\left(1-\frac{d}{\rho_1}\right)$

∴ Actual mass of the body, $m=m_1\left(1-\frac{d}{\rho_1}\right)\left(1-\frac{d}{\rho}\right)^{-1}$

$$=m_1\left(1-\frac{d}{\rho_1}\right)\left(1+\frac{d}{\rho}\right)=m_1\left[1-d\left(\frac{1}{\rho_1}-\frac{1}{\rho}\right)\right]=m_1-m_1d\left(\frac{1}{\rho_1}-\frac{1}{\rho}\right)\quad[d<<\rho]$$

So, actual mass of the body will be obtained by subtracting $m_1d\left(\frac{1}{\rho_1}-\frac{1}{\rho}\right)$ from the apparent mass. If $\rho_1=\rho$ *i.e.* the density of the body is same as the density of the standard weights, the apparent weight is the actual weight and no correction is necessary.

If $\rho_1>\rho$, the actual weight is greater than the apparent weight and if $\rho_1<\rho$, the actual weight is less than the apparent weight.

Examples : (1) *A piece of iron weighs 275 gm. ; the piece floats with 5/9 th of its volume immersed in mercury whose density is 13·59 gm/c.c. Find the volume and density of the piece of iron.*

Ans. Suppose, the volume of the piece=V c.c.

we know, $\frac{\text{Volume of the piece immersed}}{\text{Total volume}}=\frac{\text{density of the substance}}{\text{,, ,, ,, liquid}}$

or, $\frac{5}{9}.V/V=\frac{\text{density of iron}}{13{\cdot}59}$

∴ density of iron$=13{\cdot}59\times\frac{5}{9}=7{\cdot}55$ gm/c.c.

Also, the volume of the piece$=\frac{\text{weight}}{\text{density}}=\frac{275}{7{\cdot}55}=36{\cdot}4$ c.c.

(2) *Two bodies, suspended from two ends of a balance, keep the beam horizontal when immersed in water. The weight and density of one of them are 28 gm and 5·6 gm/c.c. respectively. If the weight of the other body be 36 gm, what is its density?*

Ans. Suppose the required density $=d$ gm/c.c.

Since the two bodies balance each other when immersed in water, their apparent weights are equal. Now, the volume of the first $=\frac{28}{5\cdot6}=5$ c.c.

Its apparent wt. = real wt. − wt. of equal volume of water $=28-5=23$ gm.

Similarly, the volume of the second body $=\frac{36}{d}$ c.c. Its apparent weight $=36-\frac{36}{d}$ $\quad\therefore\quad 36-\frac{36}{d}=23$ or, $\frac{36}{d}=13$ $\therefore\quad d=\frac{36}{13}=2\cdot77$ gm/c.c.

(3) *By means of a hydrostatic balance, it is found that an iron cylinder and a glass cylinder of equal radii, when completely immersed in water, balance each other. Densities of iron and glass being 7·2 gm/c.c.* and *2·55 gm/c.c. respectively, calculate the ratio of their lengths.* [*H. S. Exam. 1968*]

Ans. From the question, it is clear that the apparent weights of the cylinders are equal. Now, suppose the length of the iron and the glass cylinders are l_1 and l_2 respectively.

If the radius be r, the real weight of the iron cylinder $=\pi r^2.l_1\times7\cdot2$ gm.

So, an equal volume of water weighs $=\pi r^2l_1\times1$ gm.

[$\because$ density of water $=1$gm/c.c.]

$\therefore$ the apparent wt. of the iron cylinder

$=\pi r^2l_1\times7\cdot2-\pi r^2l_1=6\cdot2\times\pi r^2l_1$ gm .. .. (i)

Again, the real wt. of the glass cylinder $=\pi r^2l_2\times2\cdot55$ gm.

and an equal volume of water weighs $=\pi r^2l_2\times1$ gm.

So, the apparent wt. of the glass cylinder

$=\pi r^2l_2\times2\cdot55-\pi r^2l_2=1\cdot55\times\pi r^2l_2$ gm .. .. (ii)

Since, their apparent weights are equal,

$$6\cdot2\times\pi r^2l_1=1\cdot55\pi r^2l_2\quad\therefore\quad\frac{l_1}{l_2}=\frac{1\cdot55}{6\cdot2}=\frac{1}{4}.$$

(4) *A substance of volume 100 c.c. and density 0·85 gm/c.c. floats on water. Oil of density 0·8 gm/c.c. is poured on the surface of water just to submerge the substance. What volume of the body will now be under water ?*

Ans. Wt. of the substance = volume × density $=100\times0\cdot85=85$ gm.

Suppose V c.c. of the body is now under water ; $(100-V)$c.c. will, therefore, be immersed in oil. According to the condition of floatation, we may write, $85=V\times1+(100-V)\times0\cdot8=V+80-0\cdot8V$

or, $0\cdot2V=5\quad\therefore\quad V=25$ c.c.

(5) *A uniform wooden rod, 20 cm. long and 2 cm. square cross-section, carries at one end, a piece of lead of volume 1 c.c. The rod, when put in water, floats with 7·4 cm. of its length projecting outside water. If the density of lead be 11·4 gm/c.c. what is the density of wood ?*

Ans. The cross-sectional area of the rod $=2\times 2=4$ sq. cm.
The length of the rod immersed in water $=20-7\cdot4=12\cdot6$ cm.
So, the volume of the water displaced $=4\times 12\cdot6=50\cdot4$ c.c.
and the volume of water displaced by the piece of lead $=1$ c.c.
Hence, the total volume of water displaced $=50\cdot4+1=51\cdot4$ c.c.
and, wt. of the water displaced $=51\cdot4$ gm. [density of water $=1$ gm/c.c.]
Suppose, the density of wood $=\rho$ gm/c.c.
the wt. of the wooden rod $=4\times 20\times\rho=80\rho$. gm.
" " " " lead $=1\times 11\cdot4=11\cdot4$ "
Their total weight $=(80\rho+11\cdot4)$ gm.
According to the condition of floatation, $80\rho+11\cdot4=51\cdot4$
or, $80.\rho=40$ $\therefore$ $\rho=0\cdot5$ gm/c.c.

(6) *A solid displaces $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ of its volume respectively when it floats on three different liquids. Find the volume it displaces when it floats on a mixture formed of equal volumes of the aforesaid three liquids.*

Ans. The volume and density of the substance, suppose, are V and D. The densities of the liquids are d_1, d_2, and d_3 respectively.

Since the solid floats on all the liquids, we have from the condition of floatation, $VD=\frac{1}{2}Vd_1=\frac{1}{3}Vd_2=\frac{1}{4}Vd_3$ or, $D=\frac{1}{2}d_1=\frac{1}{3}d_2=\frac{1}{4}d_3$.

If we suppose that in preparing the mixture, 'v' volumes of each of the liquids are taken, then, the total mass of the mixture $=v.d_1+v.d_2+vd_3=vd_1+\frac{3}{2}v.d_1+\frac{4}{2}vd_1=\frac{9}{2}.vd_1$

$$\therefore \text{ the density of the mixture}=\frac{\frac{9}{2}.v.d_1}{3v}=\frac{3}{2}.d_1$$

Now, suppose, x be the volume of the solid immersed in the mixture. Here,

$$VD=\frac{3}{2}d_1x \quad\text{or,}\quad \frac{1}{2}V.d_1=\frac{3}{2}.d_1x \quad\therefore\quad x=\frac{V}{3}-$$

i.e. the body will float with one-third of its volume immersed in the mixture.

(7) *A hollow spherical ball whose internal and external diameters are 8 and 10 cm. respectively, is found to float in a liquid of density 1·5 gm/c.c., just fully immersed. What is the density of the material of the ball ?*

Ans. The internal and external radii of the ball are 4 and 5 cm. respectively.

The external volume of the ball $=\frac{4}{3}\pi(5)^3$ c.c. and the internal volume $=\frac{4}{3}\pi(4)^3$ c.c.

$\therefore$ the volume of the material of the ball $=\frac{4}{3}\pi(5)^3-\frac{4}{3}\pi(4)^3=\frac{4}{3}\pi\times 61$ c.c.
Suppose, the density of the material $=\rho$ gm./c.c.
So, the mass of the ball $=$ volume of the material $\times$ its density

$$=\frac{4}{3}\pi\times 61\times\rho \text{ gm.}$$

Since the ball floats just fully immersed, the mass of the liquid displaced = mass of a volume of liquid equal to the external volume of the ball $=\frac{4}{3}\pi(5)^3\times 1{\cdot}5$ gm. Hence, from the condition of floatation, we have, $\frac{4}{3}\pi(5)^3\times 1{\cdot}5=\frac{4}{3}\pi\times 61\times\rho$

$$\therefore\ \rho=\frac{125\times 1{\cdot}5}{61}=3{\cdot}07 \text{ gm/c.c. (nearly)}$$

(8) *A rod of length 6 metres has a mass of 12 kg. It is hinged at one end at a distance of 3 metres below a water surface. (i) What weight must be attached to the other end of the rod so that 5 metres of the rod are submerged? (ii) Find the magnitude and direction of the force exerted by the hinge on the rod. Sp. gravity of the material of the rod=0·5.* [*I.I.T. 1976*]

Ans. According to the problem, $AB=6$ metres, $AG=GB=3$ metres and the weight W of the rod acts through the centre of gravity G. The immersed portion $AC=5$ metres; $AO=OC=2\frac{1}{2}$ metres and the buoyancy W_1 of the displaced water acts vertically upward through the centre of gravity O of the displaced water. The point A where the rod is hinged, is at a depth 3 metres from the surface of water [Fig. 4.11]. Let the reaction at the hinge be R directed at an angle θ with the vertical.

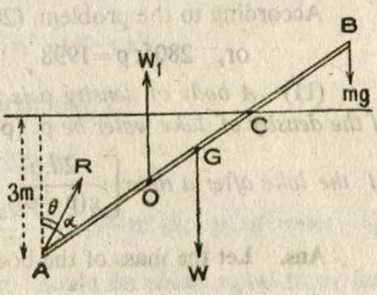


Fig. 4.11

(i) Now, the mass of the length $AC=\frac{12}{6}\times 5=10$ kg.

$$\therefore\ \text{Volume of the immersed portion}=\frac{10\times 10^3}{0{\cdot}5}=2\times 10^4 \text{ c.c.}$$

So, the weight of the displaced water $W_1=2\times 10^4$ gm.

[Density of water $=1$ gm/c.c.]

Weight of the rod $W=12$ kg

Taking moments of the forces about A, we have for equilibrium,

$$W\times AG\sin\alpha+m.g.\ AB\sin\alpha-W_1\times AO.\sin\alpha=0$$

where m is the mass attached at the point B of the rod.

$$\text{or,}\quad m.g.\ AB=W_1\times AO-W\times AG$$
$$=2\times 10^4\times\tfrac{5}{2}\times 100-12\times 10^3\times 3\times 100=1{\cdot}4\times 10^6$$

$$\therefore\ m.g.=\frac{1{\cdot}4\times 10^6}{AB}=\frac{1{\cdot}4\times 10^6}{600}\text{gm-wt}=2{\cdot}33 \text{ kg-wt.}$$

So, a weight of 2·33 kg must be attached at B.

(ii) Resolving R horizontally and vertically, we get,

$$R.\sin\theta=0 \quad\text{or,}\quad \sin\theta=0 \quad \therefore\ \theta=0$$

and $R\cos\theta=W+mg-W_1$

or $R=12+2{\cdot}33-20=-5{\cdot}67$ kg.wt. [$\cos 0°=1$]

Negative sign shows that the reaction at A is directed vertically downwards.

(9) *A buoy of volume 200 litre and density 0·95 gm/c.c., is fully immersed in sea-water of density 1·02 gm/c.c., being anchored to the sea-bottom by a light chain. What is the tension on the chain?*

Ans. Volume of displaced sea-water$=200$ litres$=200\times1000$ c.c.$=2\times10^5$ c.c. Wt. of displaced sea-water$=2\times10^5\times1{\cdot}02$ gm.$=204{,}000$ gm.$=204$ kg. Wt. of the buoy$=2\times10^5\times0{\cdot}95$ gm.$=190$ kg.

$\therefore$ tension on the chain$=(204-190)=14$ kg.

(10) *A metal sphere of volume 2000 c.c. has a cavity inside of volume 1720 c.c. A force of 2 gm-wt is necessary to immerse the sphere completely in water. Find the density of the metal.*

Ans. The volume of displaced water$=2000$ c.c. ; its mass$=2000$ gm. and its weight$=2000\times980$ dynes.

The volume of metal in the sphere$=2000-1720=280$ c.c.

The weight of the metal in the sphere$=280\times\rho\times980$ dynes. [ρ=density]

According to the problem, $(2000-280\times\rho)980=2\times980$

or, $280\times\rho=1998$ $\therefore$ $\rho=7{\cdot}1$ gm/c.c. (nearly)

(11) *A body of density ρ is kept slowly on the surface of a lake of depth d. If the density of lake water be ρ' $(\rho'<\rho)$, show that the body will reach the bottom of the lake after a time* $\left[\dfrac{2d.\rho}{g(\rho-\rho')}\right]^{\frac{1}{2}}$ [*Jt. Entrance 1966*]

Ans. Let the mass of the body be m. Its weight$=mg$ and volume$=\dfrac{m}{\rho}$.

When the body is immersed in water, the volume of water displaced$=$ Volume of the body$=\dfrac{m}{\rho}$. Wt. of displaced water$=$volume$\times\rho'\times g=\dfrac{m}{\rho}\times\rho'\times g$

$$\therefore\ \text{Apparent weight of the body}=mg-\frac{m\rho' g}{\rho}=\frac{mg}{\rho}(\rho-\rho').$$

$$\text{So, the acceleration of the body in the lake } f=\frac{\text{Apparent wt.}}{\text{mass}}$$

$$=\frac{mg(\rho-\rho')}{m\times\rho}=\frac{g(\rho-\rho')}{\rho}$$

If t be the required time, $S=\frac{1}{2}.ft^2$ or $d=\frac{1}{2}.g\dfrac{(\rho-\rho')}{\rho}.t^2$

$$\therefore\ t=\left[\frac{2\rho.d}{g(\rho-\rho')}\right]^{\frac{1}{2}}$$

(12) *Calculate the percentage of error arising from the negligence of buoyancy of air in weighing an object of density 12 gm/c.c. with brass weights of density 8 gm/c.c. by a physical balance. Density of air during experiment$=1{\cdot}2\times10^{-3}$ gm/c.c.*

[*Joint Entrance 1981*]

Ans. If m and m_1 be the actual weight and apparent weight of the body and ρ and ρ_1 the density of the body and the density of the counterpoising weights respectively, then, $m=m_1\left[1-d\left(\dfrac{1}{\rho_1}-\dfrac{1}{\rho}\right)\right]$ where d=density of air [ref. art. 4.14]

$$\text{or, } \frac{m_1}{m}=\left[1-d\left(\frac{1}{\rho_1}-\frac{1}{\rho}\right)\right]^{-1}=1+d\left(\frac{1}{\rho_1}-\frac{1}{\rho}\right)=1+1{\cdot}2\times10^{-3}\left(\frac{1}{8}-\frac{1}{12}\right)=1+0{\cdot}5\times10^{-4}$$

$$\therefore\quad \frac{m_1-m}{m}=0{\cdot}5\times10^{-4}$$

Hence the percentage error $=\dfrac{m_1-m}{m}\times100=0{\cdot}5\times10^{-4}\times100={\cdot}005\%$

(13) *A hollow sphere of radius R is made of a metal whose specific gravity is ρ. Show that the sphere will float in water if the thickness of the wall of the sphere is less than R/3ρ.*

Ans. Let the thickness of the wall of the sphere be t. The volume of the material of the sphere = Outer volume of the sphere − inner volume

$$=\frac{4\pi R^3}{3}-\frac{4\pi(R-t)^3}{3}=\frac{4\pi}{3}[R^3-(R-t)^3]$$

Weight of the sphere = volume of the material × sp. gr.

$$=\frac{4\pi\rho}{3}[R^3-(R-t)^3]\ \text{gm-wt.}$$

Weight of an equal volume of water $=\dfrac{4\pi R^3}{3}\times1$ gm-wt [Sp. gr. of water = 1]

To float in water, the weight of the sphere should be either equal to or less than that of an equal volume of water.

$$i.e.\quad \frac{4\pi\rho}{3}\,[R^3-(R-t)^3]\leqq\frac{4\pi R^3}{3}\quad i.e.\ \rho[R^3-(R-t)^3]\leqq R^3$$

$$i.e.\quad \left[1-\left(1-\frac{t}{R}\right)^3\right]\leqq\frac{1}{\rho}\quad\text{or}\quad\left(1-\frac{t}{R}\right)^3\geqq1-\frac{1}{\rho}$$

$$i.e.\quad 1-\frac{t}{R}\geqq\left(1-\frac{1}{\rho}\right)^{\frac{1}{3}}\geqq1-\frac{1}{3\rho}\quad\text{[Other factor being negligible]}$$

$$i.e.\quad \frac{t}{R}\leqq\frac{1}{3\rho}\quad\text{or}\quad t\leqq\frac{R}{3\rho}$$

Exercises

Essay type :

1. What is Archimedes' principle? Describe an experiment illustrating the principle. How can you verify it ? [*H. S. Exam. 1979, '82* '86]

2. How would you determine the volume and density of an irregularly shaped body by the application of Archimedes' principle ?

3. Clearly explain the conditions of floatation and immersion of a body. What should a body do to float at rest ? [*H. S. Exam., 1982*]

4. Discuss the stability of floating bodies and explain 'meta-centre'.

5. Describe a cartesian diver and explain its action. Do you know any modern appliance based on the principle of the cartesian diver ?

6. Is Archimedes' principle applicable to gases ? Illustrate it with suitable experiment.

Short answer type :

7. What do you mean by 'real weight' and 'apparent weight' ? Which one is greater and why ?

8. Two identical spheres—one solid and the other hollow are immersed completely in water. Which one of them will experience greater upward thrust ?

9. (a) Answer the following questions carefully :—(i) Why is it easier to move a heavy piece of stone in water ? (ii) Why is it easier to swim in sea-water than in fresh water ? [*H. S. Exam. 1980*]. (iii) Why does a ship sink more when it sails from sea-water to fresh water ? (iv) How does a ship, made of iron, float in water while a lump of iron sinks ? [*H. S. Exam 1980*]. (v) An egg sinks in pure water but floats on saline water. Why ? (vi) A toy balloon, filled with hydrogen, ascends to the roof of a room but filled with carbon dioxide, lies on the floor. Why ? (vii) Why are plimsoll lines different for sea-water and fresh water ? (viii) Explain why a lump of iron (sp. gr=7·8) sinks in water but floats in mercury (sp. gr=13·6). [*H. S. Exam. 1978*] (ix) Why is a boat likely to capsize if passengers in the boat stand up ?

(b) Answer the following questions : (i) Why does a uniform wooden stick float horizontally if it is not loaded but float vertically if enough weight is added to one end ? [*Jt. Entrance 1972*] (ii) A man sitting in a boat floating in a pool, drinks some water from the pool so that water level of the pool gets down a little. Is it true ? Explain with reason. [*I.I.T. 1980*] (iii) A ship, just floats in Bay of Bengal, sinks when it enters the Ganges. Why ?

10. 'A floating body is apparently weightless'—Explain. [*H. S. Exam. 1981*]

11. A boat, full of stone chips, floats in the middle of a swimming pool. A man, drops the chips, one by one, into the water. Will there be any change in the level of water in the pool ?

[**Hints** : W_1=wt. of stone chips; W_2=wt. of the boat. At first, the volume of water displaced $V_1=(W_1+W_2)$ c.c. In the second case, the volume of displaced water $V_2=\left(\frac{W_1}{\rho}+W_2\right)$ c.c. As $V_2<V_1$, the level will fall.]

12. A man carries a big fish in one hand and a bucket of water in the other hand. He places the fish in the bucket. Will the man now carry less weight than before ?

[**Hints** : The man carries the same weight.]

13. A leaky tramp steamer that is farely able to float in the Bay of Bengal steams up the Ganges estuary towards the Calcutta docks. It sinks before it arrives. Why ?

14. A man is sitting in a boat which is floating in a pond. If the man drinks some water from the pond, will the level of the water in the pond decrease ? [*I.I.T. 1980*]

15. A large glass bulb is balanced by a small brass weight in a sensitive beam balance. State and explain what will happen when the balance is covered by a bell jar which is then evacuated.

16. Two bodies of equal weight and volume and having the same shape, except that one has an opening at the bottom and the other is sealed, are immersed to the same depth. Is less work required to immerse one than the other ? If, so which one and why ?

17. A balloon filled with air is weighted so that it barely floats in water just immersed. Explain why it sinks to the bottom when it is submerged a short distance in water.

[**Hints** : It sinks because its volume decreases. When the balloon floats just immersed in water, the weight of the water displaced by the balloon is just equal to its own weight plus the attached weights. When it is submerged a little in water, the pressure of water around the balloon diminishes its volume. Hence it displaces less water and sinks to the bottom.]

18. A ball floats on the surface of water in a vessel exposed to atmosphere. Will the ball remain immersed at its former depth or will it sink or rise somewhat if (a) the vessel is covered and the air is removed (b) the container is covered and the air compressed ? [*H. S. Exam. 1986*]

19. A hollow glass sphere, containing atmospheric air, is weighed in air by a balance. The sphere is then broken and the broken pieces are carefully collected (none being lost) and weighed again in air. Will there be any difference in the two weights ?

20. Is Archimedes' principle valid in the case of free fall ? Does it hold in a satellite moving in a circular orbit? [*H. S. Exam. 1979*]

21. A thin-walled rubber balloon weighs equally when deflated and inflated with air. Why does it happen so ? If the weighings are done in vacuum, will they be equal ?

22. A beaker containing water is placed on the pan of a balance which shows a reading of M gm. A lump of sugar of mass m gm and volume v c.c. is now suspended by thread in such a way that it is completely immersed in water without touching the beaker and without any overflow of water. What will be the reading of the balance just when the lump of sugar is immersed ? How will the reading change as time passes on ? [*I.I.T. 1978*]

[**Hints** : At first the reading is $(M+m)$ gm ; As time passes on, no change in the reading will be observed.]

Objective type :

23. Mark the correct answer :—

(a) A block floats in a closed vessel of water. The air above the water is pumped out. The block will (i) rise, (ii) sink (iii) remain steady.

(b) A body floats in a liquid contained in a beaker. The whole system is then allowed to fall freely under gravity. The upthrust on the body due to the liquid is (i) zero (ii) equal to the weight of the liquid displaced (iii) equal to the weight of the body in air (iv) equal to the weight of the immersed portion of the body. [*I.I.T. 1982*]

(c) A piece of ice with a stone frozen in it floats on water, kept in a beaker. When the ice completely melts, the level of water (i) increases (ii) decreases (iii) remains the same.

(d) A pound of cotton and a pound of iron are counterpoised in a beam balance in vacuum. The beam will be (i) tilted towards the cotton (ii) tilted towards the iron (iii) horizontal.

(e) A ship often carries a ballast to keep its metacentre (i) above the C. G. (ii) below the C. G. (iii) undisturbed.

(f) For the stability of floating bodies the metacentre must be (i) above the *c.g.* of the floating body (ii) below the *c.g.* of the floating body (iii) at the *c.g.* of the floating body.

(g) A piece of ice floats in water in a beaker which is full to the brim. When the piece of ice melts completely, the water in the beaker (i) overflows (ii) falls down (iii) remains brimful.

(h) A piece of iron sinks in water but a big ship made of iron easily floats because (i) iron has greater density than water (ii) the ship can displace a large quantity of water (iii) iron has a much smaller volume.

(i) A leaky steamer which is just afloat in the Bay of Bengal steams up the river Ganges. It then (i) floats up (ii) remains just floating (iii) sinks.

(j) A bucket of water is suspended from a spring balance. A piece of iron held by a string is immersed in water. The spring balance gives (i) increased reading (ii) decreased reading (iii) same reading.

(k) The spring balance A reads 2 kg with a block m suspended from it. A balance B reads 5kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hanging mass is inside the liquid in the beaker, as shown in fig 4.12. In this situation (i) the balance A will read more than 2kg (ii) the balance B will read more than 5kg (iii) the balance A will read less than 2kg and B more than 5kg (iv) the balance A and B will read 2kg and 5kg respectively. [*I.I.T. 1985*]

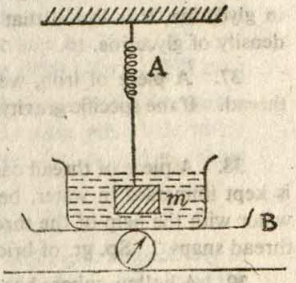


Fig. 4.12

Numerical problems :

24. The volume of a body is 36 c.c. and it can float on water with 3/4th of its volume immersed. What are the weight and density of the body ? [Ans. 27 gm. ; 0·75 gm/c.c.]

25. The mass of a piece of alloy, made of iron and copper, is 320 gm and its volume is 30 c.c. Find the volume of copper and iron. Sp. gr. of copper = 8·89 and of iron = 11·37.
[Ans. 8·5 c.c. ; 21·5 c.c.]

26. A piece of wood is 5 cm. long, 4 cm. broad and 3 cm. high. If it floats on water with 2·5 cm., of its height immersed, what will be the weight and density of the piece ?
[*H. S. Exam. 1983*] [Ans. 50 gm.; 0·83 gm/c.c.]

27. A buoy of volume 1000 litres and weighing 950 kg., is fully immersed in sea-water of sp. gravity 1·02, being anchored to the sea-bottom by a light chain. What is the tension on the chain ? [*H. S.* (*comp.*) *1963*] [Ans. 70 kg-wt.]

28. A steamer weighs 10 metric tonne. When it enters into a sweet water lake from the sea, it displaces 50 litres more water. What is the density of sea-water ? 1 litre of water weighs 1 kg.
[*H. S. Exam. 1982*] [Ans. 1·004 gm/c.c.]

29. Two spheres of equal size but of different materials are kept fully immersed in water. One sphere has been kept suspended by a thread which is experiencing a tension 1000 gm.-wt. while the other is kept tied to the bottom of the vessel by another thread which is experiencing a tension 25 gm-wt. If the volume of each sphere be 100 c.c., find the masses and the densities of the materials of the spheres. [Ans. 1100 gm. ; 11 gm/c.c. ; 75 gm ; 0·75 gm/c.c.]

30. A gold ring, set with stone, weighs 5 gm in air and 4·5 gm in water. What are the weights of gold and stone in the ring if densities of gold and stone are 19·3 gm/c.c. and 3·5 gm/c.c. respectively.
[Ans. 1·03 gm ; 3·97 gm.]

31. A substance can float on water with one-sixth of its volume projecting. What portion of its volume will project if it floats on another liquid of density 1·2 gm/c.c. ? [Ans. $\frac{11}{36}$]

32. A rectangular wooden slab is 4 ft., long, 4 ft. broad and 18 inches high. It weighs 600 lb. Density of sea-water being 65 lb/cu. ft., prove that the slab will float on sea-water. What is the minimum weight to be placed on the slab so that it just sinks ? [Ans. 960 lb]

33. The beam of a balance remains horizontal when two bodies suspended from the ends are kept immersed in water. The mass of one of them is 32 gm and its density is 8 gm/c.c., the density of the other being 5 gm/c.c. What is its mass ?
[Ans. 35 gm.]

34. A piece of wood (density = 0·9 gm/c.c.) and a piece of aluminium (density = 2·7 gm/c.c.), weighing 10 gm. when tied together are found to float just immersed in water. Calculate the volume of the piece of wood.
[Ans. 62·9 c.c.]

35. A hollow sphere of glass floats just immersed in spirit (density = 0·8 gm/c.c.). What fraction of its volume will remain inside sulphuric acid if it is floated in the acid ? Density of sulphuric acid = 1·8 gm/c.c.
[Ans. 4/9]

36. A cube of wood whose edge is 4 cm. weighs 48 gm. When this cube is floated in glycerine, it is found that the upper horizontal face is 1·6 cm. above the surface. Find the density of glycerine.
[Ans. 1·25 gm/c.c.]

37. A piece of iron, weighing 160 gm is kept immersed in water, being tied to a piece of thread. If the specific gravity of iron be 8, what will be the tension in the thread ? $g = 980$ cm/s^2.
[Ans. $1 \cdot 4 \times 10^5$ dynes (nearly)]

38. A piece of thread can withstand upto 3 kg of weight . A piece of brick, weighing 3·5 kg. is kept immersed in water, being tied to the thread. The piece of brick is slowly pulled out of water with the help of the thread. What fraction of its volume will come out of water when the thread snaps ? Sp. gr. of brick = 1·5.
[Ans. 0·8 (nearly)]

39. A hollow sphere has an internal and external diameter of 10 cm. and 12 cm. respectively. It floats in a liquid of density 1·2 gm/c.c. just fully immersed. Determine the density of the material of the sphere.
[Ans. 2·84 gm/c.c.]

40. A sphere of radius 1 cm. sinks in water but floats, just immersed, if a layer of wax 2 mm. thick be given on it. If the density of wax be 0·8 gm/c.c., what is the density of the material of the sphere ? [Ans. 1·24 gm/c.c.]

41. A sphere of glass of density 2·6 gm/c.c is given a thick coating of wax whose density is 0·8 gm./c.c. If the wax coated sphere floats in water just immersed, find the ratio of the volume of the coating and the volume of the sphere. [*Jt. Entrance 1985*] [Ans. 8]

42. A pontoon with vertical sides has a rectangular base 500 ft. by 10 ft. It is loaded so that the base is submerged to a depth of 5 ft. and 4·54 ft. of the pontoon remains above water. It then commences to leak taking 100 gallons of water per minute. How long will it take before it sinks ? 1 cu. ft. = 28·31/4·54 gallons. [Ans. 23·5 hours (nearly)]

43. A solid weighs 237·5 gm in air and 1·25 gm in an oil of sp. gravity 0·9. What is the sp. gravity of the solid ? It floats in a liquid with $\frac{1}{8}$th of its volume projecting out of the liquid. What is the sp. gravity of the liquid ? [Ans. 0·905 ; 1·13]

44. A boat is floating in a square lake of 20 metre length. There are 100 cubic shaped bricks, each of length 20 cm. If the bricks are dropped, one by one, into the water, how will the level of water in the lake change ? Density of brick = 2·5 gm/c.c. [Ans. 0·3 cm. fall]

45. A toy balloon is to be filled with hydrogen (density = 0·09 gm/litre). When empty, the balloon weighs 5·8 gm. What volume in litres of hydrogen at normal pressure is required to make the balloon just rise when the density of air is 1·25 gm/litre ? [Ans. 5 litres]

Harder problems :

46. A piece of stone of density 2·5 gm/c.c. is kept immersed in sea-water and is allowed to sink. How far will it sink in 2 sec ? Sp. gravity of sea-water = 1·025 ; $g = 980$ cm/s^2. [Ans. 1156·4 cm.]

47. An oil drop is rising in water with an acceleration $\alpha.g$ where α is a constant and g the acceleration due to gravity. Neglecting friction of water, calculate the sp. gravity of oil. $\left[\text{Ans. } \frac{1}{1+\alpha}\right]$

48. A thin cylindrical vessel of diameter 10 cm. and height 15 cm. contains 200 c.c. of water. The weight of the vessel alone is 114 gm. The cylindrical vessel with its contents are placed in a large trough. Water is now poured gradually into the trough. What is the maximum height to which water can be poured in the trough before the vessel begins to rise ? [*I.I.T. 1970*] [Ans. 4 cm.]

49. Two cylinders of same cross-section and length l but made of two materials of densities ρ_1 and ρ_2 are cemented together to form a cylinder of length $2l$. If the combination floats in water with a length $l/2$ above the surface of water, show that if $\rho_1 > \rho_2$ and the density of water be unity, then $\rho_1 > \frac{3}{4}$. Show also that the equilibrium is unstable if the heavier part of the cylinder is at the top. [*Joint Entrance 1981*]

50. The cross-sections of the two limbs of a U-tube are 1 sq.-cm. and 0·1 sq. cm. The lower parts of both the limbs contain water. What volume of oil of density 0·85 gm/c.c. must be poured in the wider limb to raise the surface of water in the narrower tube by 15 cm ? [*Joint Entrance 1976*] [Ans. 17·91 c.c.]

51. A U-tube is partly filled with water. Another liquid, which does not mix with water, is poured into one side until it stands a distance d above the water level on the other side which has meanwhile risen a distance l. If the densities of water and liquid be ρ_1 and ρ_2, show that $\frac{\rho_2}{\rho_1} = \frac{2l}{(2l+d)}$.

52. Three substances A, B and C of different material weigh 16, 20,22 gm. respectively in air and 14, 18 and 20 gm. respectively in water. Of the three, two are made of pure metal and the third is their alloy. Which one is the alloy ? Find the weight of other two substances in it. [*Jt. Entrance 1978*] [Ans. B is alloy ; $A = 5·33$ gm. ; $C = 14·67$gm]

53. A piece of cork of sp. gravity 0·25 and a metallic piece of sp. gr. 8·0 are bound together. If the combination neither floats nor sinks in alcohol of sp. gravity 0·8, calculate the ratio of the masses of cork and metal. [*Jt. Entrance 1981*] [Ans. 9 : 22]

54. Oil of sp. gravity 0·6 is standing over water in a wide-mouthed vessel. A block of wood 10 cm. on a side floats over water with its upper surface completely immersed under the oil. The lower surface of the floating block is 2 cm. below the water surface. Find the mass of the block. [*Jt. Entrance 1974*] [Ans. 680 gm.]

55. (i) A man and a stone are floating on a raft in a swimming pool 10 metres long and 5 metres wide. The stone weighs 40 kg and has a specific gravity 3·0. If the man drops the stone off from the raft into the pool, by how much will the water level on the side of the pool rise or fall ? [*Jt. Entrance 1973*] [Ans. 0·53 cm. fall]

(ii) A cylindrical vessel with cross-sectional area 4·95 sq. cm. is filled with water in which a piece of ice, containing a lead ball floats. The volume of the ice together with the lead ball is 100 c.c. and $\frac{1}{20}$th of this volume is above the water level. Find the fall of water level in the vessel when the ice melts completely. Sp. gr. of ice$=0·9$ and of lead$=11$. [Ans. 1 cm]

56. A uniform rod *AB*, 12 ft. long weighing 24 lb is supported at the end *B* by a cord and weighted at the end *A* with a 12 lb lead weight. The rod floats with one half of its length submerged in water. The buoyant force on the lead weight can be neglected. Find the tension in the cord. [Ans. 4 lb-wt]

57. A small piece of metal of mass *m* is attached to a sphere of mass *M* and radius *r*. The sphere floats just half-immersed in a liquid with the metal piece exactly at the bottom of the sphere. Show that the metacentric height of the floating sphere is $\dfrac{m.r}{m+M}$.

58. A body when immersed in a liquid floats with $\frac{1}{4}$th of its volume above the liquid surface. The body is taken to a depth of x cm. of liquid and then released. Show that the body will reach the surface after $\sqrt{\dfrac{6x}{g}}$ sec. from the time of release.

59. A body is weighed in a balance by m gm. of brass weight. If the density of the body be σ, the density of brass d and that of air ρ, then show that the real weight of the body is

$$M=m\left(1-\frac{\rho}{d}\right)\left(1+\frac{\rho}{\sigma}\right).$$

60. A balloon filled with hydrogen has a volume of 1000 litres and its mass is 1 kg. What would be the volume of a block of a very light material which it can just lift. One litre of this material has a mass of 91·3 gm. [*I.I.T. 1975*] [Ans. 3·33 litres]

5

ATMOSPHERIC PRESSURE AND AIR PRESSURE PUMPS

5.1. Atmospheric pressure :

The earth is surrounded by a gaseous envelope known as atmosphere or air. The atmosphere contains oxygen, nitrogen and many other gases. We cannot see air but we can feel it in various ways. The atmosphere, surrounding the earth, extends several miles high up. We, on the earth, are living at the bottom of a sea of air just as fishes and other aquatic animals live at the bottom of a sea of water. The living beings owe their existence to the air for obvious reasons.

Air has weight. Consequently the atmosphere exerts a pressure on the earth, to the extent of about 15 lb-wt per square inch. This pressure is exerted all over the surface of objects on the earth, including ourselves. An average-sized man, having a surface area of 16 sq. ft. will thus have a total thrust over his body of 16×144×15 lb. or something in the neighbourhood of 15 tons. But normally we do not feel such enormous load on us because our blood exerts a pressure slightly greater than the atmospheric pressure.

Like liquids, the atmosphere also exerts pressure in all directions.

5.2. Experiments to demonstrate the existence of atmospheric pressure :

(1) Take a cylinder of hard glass with both ends open. Close one end tightly by a thin rubber sheet [Fig. 5.1(a)]. Place the vessel on the receiver of an air-pump. The lower end of the cylinder being ground and well-greased, will fit air-tight on the receiver. Since the pressures of air inside and outside the vessel are equal, the rubber sheet will remain flat. The air-pump is now worked and the vessel is gradually evacuated. It will be seen that the rubber sheet, being pressed by atmosphere above, is gradually curved out. If the evacuation is carried on, the rubber sheet will finally burst with a loud report. This shows that the atmosphere exerts downward pressure.

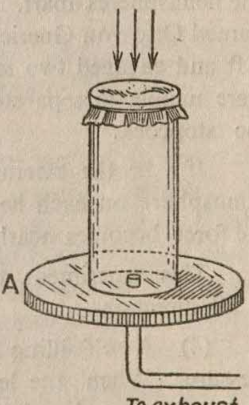


Fig. 5.1(*a*)

(2) Take a thin-walled metal can with an air tight stopper. Pour some water in the can and boil it after removing the stopper. The steam will drive out all air from the can. The stopper is then replaced tightly. Simultaneously the flame beneath the can is removed. Cold water is now sprinkled over the can. This causes some of the steam inside the can to condense. As a result, the pressure

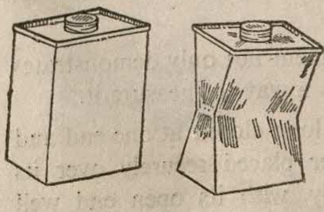


Fig. 5.1(*b*)

inside becomes low and the can is found to collapse inwards under the excess atmospheric pressure outside [Fig. 5.1 (b)]. This experiment shows that the atmosphere exerts lateral pressure.

(3) **Magdeburg hemisphere experiment** : Two hollow brass hemispheres, one of which has a stopcock, form a complete sphere when their rims are joined together [Fig. 5.1 (c)]. An air-pump can be connected to the stopcock. When the sphere contains air, the hemispheres can be easily pulled apart because the pressure outside is balanced by an equal pressure inside. On removal of the air from inside, only the external atmospheric pressure will act over the sphere which will press the hemispheres tightly together. It will then be very difficult to pull the hemispheres apart. This experiment was first performed by a German scientist named Otto von Guericke in 1654. He used two hemispheres, each of diameter 2 ft and engaged two teams of eight horses to pull the hemispheres apart. They were unable to separate the hemispheres until air had been readmitted through the stopcock.

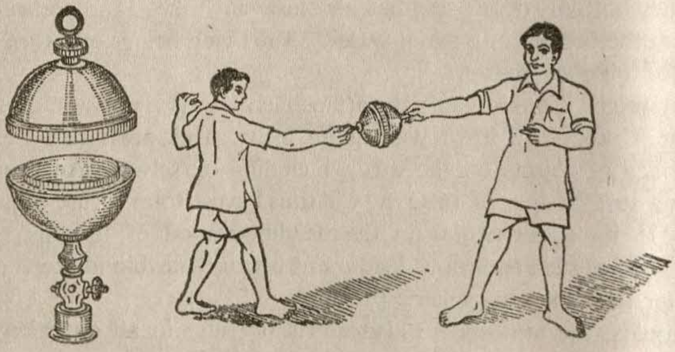


Fig. 5.1(*c*)

If r be the external radius of the sphere, the force exerted by the atmosphere on each hemisphere $=\pi r^2 \times$ atmospheric pressure. Taking $r=1$ ft. the force becomes nearly 3000 kg-wt.

This experiment, therefore, shows that the atmosphere can exert pressure in all directions.

(4) A self filling fountain pen works due to the existence of atmospheric pressure. When the lever of the fountain pen is raised, a rubber tube inside the pen is pressed and air from inside the tube is forced out. When the lever is lowered, atmospheric pressure, which is in excess over the pressure in the tube, forces some ink into the tube.

(5) **Torricelli's experiment** : Torricelli's experiment not only demonstrates the existence of atmospheric pressure but also gives us a way to measure it.

Take a stout walled glass tube about a metre long closed at one end and fill it to the top with clean mercury. With the finger placed securely over its open end, the tube is inverted and placed vertically with its open end well below the surface of some mercury in a vessel A. When the finger is removed, the mercury falls until a column of mercury stands in the tube [Fig. 5.1 (d)]. It seems at first sight that the column of mercury stands unsupported in the tube. But it is not so. Torricelli explained that the column of mercury was supported by the atmospheric pressure acting on the surface of the mercury in the vessel *A*. So,

Atmospheric pressure=weight per unit area of the mercury column.

If the experiment is carried out with tubes of different diameter, the height of the mercury column will remain the same showing that the atmospheric pressure does not depend upon the bore of the tube. Ordinarily, the height of the mercury column standing in the tube is about 76 cm. In other words, the atmospheric pressure supports a column of mercury of height 76 cm.

Since the sp. gravity of mercury is 13·6, it follows that if water were used in the above experiment, the height of the water column would have been 76×13·6 cm or about 34 ft.

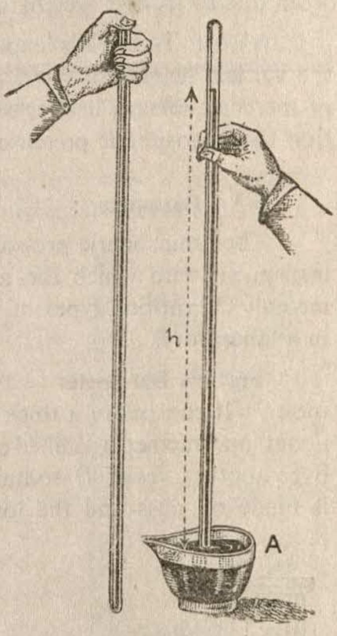


Fig. 5.1(*d*)

A few facts about Torricelli's experiment : The following are very important in connection with Torricelli's experiment :

(*i*) The space above the mercury in the tube is called a Torricellian vacuum ; it contains little mercury vapour and in this respect differs from a true vacuum.

(*ii*) Keeping the open end of the tube dipped in the mercury in the vessel *A*, if the tube be gradually inclined, the mercury column will also move gradually towards the closed end but in every position, the vertical height of the column will remain the same, because it measures the atmospheric pressure. [Fig. 5.1(e)]

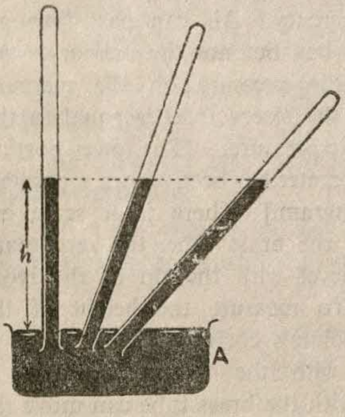


Fig. 5.1(*e*)

(*iii*) If Torricelli's experiment be performed in an enclosure and the air of the enclosure be slowly pumped out, the mercury column will fall slowly. If the air be readmitted, the column will grow in height. This conclusively proves that the column of mercury is supported by the atmospheric pressure.

(*iv*) If an opening be made at the top of the tube, air will enter through the opening and will exert pressure on the column. The pressures at the top and at the bottom of the column being thus made equal, the column will come down due to its own weight and mercury will be deposited in the vessel *A*.

(*v*) If Torricelli's experiment is performed at Calcutta and at Darjeeling, the vertical heights of mercury column in the tube will be different. The height of mercury column in Darjeeling will be less than that in Calcutta. This shows that the atmospheric pressure decreases at higher altitude.

5.3. Barometer :

The atmospheric pressure is often required in laboratory experiments. The instrument with which the atmospheric pressure is measured is called a barometer. Of various types of barometers, Fortin's barometer is generally used in a laboratory.

Fortin's Barometer : Fig. 5.2(*a*) shows a section of the Fortin's barometer. It consists of a thick walled glass tube *AB* of uniform bore, having length about one metre. It is filled completely with dry and clean mercury and inverted over another vessel *D* containing some mercury. The upper half of this vessel is made of glass and the lower half of brass. The barometer tube is protected by enclosing it in a brass tube, the upper part of which is made of glass to make the mercury meniscus visible. The tube is fitted against a wooden frame which is clamped to the wall of a room in a vertical position. An ivory pin *C* is provided to adjust the level of mercury in the reservoir *D*. The level can be raised or lowered by a screw *E* fixed at the bottom of the reservoir. When the screw *E* is turned, a leather bag increases or decreases its volume, there by lowering or raising the level of mercury. Air can pass through the leather bag but not the mercury. As a result, the pressure on the mercury surface in the reservoir *D* is equal to the atmospheric pressure. [The lower portion of the barometer has been shown separately in the diagram.] There is a scale engraved on the brass tube, the zero mark being in level with the tip of the ivory pointer. To measure the height of the mercury column correctly, a vernier *G* is provided with the scale. A screw *H* attached with the brass tube can move the vernier *G*. To read the barometer, the vernier is to be so adjusted by turning the screw *H* that the lower edge of the vernier is always tangential to the convex surface of the mercury column. For

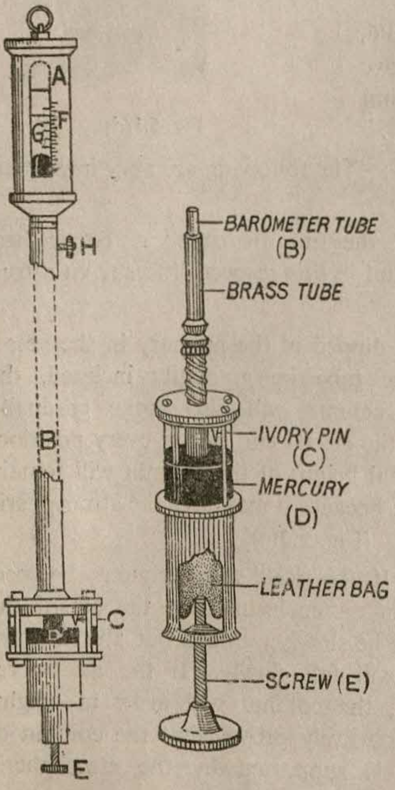


Fig. 5.2(*a*)

correct adjustment, a milk-white plate is fixed on the wooden board behind the vernier. As soon as the vernier is adjusted correctly, the milk-white plate being completely obstructed by the vernier will disappear from view. The atmospheric pressure varies with the variation of temperature. For this reason a thermometer is always provided with a barometer to note the temperature when the pressure is recorded.

To read a barometer : Before taking a reading, the screw E is adjusted until the mercury surface in the reservoir D just touches the ivory pointer C. Due to daily variation of pressure, the mercury surface may not remain in touch with the ivory pointer. When this adjustment is made, the mercury surface in the reservoir will be in the same level with the zero of the scale F. Now, by turning the screw H, the vernier is so adjusted that its lower edge is just tangential to the convex surface of the mercury column as shown in Fig. 5.2(*b*). Then the main scale and the vernier scale readings are taken from which the height of the mercury column is ascertained. This gives the atmospheric pressure at that time.

Generally, the vernier constant of a barometer is 0·005 cm. The main scale reading of the barometer shown in the diagram 5.2(*b*) is 76·4 cm. and 12th division of the vernier is found to coincide with a mark in the main scale. Hence, the vernier reading is $12 \times 0{\cdot}005 = 0{\cdot}06$ cm. So, the barometer reading$= 76{\cdot}4 + 0{\cdot}06 = 76{\cdot}46$ cm. This gives the atmospheric pressure at the time of experiment.

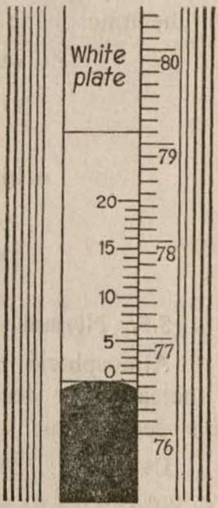


Fig. 5.2(*b*)

Advantages of using mercury in a barometer :

Several liquids, like water, glycerine, mercury etc. may be used in a barometer. Of them, mercury has the following advantages :

(*i*) The density of mercury being high, the column in the tube will be of moderate height.

(*ii*) Mercury may be available in a very pure state and its vapour exerts negligible pressure.

(*iii*) The changes of density and other physical properties of mercury due to a change in temperature are accurately known.

5.4. Magnitude of atmospheric pressure :

For ordinary laboratory purposes, atmospheric pressure is expressed in cm. of mercury. The average value of the barometric height over a long period is 76 cm. of mercury which means that the atmospheric pressure is equal to the weight per unit area of a column of mercury of height 76 cm.

(i) **Magnitude of atmospheric pressure in the C.G.S. system :** Taking the barometric height as 76 cm. at a place, the magnitude of the atmospheric pressure in the C.G.S. system may be calculated in the following way :

Atmospheric pressure, P=wt. per unit area of 76 cm. of Hg.

$=76\times13{\cdot}6$ gm-wt/sq. cm. [density of Hg=13·6 gm/c.c.]

$=76\times13{\cdot}6\times980$ dynes/sq. cm.

$=1{\cdot}013\times10^6$ dynes/sq. cm.

(ii) **Magnitude of atmospheric pressure in the F.P.S. system :** If the barometric height be taken as 76 cm. then in the F.P.S. system it is about 30 inches. So, the atmospheric pressure

P=wt. per unit area of 30 inches of Hg.

$$=30\times\frac{13{\cdot}6\times62{\cdot}5}{(12)^3}\text{ lb-wt/sq. inch.}$$

$$=14{\cdot}7\text{ lb-wt/sq. inch }\left[\text{Density of Hg}=\frac{13{\cdot}6\times62{\cdot}5}{(12)^3}\text{lb/cu. inch.}\right]$$

$=14{\cdot}7\times32$ poundals/sq. inch

$=470{\cdot}4$ poundals/sq. inch.

5.5. Normal or Standard atmospheric pressure :

Atmospheric pressure changes very often. To say whether the pressure is increasing or decreasing some specific value should be taken as the standard. This specific value is known as the normal or the standard atmospheric pressure.

Definition : *Standard or normal atmospheric pressure is defined as the pressure exerted by a column of mercury 76 cm. high at 0°C, 45° latitude and at sea-level.*

Taking the density of mercury at 0°C to be 13·596 gm/c.c. and the value of $g=980{\cdot}6$ cm/sec^2 at 45° latitude at sea-level, the normal atmospheric pressure$=76\times13{\cdot}596\times980{\cdot}6$ dynes/sq. cm.

$=1{\cdot}013\times10^6$ dynes/sq. cm.

Note that in the definition of the standard atmospheric pressure, mention has been made of a particular temperature, a particular latitude and sea-level. These are taken as standard because variation of any of these factors, severally or jointly causes a variation in the atmospheric pressure. For example, density of mercury changes with temperature and that is why 0°C is taken as the standard temperature. Similarly, the value of 'g' is different at different latitude but because the mean value of 'g' is obtained at 45° latitude, in the measurement of atmospheric pressure, 45° latitude is taken as the standard. Further, the standard of measurement of all heights being sea-level, the height of the mercury column representing the standard atmospheric pressure is taken from the sea-level.

Remarks : (i) Meteorologists express atmospheric pressure in bars and millibars.

1 bar$=10^6$ dynes/sq. cm.=1 mega dyne/sq. cm.

1 millibar$=\frac{10^6}{10^3}$ dynes/sq. cm.=1000 dynes/sq. cm.

According to this system, the standard atmospheric pressure is equal to 1·013 bars or 1013 millibars.

(ii) Pressure exerted by gases and liquids—specially when the pressures are considerable—are compared with atmospheric pressure and are usually expressed in terms of it. For example, if a gas or a liquid exerts a pressure of $1\cdot013 \times 10^6$ dynes/sq. cm., it is said that the gas or the liquid exerts a pressure of one atmosphere. In this way, pressure may be of two atmospheres, three atmospheres and so on. Hence,

$$1 \text{ atmospheric pressure} = 1\cdot013 \times 10^6 \text{ dynes/sq. cm.}$$
$$= 14\cdot7 \text{ lb/sq. inch.}$$

5.6. Height of liquid column in different barometers :

In barometer tube, different liquids may conveniently be used. Instead of mercury, sometimes water and glycerine are used as barometric liquids.

Suppose, in a barometer a liquid of density ρ gm/c.c. has been used. If h be the height of the liquid column at normal atmospheric pressure *i.e.* 76 cm. of mercury pressure, then,

pressure of liquid column = pressure of mercury column

$$\text{or} \quad h \times \rho = 76 \times 13\cdot6 \quad \text{or} \quad h = \frac{76 \times 13\cdot6}{\rho} \text{ cm.}$$

(i) In the case of *water barometer* $\rho = 1$ gm/c.c.

$$\therefore \quad h = 76 \times 13\cdot6 \text{ cm.} = 10\cdot336 \text{ metre} = 34 \text{ ft. (nearly)}$$

So, it may be said that atmospheric pressure can support a vertical column of water of height about 34 ft. or can force water to a vertical height of about 34 ft. (See Suction pump in art. 5.12).

(ii) In the case of *glycerine barometer*, $\rho = 1\cdot3$ gm/c.c.

$$\therefore \quad h = \frac{76 \times 13\cdot6}{1\cdot3} = 795 \text{ cm.} = 7\cdot95 \text{ metre (nearly)} = 27 \text{ ft. (nearly)}$$

5.7. Advantages and disadvantages of mercury, water and glycerine as barometric liquids :

(i) The heights of mercury, water and glycerine barometers are respectively, about 76 cm., 10 metres and 8 metres. The height of mercury barometer being comparatively small, it is convenient to handle. The atmospheric pressure can also be conveniently read in a mercury barometer.

(ii) As water and glycerine are not good conductors of heat like mercury, the temperature of water and glycerine barometers does not remain constant every where along the columns. For this reason, a mercury barometer gives more accurate reading than the other two.

(iii) Mercury does not stick to glass. For this reason, mercury can smoothly rise or fall along the barometer tube. But water and glycerine cannot do so because these two liquids wet the glass.

(iv) As mercury is shinning, the level of mercury becomes easily visible through glass. In the case of water and glycerine barometers, the liquids need be coloured with suitable dye.

(v) The boiling points of mercury, water and glycerine are respectively 357°C, 100°C and 290°C. At ordinary temperature, therefore, the torricellian vacuum of mercury barometer contains much less vapour. Hence much less vapour pressure is exerted on mercury column in a mercury barometer in comparison with the other two types of barometers. This enables the mercury barometer to give an accurate reading.

(vi) The freezing points of mercury, water and glycerine are −39°C, 0°C and 18°C respectively. For this reason a mercury barometer may conveniently be used in a cold country.

(vii) Water and glycerine being lighter than mercury, a slight change in atmospheric pressure will cause appreciable change in the heights of these two liquid columns compared to mercury. In other words, water and glycerine barometers are more sensitive than mercury barometer.

5.8. Height of the homogeneous atmosphere :

Considering the density of air to be uniform and same as that on the surface of the earth, the calculated theoretical height of the atmosphere which would exert the same pressure as the standard atmosphere is called the height of the homogeneous atmosphere. If this height be h cm. the weight of a column of air at N.T.P. (normal temperature and pressure) and of height h cm. and cross-section unity should be equal to the weight to 76 cm. high column of mercury of unit cross-section. Since the density of air at N.T.P. $= 1{\cdot}293\times10^{-3}$ gm/c.c.

we have, $h\times1{\cdot}293\times10^{-3}=76\times13{\cdot}59$

$$\text{or,}\quad h=\frac{76\times13{\cdot}59}{1{\cdot}293\times10^{-3}}\text{ cm.}=8\text{ kilometers (nearly)}$$

The atmosphere would, therefore, extend to the above height if the density of air remained uniform as at N.T.P., throughout this height. But we know air becomes thinner as we go up and traces of atmosphere are found even beyond 200 kilometers.

Examples : (1) *A good barometer reads 75 cm. On admitting 1 c.c. of air, the reading becomes 70 cm. Find the volume of the space above the mercury at the end. Cross-section of the barometer tube=1 sq. cm.*

Ans. Let the length of the space above mercury be x cm. after air is admitted. So, its volume $=x\times1$ c.c. The pressure exerted by air column on mercury $=(75-70)=5$ cm. of Hg.

The initial volume and pressure of air were 1 c.c. and 75 cm. of Hg respectively. According to Boyle's law, $x\times1\times5=75\times1$ or $x=15$ cm.

$\therefore$ the volume of the space above mercury $=15$ c.c.

(2) *On admitting an air bubble into a barometer tube of cross-section 1 sq. cm., the height of mercury column came down from 75 cm. to 65 cm. If the space above the mercury column be 6 cm. long before air was admitted, what would be the volume of air bubble at normal atmospheric pressure ?*

Ans. The diminution of height of mercury column $=75-65=10$ cm. So, the length of the space occupied by air $=(10+6)$ cm. $=16$ cm. ; the volume of the air $=16\times1=16$ c.c. and its pressure $=(75-65)=10$ cm. of Hg.

If the volume of air under normal atmospheric pressure (76 cm. of Hg) be x c.c., then according to Boyle's law, $x\times76=16\times10$

$$\therefore\quad x=\frac{16\times10}{76}=2{\cdot}105\text{ c.c.}$$

(3) *A tube, 6 ft. long and open at one end, is half filled with mercury. Closing the open end by thumb, the tube is inverted and then the open end is inserted into mercury kept in a reservoir. If the barometric height be 30 inches, what will the height of the mercury column in the vertical position of the tube ?*

Ans. Length of the tube = 6 ft. = 72 inches. Suppose the cross-section of the tube $=\alpha$. The volume of air occupying the half of the tube $=\frac{72}{2}.\alpha=36\ \alpha$ and its pressure = 30 inches of *Hg*.

When the tube is inverted, suppose the height of the mercury column $=h$ inches. In this condition, the length of the tube occupied by air $=(72-h)$ inches and its volume $=(72-h)\alpha$; the pressure exerted by the air $=(30-h)$ inches According to Boyle's law,

$$36.\alpha.30=(72-h).\alpha.(30-h)$$

or, $36\times30=(72-h)(30-h)$

,, $h^2-102\times h+1080=0$

or, $(h-90)(h-12)=0$

$\therefore\ h=90$ inches or 12 inches.

As the length of the tube is only 72 inches, the first solution is inadmissible. Hence the height of the mercury column in the tube = 12 inches = 1 ft.

(4) *A faulty barometer reads 28 inches and 30 inches when a true barometer reads 28·5 inches and 31 inches respectively. Find the reading of the true barometer when the faulty barometer reads 29 inches.* [*H. S. Exam. 1981*]

Ans. When the faulty barometer reads 28 inches, suppose the air column above the mercury is l inches long. The volume of the air $=l.\alpha$, where α = cross-section of the tube. The pressure of the air = true barometric height − faulty barometric height $=28{\cdot}5-28=0{\cdot}5$ inches of *Hg*.

Again, when the faulty barometer reads 30 inches, the length of air column above the mercury $=l-(30-28)=(l-2)$ inches. Its pressure $=31-30=1$ inch. Hence, from Boyle's law, we get, $0{\cdot}5\times l\times\alpha=1\times(l-2)\times\alpha$ or, $l=4$ inches.

Lastly, when the faulty barometer reads 29 inches, the length of air column above mercury $=l-(29-28)=(l-1)=4-1=3$ inches. If the barometric pressure be P inches, then the pressure of air column $=(P-29)$ inches.

$\therefore\ 0{\cdot}5\times l\times\alpha=3\alpha(P-29)$

or $0{\cdot}5\times4=3(P-29)$ $\therefore\ P=29\frac{2}{3}$ inches.

5.9. Weather forecasting ; effect of water-vapour on atmospheric pressure :

The proportion of moisture or water-vapour in the atmosphere does not always remain constant. It varies from place to place and also from time to time at a place. Now, water-vapour being lighter than air, an increase of proportion of water-vapour in a given mass of air reduces the density of the mixture and therefore, diminishes the atmospheric pressure. Thus, low barometric pressure indicates excess of moisture and forecasts rain.

A rapid fall of barometer indicates a partial vacuum in surrounding places and air from all around will rush in to fill the vacancy. This, therefore, forecasts a stormy weather.

If, on the other hand, mercury in the barometer is found to rise steadily, it indicates that water-vapour in the air is being replaced by dry air and a fine weather is in the offing.

In this way, a barometric observation may make weather forecasting in a limited way.

Various atmospheric conditions like atmospheric pressure, direction of wind, proportion of moisture etc. continually change. Meteorological office observes these changes with the help of various instruments and record the findings in a chart, known as the *weather chart*. For example, they denote the places of same atmospheric pressure by a continuous line which is known as '*isobar*'. Similarly, regions of low pressure are marked as '*cyclonic region*' and regions of high pressure as '*anticyclonic regions*'. It goes without saying that cyclonic and anticyclonic conditions are not permanent. They represent stormy and fine weathers respectively.

5.10. Determination of height by a barometer :

Barometric pressure decreases with altitude. Hence barometric readings at two stations will give the altitude between the stations.

Suppose, the altitude of a station B is H cm. from a station A. If h_1 cm. and h_2 cm. be the mercury heights of a barometer at the stations A and B respectively, then the pressure of (h_1-h_2) cm. of mercury will evidently be equal to the pressure H cm. of air column.

Let ρ and d be the average densities of mercury and air respectively. Then, the pressure of (h_1-h_2) cm. of mercury $=(h_1-h_2)\rho$ gm-wt. and the pressure of H cm. of air column $=H.d.$ gm-wt.

$$\therefore \quad H.d=(h_1-h_2)\rho \text{ or, } H=(h_1-h_2)\,\frac{\rho}{d}\text{ cm.}$$

Example : *The barometer readings at the top and bottom of a building are 29·905 inches and 29·949 inches respectively. If the densities of mercury and air are 13·53 and 0·001293 gm/c.c. what is the height of the building ?*

Ans. Since 1 inch$=2{\cdot}5$ cm., the difference between the barometric readings $=(29{\cdot}949-29{\cdot}905)\times 2{\cdot}5=0{\cdot}044\times 2{\cdot}5$ cm.

So, the pressure exerted by the mercury column$=0{\cdot}044\times 2{\cdot}5\times 13{\cdot}53$ gm-wt.,
and " " air column$=H\times 0{\cdot}001293$ gm-wt.

$$\therefore \quad H\times 0{\cdot}001293=0{\cdot}044\times 2{\cdot}5\times 13{\cdot}53$$

$$\text{or, } H=\frac{0{\cdot}044\times 2{\cdot}5\times 13{\cdot}53}{0{\cdot}001293}\text{ cm.}=1151\text{ cm.}=11{\cdot}51\text{ metre}$$

5.11. Measurement of gas pressure by a manometer :

Very often we require to measure the pressure of a certain amount of gas enclosed in a vessel. This can be done very conveniently by an open-mouthed U-tube. Such U-tubes are known as *manometers*. Ordinarily, water is taken in the U-tube for measuring gas-pressure. Since both the arms of the U-tube are open to atmosphere, the pressures on the levels of water in the two arms of the

U-tube are atmospheric and hence the heights of water columns will be equal [Fig. 5.3(*a*)]. In the figure the water levels at *A* and *C* are of same height.

Now the vessel whose gas-pressure is to be measured is connected with the arm *C* of the manometer by a piece of rubber tubing [Fig. 5.3(*b*)]. The gas will now exert pressure on the water level *C* and the level in the other arm will rise. As long as the pressure at the level *C* does not become equal to the pressure at the same horizontal level *B*, the water column on the left arm of the U-tube will go on rising. Suppose, in the steady state, the height of the water column *AB* above the horizontal level *BC* is *h* cm. Then,

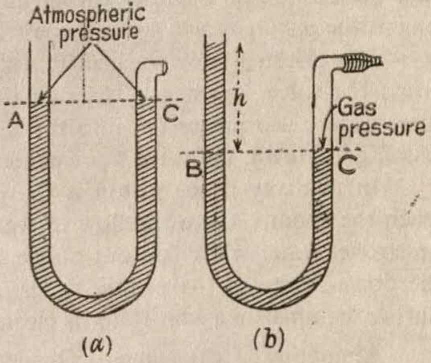


Fig. 5.3

Gas pressure=atmospheric pressure+pressure due to *h* cm. of water column.

In other words, the gas-pressure exceeds the atmospheric pressure by an amount equal to the pressure exerted by *h* cm. of water column or $h \times \rho$ gm-wt/cm^2 when ρ=density of water.

Usually, the excess pressure is expressed in terms of the height of water column, known as *head of water*. We, therefore, say, excess pressure of gas=*h* cm. of water.

For measuring high pressures, mercury is used in the manometer, while for low pressures, xylene (sp. gr=0·88) is more suitable than water.

5.12. Suction or Common pump :

Suction pump is used in a tube-well to draw water from underground levels.

Description : It consists of a cylindrical metal barrel *AB* with a side tube *E* near the top to act as a spout (Fig. 5.4). At the bottom of the barrel where it joins a pipe *CD* leading to the underground level or well, there is a valve V_1 which allows water to move up. A piston carrying a leather cup and fitted with a second valve V_2 is moved up and down inside the barrel by a handle.

Action : Figs. 5.4(a) and (b) illustrate the action of a suction pump.

Suppose, at the beginning, the piston is at the lowest position in the barrel and the two valves are closed. If the piston is now moved up, the air below the piston will expand and its pressure will fall. The valve V_1 will be forced open due to excess pressure of the air

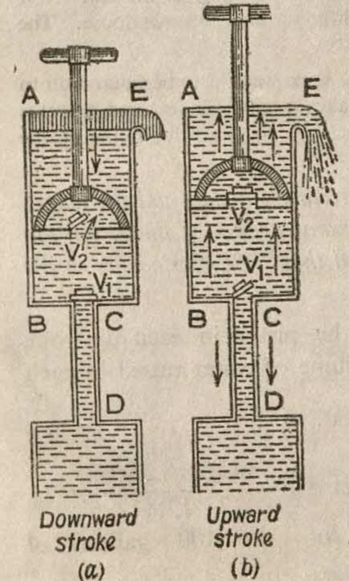


Fig. 5.4

below and some water will enter into the barrel. The valve V_2 will remain closed during this time because atmosphere exerts a greater downward pressure than the upward pressure by the air on it. In this way, water will be drawn in as long as the piston moves up and goes to the highest position in the barrel.

The piston is now pushed down. The air below it gets compressed and forces the valve V_2 open. Through the opening of the valve, air goes out and some water also passes up into the space above the piston. As long as the piston goes down, the valve V_1 remains closed due to its own weight.

In this way if the piston is moved up and down several times, water will reach the spout. Once the flow of water through the spout starts, then at every up-stroke, water will come out of the spout. It is to be remembered that during the down-stroke of the piston, water collects in the space above the piston and during the up-stroke, this water is ejected through the spout.

Limitation of the pump : Owing to the fact that the atmospheric pressure cannot support a column of water more than about 34 ft. long, it follows that 34 ft. is the maximum height to which water can be raised by a common pump. For this reason, the common pump will not draw water, if the height of the tube *CD* from the reservoir to the barrel is more than 34 ft. As a matter of fact, imperfect vacuum owing to bubbles from dissolved air forming near the top of the water column, restricts the limiting height only to 30 ft.

Notes : (1) Sometimes, in a tube-well, the pipe goes more than 34 ft, inside the earth. In that case, it is to be remembered that the underground layer of water is in communication with the rivers or lakes near by and water will come up to the level of those tanks due to the property that liquids find their own level. So, it is to be seen whether the length of the pipe from the ground level to the barrel is less than 34 ft.

(2) The piston disc moves along the barrel in an air-tight condition because it has a leather cup around it, known as 'washer'. The washer fits air-tight against the body of the barrel. If the pump is left unused for some time, the leather washer dries up and becomes loose. The pump then refuses to draw water.

(3) Sometimes it is seen that to start the pump working, some water is to be poured on to the top of the piston. This is known as '*primming*' which makes a good air seal and prevents leakage of air past the piston during the first few strokes which are necessary for filling up the barrel with water.

Example : *The barrel of a suction pump has a radius 4 inches and the length of the upward stroke of the piston is 8 inches. How many strokes of the piston are necessary to lift 100 gallons of water, it being assumed that 10% slip occurs in the pump. 1 cu. ft. of water=6·25 gallon.*

Ans. The volume of the barrel swept out by the piston in each upstroke $=\pi(2)^2\times 8$ cu. inch. Since 10% slip occurs, the volume of water raised in each upstroke$=\pi(2)^2\times 8\times 0{\cdot}9$ cu. inch$=\dfrac{\pi(2)^2\times 8\times 0{\cdot}9}{12\times 12\times 12}$cu. ft.

So, the volume of water, in gallon raised in each upstroke$=\dfrac{\pi(2)^2\times 8\times 0{\cdot}9}{12\times 12\times 12}\times 6{\cdot}25$.

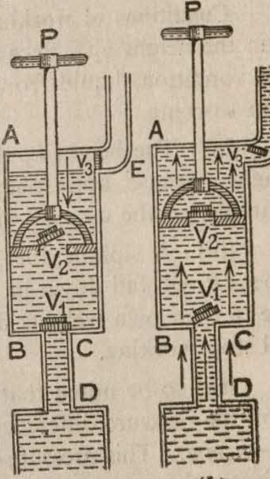
Hence, the number of strokes required to raise 100 gallons of water$=\dfrac{12\times 12\times 12\times 100}{\pi(2)^2\times 8\times 0{\cdot}9\times 6{\cdot}25}=305$ (nearly).

5.13. Lift pump :

When water is required to be lifted to any great height as is required in a tall house, this form of pump is very useful. This pump is very much like the suction pump with the difference that instead of the spout, a long delivery pipe E is led from the top of the barrel. There is a third valve V_3 at the junction of the delivery pipe and the barrel, which opens outward [Fig. 5.5].

Action : A few up and down strokes of the piston will draw water up to the level of the delivery tube E. Next upstroke of the piston will cause the accumulated water in the barrel to force open the valve V_3 and water will move up along the delivery tube. Hence by continuing the process, more water can be made to enter the delivery tube and can be lifted to any desired height.

It is to be noted that there is no limit theoretically of the height to which water can be raised by this pump because the atmospheric pressure plays no part here. Only precaution to be taken is that the delivery tube and the valve V_3 should be stout enough to bear the weight of the column of water to be lifted. In an electric pump, the piston is worked by electric power.



(a) DOWNWARD STROKE

(b) UPWARD STROKE

Fig. 5.5

5.14. The siphon :

Most people are familiar with the use of a siphon for transferring water from one vessel to another which can not be done otherwise conveniently or without disturbing the sediments in the water.

Description and action : The siphon is a bent tube made of glass, rubber or plastic tubing, in the form of inverted U. One arm of the siphon is longer than the other. Its shorter arm is dipped into the liquid to be transferred and the longer one in the vessel to be filled up. To start the siphon it must first be filled up with the liquid. After this the liquid will continue to run out so along as the end D is below the level of the liquid in the vessel V_1 [Fig. 5.6].

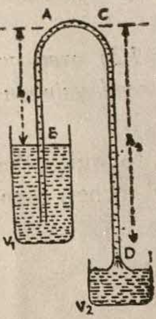


Fig. 5.6

Explanation of the action : Take two points A and C inside the liquid on the same horizontal level.

Pressure at A=Atmospheric pressure−the pressure of the liquid column $AB=P-h_1.d.g$. [P=atmospheric pressure ; d=density of the liquid ; h_1=height of the point A from the liquid level in the vessel V_1].

Similary, the pressure at $C=P-h_2.d.g$

Since, $h_1<h_2$, $(P-h_1d.g)>(P-h_2.d.g)$

i.e. the pressure at A is greater than that at C. So the liquid will move from

A to C and finally into the vessel V_2. But as soon as liquid moves from A, the atmospheric pressure on the liquid surface in the vessel V_1 will push more liquid through the shorter arm to A. In this way, liquid will continue to run out from the vessel V_1 to the vessel V_2.

[*Note* : If there be any hole in the limb CD at a point higher than the level of liquid in the vessel V_1 the siphon will not work because air will enter into the tube through the hole and will exert pressure on the liquid. If, however, the hole is at any point below the level of liquid in the vessel V_1, the siphon will work but not so fast as before.]

Conditions of working of a siphon : (i) The height h_1 must be always less than the height h_2. Because, when $h_1=h_2$, the pressure at A=pressure at C. In this condition, liquid from A will not move to the point C, and the siphon will stop working.

(ii) The height h_1 should be less than the corresponding liquid barometric height because atmospheric pressure pushes the liquid to the point A. For example, in the case of water, h_1 should be less than 34 ft.

(iii) A siphon does not operate in a vacuum. The liquid column in the tube AB will fall down in the vessel V_1 and that in the tube CD in the vessel V_2 due to their own weights and no more liquid will be pushed up. So, the siphon will stop working.

It is to be noted that the rate of flow of liquid through the siphon depends upon the pressure difference between A and C which is $(P-h_1.d.g)-(P-h_2.d.g)=(h_2-h_1)d.g$. This pressure-difference does not include atmospheric pressure which indicates that *the rate of flow of liquid is not dependent on atmospheric pressure*.

[**N.B.** Experiment shows that siphons can be made to work in a vacuum and that in certain cases, the siphon action will continue even if h_1 is somewhat greater than the corresponding liquid barometric height. The presence of atmospheric pressure, therefore, does not appear to be essential. According to a modern explanation, greater weight of the column CD pulls the shorter column AB through the cohesion of the liquid molecules and thus maintains the flow of liquid. The action is similar to that of a chain passing over a freely running pulley, which is found to run off in the direction of the longer side.

Only pure liquids will, however, siphon satisfactorily in a vacuum. Presence of dissolved gases appreciably lowers the cohesive force between the molecules and bubbles readily form under reduced pressure. In that case, atmospheric pressure is a necessary condition for maintaining the siphon action.]

Examples : (1) *It is required to siphon a liquid (sp. gr.=1·2) over an obstacle. What must be the limiting height of the obstacle which will render siphoning just possible ? Atmospheric pressure=30 inches of mercury .*

Ans. The limiting height of the obstacle is the same as that to which the liquid will be raised by atmospheric pressure. If this height be 'h' inches, then

$$h\times 1{\cdot}2\times 62{\cdot}5\times g=30\times 13{\cdot}6\times 62{\cdot}5\times g$$

$$\therefore\ h=\frac{30\times 13{\cdot}6}{1{\cdot}2}=340 \text{ inches}=28 \text{ ft. } 4 \text{ inches.}$$

(2) *A siphon is used to transfer mercury from a cylindrical vessel which is 45 inches deep and is full of mercury. The smaller arm of the siphon extends up to the bottom of the vessel. It is found that after some time, the siphon ceases to work.*

Find the volume of the vessel occupied by the mercury left behind, if the barometric height be 30 inches of mercury.

Ans. Barometric height is 30 inches of mercury—this means that atmospheric pressure can raise mercury to a height of 30 inches and no more. So, the siphon will work till the level of mercury in the vessel drops to 30 inches from the bend of the siphon. The height of mercury column left over in the vessel is, therefore $=45-30=15$ inches.

Now, the volume of the vessel $=\pi r^2\times 45$ cu. inches ($r=$radius of the vessel)
and ,, ,, ,, ,, mercury left $=\pi r^2\times 15$ cu. inches

$$\therefore \frac{\text{volume of mercury left}}{\text{,, \quad ,, the vessel}}=\frac{\pi r^2\times 15}{\pi r^2\times 45}=\tfrac{1}{3}.$$

(3) *The internal radius of the two arms of a siphon is 1 inch and their heights are 20 inches and 14 inches respectively. The shorter arm dips 6 inches in a liquid. Find the rate of flow of the liquid and also the volume of liquid coming out per second from the siphon.*

Ans. The rate of flow of liquid through a siphon depends on the difference of height (h_2-h_1) [See Fig. 5.6]. Following the laws of falling bodies, the velocity of liquid flow $v=\sqrt{2g(h_2-h_1)}$.

Here, $h_2=20$ inches and $h_1=$the length of shorter arm above the liquid level $=14-6=8$ inches. $\therefore\ h_2-h_1=(20-8)=12$ inches $=1$ ft.

So, $v=\sqrt{2\times 32\times 1}=8$ ft./s. The volume of liquid flowing out of siphon per second $=$rate of flow$\times$area of cross-section of the tube $=8\times\pi(\frac{1}{12})^2$ cu. ft $=0{\cdot}174$ cu. ft.

Application of siphon :

Automatic flush: The intermittent flush system in public lavatories is one of the applications of siphon. A section of such an automatic flush is shown in the fig. 5.7.

B is a reservoir which is fixed to the wall of the lavatory a little below the roof. A pipe, known as the flush pipe or tube is provided to the reservoir. There is a movable lid *A* to which is attached a chain (not shown in the diagram). Ordinarily, the lid prevents the liquid from reaching the top of the flush tube but as soon as the chain is pulled, the lid moves up and the liquid siphons out with great force through the flush tube. The gush of liquid prevents the lid from falling down. There is a heavy metallic ball at the end of a lever arm attached to the reservoir. As water accumulates in the reservoir, the ball floats up and controls a valve at the other end of the lever arm. When the water in the reservoir reaches a specified level, the valve closes and stops the flow of water in the reservoir. The reservoir will be emptied again if the flush chain is pulled. The valve opened

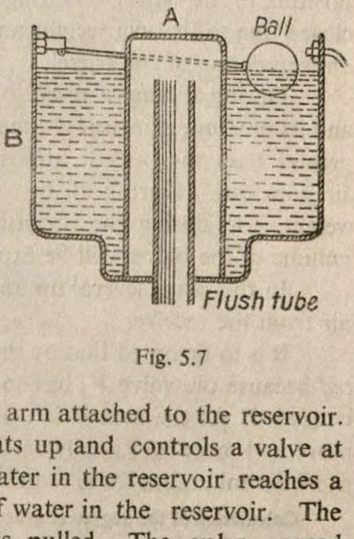


Fig. 5.7

by the fall of level arm, will readmit water in the reservoir. In this way the whole arrangement goes on working automatically.

5.15. The exhaust pump or the air pump :

This pump is used to create a vacuum in a given enclosure and was invented by Otto Von Guericke in 1650.

Description : It consists of a metallic cylinder *AB*, called the barrel of the pump through which an air-tight piston *P* can move up and down [Fig. 5.8]. *CD* is a circular plate with an opening at its centre. It is called the disc of the pump. The barrel *AB* is joined to the disc *CD* by a rubber tubing as shown in the diagram. There is a glass cover (*R*) on the disc, called the *receiver* of the pump. The space in the receiver is to be evacuated by the pump. The joint between the receiver and the disc is made air-tight by vaseline. There is a valve V_2 at the opening of the barrel and another V_1 at the piston *P*. Both the valves open outwards *i.e.* they only permit upward motion of the air.

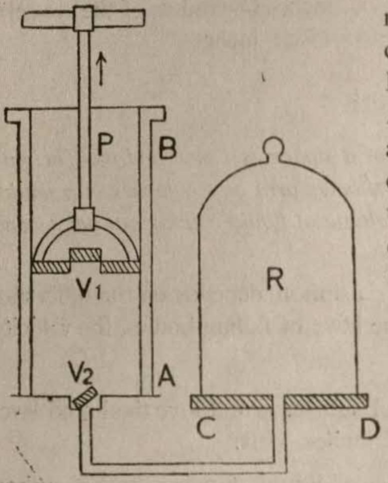


Fig. 5.8

Action : When the piston is drawn upward from the lowest position in the barrel, a partial vacuum is produced below it and the pressure there becomes less than the atmospheric pressure outside. As a result, the air initially present in the receiver (at atmospheric pressure) opens the valve V_2 and enters into the barrel of the pump. Entry of air from the receiver will continue till the piston goes to the highest position in the barrel. During this *upstroke* of the piston, the valve V_1 remains closed due to its own weight and the air in the receiver expands and occupies the entire volume of the barrel.

During the *down-stroke* of the piston, the air in the barrel gets compressed and its pressure increases gradually until the pressure of the compressed air becomes greater than the outside atmospheric pressure when the valve V_1 opens and the air goes out. During all this time, the valve V_2 remains closed due to its own weight. So during the downstroke of the piston a volume of air equal to the volume of the barrel will be expelled from the receiver.

In this way, several up and down strokes of the piston will expel most of the air from the receiver.

It is to be noted that by this pump the receiver *R* *cannot be completely evacuated* because the valve V_2 has some weight and to open it a certain minimum force is needed. As the evacuation of the receiver *R* continues, the air becomes rarefied more and more and finally small traces of air are left in the receiver which cannot exert the minimum force required to open the valve V_2.

Calculation of the degree of evacuation : The degree of evacuation is measured by the density or the pressure of the air left in the receiver after the piston makes '*n*' complete storkes (one complete stroke meaning one up and one down stroke). It can be calculated in the following way :

Let, the volume of the barrel $AB = v$.
the volume of the receiver and the rubber tubing $= V$.
initial pressure of the air in the receiver $= P$
" density " " " " " $= D$

When the piston moves from the lowest to the highest position in the barrel, the volume (V) of air occupying the receiver and the rubber tubing expands and fills up the barrel too and its volume becomes $(V+v)$. Consequently, the pressure and density of the air will be lowered. If the new density and pressure be D_1 and P_1 respectively, then from the gas laws, we can write,

$$VD=(V+v)D_1 \quad \text{or} \quad D_1=\left(\frac{V}{V+v}\right)D \quad \ldots \ldots \text{(i)}$$

$$\text{Also, } P.V=P_1(V+v) \quad \text{or} \quad P_1=\left(\frac{V}{V+v}\right)P \quad \ldots \ldots \text{(ii)}$$

During the down-storke of the piston, the air of density D_1 and pressure P_1 occupying the volume of the barrel will be expelled and only the air in the receiver and the rubber tubing will be left over. During the next upstroke, this air will again expand and will occupy a volume $(V+v)$, thereby reducing its density and pressure further. If this new pressure and density be P_2 and D_2 respectively, then we can write as before, $VD_1=(V+v)D_2$

$$\text{or,} \quad D_2=\left(\frac{V}{V+v}\right)D_1=\left(\frac{V}{V+v}\right)^2 D. \quad \text{[from eqn., (i)]}$$

Also, $P_1V=P_2(V+v)$

$$\text{Or} \quad P_2=\left(\frac{V}{V+v}\right).P_1=\left(\frac{V}{V+v}\right)^2.P \quad \text{[from eqn. (ii)]}$$

In this way, if the density and the pressure of air in the receiver be D_n and P_n respectively after 'n' complete strokes of the piston, then,

$$D_n=\left(\frac{V}{V+v}\right)^n.D \quad \text{and} \quad P_n=\left(\frac{V}{V+v}\right)^n.P.$$

It is clear that the value of $\left(\frac{V}{V+v}\right)^n$ can never be zero and hence the pressure or the density can never be zero. This means that the receiver can not be completely evacuated.

Example : *The volume of the receiver of an air exhaust pump is 1·5 litre and the pressure there is 760 mm. If the volume of the barrel be 500 c.c., what will be the pressure in the receiver after 3 strokes ?*

Ans. Volume of the receiver $= V$; Volume of the barrel $= v$ and no. of strokes $= n$; The final pressure $P_n=\left(\frac{V}{V+v}\right)^n.P$ where P is the initial pressure.

Here $V=1·5$ litre ; $v=500$ c.c. $=0·5$ litre ; $n=3$. Hence,

$$P_n=\left(\frac{1·5}{1·5+·5}\right)^3\times 760=(0·75)^3\times 760=320·6 \text{ mm. of } Hg.$$

5.16. Rotary pump :

This pump was invented by Gaede. It can produce high degree of vacuum. Fig. 5.9 shows a section of the pump.

It consists of a stout cylinder made of steel and is known as the *stator*. Inside it, a solid cylinder, known as the *rotor* can revolve eccentrically. A partition, V, known as *vane* always presses against the rotor by spring action, which separates the air or the gas already present in the cylinder from the air or the gas which enters the cylinder subsequently. There is an entrance tube I which connects the cylinder C with the vessel to be evacuated. There is also an exit tube (O) attached to the stator. There is a valve arrangement, opening outward, at the junction of the exit tube and the stator. The whole arrangement is kept immersed in an oil in order to prevent leakage. A special type of valve prevents entry of oil in the evacuated vessel when the pump stops working. If the rotor is whirled in the direction of the arrow with a high speed by a separate electric motor, the air from the vessel to be evacuated is readily withdrawn.

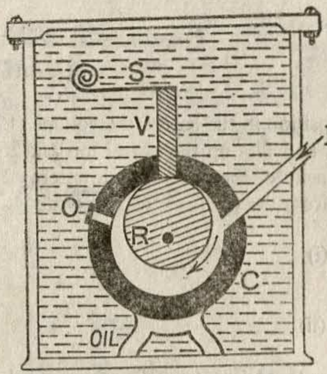


Fig. 5.9

5.17. Air condensing or compression pump :

By this pump, an enclosure may be filled up by air or any other gas. Its function, therefore, is exactly opposite to the function of an exhaust pump.

Description : The description of this pump is exactly the same as that of an exhaust pump with the only exception that the valves open in the opposite direction *i.e.* they allow air to go to the receiver and not in the opposite direction.

Action : Fig. 5.10 shows the section of an air compresion pump. When the piston P during upstroke moves from B to A, the valve V_2 opens because the atmospheric pressure is greater than the pressure of air in the barrel. As a result, air from outside enters into the barrel through the opening of the piston P. All along this time, the valve V_1 remains closed due to its own weight. If now the piston P is pushed down, the air below it will be compressed. The pressure of the compressed air will gradually increase as the piston goes down, until the compressed air forces the valve V_1 open and passes into the receiver R, which is connected to the pump by a rubber tubing.

During this time, the valve V_2 remains closed for obvious reasons.

If, in this way, the piston is moved up and down several times the receiver R will be filled up by air. When the desired compression is attained, the receiver is detached from the pump after closing its mouth by a key.

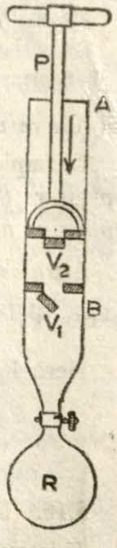


Fig. 5.10

The football inflator, the bicycle pump, the pump used in a stove are examples of air compression pump.

Calculation of the degree of compression : The degree of compression is measured by the density or the pressure of air occupying the volume of the receiver after 'n' complete strokes of the piston are over. As before, suppose,

the volume of the cylinder from A to $B=v$

and the volume of the receiver R together with the rubber tubing $=V$

Initial density of air in the receiver $=D$

,, pressure ,, ,, ,, ,, ,, $=P$

Now, when the piston goes from the lowest to the highest position in the barrel, outside air of density D fills up the barrel. The volume and mass of this air are v and vD respectively. During the next downstroke, this mass of air enters the receiver and its volume, then, will be V. The mass of air initially present in the receiver $=V.D$. So, the total mass of air in the receiver after one complete stroke of the piston $=vD+V.D$.

So, after n strokes the mass of air that will be collected in the receiver $=n.v.D+V.D$. Its volume, evidently, is equal to the volume of the receiver $=V$. So, the density D_n of the air is given by $D_n.V=n.v.D+V.D=(nv+V)D$

$$\therefore \quad D_n=\left(\frac{nv+V}{V}\right)D=\left(1+n\frac{v}{V}\right)D$$

Since density and pressure are proportional to each other, if P_n be the final pressure and P the initial pressure of air in the receiver, we can write, $P_n=\left(1+n.\frac{v}{V}\right)P$.

Examples : (1) *A vessel is being filled up by air with help of an air-compression pump. The volume of the barrel of the pump is $\frac{1}{20}$ of the volume of the vessel. How many strokes will make the pressure in the vessel double than before ?*

Ans. If v = the volume of the barrel and V = the volume of the vessel, then the final pressure P_n is given by, $P_n=\left(1+\frac{n.v}{V}\right).P$ [n = No. of strokes]

Here, $P_n=2P$ and $v=\frac{V}{20}$. $\therefore \quad 2P=\left(1+n.\frac{1}{20}\right)P$ or $n=20$.

(2) *The volume of an air-compression pump is 0·2 litre. The pressure of air in a 20 litre vessel is equal to the atmospheric pressure. If the pump be worked for 50 strokes, what will be the pressure in the vessel in terms of atmospheric pressure ?*

Ans. We know, $P_n=\left(1+\frac{n.v}{V}\right)P$. Here, volume of the barrel $(v)=0{\cdot}2$ litre ; volume of the vessel $(V)=20$ litre ; n = No. of strokes $=50$.

$$\therefore \quad P_n=\left(1+\frac{50\times 0{\cdot}2}{20}\right)P.=\frac{3}{2}.P.$$

So, the final pressure in the vessel is $\frac{3}{2}$ times the atmospheric pressure.

5.18. Applications of air compression pump :

(i) **Primus stove :** *A* is an oil reservoir, part of which is filled up by kerosene oil [Fig. 5.11]. An air-compression pump *P* is attached with the reservoir, *K* is an air-tight tap. When the tap is closed, the air becomes entrapped in the reservoir. With the working of the compression pump, the air-pressure increases which forces the oil up along a tube *N* till it reaches a jet tube *j*. There is a wire-gauge coiled in the jet tube which is pre-heated. Kerosene oil, reaching the jet tube, gets evaporated. This heated kerosene vapour then, arrives at the burner *B* and burns. If the stove is required to be put out, the tap *K* is opened. This diminishes the air pressure in the reservoir and no more oil goes up the tube *N*. The stove then slowly extinguishes.

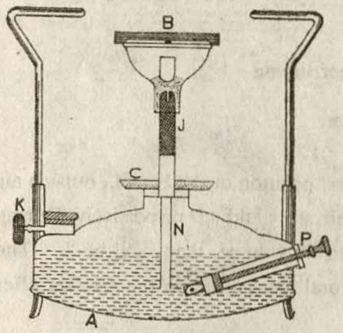
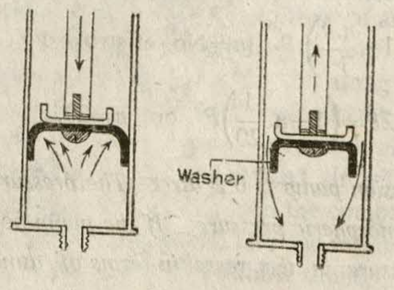


Fig. 5.11

(ii) **Bicycle pump :** The bicycle pump is an illustration of air-compression pump. It has a cup-shaped leather washer attached to the piston-head which acts as a valve. During the up-stroke of the piston, air passes through the gap between the washer and the wall of the barrel [Fig. 5.12 (*ii*)]. During the down stroke, the air is compressed and the compressed air presses the washer tightly against the barrel, closing the gap completely and thereby preventing the leakage of air [Fig. 5.12(*i*)]. When the air in the barrel is sufficiently compressed, it forces open the valve in the tube. This valve consists of a narrow metal tube with a small hole in the side covered by a thin rubber tube [Fig. 5.13]. This arrangement allows air to flow only in one direction.



(i) Fig. 5.12 (ii)

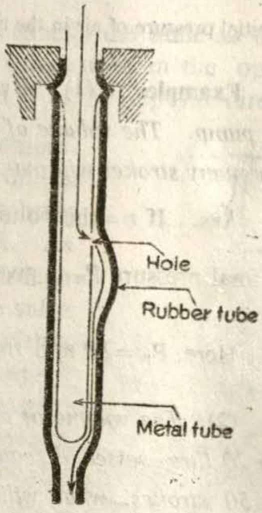


Fig. 5.13

(iii) **Soda-water machine :** It is an example of compression pump. Carbon dioxide gas is compressed into the soda water bottle containing water. The gas mixes with water and the high pressure inside the bottle forces the glass ball inside it to close the mouth tightly.

Example : *A bicycle pump is used to pump air into the tyre of a bicycle. The volume of the tyre is 2 litres. If the cross sectional area of the pump be 5 sq. cm. and the length of each stroke of the piston be 20 cm., what will be the air pressure in the tyre after 40 strokes ? Initial pressure of air in the tyre was 75 cm. of Hg.*

Ans. We know, $P_n=\left(1+n\frac{v}{V}\right)P$

Here, P=initial pressure=75 cm. of Hg ; n=number of strokes=40 ;

v=volume of the pump=5×20 c.c. ; V=volume of tyre =2 litres=2000 c.c.

Hence, $P_n=\left(1+\frac{40\times 5\times 20}{2000}\right)\times 75$ cm. of Hg=225 cm. of Hg.

Exercises

Essay type :

1. "The atmosphere exerts pressure"—explain the statement with suitable experiments [*H. S. Exam 1978*]. What is the similarity between air pressure and liquid pressure ?

2. Describe Torricelli's experiment. How can atmospheric pressure be measured by this experiment ?

3. What is a barometer ? Describe and explain the action of a Fortin's barometer. If the height of water barometer be 32 ft., what should be the height of a glycerine barometer. Sp. gravity of glycerine=1·25. [Ans. 25·6 ft.]

4. (*a*) What do you mean by standard atmospheric pressure ? (*b*) Describe the construction and action of a Fortin's barometer. (*c*) The height of mercury column of a barometer is found to be 76 cm. at a place. Find the atmospheric pressure in c.g.s. system at that time. g= 980 cm/s^2 at that place.

5. What do you mean by the height of homogeneous atmosphere ? What is its magnitude ?

6. How can weather forecasting be made from barometric observation ? Mention the advantages and disadvantages of water barometer and mercury barometer. What are the advantages of using mercury as a barometric substance ?

7. Describe a suction pump. Why is it that this pump cannot draw water more than 30 ft. high ? What is primming ? Why is it necessary ?

8. Describe how a lift pump works. Explain, with a neat diagram, its different parts. How far can this pump raise water ?

9. Mention the defects of suction pump used for drawing water . Explain the action, with a diagram, of a pump which can raise water to a height greater than the height of a water-barometer. [*H. S. Exam 1980*]

10. What is a siphon ? Explain its action. What are the conditions of its working ? [*H. S. Exam 1978*]

11. What is an exhaust pump ? Describe it and explain its action. What is its limitation ?

12. Explain the action of an air-condensing pump. Mention some of its applications.

13. Explain how a bicycle pump acts as an air-compression pump. What is the function of the cup-shaped leather washer ?

Short answer type :

14. What is Torricellian vacuum ? Is it perfectly vacuum ? Explain with reasons what happens in the following cases :—(i) A glass tube 50 inches long and closed at one end, is filled with mercury and then inverted over a trough of mercury with the open end dipping into the mercury of the trough, in a vertical position (ii) The glass tube is gradually inclined to the vertical (iii) A wider tube is taken.

15. "Atmosphere exerts a pressure of about 15 lb per sq. inch"—explain the statement clearly.

16. "The atmospheric pressure at a place is 760 mm. of mercury"—What do you understand by this statement ? *[H. S. Exam 1978]*

17. "The standard atmospheric pressure is equal to the pressure due to 76 cm. of mercury at 0°C, 45° latitude and mean sea-level." Briefly explain why it is necessary to mention the temperature, latitude, and height relative to sea-level in the above definition.

18. Some air is believed to have entered into the torricellian vacuum of a barometer. How can you be sure of that ?

19. What will you conclude if you observe (i) that the height of mercury in the barometer rises steadily (ii) that the height of mercury in the barometer drops suddenly ? *[H. S. Exam 1980]*

20. A piece of iron, weighing 1 kg and a piece of ordinary wood, also weighing 1 kg, are removed from air to vacuum. Will there be any difference in their weights ? Give reasons in support of your answer.

21. Explain why water collected at the bottom of a floating boat cannot be siphoned out by the side of the boat ?

22. If the barometric pressure is increased, will the rate of flow of liquid through a siphon increase ? *[H. S. Exam 1982]*

23. Answer the following questions :—(i) Sometimes a suction pump does not work properly. But when some water is poured on to the top of the piston, the pump starts working. Why ?

(ii) Are the limitations of the suction pump applicable to a lift pump ?

(iii) What happens when there is a small hole in one of the arms of a siphon ?

(iv) Can an enclosure be completely evacuated by an exhaust pump ?

(v) Can a siphon transfer a heavy liquid ?

(vi) What modifications will render an air exhaust pump into a bicycle pump ?

(vii) Is siphon action possible on the surface of the moon ?

Objective type :

24. Fill up the blanks in the following cases, selecting suitable words from the right-hand column :

(i) Torricelli's experimental arrangement suitably modified gives us a —.

(ii) Of all the liquids used in a barometer, — is the best.

(iii) A rapid fall of barometer indicates a —.

(iv) A tube-well uses a — pump.

(v) The piston disc in a suction pump moves along the barrel in an air-tight condition because it has a leather —.

(vi) The rate of flow of liquid through a siphon does not depend on —.

(vii) A bicycle pump is an example of air — pump.

(i) suction

(ii) barometer

(iii) compression

(iv) mercury

(v) washer

(vi) storm

(vii) atmospheric pressure.

Numerical problems :

25. Find the magnitude of the atmospheric pressure in the C.G.S. system. 'g' at the place=980 C.G.S. unit and the density of mercury=13·6 gm/c.c. *[H. S. Exam. 1961]*

26. If the height of water barometer be 32 ft., what will be the height of glycerine barometer ? Sp. gr. of glycerine=1·25. [Ans. 25·6 ft.]

27. The volume of a certain quantity of air is 250 c.c. when the mercury column in the barometer is 75 cm. high. Next day the volume of the air becomes 260 c.c. What is the barometer height then ? [Ans. 72·11 cm.]

28. The height of a barometer is 30 inches and the torricellian vacuum above mercury column is 1 inch long. A quantity of air which occupies 1 inch of the barometer tube under atmospheric pressure is introduced into the barometer. What will be the height of mercury column in the barometer now ? [Ans. 25 inches]

29. When a faulty barometer reads 28·5 inches and 29·5 inches, a true barometer reads 29·5 inches and 30·7 inches respectively. What is the true barometric height when the faulty barometer reads 29·9 inches ? [Ans 31·2 inches]

30. A glass tube of uniform bore, with one end open, is half filled with mercury. Keeping the open end closed, the tube is inverted and held vertically with the open end dipping into mercury kept in a reservoir. The height of the mercury column in the tube is 1 ft. If the barometric height at that time is 30 inches find the length of the tube. [Ans. 6 ft]

[*Hints* : The volume of air $=\frac{l}{2}.\alpha$ cu. ft. and pressure=30 inches. When inverted, volume $=(l-1)\alpha$ cu. ft. and pressure=(30 − 12)=18 inches

$$\therefore \quad \frac{l}{2}.\alpha\times 30=(l-1)\alpha\times 18 \quad \therefore \quad l=6 \text{ ft.}]$$

31. Kerosene of sp. gravity=0·8 is to be transferred from a tank 30 metres high to another vessel by siphon arrangement. The siphon is kept hanging with its bend resting against the edge of the tank. After sometime, the siphoning is found to have stopped although the smaller arm of the siphon is still dipping in the kerosene. Explain it and find the level of the kerosene left in the tank. The barometric height corresponds to 11 metres of water column. [Ans. 16·25 metres]

32. It is required to siphon kerosene (sp. gr.=0·8) over an obstacle . What must be the limiting height of the obstacle which will render siphoning just possible ? Atmospheric pressure=30 inches of mercury. [Ans. 42·5 ft.]

33. A cylinder, 1 metre long, has an area of cross-section of 30 sq. cm. 85 cm. length of the cylinder is full of mercury. To bring out mercury from the cylinder, a siphon is kept hanging with its bend resting against the edge of the cylinder. What volume of mercury will be siphoned out ? Atmospheric pressure=75 cm. of *Hg*. [Ans. 15 litre]

34. The volume of the receiver of an air pump is six times that of the barrel. Find the number of strokes of piston required to reduce the density of air to $\frac{216}{343}$ of the original value.

[Ans. 3]

35. The volume of the receiver of a condensing pump is 20 times that of the barrel. Find after how many strokes of the piston the pressure of the air inside the receiver will be increased from one to three atmospheres. [Ans. 40]

36. The volume of a bicycle tube is 100 cu. inches and that of its barrel is 10 cu. inches. How many strokes will be necessary to make the pressure of the tube double that of atmosphere ? It may be assumed that when inflated, the volume of the tyre increases by $\frac{1}{10}$. [Ans. 11]

Harder problems :

37. A barometer contains some air above the mercury whose height is 70 cm. The top of the barometer is 80 cm. above the mercury level in the cistern and the atmospheric pressure is 75 cm. of *Hg*. What is the atmospheric pressure when this barometer reads 67·5 cm. ? [Ans. 71·5 cm.]

38. An ideal gas is trapped between a mercury column and the closed lower end of a narrow vertical tube of uniform bore [Fig 5.14]. The upper end of the tube is open to atmosphere. (Atmospheric pressure$=76$ cm of *Hg*). The lengths of the mercury and the trapped gas columns are 20 cm and 43 cm respectively. What will be the length of the gas column when the tube is tilted slowly in a vertical plane through an angle of 60° ? Assume contant temperature all through. [*I.I.T. 1972*] [Ans. 48 cm]

[**Hints** : When vertical, the pressure on the gas$=(76+20)$ cm. and volume of the gas$=43.\alpha$. When inclined, the pressure$=76+20.\cos 60°$ and volume$=x.\alpha$.]

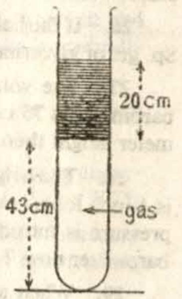


Fig. 5.14

39. A horizontal metal cylinder closed at one end is fitted with an air-tight but smoothly movable piston. The enclosed volume contains air and a few drops of water. Find the pressure inside if the piston is slowly withdrawn to double the enclosed volume, assuming that a few drops of water are still left in the cylinder. Neglect the volume occupied by the water drops. Room temperature$=30°C$; atmospheric pressure$=76$ cm of *Hg* ; saturated vapour pressure of water at $30°C=31{\cdot}83$ mm of *Hg*. [*I.I.T. 1968*] [Ans. 39·59 cm of *Hg*]

40. At what depth in a tank the pressure of water will be twice the atmospheric pressure ? [Ans. 10·2 metres]

41. The two arms of a siphon are respectively 20 inches and 14 inches long. The siphon tube has a uniform bore of internal radius $3\sqrt{\pi}$. The shorter arm is dipped 6 inches in a liquid. What volume of the liquid will come out of the siphon per second ?

[*Jt. Entrance 1972*] [Ans. 0·5 cu. ft.]

42. The cross-sectional areas of two vessels *A* and *B* are respectively 75 sq. cm. and 150 sq. cm. The vessel *A* is 4 cm. higher than the vessel *B*. 1·5 litres of water are poured in the vessel *A* and 0.35 litres in *B*. What is the maximum amount of water that can be siphoned out from the vessel *A* to the vessel *B* ? [Ans. 0·86 litre]

43. A rubber bladder is in a completely deflated state. When the air pressure in the bladder becomes equal to and double than the atmospheric pressure, its volume becomes 1000 c.c. and 1200 c.c. respectively. An air compression pump, whose barrel has an effective volume of 200 c.c. is connected to the bladder. How many strokes of the pump will be necessary to make the air-pressure in the bladder equal to and double than the atmospheric pressure ? [Ans. 5 ; 12]

44. An exhaust pump has a barrel of volume 100 c.c. and the volume of the receiver is 500 c.c. If the initial pressure of the air in the receiver is 76 cm. of mercury, calculate its value after 10 complete strokes. [*Jt. Entrance 1978*] [Ans. 12.16 cm. of *Hg*]

6

SURFACE TENSION AND VISCOSITY*

6.1. Surface tension :

The cohesive forces between molecules of a liquid endow the liquid surface with a property known as *surface tension.* In manifestation of this property a liquid surface always behaves like a stretched elastic membrane and tends to contract to the smallest possible area. Many common examples may be cited in favour of this property of water.

A little water spilled on a very clean plate wets it and spreads over the surface because the plate attracts the water molecules strongly. But in a dirty dish even the thinnest film of grease is enough to weaken the attraction of the dish and the cohesion of the water molecules makes them collect in separate drops. Falling rain drops, being free from the deforming effect of nearby things, always assume spherical shapes due to surface tension.

When a spider moves over the surface of water, we notice fine depressions at points where the feet of the spider touch the surface of water, as if the water surface is stretched like a rubber membrane.

A grease free knitting needle when carefully placed over the surface of water is found to be supported by the surface with a slight depression.

All these examples show that free surface of a liquid is always in a stretched condition and tends to contract its surface area. The following simple experiments also lend support to the phenomenon of surface tension.

In fig. 6.1(*a*) is shown a soap film formed in a circular metal frame. A wet cotton loop is placed on it. The film, in trying to decrease its surface area, pulls inward and at every point the pull is along the perpendicular to the frame there.

The cotton loop when placed on the film does not take up any particular form. The film is both inside and outside it. The outside film pulls it outward and the inside film inward so that there is no resultant force on it due to surface tension. On piercing the film inside it, the inward pull on it is removed, but the outside film continues to pull it outward along the perpendicular to the loop at every point. The outside film tries to decrease its surface area and thereby the loop becomes circular [Fig. 6.1(*b*)]. This clearly shows that the film between the frame and the loop is in a stretched condition and tends to decrease its surface area.

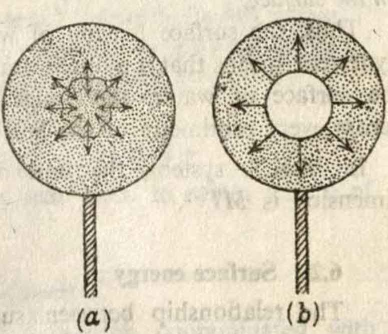


Fig. 6.1

Another experiment may be done in

[* Excluded from the syllabus of H. S. councils of W. Bengal & Tripura] the following way : Take a metal frame as above and fasten two pieces of wet cotton thread *AB* and *CD* to the frame so that the threads hang loosely. Now dip the frame in a soap-solution. A soap film will stick all over the frame and the pieces of threads will be found to hang on the film loosely as before. On piercing the portion of film lying between the thread *CD* and the frame, the portion will vanish but the rest of the film will remain in tact and at the same time, the thread *CD* will be tightened into an arc of a circle as shown in the figure 6.2.

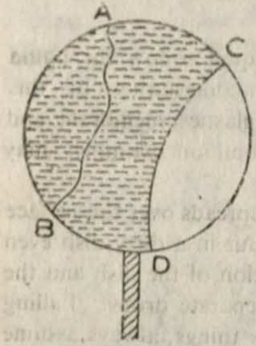


Fig. 6.2

This shows that the film remaining in the frame tries to contract its surface area and a force acts tangential to its surface. The other thread *AB*, however, remains as loose as before because it experiences equal and opposite forces on both sides.

From the above experiments, it may be said that there is a tension on the free surface of a liquid which behaves like a stretched elastic membrane. The only difference between the behaviours of an ordinary stretched membrane and the free surface of a liquid is that the surface area of an ordinary membrane increases with the increase of tension while for a liquid the surface the tension does not depend upon the surface area. This tension on the free surface of a liquid is called *surface tension*.

Suppose a line *AB* is drawn on the free surface of liquid (Fig. 6.3). Now due to the surface tension, the liquid molecules on one side of the line *AB* try to go apart from the molecules on the other side. Hence, the surface tension of a liquid may be defined in the following way :

Definition : *The surface tension is defined as the tangential force on the surface acting normally per centimetre across any line drawn on the surface.*

Thus, the surface tension of water is 72 dynes/cm. means that if a line is drawn on the the surface of water, the tangential force across every centimetre of the line perpendicular to it is 72 dynes.

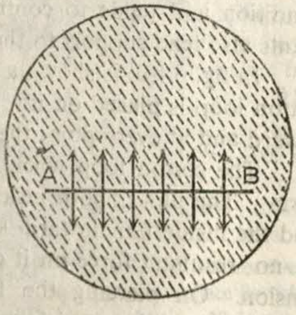


Fig. 6.3

In M.K.S. system, the unit of surface tension is Newton per metre. Its dimension is MT^{-2}.

6.2. Surface energy :

The relationship between surface tension and surface energy can be illustrated by considering the experiment described below.

A thin wire is bent into right angles twice to give a framework *GBCH*. Another straight piece of wire *AD* can slide without friction along the horizontal

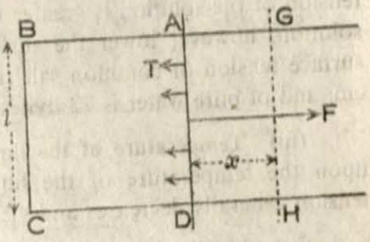


Fig. 6.4

arms BG and CH [Fig. 6.4]. If the frame-work is dipped in soap-solution and then taken out a thin film will stick in the space $ABCD$. As a result, surface tensional forces will act at every point on the wire AD, perpendicular to it and tangential to the surface of the film. The wire AD will slowly move towards the arm BC of the frame-work. To keep the wire AD fixed to its position, an equal force is to be applied on it in the opposite direction. If the surface tension of the liquid film be T, then force per unit length of the wire AD in the direction as shown in the figure is T. If the length of the wire AD is l, then the total force acting on the wire AD in the direction of the arm $BC=2lT$ (the film has two surfaces—upper and lower—each producing a force$=lT$.)

So, to keep the wire steady, the opposite force (F) that need be applied on the wire AD is given by $F=2lT$. If the movable wire is shifted to a distance x perpendicular to its length to the position GH, the work done against the surface tension forces$=F.x=2lT.x$.

The surface area of the film—both upper and lower—will, thereby, have an increment lx each, and the total increase in the surface area$=2lx$.

Thus, the work required to increase the surface area of film by unity$=\dfrac{2lT.x}{2l.x}=T$.

Similarly, if the surface area of the liquid film be allowed to shrink by unity, the work available is also T. This work remains stored in the film as its potential energy and is called the surface energy of the liquid. So, surface tension of a liquid is *numerically* equal to the surface energy per unit area.

Example : *Two small spherical drops of water of radius r are brought close to each other. From the consideration of surface energy, show that the drops coalesce.*

Ans. Volume of each drop$=\frac{4}{3}\pi r^3$ and surface area of each drop$=4\pi r^2$

Total surface energy of the two drops $E_1=4\pi r^2.T+4\pi r^2T=8\pi r^2T$ where T is the surface tension of water. If the two drops coalesce, let R be the radius of the single large drop. Then, the volume of the single drop$=\frac{4}{3}\pi R^3$

$$\therefore\ \tfrac{4}{3}\pi R^3=2\times\tfrac{4}{3}\pi r^3 \quad \text{or} \quad R=(2)^{\frac{1}{3}}.r$$

Hence, the surface energy of the single drop $E_2=4\pi(2)^{\frac{2}{3}}.r^2.T$

Clearly, $E_2<E_1$. As every mechanical system tries to attain a state of minimum energy, the drops would coalesce.

6.3. Factors influencing surface tension of a liquid :

(i) **Contamination** : If the surface of a liquid be contaminated with impurities, its surface tension is lowered appreciably. For example, if oily or greasy substances be left on the surface of water, they will float on the surface and will lower the surface tension of water to a great extent.

(ii) **Presence of dissolved substances** : For most inorganic salts, the surface tension of the solution is greater than that of pure water. Organic compounds in solution, however lower the surface tension of the solution. For example, the surface tension of common salt (inorganic substance) solution is about 83 dynes/cm. and of pure water is 72 dynes/cm.

(iii) **Temperature of the liquid** : The surface tension of a liquid depends upon the temperature of the liquid. If the temperature increases, the surface tension generally decreases and vice versa.

(iv) **Medium above the liquid** : If the medium above the liquid changes, its surface tension also changes.

6.4. Some phenomena in connection with surface tension :

(a) When a piece of camphor is placed on clean water, it darts about in different directions. The surface tension of camphor solution of water is smaller than that of pure water and as camphor dissolves more quickly at some points than others, the force due to surface tension exerted on it by the surrounding water is not the same in all directions.

If, now, a drop of oil be spilled on the surface of water, the haphazard motion of the piece of camphor at once stops. The reason is that the oil lowers the surface tension of water to such an extent that solution of camphor in water cannot make appreciable difference in the surface tension between the oily water and the camphor solution. As a result, the piece of camphor is acted on by equal forces on all sides and it remains at rest.

(b) If a small paint brush be dipped into water, the hairs of the brush are seen to stand apart but on raising the brush out of water, the hairs cling together. Thin films of water sticking between each pair of hairs tend to shrink owing to surface tensional forces. As there is no surface tensional force inside a liquid, the hairs do not form a tuft when the brush is dipped in water.

(c) Umbrella cloth, waterproof materials, tents etc. are processed in a special way. They possess pores through which air can pass but when rain water falls on it, they roll down and do not penetrate. If the inside of a tent during rain is touched, the water surface is broken and the tent begins to leak. This is why every camper should learn that it is not advisible to touch the inside of the canvas when it is raining.

(d) Sprinkle some fine chalk dust on the surface of water and a few drops of alcohol. Particles of chalk will be found to be drawn away to the sides from the part of water surface where alcohol falls. The surface tension of the solution of alcohol is smaller than that of pure water and so the stronger liquid film drags the powder away.

(e) If a small quantity of oil be poured over water, the oil spreads and covers almost the whole surface of water. Pure water has a surface tension much larger than the oil which, therefore, feels a tension tangential to the surface. This helps the oil to spread over.

(f) In ancient times, sailors used to spread oil over sea to calm waves during a storm. Oil moves forward being driven by wind, leaving comparatively pure water behind. Surface tension of pure water is larger than that of oil. So pure water exerts a hind-pull which prevents waves from becoming large in size.

6.5. Pressure difference in a curved liquid surface :

We can show in the following way that the magnitude of curvature of a liquid surface or of a bubble is related to the surface tension of the liquid.

Consider a spherical bubble formed in a liquid. Clearly the bubble has two surfaces—one inside and the other outside. The molecules on both the surfaces will feel an inward force due to surface tension. The pressure inside the bubble is evidently greater than the pressure outside otherwise the bubble will not be able to maintain its spherical shape. The excess internal pressure tries to bulge out the bubble while the surface tensional forces try to minimise the surface area. When these two forces balance the bubble becomes steady.

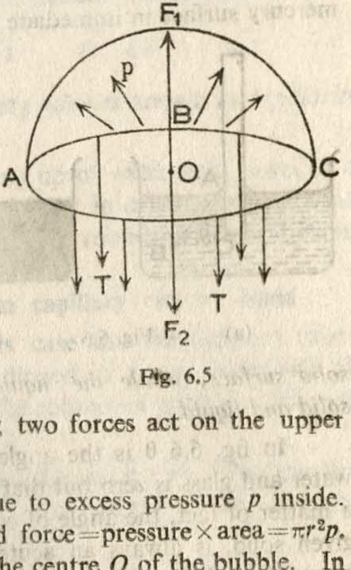


Fig. 6.5

Suppose r=radius of the bubble ; p=excess internal pressure and T=surface tension of the liquid. Imagine the spherical bubble to be divided into two hemispheres by a (horizontal) plane ABC through its centre and consider the equilibrium of the upper hemisphere [Fig. 6.5]. The following two forces act on the upper hemisphere.

(i) Total upward force on the plane ABC due to excess pressure p inside. Area of the surface $ABC=\pi r^2$. So, the upward force=pressure × area$=\pi r^2 p$. Due to symmetry, this upward force acts through the centre O of the bubble. In fig. 6.5 F_1 represents this upward force.

(ii) Downward forces along the circumference ABC due to surface tension. Now, length of the circumference$=2\pi r$. Since the bubble has two surfaces, the total downward force F_2 due to surface tension$=2\times 2\pi rT$. This force evidently acts through the centre O of the bubble downward. For the equilibrium of the bubble, $F_1=F_2$ or $4\pi rT=\pi r^2 p$ $\therefore\ p=\frac{4T}{r}$

If the atmospheric pressure outside the bubble be π, the total pressure inside$=\pi+p=\pi+\frac{4T}{r}$.

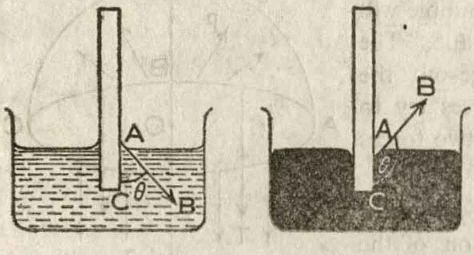
Example : *Assuming that the surface tension of rain-water is 72 dynes/cm., find the difference of pressure between inside and outside of a rain drop of diameter 0·02 cm.*

Ans. The rain drop having only one surface, the excess pressure inside is given by $p=\frac{2T}{r}=\frac{2\times 72}{0\cdot 01}=14400$ dynes/cm^2.

6.6. Angle of contact :

When a glass plate is dipped vertically in water, it is found that the water surface in immediate contact with the glass plate is not horizontal like the surface elsewhere, but is curved upward along the wall of the plate [Fig. 6.6(a)]. Such thing happens when the solid and the liquid are such that *the liquid wets the solid as* in the case of glass and water.

On the other hand, if the glass plate be dipped vertically in mercury, the mercury surface in immediate contact with the glass plate is found to be depressed [Fig. 6.6(b)]. Such thing happens when the solid and the liquid are such that *the liquid does not wet the solid.*



(a) Fig. 6.6 (b)

In general, we may say, that when a liquid meets a solid, the liquid surface near its plane of contact with the solid is curved.

Definition : *The angle between the tangent to the curved liquid surface at the point of contact and the solid surface, inside the liquid, is called the angle of contact for that pair of solid and liquid.*

In fig. 6.6 θ is the angle of contact. The angle of contact between pure water and glass is zero but that between ordinary water and glass is about 8°. As a matter of fact, the angle of contact between a solid and a liquid which wets the given solid, is always an acute angle. On the other hand the angle of contact between a solid and a liquid which does not wet the given solid, is always obtuse.

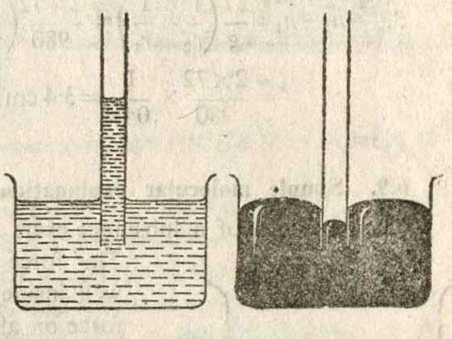
The angle of contact, however, depends on the following factors : (i) the nature of the solid and the liquid (ii) the medium above the free surface of the liquid ; for example, the angle of contact between glass and mercury with air above the mercury surface is different from that between the mercury and glass with a layer of water above the mercury surface. (iii) The angle of contact does not depend on the inclination of the solid with the liquid surface.

6.7. Capillarity or Capillary action :

If a piece of ordinary glass tube, on being heated, be drawn out into a tube of fine bore, the tube is called a capillary tube.

If one end of such a capillary tube be dipped in water, the liquid will rise inside the tube until it stands at some height above the level in the dish. Further, since water molecules are attracted by glass and water wets glass, the water surface curves upward along the glass, presenting a concave surface [Fig. 6.7(*a*)]. Same thing happens when a glass capillary tube is dipped in copper sulphate solution, alcohol etc *i.e.* in the liquids which wet glass.

But if the same capillary tube is dipped in mercury or in any other liquid, which does not wet glass, the mercury level in the tube goes below the level in the dish *i.e.* there is a depression of mercury column in the tube. Further the mercury meniscus in the tube becomes convex [Fig. 6.7(*b*)].



(a) Fig. 6.7 (b)

This ascent or descent of liquid in a capillary tube is known as '*capillarity*' and it is due to the surface tension of the liquid.

Capillary action is responsible for the soaking up of water by a towel or of ink by a blotter. It causes the oil to rise along a wick in a lamp or subsurface water in the soil to the surface and to some extent, in the roots and stems of plants.

6.8. Measurement of surface tension from capillary rise of liquid :

We consider water as the liquid in this case and the capillary tube is made of glass. When one end of such a tube is dipped in water, water will rise to a certain height in the tube till the weight of the column of water is equal to the upward force exerted by the surface tension.

As water wets the surface of glass, the angle of contact is zero. The meniscus of water in the tube may be taken as hemispherical. [Fig. 6.8].

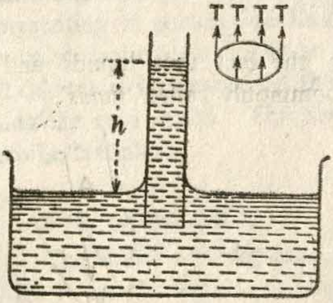


Fig. 6.8

The upward pull due to the surface tension $=2\pi r \times T$, where $r=$raduis of the bore of the tube and $T=$surface tension of water.

The weight of h column of water $=\pi r^2 h.g$

$$\therefore \quad 2\pi r \times T = \pi r^2 h.g$$

$$\text{or} \quad T = \frac{r.h.g}{2}$$

Measuring r and h, T may be determined experimentally.

Example : *A U-tube is such that the diameter of one limb is 0·4 mm and that of the other is 0·8 mm. Find the difference in the levels of water in the limbs. Surface tension of water=72 dynes/cm.*

Ans. In a capillary tube, we know, $T=\frac{r.h.g}{2}$ or $h=\frac{2T}{r.g}$

In the case of the U-tube, $h_1=\frac{2T}{r_1 g}$ and $h_2=\frac{2T}{r_2 g}$

$$\therefore \quad h_2-h_1=\frac{2T}{g}\left(\frac{1}{r_2}-\frac{1}{r_1}\right)=\frac{2\times 72}{980}\left(\frac{1}{\cdot 02}-\frac{1}{\cdot 04}\right)$$

$$=\frac{2\times 72}{980}\times\frac{1}{0\cdot 04}=3\cdot 4 \text{ cm.}$$

6.9. Simple molecular explanation of surface tension :

The molecules of a substance exert a force of attraction upon one another. The force is called *cohesive force*. A molecule, well inside a liquid, experiences equal cohesive force on all sides, being exerted by the surrounding molecules and therefore remains in equilibrium [Fig. 6.9]. But a molecule just on the free surface of the liquid, is attracted by other molecules below it and experiences an unbalanced force F towards the interior of the liquid due to which the molecule has a tendency of going inside. Consequently, the molecule has to do some work against this inward force when it tries to, go to the surface from inside. This work remains stored up in the molecule as its potential energy.

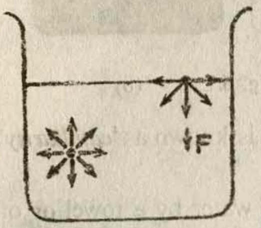


Fig. 6.9

But we know that a mechanical system has always a tendency of attaining such a state in which it has minimum potential energy. For this reason, the free surface of the liquid, in its bid to minimise the potential energy, tends to minimise its surface area. This endows the surface with a tension which acts tangential to the surface of the liquid and is known as its surface tension.

6.10. Motion of a liquid :

For some common properties—specially for the fact that liquids and gases can flow from one place to another—they are commonly called '*fluids*'.

When a fluid flows steadily, the magnitude and direction of the velocity at every point of the fluid remain unchanged. Suppose a liquid is flowing through a tube and its velocities at points A, B, C, etc. are respectively V_1, V_2, V_3. etc. [Fig. 6.10]. If the magnitudes and directions of the velocities at those points remain unchanged during the flow of the liquid, then the flow will be called a *steady flow*. In other words, in a steady flow of liquid each particle follows exactly the same path and has the same velocity as its predecessor. Such flow of liquid is also known as *orderly* or *stream-line flow*. In such a case if we consider a line along which

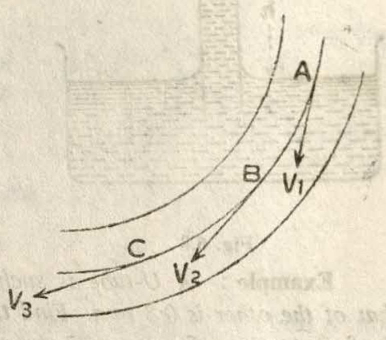


Fig. 6 10

a particle of the liquid moves, the line is called a *stream line*. More correctly, a *stream-line may be defined as a curve, the tangent to which at any point gives the direction of flow of the liquid at the point.* Obviously the stream-lines may be straight or curved along which the liquid particles move in an orderly fashion. For example, the stream-lines of a liquid flowing through a tube of uniform cross-section are straight lines parallel to the axis of the tube [Fig. 6.11].

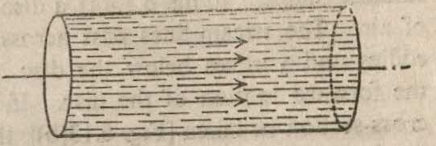
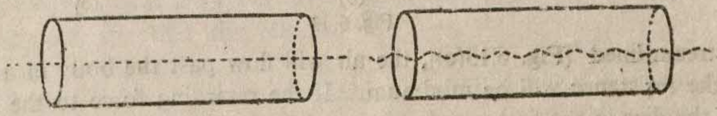


Fig. 6.11

It is to be remembered that the stream-line motion holds good so long as the velocity of the liquid does not exceed a particular limit, known as the *critical velocity* of the liquid. If the velocity goes beyond the critical value, the motion no longer remains steady. Eddies and vortices are created here and there and the path and velocity of liquid particles constantly change. Such irregular motion of liquids are called '*turbulent*' motion.

Experiment : Stream-line and turbulent motion of a liquid can be easily demonstrated by the following experiment :

Water is flowing through a narrow tube of uniform cross-section. There is arrangement for the liquid to flow with a gradually increasing velocity. A small jet of colouring matter like potassium permanganate is introduced into the tube. As long as the velocity remains below its critical value, we see only a thin streak of the colouring matter along the axis of the tube [Fig. 6.12 (a)] representing a stream-line flow. When the velocity reaches the critical value, the colouring matter moves in a zig-zag path [Fig 6.12(b)] and later, when the critical velocity is exceeded the colouring material spreads out in all directions filling the entire tube. This shows that the motion is no longer orderly but has become turbulent.

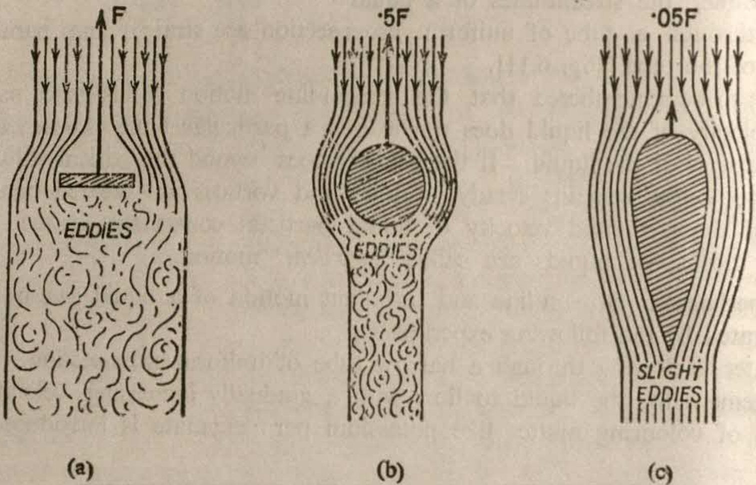


(a) Fig. 6.12 (b)

6.11. Streamlining :

The bodies of aeroplanes, torpedoes and ships are streamlined. Automobiles, if they are to travel at high speeds, should be streamlined to make more efficient use of fuels. *Streamlining means shaping a body to the streamlines of the fluid through which it moves so that the retarding force may be greatly reduced.* This is particularly effective at high speeds where the conditions of turbulent flow would otherwise predominate. Experiment shows that a long pointed tail and a round or pointed nose are both effective in cutting down resistance. A streamlined body does not possess any abrupt bend or fold. The body of a fish is streamlined ; that is why fishes can pass through water without much resistance.

Fig. 6.13 shows how air-resistance can be minimised by giving a body a suitable shape. In fig. 6.13(a), a disc has been held perpendicularly to the flow of air. The stream-lines pass across the edge of the disc and break down into eddies and vortices below the disc. This produces a great retarding force to the forward motion of the disc. If, instead of a disc, a round sphere of same cross-section be taken [Fig. 6.13(b)], the air-flow behind the sphere will be stream-lined to some extent, and the irregularities will be reduced. If the shape of the body be stream-lined [Fig. 6.13(c)], the air will flow past the body in a smooth way and the resistance will be minimum. If the retarding force to the forward motion of the disc in the first case be assumed *F*, then in the second case, it will be nearly half and in the case of a stream-lined body it is only 0·05 *F i.e.* 20 times less.



(a) (b) (c)

Fig. 6.13

6.12. Viscosity :

When a liquid flows in a stream-line motion over a rigidly fixed horizontal plane, the layer in contact with the plane is found to have no motion due to adhesion and the other layers, parallel to the plane, have velocities proportional to their distances from the plane *i.e.* the greater the distance of a layer from the horizontal plane, greater is its velocity.

Now, let us consider two consecutive layers of the moving liquid. Since the upper layer is moving faster than the lower layer, upper layer will experience a dragging force exerted by the slow moving layer which will hinder the forward motion of the upper layer. The upper layer, in its turn, will try to accelerate the slow moving lower layer. Thus, the two layers will tend to destroy the relative motion between them as if a dragging force is acting tangentially on the layers. In order to maintain the relative motion, a force is to be applied from outside to overcome the internal dragging force, otherwise the relative motion of each consecutive layers will be destroyed and the liquid will finally come to rest.

The property of a liquid by virtue of which it opposes a relative motion of its layers is called its viscosity. It is a general property of a fluid.

It may be pointed out here that the friction between two solid surfaces bears resemblance with the viscosity of a liquid. This is why viscosity is sometimes referred to as '*internal friction*' of a liquid. Whenever there is a relative motion between the layers of a liquid, viscosity comes into play. It does not exist when the liquid is at rest. We find that a mass of liquid, when stirred, comes to rest after sometime. This is due to the internal friction or the viscosity between the layers of the liquid.

Different liquids are differently viscous. For example, honey is more viscous than water, tar is more viscous than honey. Take equal quantities of water, honey and tar in three vessels. Tilt the vessels a little. It will be seen that the mobility of tar is minimum. Honey is more mobile than tar, while water is the most mobile liquid of the three. Tar will come to rest first, then honey and water will come to rest at last. Viscosity plays a very important part in lubricant used to minimise friction between different parts of a machinery. The heavier the machinery, the more viscous should be the lubricant.

6.13. Coefficient of viscosity :

Consider a column of liquid flowing in a stream-line manner over a fixed horizontal surface*CD* [Fig. 6.14]. The layer of liquid in contact with the fixed surface is stationary; it has no velocity. The next upper layer has some motion. In this way, if we go upwards from layer to layer, we will see that the velocity of the layers is gradually increasing. If a layer *AB* at a distance *x* from the fixed surface *CD*, has velocity *v*, then the ratio *v*/*x* is called the *velocity gradient*. Now considering the layer *AB* and a layer immediately below it, we see that the comparatively slower layer below tends to retard the motion of the upper layer while faster upper layer tends to accelerate the motion of the lower layer. The two layers together tend to destory their relative motion as if there is a backward dragging tangential force. This is known as *viscous drag*. According to Newton, the backward tangential force *F* in the case of stream-line motion of a liquid is (*i*) proportional to the area of the liquid layer, *i.e.* $F \propto A$ end (*ii*) proportional to the velocity gradient of the layer *i.e.* $F \propto v/x$.

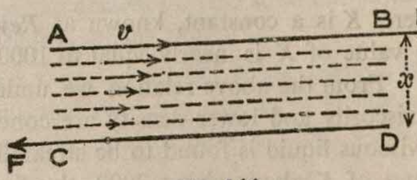


Fig. 6.14

From this, we have $F \propto A.\frac{v}{x}$ or $F=\eta.A\frac{v}{x}$. where η is a constant known as the *coefficient of viscosity*. It depends upon the nature of the liquid.

If we assume that $A=1$ and $v/x=1$, then $\eta=F$. ; this gives us a definition of the co-efficient of viscosity.

Definition : *The coefficient of viscosity of a liquid is defined as the tangential force per unit area required to maintain a unit velocity gradient.*

In the *C.G.S.* system, its unit is '*Poise*'.

The dimension of η can be easily calculated in the following way :

$$\eta=\frac{F}{A}\Big/\frac{v}{x}=\frac{MLT^{-2}}{L^2}\Big/\frac{L}{T.L}=ML^{-1}T^{-1}$$

6.14. Critical velocity and Reynold's number :

It has been mentioned earlier that as long as the velocity of a liquid is below a limiting value, the motion of the liquid through a tube is steamline. This limiting value of the velocity is called *critical velocity.* If the flow-velocity of the liquid exceeds the critical velocity, the motion becomes turbulent. Osborne Reynold first established experimentally that the critical velocity v_c of a particular liquid depends on the following factors :

(*a*) $v_c \propto \eta$ *i.e.* the critical velocity is proportional to the coefficient of viscosity of the liquid.

(*b*) $v_c \propto \frac{1}{\rho}$ *i.e.* the critical velocity is inversely proportional to the density of the liquid.

(*c*) $v_c \propto \frac{1}{r}$ *i.e.* the critical velocity is inversely proportional to the radius of the tube. Combining all factors, it may be stated that $v_c \propto \frac{\eta}{\rho . r}$ or $v_c = K . \frac{\eta}{\rho . r}$, where K is a constant, known as *Reynold's number.* For a very narrow tube, the value of K is nearly equal to 1000.

From the above relation we understand that finer bore, higher coefficient of viscosity and lower density are conducive to steady flow of liquid. The flow of viscous liquid is found to be streamline when K lies between 0 and 2000. For values of K above about 3000, the flow is turbulent while for K lying between 2000 and 3000 the flow is unstable and may change from one type to another.

6.15. Flow of a liquid through a narrow tube ; Poiseuille's equation :

We have seen in the previous article that when a viscous liquid flows through a tube, the flow is steady and streamline depending upon the radius of the tube and the average velocity of the liquid. Poiseuille showed that the force required to maintain a streamline flow of liquid through a tube of length l and radius r is given by $F = 8\eta . l \frac{V}{r^2}$, where V=volume of liquid coming out per sec. through the tube. This equation is known as Poiseuille's equation.

6.16. Motion of a body through a viscous medium : Stokes' law :

So far we have discussed the flow of a liquid through a tube. Let us now consider the motion of a body through a viscous liquid at rest. When a body falls in a liquid, the layer of the liquid in contact with the body is carried along with it. This tends to produce a relative motion among the layers of the liquid but the viscosity of the liquid opposes the relative motion. The opposing force due to viscosity increases with the velocity of the body until it becomes equal to, if the body is very small, the effective weight of the body. The body then moves with a *constant* velocity, called its *terminal velocity*.

For a small *spherical* body of radius a falling with a terminal velocity v_1 Stokes' had shown that the retarding force F acting on the body is given by

$$F=6\pi\eta a.v_T \quad \cdot\cdot \quad \cdot\cdot \quad (i)$$

where η is the coefficient of viscosity of the liquid. This is known as Stokes' law.

Now, for a spherical body, its weight$=\frac{4\pi}{3}a^3.\rho.g$ where ρ=density of the body. The upthrust of the displaced liquid$=\frac{4}{3}\pi a^3.\sigma g$, where σ=density of the liquid. Hence, the resultant downward force or, the effective weight$=\frac{4}{3}\pi a^3.\rho g-\frac{4}{3}\pi a^3\sigma g=\frac{4}{3}\pi a^3(\rho-\sigma).g \quad \cdot\cdot \quad \cdot\cdot \quad (ii)$

When the body attains terminal velocity, the equations (i) and (ii) are equal, so that

$$6\eta\pi a v_T=\frac{4}{3}\pi a^3(\rho-\sigma).g$$

$$\text{or, } v_T=\frac{2}{9}.\frac{a^2(\rho-\sigma)}{\eta}.g \quad \cdot\cdot \quad \cdot\cdot \quad (iii)$$

Equation (iii) shows that the terminal velocity v_T depends on the following factors:

(*a*) directly upon the square of the radius of the spherical body ;

(*b*) directly upon the difference between the densities of the solid and liquid;

(*c*) inversely upon the coefficient of viscosity of the liquid.

If the density of the liquid σ be greater than the density of the body ρ, the terminal velocity v_T will be negative, meaning that the body will move up instead of falling down. The best example of this is the upward motion of air bubbles in water.

Example : *Find the viscosity of glycerine if the terminal velocity of a steel ball 2 mm in diameter, falling through glycerine be 1·8 cm/sec. Density of steel =8 gm/c.c. and of glycerine=1·3 gm./c.c.*

Ans. We know, $v_T=\frac{2}{9}\times a^2\times\frac{(\rho-\sigma)}{\eta}\times g$

$$\text{or, } 1{\cdot}8=\frac{2}{9}\times\frac{(0{\cdot}1)^2\times(8-1{\cdot}3)\times980}{\eta}$$

$$\text{or, } \eta=\frac{2}{9}\times\frac{(0{\cdot}1)^2\times6{\cdot}7\times980}{1{\cdot}8}$$

$$=8{\cdot}1 \text{ poise (nearly)}$$

Exercises

Short answer type :

1. What do you understand by the surface tension of a liquid ? What is its unit ?

2. Mention a few illustrations to show that the free surface of a liquid is always under a tension.

3. Answer the following questions :

(a) Why does a piece of camphor dart in every direction when put on the surface of water ?

(b) Why do the sailors spread oil over the sea to calm the waves ?

(c) Why does a small amount of mercury assume spherical shape ?

(d) Why does oil ascend through a wick in a lantern ?

4. When common salt is dissolved in water, the solution has a higher surface tension but in a soap-solution, the surface tension has a smaller value. Why ?

5. How does surface tension of a liquid depend upon the temperature ?

6. What is capillary action ? Give some common examples of capillary action.

7. What is streamlining ? What is its advantage ?

8. Distinguish between stream-line motion and turbulent motion of a liquid.

9. Define the term 'viscosity' and 'coefficient of viscosity'.

10. Three vessels contain equal quantities of milk, molasses and tar. Which liquid will come to rest first when the vessels are tilted once and why ?

Numerical problems :

11. One thousand drops of water, each of diameter 0·2 mm, combine to form a single drop. Calculate the loss of energy in forming the larger drop. Surface tension of water=72 dynes/cm. [Ans. 82·79 ergs]

12. Two soap bubbles of radii 4 cm. and 5 cm. are joined together so as to have a common surface. Find the radius of this surface. [Ans. 20 cm.]

13. A capillary tube 0·5 mm. bore stands vertically in a wide vessel containing a liquid of surface tension 30 dynes/cm. The liquid wets the tube and has a specific gravity of 0·8. Calculate the rise of the liquid in the tube. [Ans. 3·06 cm.]

14. A flat plate of area 20 sq. cm. is separated from a large plate by a layer of glycerine 2 mm. thick. If the coefficient of viscosity of glycerine be 20 poise, what force is required to keep the plate moving with a velocity of 1·5 cm/*s* ? [Ans. 3000 dynes]

[**Hints** : Apply the formula $F=\eta.A.\frac{v}{x}$]

HEAT

1

HEAT AND TEMPERATURE

1.1. Heat :

All of us have some idea about heat. We get heat from the sun or from a fire. Our familiarity with heat grows almost from our childhood. Yet, heat is more of perception than of description. We feel its existence in terms of physical changes like hotness or coldness, expansion or contraction of a body, etc. Heat is, thus, an invariable associate of material bodies. Our common experience is that a body becomes hot when it absorbs heat and it becomes cold when it gives up heat.

Definition : *Heat may be regarded as a form of energy whose absorption makes a body hot and abstraction makes a body cold.*

1.2. Nature of heat :

If we carefully observe the developement of heat in a body, we will find that some form of energy is being spent for it. For example, burning of coal produces heat. Here, the chemical energy stored in coal is spent for the development of heat.

We know that heat is produced when two pieces of stones are rubbed against each other. In the process of rubbing, some mechanical energy is spent, which is responsible for the production of heat.

When electric current passes through the filament of an electric lamp, light is produced and at the same time the lamp becomes hot. Here, heat is produced in exchange of electric energy.

Thus, we see that some form of energy is necessary for the production of heat. For this reason, *heat is regarded as an energy.*

Different forms of heat : Heat may have three different forms; viz

(i) Sensible heat
(ii) Latent heat
(iii) Radiant heat :

Sensible heat : Heat which causes a change of temperature in a body is called the sensible heat. The change of temperature due to sensible heat may be ascertained by a thermometer.

Latent heat : Heat which causes change of state of a body without any change of temperature is called the latent heat. For example, latent heat of fusion of ice, latent heat of steam etc.

Radiant heat : Heat which comes to us from a source (for example, the sun) by the process of radiation is called the radiant heat. Radiant heat is similar to light.

1.3. Temperature :

All of us have feelings of hotness and coldness. When we touch a piece of ice, we feel cold. On the other hand, if we touch a boiling kettle, we feel hot.

Definition : *Temperature of a body expresses the degree of hotness of the body. It also represents a thermal condition of the body from which we know whether the body will absorb heat from another body or will part heat to another body.*

In this respect temperature of a body may be compared to the level of a liquid. We know, that water flows of its own from higher to lower level and never in the opposite direction. Level of water, therefore, gives us an indication of the direction of its flow. Similarly temperature of a body indicates the direction in which heat will flow. When heat flows from a body *A* to another body *B*, we say that the temperature of *A* is higher than that of *B*. A reverse flow of heat will indicate that the body *B* has a temperature higher than that of *A*.

1.4. Difference between heat and temperature :

(1) Heat is a form of energy but temperature represents a thermal condition of a body.

(2) When a body absorbs heat, its temperature generally rises and when it parts with its heat, its temperature falls. Heat, thus, may be regarded as the cause and temperature its effect.

(3) Level in hydrostatics and temperature in heat are analogous.

(4) Equal temperature in two bodies does not necessarily mean equal heat-contents in them. Similarly, equal heat-contents in two bodies do not mean equality of temperature in them.

1.5. Thermal equilibrium ; the Zeroth law of thermodynamics :

When two bodies, in contact, have same temperature, there is no net flow of heat between them. Under these circumstances, the bodies are said to be in *thermal equilibrium.* Experimentally it is found that when two bodies *A* and *B* are each in thermal equilibrium with a third body *C*, then *A* and *B* are also in thermal equilibrium with each other. For example, we want to ascertain whether *A* and *B* are in thermal equilibrium. In practice, we do it by bringing, in turn, a thermometer, say *C*, in contact with *A* and *B*. If the thermometer reading is the same in two cases, we conclude that *A* and *B* are in thermal equilibrium. We can formulate the above fact in the form of a law which states that when two bodies *A* and *B* are each in thermal equilibrium with a third body *C*, then *A* and *B* are also in thermal equilibrium with each other. This law is referred to as the **Zeroth law** of thermodynamics and was first established by R. H. Fowler. *The law thus enables temperature to be defined as that thermal property of a body which decides whether or not it is in thermal equilibrium with another body.*

1.6. Thermometers :

Temperature is something we can perceive by our sense ; we can distinguish hot bodies from cold ones by the sense of touch. By touch we can easily arrange bodies in order of their hotness, indicating that *A* is hotter than *B*, *B* than *C* etc. Yet, our temperature sense is neither accurate nor infalliable. An inhabitant of cold country will feel extremely hot when he comes to a tropical country. But we, the people of tropical country, being used to it, will not feel so hot here. When we go to any cold country, we feel it colder than the people of that country.

Place three bowls in a row—the first containing cold water, the second lukewarm water and the third hot water. Put your left hand in the cold water and your right hand in the hot. After a few seconds, remove and plunge both of your hands at once in the middle bowl. The lukewarm water in it will seem hot to your left hand and at the same time, cold to your right although you know it to be at the same temperature throughout.

So, the judgement of sense organ is not correct. Moreover, accurate measurement of temperature can not be done by senses. For this, instruments are necessary. The instruments which measure temperature correctly are called **thermometers.**

Different types of thermometers have been constructed utilising different properties of solid, liquid and gas. They are as follows :

(*i*) *Liquid thermometers* : The basic principle of liquid thermometers is the change of volume of a liquid due to change of temperature. Mercury thermometer, alcohol thermometer etc. are the best examples of liquid thermometers.

(*ii*) *Gas thermometers* : The volume of a certain mass of gas changes with the temperature when its pressure is kept constant. On the other hand, the pressure of a given mass of gas changes with temperature, when its volume is kept constant. Utilising these properties of gas, different types of gas thermometers have been devised.

(*iii*) *Electrical resistance thermometer* : The resistance of a conductor of electricity depends on temperature. The resistance, in general, increases as the temperature rises and decreases as the temperature falls. This change is found to be very regular in the case of platinum. This has given rise to a kind of temperature measuring device known as platinum resistance thermometer.

(*iv*) *Thermo-electric thermometers* : If two different metals are joined end to end and a temperature difference is maintained at the two ends, a current flows through the circuit. Such a circuit is called a thermo-couple. A thermo-couple can also act as a temperature recording instrument.

1.7. Mercury thermometers :

The thermometer in which mercury is used as a thermometric substance, is called a mercury thermometer. It is used widely in laboratories. There are some *advantages* of mercury being used as a thermometric substance.

Advantages of mercury as a thermometric liquid :

(1) The change of volume of mercury due to change of temperature is very regular within a wide range of temperature.

(2) Compared with other liquids, mercury absorbs much less heat from a body in order to attain its temperature. As a result, no appreciable change of temperature takes place in the body but the thermometer registers its temperature correctly.

(3) For a certain difference of the temperature the volume expansion of mercury is appreciable. It is, therefore, very sensitive.

(4) Mercury boils at 350°C and solidifies at -39°C. It, therefore, remains in the liquid state within a wide range of temperature and can record any temperature within this range.

(5) Mercury may easily be obtained in a pure state.

(6) Pure mercury does not wet glass. So, it will not stick to the glass wall of the thermometer tubing.

(7) Mercury being opaque and shining shows its meniscus prominently against the walls of the glass tube.

Description of mercury thermometer :

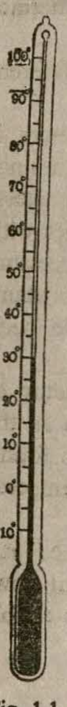


Fig. 1.1

Fig. 1.1 shows a mercury thermometer commonly used in laboratories. It consists of a thick-walled glass tube of uniform capillary bore. At one end of the tube, there is a cylindrical bulb. The other end is sealed. Mercury fills the whole of the bulb and a part of the capillary tubing. There is a scale graduated on the glass wall which denotes the temperature of a body. The bulb is brought in contact with the body whose temperature is to be measured and the mercury receiving heat from the body, expands. The mark on the scale against which the mercury thread stands, denotes the temperature of the body.

A few important facts : If you carefully observe a thermometer, you will see a small bulb at the top of the highest graduation of the thermometer [Fig. 1.1]. It is a safety arrangement for the thermometer. If ever the thermometer bulb be heated too much so that the mercury thread goes past the highest graduation, the mercury will be collected in this small bulb, otherwise, the thermometer might be damaged due to the excessive pressure of the expanding mercury thread.

While constructing a thermometer, care should be taken so that the thermometer may be *sensitive* and *quick-acting*. By sensitive, we mean that the liquid thread in the thermometer should advance appreciably due to a slight difference in temperature. By quick-acting, we mean that the thermometer should be capable of indicating temperature difference or change quickly.

Sensitivity can be achieved by taking a *bulb of large size*; for, a large-sized bulb holds a greater volume of liquid which expands more for every degree change of temperature. A capillary tube of *fine bore* also increases the sensitivity. Besides, a liquid of high thermal expansion coefficient helps to increase the sensitivity of a thermometer.

A thermometer can be made quick-acting if (i) the bulb of the thermometer is made of thin-walled glass, (ii) the bulb is small in size and (iii) the liquid is a good conductor of heat.

1.8. Thermometric scales :

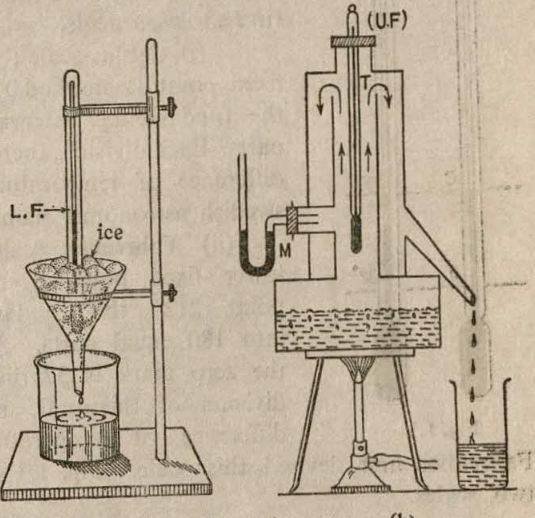
In order to specify a scale for a thermometer, two fixed and easily obtainable temperatures must be chosen. These are known as *fixed points* of a thermometer.

Definition of fixed points : *The temperature at which pure ice melts is taken as the lower fixed point.* This is also known as freezing point or ice point.

The temperature of steam from water boiling under standard atmospheric pressure of 760 mm. of mercury is taken as the upper fixed point. It is also known as the boiling point or steam point.

Determination of fixed points :

To find out the lower fixed point of the thermometer, an arrangement should be made as shown in fig. 1.2(a).



(a) Fig. 1.2 (b)

The bulb of the thermometer with a portion of its stem is inserted into a loosely packed pounded ice taken in a big funnel. The arrangement should be allowed to stay till the mercury meniscus in the stem comes to a steady position. A fine mark is made on the stem at this position. This is called the *lower fixed point* (L.F.) or the *ice-point* of the thermometer. This means that when the temperature of the bulb of the thermometer with its contents assumes the temperature of the melting ice, the mercury thread stands up to the said mark on the stem.

To find out the upper fixed point, an arrangement should be made as shown in fig. 1.2(b). The special type of apparatus that is used for the purpose is called the *hypsometer*, which is a steam boiler with a double-chambered cylinder. A manometer *M* is provided at one side and an outlet for the steam at the other side. The thermometer *T* is introduced in the dry steam to enable it to assume the temperature of true boiling point of water (*i.e.* steam point) under atmospheric pressure.

The water in the hypsometer should be kept boiling till the mercury thread in the thermometer becomes stationary. A fine mark is given on the stem at the position of the stationary meniscus. This mark (U.F.) is considerded as the *upper fixed point* or *steam point* of the thermometer provided the atmospheric pressure is normal.

[**N.B.** If the barometer reading is other than 760 mm (normal atmospheric pressure), the following calculation is necessary for getting the true steam point under normal atmospheric pressure.

For example, the barometer reads 746 mm at the time of experiment. From the boiling point and pressure chart of steam, it is found that the true boiling point corresponding to 746 mm pressure is 99·5°*C*. Suppose the length between the experimentally found fixed marks is 20 cm.

Then the correct length between the lower fixed point and the true upper fixed point corresponding to celsius scale is given by,

$$\frac{x}{20}=\frac{100}{99{\cdot}5} \text{ or } x=\frac{100\times20}{99{\cdot}5}=20{\cdot}1 \text{ cm (nearly)}$$

Thus the correct marking for the upper fixed point should be at a distance of 20·1 cm. from the lower fixed point.]

The difference in temperature between the upper and the lower fixed points of a thermometer is called its **fundamental interval.** Different thermometric scales have been devised by dividing the fundamental interval in different ways. Two scales mainly used in different countries are : (i) *Celsius scale* and (ii) *Fahrenheit scale.*

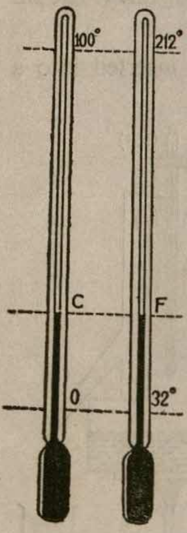


Fig. 1.3

(i) **Celsius scale :** According to this scale, the lower fixed point is marked 0° and the upper fixed point 100° the fundamental interval being divided into 100 equal parts. Each division, therefore, represents one degree Celsius difference of temperature. This scale was suggested by a swedish astronomer named Celsius.

(ii) **Fahrenheit scale :** According to this scale, the lower fixed point is marked 32° and the upper fixed point 212° the fundamental interval being divided into 180 equal parts. According to the scale, therefore, the zero mark is 32 divisions below the ice-point. Each division of this scale represents one degree Fahrenheit difference of temperature. A British scientist named Fahrenheit first devised this scale. Fig. 1.3 shows a comparative view of the two scales.

(iii) **Reaumer scale :** In certain European countries there is a third scale in use. It is called Reaumer scale, according to which the lower fixed point is marked 0° degree and the upper fixed point 80° degree. The fundamental interval, in the scale, is divided into 80 equal parts.

Relation between the Celsius and the Fahrenheit scales :

Suppose a temperature reads C on the Celsius scale and F on the Fahrenheit scale (Fig. 1.3). Their relation may be found out in the following way :

Since the fundamental interval is divided into 100 and 180 degrees respectively on the Celsius and the Fahrenheit scales, it follows that,

100 Celsius degrees = 180 Fahrenheit degrees.

$\therefore$ 1 " " $=\frac{180}{100}=\frac{9}{5}$ " "

$\therefore$ C " " $=\frac{9}{5}C$ Fahrenheit degrees.

Now, since 0° in the Fahrenheit scale is 32 divisions below the ice-point *i.e.* below 0° of the Celsius scale, the reading F will be,

$$F=\frac{9}{5}C+32. \text{ or, } \frac{F-32}{9}=\frac{C}{5}$$

This is the relation between the Celsius and the Fahrenheit scales.

Alternatively, from fig. 1.2, we can write, $\dfrac{C-0}{100-0}=\dfrac{F-32}{212-32}$

or $\dfrac{C}{100}=\dfrac{F-32}{180}$ or $\dfrac{C}{5}=\dfrac{F-32}{9}$

Examples : (1) *The temperature on a certain day is 94° Fahrenheit. What will the temperature be in the Celsius scale?*

Ans. We know, $\dfrac{C}{5}=\dfrac{F-32}{9}$ Here, $F=94°$; So $\dfrac{C}{5}=\dfrac{94-32}{9}=\dfrac{62}{9}$

$\therefore \quad C=\dfrac{62\times5}{9}=34{\cdot}4°$

(2) *The temperature of a body rises by 25°C. How much is this increase in degrees Fahrenheit ?* [*H.S. Exam. 1964*]

Ans. We know, a temperature difference of $100°C$=a temp. diff of $180°F$.

$\therefore$ „ „ „ „ $1°C=\frac{180}{100}°F$

or, „ „ „ „ $25°C=\dfrac{180\times25}{100}=45°F$

So, the increase in the Fahrenheit degrees is 45.

(3) *An unspecified thermometer reads —20° at the ice-point and 80° at the steam point . Calculate what this thermometer will read corresponding to 50°C.*

From fig. 1.4 we can write, $\dfrac{50-0}{100-0}=\dfrac{t-(-20)}{80-(-20)}$

or $\dfrac{50}{100}=\dfrac{t+20}{100}$ $\therefore \quad t=30°$

(4) *The lower and upper fixed points of a thermometer are 0·5°C and 101°C respectively. What would be the reading of this thermometer corresponding to 30°C?* [*H.S. Exam. 1980*]

Ans. Referring to fig. 1.4, we can write,

$\dfrac{30-0}{100-0}=\dfrac{t-0{\cdot}5}{101-0{\cdot}5}$ or $\dfrac{30}{100}=\dfrac{t-0{\cdot}5}{100{\cdot}5}$

$\therefore \quad t=30{\cdot}65°$

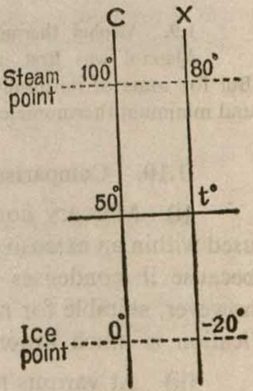


Fig. 1.4

(5) *A thermometer has its fundamental interval divided into 80 equal parts and another in 120. If the lower fixed point of the first is marked 0 and of the second 60, what is the temperature shown by the first when it is 100° by the second ?*

Ans. Suppose the temperature shown by the first thermometer is t_1 and that shown by the second is t_2.

Now, we can write, $\dfrac{t_1-0}{80}=\dfrac{t_2-60}{120}$

Here, $t_2=100^\circ$; So, $\frac{t_1}{80}=\frac{100-60}{120}=\frac{40}{120}$

$$\therefore \quad t_1=\frac{40\times 80}{120}=26{\cdot}6^\circ \text{ (nearly)}$$

(6) *A Celsius thermometer reads 1·5°C in melting ice and 98·5°C in steam coming out of boiling water at 747 mm. pressure. What will be the correct reading in Fahrenheit scale when the Celsius thermometer gives a reading of 20°C? The boiling point of water at 734 mm. pressure is 99°C.*

Ans. At normal atmospheric pressure (*i.e.*, 760 mm. of mercury) the boiling point of water $=100^\circ C$. It is given that at 734 mm. pressure, the boiling point $=99^\circ C$. So, for a difference of $(760-734)=26$ mm. of pressure, the boiling point changes by $=(100-99)=1^\circ C$.

Hence, for a difference of $(747-734)=13$ mm. of pressure, the boiling point changes by $=0{\cdot}5^\circ C$. *i.e.*, the boiling point of water at 747 mm. pressure is $99+0{\cdot}5=99{\cdot}5^\circ C$.

Now, when the Celsius scale shows a difference of $(99{\cdot}5-0)=99{\cdot}5^\circ C$, the faulty thermometer shows a difference of $(98{\cdot}5-1{\cdot}5)=97^\circ C$.

So, when the faulty thermometer registers a difference of $(20-1{\cdot}5)=18{\cdot}5^\circ C$, the Celsius scale shows a real increase $=\frac{99{\cdot}5}{97}\times 18{\cdot}5=18{\cdot}97^\circ C$.

i.e., the temperature according to Celsius scale $=18{\cdot}97^\circ C$.

Now, we know, $\frac{C}{5}=\frac{F-32}{9}$; Here $C=18{\cdot}97^\circ C$

$$\therefore \quad \frac{18{\cdot}97}{5}=\frac{F-32}{9} \quad \text{or,} \quad F=66{\cdot}14^\circ$$

1.9. Alcohol thermometer :

Alcohol was first used as a thermometric liquid for the measurement of temperature. But for some of its disadvantages, it is no longer used as such except in the case of maximum and minimum thermometer.

1.10. Comparison between mercury and alcohol as thermometric liquid :

(i) Mercury condenses at $-39^\circ C$ and boils at $357^\circ C$. It, therefore, can be used within an extensive range of temperature. For alcohol, the range is narrower, because it condenses at $-130^\circ C$ and boils at $78^\circ C$. Alcohol thermometer is, however, suitable for measurement of low temperature because its point of solidification is much lower than that of mercury.

(ii) At various tempeartures, the expansion of mercury is very regular but the expansion of alcohol is not so. For this reason, the graduation of mercury thermometer is an easy affair while that for alcohol thermometer is difficult and costly.

(iii) Being opaque and shining, mercury can be easily seen through glass. Alcohol is transparent and to make it visible a dye is added to it.

(iv) Mercury has a lower specific heat (0·033) than alcohol (0·6) but its specific gravity (13·6) is much higher than that of alcohol (0·8). For this reason, volume for volume, mercury has slighty less thermal capacity than alcohol. As a result, alcohol thermometer will absorb more heat than mercury thermometer from the body whose temperature is to be measured. From the view point of the principle of thermometry, it is regarded as a disadvantage for the alcohol thermometer.

(v) Mercury is a good conductor of heat but alcohol is not.

For this reason mercury thermometer is quick-acting but alcohol thermometer is slow-acting.

(vi) Mercury does not wet glass, so its motion along the bore of the thermometer is jerky and not smooth. Alcohol, on the other hand, moves smoothly because it wets glass. But this creates a difficulty. With the fall of temperature, when the alcohol thread comes down, a layer sticks to the wall of the bore and it causes some error in the reading.

(vii) The coefficient of expansion of mercury is $1{\cdot}8\times10^{-4}$ but the coefficient of alcohol is $1{\cdot}04\times10^{-3}$. For this difference, a mercury thermometer is less sensitive than an alcohol thermometer.

1.11. Range of temperature :

Nature has a limit to the lowest possible temperature that human beings can measure and it is about −273°C which is called the ***absolute zero*** (vide chapter 4). We cannot conceive of any temperature lower than this. But there seems to be no limit for higher temperature. It is, however, difficult to maintain in the extensive space of a laboratory temperature higher than 3000°C for long but it has been possible to create in a small volume and for a very short time a temperature as high as 20,000°C. By the explosion of atomic bomb a temperature of several millions of degrees Celsius has been created. Measurement shows that the temperature on the surface of the sun is about 6000°C but the temperature at the core of the sun is nearly 20 million degree Celsius. There are stars in the sky which are even hotter than the sun.

[N.B. Heat is a form of energy. Mechanical and electrical energies are measured in the units of Joule. For this reason, according to an international agreement, the unit of heat energy has been fixed as Joule, instead of calorie, from 1948. 1 Calorie=4·1855 Joules].

Exercises

Essay type :

1. What is a thermometer ? Which liquid is generally used in a thermometer and why ?

2. How are the fixed points of a thermometer determined experimentally ? What is fundamental interval ?

3. How many thermometric scales are in use generally ? What is the relation between them ? [*H. S. Exam. 1981*]

Short answer type :

4. What is the difference between heat and temperature ?

5. What purpose is served by the small bulb at the upper end of the capillary of a thermometer ?

6. How would a thermometer behave if the liquid in it expands less than the bulb ?

7. Two thermometers are given—one with a fine bore and the other with a small bulb. Which one will be more sensitive and which one quick-acting ?

8. Two thermometers are identical in all respects except that one has a spherical bulb while the other a cylindrical bulb. Which one is quick acting ?

9. What are the advantages of using mercury as a thermometric liquid ? [*H. S. Exam. 1981*],

10. Compare mercury and alcohol as thermometric liquids.

11. At what common temperature would a block of wood and a block of metal feel equally cold or equally hot when touched ?

[**Hints** : Only at the body temperature a block of wood and a block of metal will appear equally cold or equally hot when touched.]

12. Two thermometers are hung up in a room. One reads 15° and the other reads 59°. Why is this difference ?

Numerical problems :

13. On a certain winter day in Darjeeling the minimum temperature was found to be 30° Fahrenheit. What was it in Celsius scale ? [Ans. −1·11°]

14. The minimum temperature so far attainable is −270° Celsius. What is it on Fahrenheit scale ? [Ans. −454°]

15. Find the temperature which will be expressed by the same number both on the Fahrenheit and the Celsius scales. [*H. S. Exam. 1960*] [Ans. −40°]

16. The boiling point and the freezing point of a thermometer are 160° and 15° respectively. What would be the temperature on the Celsius and the Fahrenheit scales, when it shows a temperature of 73° ? [Ans. 40°C ; 104°F]

17. The freezing point on a thermometer is marked 20° and the boiling point 150°. What reading would this thermometer give for a temperature of 45°C ? [Ans. 78·5°]

18. Find out the temperature when the degrees of the Fahrenheit thermometer will be 5 times as the corresponding degrees on the Celsius thermometer. [*H. S.* (*Comp.*) *1963*] [Ans. 10°C and 50°F]

19. A thermometer of uniform bore is divided equally in degrees. If read 20° at the melting ice and 80° at 100°C. What will it read at 100° Fahrenheit ? [Ans. 42·66°]

20. The scale of a mercury thermometer is graduated in millimetres. It reads 10·6 mm. and 208·6 mm. at two standard fixed points. What reading will this thermometer give in a liquid of temperature 72°F ? [Ans. 54·6 mm.]

Harder Problems :

21. Two thermometers—one reading in Celsius and the other Fahrenheit, were successively dipped into two baths. In both cases, the difference in the readings between the two thermometers was found to be 20. If the baths were at different temperatures, find their temperatures in Celsius scale. [*Jt. Entrance 1976*] [Ans. −65° ; −15°]

[**Hints** : From the condition of the problem. $C-F=20$ or $F-C=20$

From (i) we get, $\frac{F+20}{5}=\frac{F-32}{9}$ and from (ii) $\frac{F-20}{5}=\frac{F-32}{9}$

Solving $C=-65°$ and $-15°$.]

22. A centimetre scale is attached along a thermometer tubing of uniform bore. In ice, the thermometer reads 7·3 cm., in steam 23·8 cm. and in a freezing mixture 3·5 cm. Find the temperature of the freezing mixture in Celsius scale. [Ans. −23·03°C]

23. A centigrade thermometer reads 0·5°C in melting ice and 98°C in dry steam at 74·8 cm. of *Hg* pressure. Find the correct temperature when this thermometer reads 14°C, given the boiling point of pure water at 73·6 cm. of *Hg* is 99°C. [Ans. 13·77°C]

24. The interval between the ice and steam points of two thermometers *A* and *B* are divided into 60 and 120 equal parts respectively and their lower fixed points are 10° and 0° respectively. What will be the reading of *A* when *B* reads 100° ? [Ans. 60°]

2

CALORIMETRY

2.1. Calorimetry :

Heat being a physical quantity, is measurable. The method in which a quantity of heat either absorbed or given up by a body, is measured is known as *calorimetry.* The vessel used for such measurement is called a *calorimeter* which is usually a cylindrical vessel made of copper (Fig. 2.1), provided with a stirrer which is also made of same material. The stirrer is necessary for stirring the liquid which allows the liquid in the calorimeter to attain same temperature throughout.

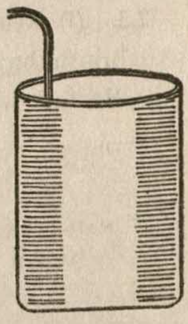


Fig. 2.1

2.2. Units of measurement of heat :

The following are the different units of heat :

(i) **Calorie :** It is the C.G.S. unit of heat. It is defined as the heat required to raise the temperature of 1 gm. of water through 1 degree Celsius.

(ii) **British thermal unit :** It is the F.P.S. unit of heat. It is defined as the heat required to raise the temperature of 1 lb of water through 1 degree Fahrenheit. This unit is widely used in England. It is abbreviated as Btu.

(iii) **Therm :** It is the commercial heat unit prevalent in England. The consumers of gas are charged according to therm.

1 Therm=1,00,000 British thermal unit.

So, Therm may be defined as the heat required to raise the temperature of 1,00,000 lb. of water through 1 degree Fahrenheit.

(iv) **Celsius heat unit or Chu. :** This unit is a mixture of C.G.S. and F.P.S. units of heat and is generally used in engineering and technology.

Chu is defined as the heat required to raise the temperature of 1 lb of water through 1 degree Celsius. Note that the mass of water has been expressed in the F.P.S. unit while the temperature is expressed in the C.G.S. unit. This is why the unit is called mixed unit.

(v) **Mean calorie and 15°C calorie :** Calorie has been defined as the heat required to raise the temperature of 1 gm. of water through $1°C$ but no mention has been made as regards the range of $1°C$ *viz* whether from $0°C$ to $1°C$ or from $20°C$ to $21°C$ or any other range. As a matter of fact, the heat required to raise the temperature of 1 gm. of water from $0°C$ to $1°C$ will be different from the heat required to raise the temperature of the same quantity of water say, from $20°C$

to $21^\circ C$. In other words, if we use $1^\circ C$ difference of temperature from various parts of the temperature scale, results will not be the same. To remove this difficulty, the mean calorie has been defined. It is as follows :

The amount of heat necessary to raise the temperature of 1 gm. of water from $0^\circ C$ *to* $100^\circ C$ *divided by 100 is taken as the mean calorie.* It has been found experimentally that this heat is same as that required to raise the temperature of 1 gm. of water from $14{\cdot}5^\circ C$ to $15{\cdot}5^\circ C$. For this reason, this particular amount of heat is regarded as a unit and is called $15^\circ C$ calorie.

[N.B. Heat is a form of energy. Mechanical and electrical energies are measured in the units of Joule. For this reason, according to an interational agreement, the unit of heat energy has been fixed as Joule, instead of Calorie from 1948. 1 Calorie=4·1855 Joules.]

2.3. (i) Relation between calorie and Btu. :

1 British thermal unit

=Heat reqd. to raise the temperature of 1 lb of water thro'. $1^\circ F$

= ,, ,, ,, ,, ,, ,, 453·6 gm. ,, ,, ,,

[∵ 1 lb=453·6 gm.]

= ,, ,, ,, ,, ,, ,, 453·6 gm. ,, $\frac{5}{9}{}^\circ C$

$[\therefore\ 1^\circ F = \frac{5}{9}{}^\circ C]$

$= 453{\cdot}6 \times \frac{5}{9}$ calories = 252 calories

Hence, 1 Btu. = 252 calories.

(ii) Relation between calorie and Chu.

1 Chu. $= 1\ \text{lb} \times 1^\circ C = 453{\cdot}6 \times 1^\circ C = 453{\cdot}6$ calories.

Unit	Quantity of water	Rise of temp.	Relation with Calorie
(a) Calorie	1 gm	1°C	=1 calorie
(b) Kg. calorie or kilo-calorie	1 kg	1°C	=1000 ,,
(c) Btu	1 lb	1°F	=252 ,,
(d) Therm	10^5 lb.	1°F	$=2{\cdot}52 \times 10^7$,,
(e) Chu	1 lb	1°C	=453·6 ,,

2.4. Specific heat :

If we take equal masses of different materials like lead, iron, copper etc. and warm them by the same burner separately for the same time, we shall see that the temperatures of different materials are different. Since the rate of supply of heat is the same in all cases, it is clear that different materials of equal mass have different capacity for absorbing heat. The following simple experiment will illustrate it.

Experiments : (1) Take several spheres of equal mass but of different materials like lead, copper, iron etc. Warm them to the same temperature and then quickly put them on a plate of wax. After some time, you will see that the spheres have melted different quantities of wax. Some may melt the whole thickness of wax and fall down while the others may melt half or one quarter of thickness and so on. Although the masses of all the spheres are equal and all of them have gone through the same range of temperature (*i.e.* from the same initial temperature to the temperature of melting wax), yet, they gave up different quantities of heat. It is clear, therefore that the *absorption or rejection of heat by a body does not entirely depend upon the mass or the change of temperature.*

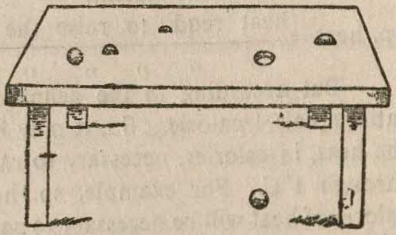


Fig. 2.2

(2) Take equal quantities of milk and water in two identical kettles. Place them side by side on the same oven. After sometime, if you note the temperature of the liquids by a thermometer, you will see that the temperature of milk is higher than that of water. If you observe two thermometers dipped in the two liquids, you will see that the mercury column in the thermometer dipped in milk is rising quicker than the other thermometer. Though the rate of supply of heat is the same in both cases, yet the temperatures attained by equal quantities of water and milk are not the same. So, *the change of temperature of a body does not entirely depend upon the mass or the quantity of heat supplied.*

We can conclude from the above experiments that the amount of heat absorbed or given up by a body does not entirely depend upon the mass of the body or the change of temperature of the body. It also depends upon another property of the body, known as the *specific heat.*

In the first experiment described above, the different spheres gave up different quantities of heat because the specific heats of the materials were not the same. In the second experiment, milk and water had different rise of temperature because their specific heats are different.

2.5. Definition of specific heat :

Specific heat of a substance (liquid or solid) is the ratio of the heat required to raise the temperature of a certain mass of the substance through a certain range of temperature to the heat required to raise the temperature of an equal mass of water through equal range of temperature.

In determining the specific heat of solids, or liquids, water is taken as the standard of comparison.

If the mass of the substance taken be unity and the range of temperature 1°, then from the above definition of the specific heat, we can write,

$$\text{sp. heat} = \frac{\text{heat reqd. to raise the temp. of unit mass of the substance thro' } 1^\circ}{\text{,, ,, ,, ,, ,, ,, ,, ,, ,, ,, ,, water thro' } 1^\circ}$$

Specific heat, thus, being a ratio of two heats, is a pure number and has no unit.

In the C. G. S. system, the unit of mass is gram and that of temperature is Celsius. So, in this system,

$$\text{sp. heat}=\frac{\text{heat reqd. to raise the temp. of 1 gm. of the substance thro' }1^\circ C}{\text{,, ,, ,, ,, ,, ,, ,, ,, ,, ,, water thro' }1^\circ C}$$

But according to the definition of calorie, the denominator of the above ratio equals 1 calorie. So, it may be said that the specific heat of a substance is the heat, in calories, necessary to raise the temperature of 1 gm. of the substance through 1°*C*. For example, sp. heat of copper is 0·09—this means that 0·09 calorie of heat will be necessary to raise the temperature of 1 gm. of copper through 1°*C*.

In the F. P. S. system, mass is expressed in pounds and temperature in Fahrenheit. So, in this system,

$$\text{sp. heat}=\frac{\text{Heat necessary to raise the temp. of 1 lb of the substance thro' }1^\circ F}{\text{,, ,, ,, ,, ,, ,, ,, ,, water thro' }1^\circ F}$$

But according to the definition of Btu, the denominator of the above ratio equals 1 Btu. So, it may be said that the specific heat of a substance is the heat, in British thermal unit necessary to raise the temperature of 1 lb of the substance through 1°*F*. For example, sp. heat of copper is 0·09—this means that 0·09 Btu. of heat will be necessary to raise the temperature of 1 lb of copper through 1°*F*.

Modern nomenclature :

Now-a-days, specific heat of a body is nomenclatured as *specific heat capacity* and is defined as the heat required to raise the temperature of unit mass of the substance by 1 degree. According to this definition, the specific heat capacity is no longer a pure number. It has a unit. In the C.G.S. system, specific heat capacity is defined as the heat necessary to raise the temperature of 1 gm. of the body by 1°*C* and its unit is calorie per gm. per degree celsius or $\text{Cal gm}^{-1}{}^\circ\text{C}^{-1}$. Thus specific heat capacity of water is 1 $\text{Cal gm}^{-1}{}^\circ\text{C}^{-1}$.

In the F.P.S. system, specific heat capacity is defined as the heat necessary to raise the temperature of 1 lb of the body by 1°*F* and its unit is Btu per pound per degree Fahrenheit or $\text{Btu lb}^{-1}{}^\circ\text{F}^{-1}$. Thus specific heat capacity of water is 1 $\text{Btu lb}^{-1}{}^\circ\text{F}^{-1}$.

In the M.K.S. system, specific heat capacity is defined as the heat required to raise the temperature of 1 kg of the body by 1°K and its unit is joule per kilogram per kelvin or $\text{JKg}^{-1}{}^\circ\text{K}^{-1}$. Thus specific heat capacity of water is 4200 $\text{Jkg}^{-1}{}^\circ\text{K}^{-1}$.

In this book, however, the old nomenclature with original definition, has been retained.

2.6. Amount of heat either absorbed or given out by a body for a rise or a fall of temperature :

If *s* be the sp. heat of a substance, then we know,

1 gm. of the substance for 1°*C* rise or fall of temperature, absorbs or gives out *s* calories of heat.

∴ m gm. of the substance for 1°C rise or fall of temperature absorbs or gives out $m.s.$ calories of heat.

or, m gm. of the substance for t°C rise or fall of temperature absorbs or gives out $m.s.t.$ calories of heat.

If H be the heat absorbed or given out by the body then

$$H=m.s.t. \text{ calories.}$$

i.e., Heat gained or lost=mass×sp. heat×rise or fall of temperature.

If the temperature of the substance before gaining heat be t_1 and the final temperature be t_2, then the rise of temperature$=t_2-t_1$. In this case $H=m.s.(t_2-t_1)$ calories.

Similarly, if the initial temperature be t_1 and the final temperature after giving out heat be t_2, then the fall of temperature$=t_1-t_2$. In this case, $H=m.s.(t_1-t_2)$ calories.

Example : (1) *A substance made of copper weighs 180 gm. Sp. heat of copper is ·09. How much heat is required to raise the temperature of the substance from 25°C to 95°C ?*

Ans. Here, $m=180$ gm. ; $s=\cdot09$; $t_1=25°C$; $t_2=95°C$

We know, $H=m.s.(t_2-t_1)$

$=180\times\cdot09\times(95-25)=180\times\cdot09\times70=1134$ calories.

(2) *How much heat is required to raise the temperature of 2·5 lb of alcohol from 68°F to its boiling point 173°F ? Sp. heat of alcohol=0·6.*

Ans. Here, $m=2\cdot5$ lb ; $s=0\cdot6$; $t_1=68°F$; $t_2=173°F$

From $H=m.s.(t_2-t_1)$

we have, $H=2\cdot5\times0\cdot6(173-68)=2\cdot5\times0\cdot6\times105=157\cdot5$ Btu.

(3) *If a therm of a gas costs 1s. 3d., find the cost of heating 50 gallons of water from 40°F to 200°F. 1 gallon of water weighs 10 lb.*

Ans. Since 1 gallon of water weighs 10 lb, the mass of 50 gallons$=50\times10=500$ lb. Now, the reqd. heat=mass of water×rise of temp.

$$=500\times(200-40)=500\times160 \text{ Btu}$$

$$=\frac{500\times160}{100,000} \text{ therm}=0\cdot8 \text{ therm.}$$

∴ Cost=1s. 3d×0·8=12d.

[**N.B.** Note the different units of the quantities involved in the above examples.]

2.7. Thermal capacity of a body :

The thermal capacity of a body is defined as the quantity of heat required to raise its temperature by 1 degree.

In the C.G.S. system, the thermal capacity of a body is the heat, in calories, necessary to raise its temperature by 1°C.

If the mass of the body be 'm' gm. and the sp. heat of its material 'S', then its thermal capacity C, according to the definition, is given by $C=m\times S\times1$ cal $=m.S.$ calorie.

In the F.P.S. system, the thermal capacity of a substance means the quantity of heat, in Btu necessary to raise its temperature through 1°F. If the mass of the body be m lb and the sp. heat of its material 'S', then its thermal capacity $C = m \times S \times 1$ Btu $= m.S.$ Btu.

So, Thermal capacity = mass × sp. heat.

From thermal capacity of a body ,we can fix an alternative definition of the specific heat of its material. *Sp. heat of a substance may, alternatively, be defined as its thermal capacity per unit mass.*

[N.B. Now-a-days, thermal capacity of a body is called the heat capacity of the body and specific heat capacity of a body is alternatively defined as the heat capacity per unit mass of the body.]

2.8. Water equivalent of a body :

Definition : *It is defined as the mass of water in grammes that requires for its 1°C rise in temperature, the same quantity of heat as required by the body for its 1°C rise in temperature.*

In short, it may be defined as that mass of water which has the same thermal capacity as the body. This term has practically gone out of use.

The water equivalent of a calorimeter is say, 10 gm. This means that the heat necessary to raise the temperature of the calorimeter through 1°C will raise the temperature of 10 gm. of water through 1°C. In other words, in respect of the heat absorbed or given up, the calorimeter may be replaced by 10 gm. water.

Suppose the mass of a body is m gm. and the sp. heat of its material is S. Then, the heat necessary to raise its temperature through $1°C = m \times S \times 1$ calorie. But we know,

1 calorie of heat will raise the temp of 1 gm. of water through 1°C

∴ $m \times S$ „ „ „ „ „ $m \times S$ gm. „ „ „

Hence, the water equivalent of the body, $W = m \times S$ gm. Similarly, in the *F.P.S.* system, the water equivalent of a body is defined as the mass of water, in lb, that requires for its 1°F rise in temperature the same quantity of heat as required by the body for its 1°F rise in temperature. In this system also, the water equivalent $W = m \times S$ lb.

2.9. Difference between thermal capacity and water equivalent :

(1) The magnitudes of both of them are equal, being the product of the mass of the body and the sp. heat of its material.

(2) Thermal capacity denotes some amount of heat and hence is expressed in calories or Btu. Water equivalent on the other hand, denotes some amount of water and hence is expressed in grammes or pounds.

Examples : (1) *A copper calorimeter weighs 75 gm. If the sp. heat of copper is ·09, what are the thermal capacity and the water equivalent of the calorimeter ?*

Ans. Here $m = 75$ gm. ; $S = \cdot 09$.

Hence, thermal capacity $C = m \times S$ calories $= 75 \times \cdot 09$ calories

$= 6 \cdot 75$ calories

and water equivalent $W = m \times S$ gm. $= 75 \times \cdot 09$ gm. $= 6 \cdot 75$ gm.

(2) *Two substances X and Y have their densities in the ratio 3 : 4 and specific heats in the ratio 4 : 5. If the volumes taken be in the ratio 2 : 3, find the ratio of their thermal capacities.*

Ans. Suppose the density, specific heat and volume of the substance X be respectively d_1, s_1 and V_1. Likewise those for the substance Y are d_2, s_2 and V_2.

The thermal capacity of X is $C_1 = V_1.d_1.s_1$ and

" " " " Y " $C_2 = V_2.d_2.s_2$

$$\therefore \frac{C_1}{C_2} = \frac{V_1}{V_2} \cdot \frac{d_1}{d_2} \cdot \frac{s_1}{s_2} = \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} = \frac{2}{5}$$

2.10. Principle of calorimetry :

If we put hot and cold substances in contact and take care to avoid heat transfer with the surroundings, everything will finally come to a common temperature. Then, provided there is no conversion of heat to other forms of energy ,(or vice versa), it will be true that,

Heat given up by the hot bodies=heat taken in by the cold bodies. This is the basic principle of calorimetry.

To put the principle in practice the condition envisaged in the principle *viz*, there should not be any heat transfer with the surroundings must be realised. For this, the calorimeter should be protected against any loss of heat due to conduction, convection and radiation. If the surface of the calorimeter is polished, the loss of heat due to radiation is minimised. The calorimeter should be surrounded by heat insulating jacket in order to prevent loss of heat by cold convection current of air. The calorimeter should be placed upon a non-conducting substance like cork and its open mouth should be covered by a non-conducting lid. This will reduce loss of heat due to conduction and evaporation. Further the body under investigation, should be heated for a long time so that it may attain a steady temperature. Care should be taken to transfer the hot body quickly into the calorimeter, to stir the liquid of the calorimeter constantly, to cut off and insulate the calorimeter completely from the surroundings etc. Furthermore, the solid and the liquid must be so selected that they do not react chemically with each other. Chemical reaction always absorbs or evolves heat which can not be accounted for in calorimetric calculations.

It is neither convenient nor advisable to use water in calorimetric measurement for water has a very high specific heat compared to other liquids. For a certain quantity of heat, water will have a smaller rise of temperature than other liquids which will make the measurements erroneous. Further, the boiling point of water being only 100°C, the solid introduced into the water, can not be heated to a high temperature, because in that case some water may be vaporised. Inspite of these drawbacks, water is, howver frequently used in ordinary calorimetric measurements because it is available easily in large quantities.

2.11. Determination of water equivalent of a calorimeter :

Take a dry and clean calorimeter with a stirrer and weigh it. Pour some cold water into the calorimeter and weigh it again. Note the temperature of cold water by a thermometer. Place the calorimeter in a wider jacket and fill up the space between the calorimeter and the jacket by cotton, wool or any other non-conducting substance. It will prevent loss of heat from the calorimeter. Take some hot water in another vessel and note its temperature. Pour the hot water quickly in the calorimeter and stir the mixture well. The temperature of the mixture in the calorimeter will rise. Note the final steady temperature of the mixture. Weigh the calorimeter with its contents again when it returns to the room-temperature.

Suppose,

the water equivalent of the calorimeter $= W$ gm.
the mass of empty calorimeter $= m_1$ gm.
the mass of calorimeter + cold water $= m_2$ gm.
the mass of calorimeter + cold water + hot water $= m_3$ gm.
the temperature of cold water $= t_1{}^\circ C$
" " " hot " $= t_2{}^\circ C$
the final steady temp. of the mixture $= t^\circ C$

Here, the mass of cold water taken $= (m_2 - m_1) = m$ gm. (say)
and " " " hot " " $= (m_3 - m_2) = M$ gm. (say)

Now, heat given up by hot water = mass of hot water × 1 × fall of temp.
$= M(t_2 - t)$ cal. [Sp. heat of water = 1]

Heat taken in by cold water = mass of cold water × rise of temperature $= m(t - t_1)$ cal.

Heat taken by calorimeter = water equivalent of the calorimeter × rise of of temp. $= W(t - t_1)$ cal.

Since, heat gained = heat lost.

We have, $W(t-t_1) + m(t-t_1) = M(t_2 - t)$

or, $W(t-t_1) = M(t_2-t) - m(t-t_1)$

$$\therefore \quad W = \frac{M(t_2-t)}{t-t_1} - m \text{ gm.}$$

All the quantities on the right hand side of the above equation being known, W can be calculated.

Examples : (1) *A calorimeter contains 140 gm. of water at 15°C. 150 gm. of water at 35°C are mixed with it and the mixture attains a final temperature of 25°C. Calculate the water equivalent of the calorimeter.*

Ans. Suppose, W = water equivalent of the calorimeter.

Heat given up by the hot water $= 150 \times (35-25) = 150 \times 10 = 1500$ cal.
" taken " " cold " $= 140 \times (25-15) = 140 \times 10 = 1400$ "
" " " " calorimeter $= W \times (25-15) = 10W$ cal.

Since, heat gained = heat lost.

We have, $10W + 1400 = 1500 \quad \therefore \quad W = 10$ gm.

(2) *An iron saucepan contains 100 gm. of water at 25°C. 50 gm. of water at 60°C are poured into the pan and the resultant temperature is found to be 35°C. Calculate the water equivalent of the pan assuming no loss of heat by radiation or otherwise. If the mass of the pan be 238 gm., what is the sp. heat of iron ?*

Ans. Suppose W=water equivalent of the saucepan.

Now, heat lost by the hot water=mass of hot water × fall of temp.

$$=50\times(60-35)=1250 \text{ cal.}$$

Heat gained by the pan=Water equivalent of the pan × rise of temp.

$$=W(35-25)=10W \text{ cal.}$$

" " " " water in the pan=mass of water × rise of temp.

$$=100(35-25)=1000 \text{ cal.}$$

Since heat gained=heat lost

We have, $10W+1000=1250$ or, $10W=250$ $\therefore$ $W=25$ gm.

Again, water equivalent W=mass of the pan × its sp. heat

$$\therefore\ 25=238\times S \quad \text{or,}\ S=\tfrac{25}{238}=0{\cdot}105 \text{ (nearly)}$$

(3) *The densities of two substances are as 2 : 3 and their specific heats are 0·12 and 0·09 respectively. Compare their thermal capacities per unit volume.*

Ans. Suppose the densities of the substances are 2ρ and 3ρ.

Now, masses of the substances per unit volume are $1\times2\rho$ and $1\times3\rho$ respectively.

Hence, the thermal capacity of the first body$=1\times2\rho\times0{\cdot}12$

and " " " " " second body$=1\times3\rho\times0{\cdot}09$

$$\therefore \text{ Their ratio}=\frac{1\times2\rho\times0{\cdot}12}{1\times3\rho\times0{\cdot}09}=\frac{2\times12}{3\times9}=\frac{8}{9}$$

(4) *A bath tub in a bathroom contains 50 kg. of water at 50°C. Hot and cold water taps are then turned on to supply 20 kg per minute each at temperature of 55°C and 2°C respectively. How long will it be before the temperature of the tub has come down to 30°C ? Assume complete mixing of water and no loss of heat by radiation or otherwise.*

Ans. Let the required time be t minutes. The amount of hot and cold water supplied during this time$=20\times t$ kg each.

Heat lost by the hot water supplied by the tap

$$=20\times t\times(55-30)=20\times25\times t \text{ units.}$$

Heat lost by the water in the tub$=50\times(50-30)=50\times20$ "

Heat gained by the cold water supplied by the tap

$$=20\times t\times(30-2)=20\times28\times t \text{ units.}$$

Since, heat lost=heat gained,

we have, $20\times25\times t+50\times20=20\times28\times t$

or, $500\,t+1000=560.\,t$

$$\therefore\ t=\tfrac{1000}{60}=16\text{m. } 40 \text{ sec.}$$

2.12. Determination of specific heat of a solid by the method of mixtures :

In order to determine the specific heat of a solid correctly avoiding loss of heat by conduction etc, Regnault's apparatus is used. In fig. 2.3, a section of the apparatus has been shown.

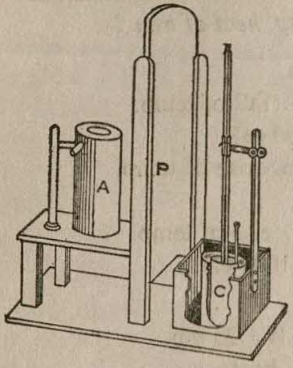


Fig. 2.3

Description of the apparatus : *A* is a steam-heater. It has been shown separately in fig. 2.4. It consists of two cylinders *A* and *B* (Fig. 2.4). The lower end of the cylinder *A* can be closed by a lid, the upper end being closed by a piece of cork. Through one hole in the cork, a thermometer (*T*) is inserted and through the other, a piece of thread passes into the cylinder. Steam enters through the upper end of the cylinder *B*, passes between the cylinders and goes out through an outlet at the bottom of the cylinder *B*. By this arrangement, the solid under test will attain steam temperature without directly coming in contact with the steam.

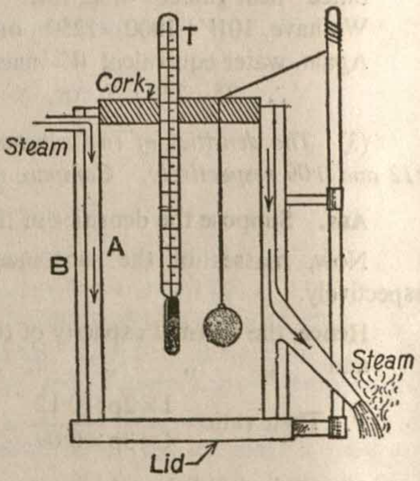


Fig. 2.4

Fig. 2.3 shows the complete arrangement for the experimental procedure. *A* is the steam-heater described earlier. *C* is a calorimeter placed in a wider copper vessel, the intervening space being packed by felt or any other non-conducting material. This will help the calorimeter to retain its heat. There is a wooden partition *P* to cut off any thermal contact between the calorimeter and the steam-heater. The partition can be raised or lowered.

Procedure : Take a suitable piece of the solid (say, copper) and weigh it. Tie a piece of thread with it and suspend it inside the steam-heater as shown in the diagram. [Fig. 2.4]

In the meantime, weigh a dry and clean calorimeter with its stirrer. Fill two-thirds of the calorimeter with cold water and weigh it again. Note the temperature of the water. By this time, the solid will have attained the steam-temperature. By raising the partition *P*, bring the calorimeter just below the steam-heater (Fig 2.3). Remove the lid at the bottom of the steam-heater and drop the solid into the calorimeter by snapping the thread. Remove the calorimeter at once to the right of the partition which is also immediately lowered. Stir the water in the calorimeter well and note the final steady temperature attained by the calorimeter and its contents.

Calculation : Suppose, the mass of the piece of solid$=M$ gm.

The mass of the empty calorimeter$=m_1$ gm.

„ „ „ „ „ „ +cold water$=m_2$ gm.

Initial temperature of cold water $=t_1{}^\circ C$

" " " the solid $=t_2{}^\circ C$

Final steady temp. of the calorimeter and it contents $=t\,{}^\circ C$

Sp. heat of the solid $=S$.

Here, the hot solid will give up heat which the calorimeter and water will take in. Now, heat given up by the solid $=M\times S\times(t_2-t)$ cal.

and heat taken up by the calorimeter $=m_1S_1\times(t-t_1)$ cal.

[S_1=sp. heat of the material of the calorimeter]

Heat taken up by cold water $=(m_2-m_1)(t-t_1)=m(t-t_1)$ cal.

[m=mass of cold water taken $=m_2-m_1$]

We know, heat gained = heat lost

$$\therefore\quad m(t-t_1)+m_1S_1(t-t_1)=M.S.(t_2-t)$$

$$\text{or,}\quad M.S.(t_2-t)=(t-t_1)(m+m_1S_1)$$

$$\therefore\quad S=\frac{(t-t_1)(m+m_1S_1)}{M(t_2-t)}$$

If W be the water-equivalent of the calorimeter then $W=m_1S_1$. In that case,

$$S=\frac{(t-t_1)(m+W)}{M(t_2-t)}$$

Sources of error and precautions :

(1) Some heat is lost when the hot solid body is transferred from the steam-heater to the calorimeter. This makes the result a bit erroneous.

(2) Some heat is also lost due to conduction and radiation, in spite of the use of Regnault's special apparatus. Loss of heat due to radiation may be avoided by initially taking the temperature of the calorimeter with its contents 3° or 4°C below the room-temperature by placing the calorimeter in ice-cold water for some time.

(3) Sensitive thermometers should be used to record the initial and final temperatures of water.

(4) While dropping the heated solid body into the water of the calorimeter, care should be taken not to splash any water.

(5) Solid body should be such that it does not dissolve in water as otherwise some latent heat will be involved in the solution which will not come in the calculation.

(6) There should not be any chemical reaction between the solid and the water because in every chemical reaction some heat is either absorbed or evolved which cannot be accounted for in the calculation.

2.13. Determination of specific heat of a liquid by the method of mixtures :

To determine the sp. heat of a liquid by the method of mixtures the same process as described above is to be adopted, the only difference being that the water in the calorimeter is to be replaced by the liquid under test. Moreover, a piece of solid is to be selected such that its specific heat is known and that it has no chemical reaction with the given liquid.

Suppose, the sp. heat of the liquid $=S_2$

Mass of the piece of solid $=M$ gm.

" " " calorimeter $=m_1$ gm.

" " " liquid taken $=m$ gm.

Initial temp. of the liquid $=t_1{}^\circ C$

" " " " solid $=t_2{}^\circ C$

Final temp. of the calorimeter, liquid and solid $=t^\circ C$

Sp. heat of the solid $=S$.

Here, heat lost by the hot solid body $=M\times S\times(t_2-t)$ cal.

and " gained by the calorimeter and cold liquid

$=(m_1S_1+mS_2)(t-t_1)$ cal

[$S_1=$Sp. heat of the material of the calorimeter]

We know, heat lost $=$ heat gained.

So, $M.S.(t_2-t)=(m_1S_1+mS_2)(t-t_1)$

$$\therefore\ m_1S_1+mS_2=\frac{M.S.(t_2-t)}{(t-t_1)}$$

$$\text{or,}\quad S_2=\frac{M.S.(t_2-t)}{m(t-t_1)}-\frac{m_1S_1}{m}$$

All the quantities on the R.H.S. of the above equation being known, S_2 can be calculated.

Examples : (1) *A solid weighs 500 gm. and is at 100°C. It is dropped into 100 gm. of water at 12°C. If the water-equivalent of the calorimeter be 10 gm., calculate the sp. heat of the solid, final temperature of the mixture being 49°C.*

Ans. Suppose, the sp. heat of solid $=S$.

Heat lost by the body $=$ mass of the body × its sp. heat × fall of temp.

$=500\times S\times(100-49)$ cal $=25500\times S$ calories

Heat gained by water $=100\times1\times(49-12)$ cal $=3700$ cal.

Heat gained by the calorimeter $=$ its water equivalent × rise of temp.

$=10\times(49-12)$ cal $=370$ cal.

Since, heat lost $=$ heat gained, we have,

$25500\times S=3700+370=4070$

$$\therefore\ S=\frac{4070}{25500}=0{\cdot}16\ \text{(approx)}$$

(2) *The heat required to raise 3 kg. of copper from 0°C to 10°C raises 1 kg. of lead from 10° to 100°C. If the sp. heat of copper be ·093, find that of lead.*

Ans. Let S be sp. heat of lead.

Now, heat necessary for raising the temperature of 3 kg. of copper through $10^\circ C=3000\times{\cdot}093\times10$ cal. [3 kg $=3000$ gm]

Again heat necessary for raising the temp. of 1 kg. of lead from $10^\circ C$ to $100^\circ C=1000\times S\times(100-10)=1000\times S\times90$ cal.

According to the question, $1000\times S\times90=3000\times{\cdot}093\times10$

$$\text{or,}\ S=\frac{3000\times{\cdot}093\times10}{1000\times90}={\cdot}031$$

(3) *A calorimeter contains 85 gm of water at 16°C. A piece of marble weighing 80 gm heated to 100°C is dropped into the water. The final temperature of water is 29·8°C. Calculate the sp. heat of marble. The water-equivalent of the calorimeter=4·53 gm.*

Ans. Heat gained by the calorimeter $=4{\cdot}53(29{\cdot}8-16)$ cal.

" " " " water $=85(29{\cdot}8-16)$ cal.

" lost by the heated marble $=80\times S\times(100-29{\cdot}8)$ cal.

[S=Sp. heat of marble]

Since, heat gained=heat lost,

we have, $4{\cdot}53(29{\cdot}8-16)+85(29{\cdot}8-16)=80\times S\times(100-29{\cdot}8)$

or, $(29{\cdot}8-16)(4{\cdot}53+85)=80\times S\times(100-29{\cdot}8)$

or, $13{\cdot}8\times85{\cdot}93=80\times S\times70{\cdot}2$

$$\therefore\quad S=\frac{13{\cdot}8\times85{\cdot}93}{80\times70{\cdot}2}=0{\cdot}21 \text{ (approx)}$$

(4) *An alloy contains 60% copper and 40% nickel. A piece of the alloy weighing 50 gm is heated to 80°C and is dropped into a calorimeter of water equivalent 10 gm. The calorimeter contains 90 gm of water at 10°C. Find the final temperature of the mixture. Sp. heat of copper=0·09 and sp. heat of nickel=0·11.* [*H.S. Exam. 1978*]

Ans. Mass of copper in the piece $=\frac{60}{100}\times50=30$ gm. $\therefore$ the mass of nickel $=50-30=20$ gm.

Heat given up by the piece of alloy $=(30\times0{\cdot}09+20\times0{\cdot}11)(80-t)$ cal where t=final temperature.

Heat absorbed by the calorimeter and water $=(10+90)(t-10)=100(t-10)$ cal.

$\therefore\quad 100(t-10)=(30\times0{\cdot}09+20\times0{\cdot}11)(80-t)=4{\cdot}9(80-t)$

or, $100t-1000=392-4{\cdot}9t\quad\therefore\quad t=13{\cdot}2$°C (nearly)

(5) *Three liquids A, B, C, are given. 4 gm. of A at 60°C and 1 gm. of C at 50°C have, after mixing, a temperature of 55°C. A mixture of 1 gm of A at 60°C and 1 gm of B at 50°C shows a temperature 55°C. What would be the temperature of 1 gm. of B at 60°C and 1 gm. of C at 50°C ?*

Ans. Suppose, S_A, S_B, and S_C are the sp. heats of the liquids A, B and C respectively and 't' the required temperature. We have,

in the first case, $4\times S_A\times5=1\times S_C\times5$ ..(i)

" " second " $1\times S_A\times5=1\times S_B\times5$ ..(ii)

" " third " $1\times S_B\times(60-t)=1\times S_C\times(t-50)$ ..(iii)

Solving these equations, we have $t=52$°C.

(6) *Three liquids A, B and C are at temperatures 15°C, 25°C and 35°C respectively. A mixture of equal quantities of the liquids A and B has a common temperature 21°C while a mixture of equal quantities of the liquids B and C has a common temperature 32°C. Show that the ratio of the sp. heats of the liquids A and C is 2 : 7.*

Ans. Suppose, '*m*' gm of the liquids *A* and *B* are mixed. In this case, the liquid *A* will gain heat while the liquid *B* will give it up. So,

$$mS_A\,(21-15)=m.S_B.(25-21) \text{ or, } 6S_A=4S_B \text{ or, } \frac{S_A}{S_B}=\frac{2}{3} \quad ..\text{(i)}$$

Again suppose, *M* gm of the liquids *B* and *C* are mixed. In this case, the liquid *C* will give up heat which the liquid *B* will absorb. So, $M.S_B(32-25)=$

$$M.S_C(35-32) \text{ or, } 7.S_B=3.S_C \text{ or, } \frac{S_B}{S_C}=\frac{3}{7} \quad ..\text{(ii)}$$

Multiplying the equations (i) and (ii), we get $\frac{S_A}{S_C}=\frac{2}{3}\times\frac{3}{7}=\frac{2}{7}$

Specific heat of some solids and liquids.

Solid	Sp. heat	Liquid	Sp. heat
Brass	0·092	Alcohol	0·6
Copper	0·093	Kerosene	0·45–0·5
Glass	0·156	Mercury	0·033
Iron	0·117	Mustard oil	0·5
Marble	0·22	Turpentine	0·42
Ice	0·51	Olive oil	0·47

2.14. Dulong and Petit's law : Atomic heat :

An inspection of the table of specific heats given above, reveals that the specific heats of solids vary widely from one element to another. However quite a different story emerges if we find the products of the atomic weights of elements in solid states and their specific heat. Experimenting with different elements (in solid state), Dulong and Petit established an empirical law in this connection in 1819. The law is known as Dulong and Petit's law which states that *the product of the atomic weight of a solid element and its specific heat is very nearly a constant.* The product is called the *atomic heat*. The average value of this constant in the cases of different elements is 6·4—the maximum value in some cases being 6·76 and the minimum being 5·7. But some elements like boron, carbon and silicon are exception to this law. The atomic heat of boron is 2·75, of carbon 1·46 and of silicon 4·98 [See table below]

Table of atomic heats of some solid elements

Elements	Sp. heat	At. wt.	Atomic heats
Aluminium	0·215	26·98	5·82
Carbon	0·121	12·0	1·46
Copper	0·0923	63·54	5·86
Lead	0·0305	207·19	6·32
Silver	0·0564	107·87	6·09
Boron	0·2544	10·81	2·75
Silicon	0·177	28·09	4·98
Tungsten	0·0321	183·85	5·92

2.15. Application of calorimetry in the measurement of high temperature :

Sometimes it becomes necessary to measure a high temperature like the temperature of a furnace or of a flame. It is not possible to use thermometers directly in such cases. Calorimetry may be conveniently applied to measure such high temperatures with moderate accuracy. In this method, a piece of solid is so selected that its melting point is higher than the temperature to be measured so that it does not melt when it is placed in the furnace or in the flame. Furthermore, its sp. heat should be known. The method is almost same as that described in connection with the determination of the sp. heat of solid (see art. 2.12), the only difference being that the solid is to be put in the furnace instead of in the steam-heater, in order to attain the temperature of the furnace. Then, following the process described in art. 2.12, we can find the temperature t of the furnace by using the final formula of that article. The example below may serve as a good illustration of the method.

Example : *In order to determine the temperature of a furnace, a platinum ball weighing 80 gm is introduced into it. When it has acquired the temperature of the furnace, it is transferred quickly to a calorimeter containing water at 15°C. The temperature rises to 20°C. If the weight of water, together with the water-equivalent of the calorimeter be 400 gm, calculate the temperature of the furnace, sp-heat of platinum being 0·0365.*

Ans. Let the temperature of the furnace be $t°C$. So the initial temperature of the ball is also $t°C$. Now, heat lost by the hot ball = mass of the ball × sp. heat of platinum × fall of temperature.

Hence, heat lost $=80\times{\cdot}0365\times(t-20)=8\times{\cdot}365\times(t-20)=2{\cdot}92t-58{\cdot}4$ cal.

Heat gained by the calorimeter and water = water-equivalent × rise of temp.

$$=400\times(20-15)=400\times5=2000 \text{ cal.}$$

Since, heat lost = heat gained

We have, $2{\cdot}92t-58{\cdot}4=2000$ or, $2{\cdot}92t=2058{\cdot}4$

$$\text{or,}\quad t=\frac{2058{\cdot}4}{2{\cdot}92}=704{\cdot}9°C \text{ (approx)}$$

2.16. Effects of high specific heat of water :

Specific heat of water is 1 and is much higher than the specific heats of other liquids and solids. Consequently, water will require much more heat for 1°C rise or fall of its temperature than an equal amount of any other substance, solid or liquid, for the same purpose. For this reason water acts as a sort of storehouse for heat. It is a good coolant and at the same time a good heater. As a coolant, water is used in automobile radiator and in steam engine and for heating purposes, hot water bags or bottles are frequently used. In cold countries, rooms of a building are kept hot by circulating hot water through pipes.

The fact that the sp. heat of water is considerably larger than that of other substances helps to reduce the daily and seasonal temperature variations of islands and of places on the sea-coast. When the sun is strong the ocean takes up a great deal of heat, but does not rise much in temperature. When the sun is weak or at night this heat is given up to the surroundings, thus preventing the temperature from falling as much as it otherwise might. As a result, the summer and winter temperatures of islands or coastal regions are more equable than those places of farther inland. Owing to its high specific heat, the sea warms up much more slowly than the land during the summer and consequently the winds from the sea have a cooling effect. During winter, however, warm sea winds make the weather comfortable. The high specific heat of water is also responsible for land and sea-breezes.

2.17. Calorific values or heating values of fuels :

Fuels mean the substances by burning which we get heat and which are available in quantities sufficient to be of commercial value. Fuels may be of three different varieties : (i) Solid fuels, such as coal, timber etc. (ii) Liquid fuels, such as petroleum, paraffin oil, kerosene oil etc. (iii) Gaseous fuels such as coal gas, natural gas, carbon monoxide etc.

Calorific values or the heating values of solid and liquid fuels mean the amount of heat developed due to complete combustion of unit quantity of the fuel. When the mass of the fuel taken is 1 lb and the heat developed is expressed in Btu, the calorific value of the fuel is expressed in Btu per lb. For example, 'the calorific value of coal is 15000 Btu per lb' means that by burning 1 lb of coal completely, the heat produced is 15000 Btu. Similarly, the calorific value of petroleum is 20,200 Btu per lb means that by complete combustion of 1 lb of petroleum, 20,200 Btu of heat will be available.

The calorific value of gaseous fuels means the amount of heat developed due to complete combustion of unit volume of gas under certain conditions of temperature and pressure . For example, the calorific value of the gaseous fuel supplied by Inden gas company is 500 Btu/cu.ft. under 60°*F* and 30 inches of pressure means that on complete combustion of 1 cu.ft of the gas at 60°*F* and 30 inches pressure, 500 Btu of heat is available.

In C.G.S. unit, the calorific values are expressed in cal/gm in the case of solid and liquid fuels and in cal/litre in the case of gaseous fuels.

Table for calorific values of some fuels

Fuels	Cal/ gm.	Btu/lb.	Fuels	Cal/gm.	Btu/lb.	Fuels	Cal / litre	Btu/ cu.ft.
Purest coal	8825	15,900	Petrol	11,350	20,400	London gas supply (at 60°*F* and 30 inch pressure)	4410	500
Gas coke	6100	11,000	Paraffin oil	11,200	20,100			
Wood	4450	8,000	Crude oil	25	46			

Exercises

Essay type :

1. Describe briefly how you would determine in the laboratory the specific heat of a solid insoluble in water. Does the relation "heat lost=heat gained" hold good if the solid and the liquid in the calorimeter react chemically. Explain. [*H. S. Exam. 1979*]

2. Suppose you are given a thermometer reading only from 50°C to 100°C, and some water of which the temperature is below 20°C. Describe an experiment to determine roughly the temperature of water, without using any other thermometer.

3. Explain how calorimetric principle can be applied in determining the high temperature of a furnace.

4. Which method would you adopt to determine the specific heat of a liquid ?

5. Explain the fact that the presence of a large body of water nearby, such as a sea or an ocean, tends to moderate the temperature extremes of the climate on adjacent land.

Short answer type :

6. Write the correct definitions of the following (i) Specific heat (ii) Calorie (iii) British thermal unit (iv) Therm (v) Thermal capacity (vi) Water-equivalent. Define calorie. What is a calorimeter ?

7. Define specific heat. Does it depend on (i) the unit of mass (ii) the unit of temperature ? If so, how ?

8. Discuss the basic principle of calorimetry. What precautions should be taken in a calorimetric experiment ? [*H. S. Exam. 1980*]

9. Why does 1 lb of iron at 100°C melt more ice than 1 lb of lead, also, at 100°C ?

10. Ice melts into water. Do water and ice have the same specific heats ?

11. Will the temperature be different if equal quantities of heat be supplied to equal masses of different materials ?

12. Two identical kettles containing equal masses of milk and water are placed side by side on an oven. Milk is found to have a rapid rise of temperature. Explain the reason.

13. 'Sp. heat of lead is 0·03'—explain the statement fully.

14. What are thermal capacity and water equivalent ? What are the differences between the two ?

15. In experiments by the method of mixtures in a calorimeter, we assume that the heat lost by the hot body is equal to the heat gained by the calorimeter and its contents. In order that the relation may hold, no heat must be allowed to enter or leave the calorimeter or be developed or absorbed inside. Will then, the relation hold if (i) the calorimeter contains water and the solid is sugar, (ii) the solid and the liquid in the calorimeter react chemically (iii) the calorimeter is kept on a table and is exposed to the air ? Explain your answer briefly.

16. Why is water not considered as a good calorimetric liquid ?

17. What is the effect of high specific heat of water on the climate of sea-shore ? What is the advantage of taking water as the hot substance in a hot water bottle ? [*H. S. Exam. 1982*]

Objective type :

18. Put a tick mark against the correct answer :—

(a) The heat required to raise the temperature of 1 lb of water through 1° Celsius is called a Calorie/Btu/Chu.

(b) Absorption or rejection of heat by a body does not entirely depend on the mass or change of temperature but also on specific gravity/specific heat/thermal capacity.

(c) Heat required to raise the temperature of 1 lb of a substance whose specific heat is 0·09 through 1°F is 0·09 Btu/0·09 cal/0·09 × density of the substance.

(d) The water equivalent of a calorimeter being 10 gm., the heat necessary to raise the temperature of the calorimeter through 1°C will raise the temperature of 10 gm/1 gm/100 gm. of water through 1°C.

(e) Water is not regarded as a good calorimetric liquid because water has low heat conductivity/high specific heat/easy availability.

Numerical problems :

19. A room contains 50 litres of air at 40°C weighing 1·3 gm/litre. How much heat is required to raise the temperature of the air to 50°C ? Sp. heat of air=0·24. [Ans. 156 cal.]

20. A vessel contains 40 gm. of water at 12°C. Into this water are added 50 gm. of water at 80°C. The final temperature of the mixture is 46°C. Calculate the water-equivalent of the vessel. [Ans. 10 gm.]

21. A body of mass 100 gm. is heated to 122°C and is quickly immersed in 300 gm. of water at 28°C contained in a copper calorimeter of mass 50 gm. The final common temperature is 30°C. If the sp. heat of copper be 0·09, calculate that of material of the body. [*H. S.* (*Comp.*) *1962*] [Ans. 0·066]

22. How much water at 100°C should be added to water at 20°C to provide a bath of 24 gallons at 40°C? How much more water at 20°C is required to reduce the temperature to 35°C? [Ans. 6 gallons; 8 gallons]

23. 80 gm. of iron at 100°C are dropped into 200 gm. of water at 20°C. The water was contained in an iron vessel weighing 50 gm. Calculate the temperature of the mixture. Sp. heat of iron=0·12. [Ans. 23·5°C]

24. A copper vessel of water-quivalent 60 gm. contains 600 gm. of water at 30°C. A bunsen burner, adjusted to supply100 calories per sec. is used to heat the vessel. Calculate the time taken to raise the water to the boiling point. [Ans. 7 min. 42 sec.]

25. A ball of platinum whose mass is 200 gm. is removed from a furnace and immersed in 150 gm. of water at 0°C. Supposing the water to gain all heat the platinum loses and if the temperature of water rises to 30°C determine the temperature of the furnace. Sp.heat of platinum =·031. [Ans. 755·8°C]

26. 200 gm. of lead are heated up to 100°C and dropped into a vessel containing 200 gm. of a liquid of sp. heat 0·5. If the initial temperature of the liquid were 0°C, find its final temperature, assuming that the vessel does not absorb any heat. Sp. heat of lead=0·03. [Ans. 5·66°C]

27. Which one will require more heat—to raise 500 gm. of water from 35°C to 95°C or to raise 4 lb. of water from 100°F to 212°F? [Ans. Second one]

28. A liquid of sp. heat 0·54 and temperature 29°C was mixed with another liquid of sp. heat 0·36 and temperature 11°C. The final temperature of the mixture was 17°C. In what proportion were the liquids mixed.? [Ans. 1 : 3]

29. Two liquids *A* and *B* are at temperatures of 75°*C* and 15°*C*. Their masses are in the ratio of 2:3 and their specific heats are in the ratio of 3:4. Calculate the resultant temperature of the mixture when the two liquids *A* and *B* are mixed. [Ans. 35°*C*]

30. The temperature of three liquids of equal mass are 15°C, 25°C and 35°C respectively. When the first two liquids are mixed, the temperature of the mixture is 21°C. When the second and third liquids are mixed, the mixture has a temperature of 32°C. What will be the temperature of the mixture when the first liquid is mixed with the third. [Ans. 30·55°C]

31. A calorimeter of water equivalent 15 gm. contains 60 gm. of water at 40°C. Due to the loss of heat by radiation, its temperature falls to 30°C in 5 minutes. If the same calorimeter is filled with 50 gm. of another liquid, the time in which the same drop of temperature takes place is found to be 2 minutes. Calculate the specific heat of the liquid assuming that the rate of loss of heat due to radiation is same in both cases. [Ans. 0·3]

32. A piece of metal, weighing 70 gm. is heated and transferred to 116 gm. of water, kept in a calorimeter of water-equivalent 10 gm. If the fall of temperature of the metal piece be 15 times the rise of temperature of the water, find the specific heat of the metal.
[*Jt. Entrance 1976*] [Ans. 0·12]

33. Three liquids of equal mass are mixed together. The liquids have specific heats S_1, S_2 and S_3 and initial temperatures θ_1, θ_2 and θ_3 respectively ($\theta_1>\theta_2>\theta_3$). Calculate the final temperature of the mixture. $\left[\text{Ans. } \dfrac{S_1\theta_1+S_2\theta_2+S_3\theta_3}{S_1+S_2+S_3}\right]$

Harder peoblems :

34. A thermometer of mass 55 gm. and sp. heat 0·2, gives 15°C as the room temperature. When dipped in some water, kept in a calorimeter, it reads 44·4°C. If the water equivalent of the calorimeter be 50 gm. and mass of water taken be 250 gm., find the temperature of water before the insertion of the thermometer. [*Jt. Entrance 1974*] [Ans. 45·5°C]

35. A steel ball of mass 10 gm. and sp. heat 0·1 is heated in a furnace and is quickly transferred to a copper vessel of mass 200 gm. and sp. heat 0·09 (temperature=50°C) which is immediately put inside a calorimeter of water equivalent 20 gm. The calorimeter contains 180 gm. of water at 20°C. A thermometer inserted into the calorimeter gives final temperature of 26°C. Calculate the temperature of the furnace and show, by calculation whether there is any local boiling in the calorimeter. [*Jt. Entrance 1978*] [Ans. 794°C, No local boiling]

36. 60 cu. ft. of water meant for bath is heated by a gas burner. The heat given out by burning 1 cu. ft. of the gas is 600 Btu and the cost of the gas is 95 paisa per 1000 cu. ft. What will be the cost of heating the water from 55°F to 100°F assuming 70% of the heat given out by the burner is delivered to water ? [*Jt. Entrance 1981*] [Ans. 38 paise]

37. 6 cu.ft. of water is heated in a bath tub on a gas burner to raise it from $55°F$ to $100°F$. The heat of combustion of the gas is 600 Btu per cu. ft. and it costs one rupee per thousand cubic foot. What is the cost of heating the water in the bath tub if 70% of the fuel heat is utilised in heating the same ? [*Jt. Entrance 1985*] [Ans. 4 paisa]

[**Hints :** Total heat reqd. $=\frac{100}{70}\times 6\times 62{\cdot}5\times(100-55)$ Btu. $=\frac{168750}{7}$ Btu.

Gas reqd. $=\frac{168750}{7\times 600}$ cu. ft. $\therefore$ Cost $=$ Rs. $\frac{168750}{7\times 600\times 1000}=4$ paisa]

38. A copper calorimeter with a copper heating element fitted inside weighs 300 gm. It is filled to a certain mark with 200 gm of a liquid. With 41 watt electrical input to the heater, the temperature rises from 20°C to 45°C in 10 minutes. When 140 gm of the liquid is replaced by 1250 gm of copper such that the liquid level remains the same and the calorimeter heater is supplied with the same electrical input as before, the temperature rises from 20°C to 45°C in 9 min. 5 sec. It is found that a steady temperature of 45°C can be maintained in either case with an electrical input of 2 watt. The room temperature remains constant at 20°C throughout the experiment. Calculate the sp. heats of (*i*) copper and (*ii*) liquid. [*Jt. Entrance 1980*] [Ans. (*i*) 0·095 (*ii*) 1·0]

39. Find the increase in energy per atom of aluminium when temperature of a piece of aluminium increases by $1°C$. 27 gm. of aluminium contain 6×10^{23} atoms and specific heat of aluminium $=0{\cdot}2$. [*I.I.T. 1965*] [Ans. $3{\cdot}78\times 10^{-16}$ erg]

40. A piece of iron and a piece of copper, each of mass 100 gm. are joined together. The combined mass is heated to 100°C. It is then dropped into water in a calorimeter of mass 100gm. containing 132·4 gm. of water at 20°C. The resulting temperature is 30°C. The experiment repeated with a mass of water 294 gm taken in the calorimeter results in a temperature of 25°C. Calculate the sp. heat of copper and iron. [Ans. Cu $=0{\cdot}09$; Fe $=0{\cdot}112$]

3

EXPANSION OF SOLIDS

3.1. Expansion of solid :

Solids, in general, expand when heated. Copper, iron, brass and other metallic substances show marked expansion when heated. The reasons of expansion of a solid are as follows. The model of a crystalline solid may be likened with that of a bed spring [Fig. 3.1]. The atoms of the solid are held together in a regular array by electrical forces which can be compared with the forces exerted by a set of springs connecting the atoms. These 'springs' are evidently quite stiff and there are about 10^{22} of such 'springs' per c.c. of the solid body. The atoms of the solid, we know, are always vibrating. When the temperature of the body is increased, the amplitude of vibration of the atoms increases. Hence, the average distance between atoms also increases as if the springs undergo elastic expansion. This evidently leads to an expansion of the whole solid body as its temperature rises. The change in *any* linear dimension, such as its length, breadth or thickness, is called its linear expansion, while the change in area and volume of the body are called superficial and volume expansion respectively. Solids may therefore have expansion in three ways, *viz.* (i) expansion in length or linear expansion, (ii) expansion in surface area or superficial expansion and (iii) expansion in volume or cubical expansion. The following simple experiments illustrate the three different ways of expansion of solids.

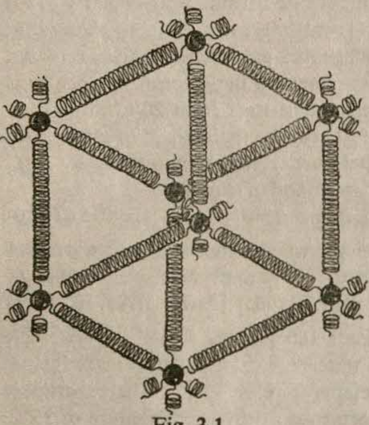


Fig. 3.1

(1) **Bar and Gauge experiment** : *A* is an iron rod with a wooden handle and *B* is metal gauge. At room temperature, the rod *A* just fits in the gauge *B* (Fig. 3.2). But when the rod *A* is heated for sometime, it will not fit into the gauge any more. The rod *A* will fit again when it returns to the room temperature. This shows that the rod *A*, on being heated, has expanded in length.

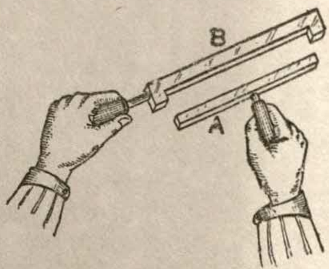


Fig. 3.2

(2) **Ball and ring experiment** : *A* is a hollow brass sphere which can just pass through the ring *B* when they are at room-temperature. On heating the sphere

it will be seen that the sphere is no longer passing through the ring (Fig. 3.3). The sphere will again pass through the ring when it comes back to the initial room temperature. So, the experiment shows that the volume of the sphere has increased due to heating.

On account of the volume expansion of the sphere, the surface area has also increased. So, it may be said that solids, when heated expand in surface area.

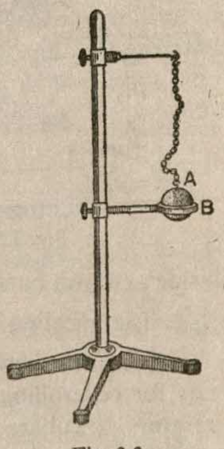


Fig. 3 3

3.2. Different materials expand differently :

Different materials, on being heated equally, are found to expand differently. Brass, for example, expands about one and a half times as much as steel ; aluminium expands about twice as much as steel. The following experiments may be performed to illustrate it.

(1) **Ferguson's experiment** : A metallic rod *PQ* is supported horizontally on two stands *A* and *B* (Fig. 3.4), the end *Q* being in contact with a screw which does not allow the end *Q* to expand. The other end *P* is in touch with a pointer which is pivoted to a vertical rod at the point *O*. The pointed end of the pointer can move over a graduated scale. When the screw at the end *Q* is worked, the end *P* presses against the pointer which, then, moves along the scale. At the start of the experiment, the pointer is to be made coincident with the zero mark of the scale by working the screw. If, now, the rod is heated by Bunsen burners, the pointer will be found to move slowly towards the right on the scale, showing that the rod is expanding in length.

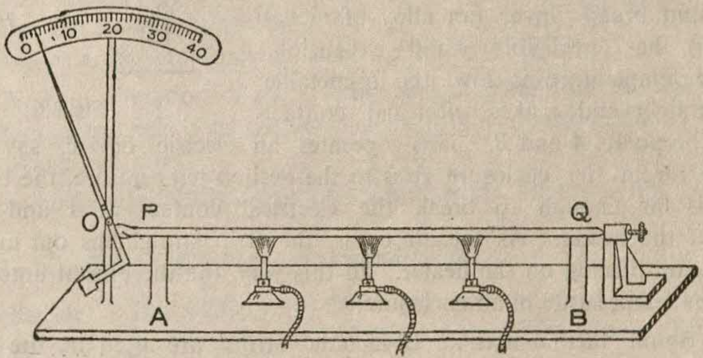


Fig. 3.4

If the above experiment be carried out with rods of same length but of different substances and heated through the same range of temperature, the pointer will give different readings showing that their expansions are not equal.

(2) **The buckling of a bi-metallic strip** : The difference in the expansions of different metals may be shown by riveting together a strip of brass and an equal strip of iron. On heating this bi-metallic strip, it bends so that the brass is on the outside of the curve (Fig. 3.5). The brass thus becomes longer than the iron, showing that it expands more than iron for the same temperature change. If the strip be cooled, it again bends in the form of an arc but this time brass goes inside and iron outside of the curve.

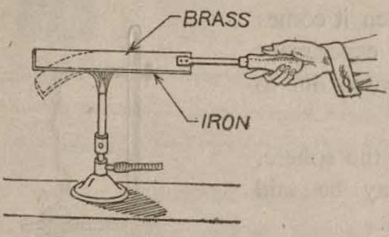


Fig. 3.5

The buckling of bi-metallic strips has several practical applications.

(*a*) **Thermostat** : Bi-metallic strips are used in the construction of thermostats for controlling inside temperature of various electrical apparatus like refrigerators, incubators, hot-house etc. It is an automatic electric switch which closes when the temperature reaches one desired value and opens when it reaches another. One type of thermostat is shown in fig. 3.6.

Ordinarily it consists of a bi-metallic strip of invar and brass. Invar (an alloy of nickel and steel) has negligibly small expansion. When the temperature is low, the bi-metallic strip is straight and makes electrical contact between the points *A* and *B*. This operates an electric device, say a heater. When the air in the enclosure rises to the desired temperature, the bi-metallic strip bends far enough to break the electrical contact at *A* and *B*, thus turning off the heater. As the air cools, the strip straightens out and makes contact, again turning on the heater. In this way, the thermostat automatically controls the temperature of an enclosure.

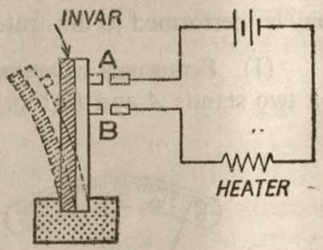


Fig. 3.6

(*b*) **Spiral thermometer** : Bi-metallic strips are used in the construction of a solid maximum-minimum thermometer, known as spiral thermometer (Fig. 3.7). It consists of a composite strip with steel outside and brass inside bent in a spiral, one end of which is fixed on a suitable frame and the other is held free between two pinpoints *P* and *Q*. There are two pointers *AB* and *CD* which are so adjusted that any pressure on *P* by the spiral-end in the direction from right to left will enable the pointer *AB* to move over a scale graduated in celsius or fahrenheit degree. Similarly, any pressure on *Q* in the direction from left to right will allow *CD* to move along the scale.

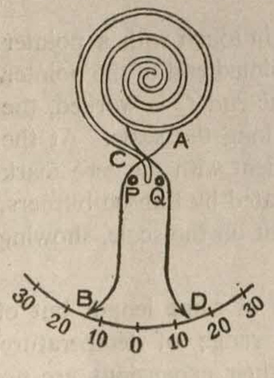


Fig. 3.7

Since the spiral is a bi-metallic strip with more expansible metalbrass inside, a rise of temperature is equivalent to unwinding of the spiral. The free end of the spiral, therefore, presses *P* which results in the movement of the pointer *AB* over the scale. *AB* is held finally in position due to gentle friction applied to it and therefore it reads maximum temperature. Similarly, a lowering of temperature is equivalent to winding of the spiral, which results in the movement of the pointer *CD* over the scale, giving the minimum temperature.

3.3. Enormous force is developed due to expansion or contraction :

The following experiment shows that expansion or contraction may develop enormous force.

A wrought iron bar with a screw (*S*) at one end and a hole at the other is placed horizontally in the grooves of two uprights made of cast iron. A short cast iron bolt (*P*), about ½ inch in diameter, is inserted through the hole of the iron bar. The screw *S* is then tightened after heating the bar for sometime. If the bar be now cooled by sprinkling cold water, the cast-iron bolt *P* will snap. The cooling produces contraction of the bar in length, which, in its turn, develops enormous force sufficient to break the bolt.

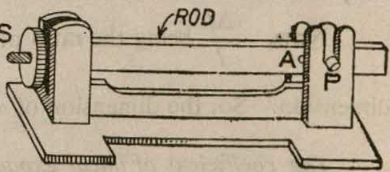


Fig. 3.8

A slight alteration of the arrangement together with the insertion of the bolt *P* through the hole *A* on the bar will show that expansion due to heating may also develop a big force.

[For mathematical discussion on thermal stress, see art. 3.14].

3.4. Coefficient of linear expansion of solids :

Definition : *The coefficient of linear expansion of a substance is the increase in length per unit original length of the substance (in the form of a rod) per degree rise in temperature.*

[**N.B.** For an exact definition, the rise of temperature should be from 0° to 1°. But if the initial temperature be other than 0°, and if the temperature is then raised by 1°, the result will not be very much different from the actual value of the linear coefficient. Moreover, it is not parctically possible to get 0° as the initial temperature in all cases. For these reasons, it is not necessary to mention 0° as the initial temperature in the definition of the coefficient.]

Suppose, the length of a rod at temperature $t_1°$ is l_1 and at a higher temperature $t_2°$ it is l_2. Hence, for $(t_2-t_1)°$ rise of temperature, the increase in length$=l_2-l_1$.

Or, for $(t_2-t_1)°$ rise of temperature, the increase in length per unit original length$=\dfrac{l_2-l_1}{l_1}$

or, for 1° rise of temperature, the increase in length per unit original length$=\frac{l_2-l_1}{l_1(t_2-t_1)}$. If α be the mean coefficient of linear expansion, over the range t_1 to t_2 then according to its definition,

$$\alpha=\frac{l_2-l_1}{l_1(t_2-t_1)}=\frac{\text{Increase in length}}{\text{Original length}\times\text{rise of temperature}}=\frac{\Delta l}{l.\Delta t}.$$

$$\text{or,}\quad l_2-l_1=\alpha.l_1(t_2-t_1)\quad\therefore\quad l_2=l_1\{1+\alpha(t_2-t_1)\}$$

If the initial and final temperatures be 0° and $t°$ respectively then,

$$l_t=l_0(1+\alpha t)$$

When solid is subjected to a small change of temperature about a mean value t, the coefficient of linear expansion α in the neighbourhood of t, may, therefore, be defined by the equation, $\alpha=\frac{1}{l}\cdot\frac{\Delta l}{\Delta t}$, where l is the length of the bar at the temperature t.

Now, $\frac{\Delta l}{l}$ being the ratio of two lengths, is a pure number, but Δt has some dimension. So, the dimension of α is $[\alpha]=\frac{[\text{length}]}{[\text{length}]\times[\text{temp}]}=[\text{temp}]^{-1}$

The coefficient of linear expansion, therefore, does not depend upon the unit of length but it does depend upon the unit of temperature.

Thus, the coefficient of linear expansion of iron is ·000012 per °C means that an iron rod, 1 cm. or 1 ft. or 1 yard long will increase in length by ·000012 cm. or ·000012 ft. or ·000012 yard respectively due to 1°C rise in temperature. If, however, the temperature be given in fahrenheit scale, the value of the coefficient will be different. Since $1°F=\frac{5}{9}°C$, the coefficient of linear expansion of iron will be $\cdot000012\times\frac{5}{9}=\cdot0000067$. In general, it may be written, $\alpha_F=\frac{5}{9}\times\alpha_C$.

Table for the coefficient of linear expansion of some solids

Solids	per °C	per °F
Brass	$1{\cdot}9\times10^{-5}$	$1{\cdot}1\times10^{-5}$
Iron	$1{\cdot}2\times10^{-5}$	$6{\cdot}7\times10^{-6}$
Steel	$1{\cdot}1\times10^{-5}$	$6{\cdot}1\times10^{-6}$
Copper	$1{\cdot}7\times10^{-5}$	$9{\cdot}7\times10^{-6}$
German-silver	$1{\cdot}8\times10^{-5}$	$1{\cdot}0\times10^{-5}$
Invar (Nickel-steel alloy)	$9{\cdot}0\times10^{-7}$	$5{\cdot}0\times10^{-7}$

Examples : (1) *A copper bar is 2 metre long at 0°C. If its temperature is raised to 200°C, its length becomes 200·68 cm. What is the coefficient of linear expansion of copper ?*

Ans. Here, l_1=2 metres=200 cm. ; l_2=200·68 cm ; $t_1=0°C$; $t_2=200°C$.

$$\text{Hence, } \alpha=\frac{l_2-l_1}{l_1(t_2-t_1)}=\frac{200{\cdot}68-200}{200(200-0)}=\frac{0{\cdot}68}{200\times200}$$
$$=\tfrac{68}{4}\times10^{-4}=17\times10^{-4}\text{ per }^\circ C.$$

(2) *A metal bar is 8 ft. long at 68°F. How much expansion in length would take place at 110°F ?* (α *for metal*$=9{\cdot}4\times10^{-6}$ *per* °*F*)

Ans. We know, $\alpha=\dfrac{\text{Increase in length}}{\text{Original length}\times\text{rise in temp.}}$

or, $9{\cdot}4\times10^{-6}=\dfrac{\text{Increase in length}}{8(110-68)}$

$\therefore$ Increase in length$=9{\cdot}4\times8\times42\times10^{-6}=31{\cdot}58\times10^{-4}$ ft.

(3) *What must be the length of a zinc rod at 59°F, if its length is to increase by 5 mm. when the temperature is raised to 100°C ?* (α *for zinc*$=2{\cdot}9\times10^{-5}$ *per* °*C*).

Ans. First the temperature is to be converted into celsius scale. Now, we know, $\dfrac{C}{5}=\dfrac{F-32}{9}$: Here, $F=59^\circ$; So, $\dfrac{C}{5}=\dfrac{59-32}{9}=3 \quad \therefore \quad C=15^\circ$

Again, we know, increase in length$=$original length$\times$coeff$^n\times$diff. in temp.

$\therefore \quad 0{\cdot}5=l\times2{\cdot}9\times10^{-5}(100-15)$ [l=original length]

or, $0{\cdot}5=l\times2{\cdot}9\times10^{-5}\times85$

or, $l=\dfrac{0{\cdot}5}{2{\cdot}9\times10^{-5}\times85}=\dfrac{0{\cdot}5\times10^6}{29\times85}=202{\cdot}9$ cm.

[N. B. Carefully note the different units used in the above examples.]

3.5. Coefficient of surface or superficial expansion :

Definition : *The coefficient of surface expansion of a substance is the increase in surface area per unit original surface area of the substance (in the form of a plate) per degree rise in temperature.*

Suppose, the surface area of a plate is S_1 at temperature t_1° and at a higher temperature t_2, it is S_2.

So, for $(t_2-t_1)^\circ$ rise in temperature, the increase in surface area$=S_2-S_1$

„ „ „ „ „ „ „ „ in surface area per unit original surface area$=\dfrac{S_2-S_1}{S_1}$

or, „ 1° rise in temperature, the increase in surface area per unit original surface area$=\dfrac{S_2-S_1}{S_1(t_2-t_1)}$

If β be the coefficient of surface expansion of the substance then according to its definition, $\beta=\dfrac{S_2-S_1}{S_1(t_2-t_1)}=\dfrac{\text{Increase in area}}{\text{original area}\times\text{rise in temperature}}=\dfrac{\Delta S}{S.\Delta t}$

or, $S_2-S_1=\beta\, S_1(t_2-t_1)$

or, $S_2=S_1\{1+\beta(t_2-t_1)\}$

[N. B. Here, also, for an exact definition, the initial temperature should be 0°. But for ordinary purposes, the initial temperature may be other than 0°.]

If the initial temperature be 0° and final temperature $t°$, then, $S_t = S_0(1+\beta.t)$

Like linear expansion, β may be expressed by the equation, $\beta = \frac{1}{S}\ \frac{\Delta s}{\Delta t}$

$\frac{\Delta s}{S}$ being a ratio, β does not depend on the unit of area but it depends on the unit of temperature.

Thus, the coefficient of surface expansion of iron is ·000024 per °C means that an iron plate, 1 sq. cm. or 1 sq. ft. or 1 sq. yard in area, will increase by ·000024 sq. cm. or ·000024 sq. ft. or ·000024 sq. yard respectively for 1°C rise in temperature. But if the temperature is expressed in Fahrenheit, the value will be different. According to Fahrenheit scale, its value is $\frac{5}{9} \times {\cdot}000024 = {\cdot}0000134$. In general, it may be written $\beta_F = \frac{5}{9} \times \beta_C$.

Example : *A glass window measuring 2 metre*^{2} *was fitted in a wooden frame with an allowance of 24 cm*^{2} *expansion in area. What is the highest temperature difference expected for which the above allowance was kept ? Coefficient of linear expansion of glass*$=9\times10^{-6}/°C$.

Ans. Initial area of the window$=2\times2=4$ sq. metre.
Expected expansion in area$=24\ \text{cm}^2=24\times10^{-4}$ sq. metre.
Coefficient of superficial expansion$=2\times$linear coeff$^n=2\times9\times10^{-6}$
$=18\times10^{-6}/°C$

Now, expansion in area$=$Initial area$\times$coeff$^n\times$diff. in temperature

or, $24\times10^{-4}=4\times18\times10^{-6}\times$diff. in temperature.

$$\therefore \text{ highest temperature diff.} = \frac{24\times10^{-4}}{4\times18\times10^{-6}} = 33{\cdot}3°\text{C (nearly)}$$

3.6. Coefficient of volume or cubical expansion :

Definition : *The coefficient of volume expansion of a substance is the increase in volume per unit original volume of the substance (in the form of a sphere, say) per degree rise in temperature.*

Suppose, the volume of a metallic sphere is V_1 at temperature $t_1°$ and at a higher temperature $t_2°$, it is V_2.

Then, for $(t_2-t_1)°$ rise in temperature, the increase in volume$=V_2-V_1$

∴ ,, ,, ,, ,, ,, ,, ,, ,, ,,

per unit original volume$=\frac{V_2-V_1}{V_1}$

∴ for 1° ,, ,, ,, ,, ,, ,, ,,

per unit original volume$=\frac{V_2-V_1}{V_1(t_2-t_1)}$

If γ be the coefficient of volume expansion, then according to its definition,

$$\gamma = \frac{V_2-V_1}{V_1(t_2-t_1)} = \frac{\text{Increase in volume}}{\text{original volume}\times\text{rise of temp.}} = \frac{\Delta V}{V.\Delta t}$$

or, $V_2 - V_1 = V_1\gamma.(t_2 - t_1)$

or, $V_2 = V_1\{1 + \gamma(t_2 - t_1)\}$

If the initial and final temperatures be 0° and t° respectively, then

$$V_t = V_0\{1 + \gamma.t\}$$

As before, the coefficient of volume expansion does not depend upon the unit of volume but depends upon the unit of temperature. Thus, the coefficient of volume expansion of iron is ·000036 per °C means that a sphere of iron, 1 c.c. or 1 cu. ft. or 1 cu. yard in volume would expand by ·000036 c.c. or ·000036 cu. ft. or ·000036 cu. yard respectively for 1°C rise in temperature. According to fahrenheit scale, however, the value of the coefficient is $\frac{5}{9} \times \cdot 000036 = \cdot 00002$. In general, it may be written $\gamma_F = \gamma_C \times \frac{5}{9}$.

3.7. Relation between the three coefficients of expansion :

Consider a rectangular plate with length a and breadth b at a certain temperature. Its area at that temperature $S_1 = a.b.$

If the temperature be changed (say increased) by θ, the length of the plate will be $a(1+\alpha\theta)$ and breadth $b(1+\alpha.\theta)$ where α=co-efficient of linear expansion.

The present area becomes $S_2 = a(1+\alpha\theta)b(1+\alpha\theta)$

$$= ab(1+\alpha\theta)^2 = S_1(1+2\alpha\theta+\alpha^2\theta^2)$$

$$= S_1(1+2\alpha\theta) \quad .. \quad \text{(i)}$$

[α being very small, $\alpha^2\theta^2$ may be neglected.]

If the coefficient of superficial expansion be β, then according to its definition

$S_2 = S_1(1+\beta\theta)$ .. (ii)

Comparing eq^n. (i) and (ii), we see that $\beta = 2\alpha$

Now consider a rectangular block of length, breadth and height equal to a, b and c respectively. Its volume $V_1 = a.b.c.$

If the temperature of the block be increased by θ, the sides will be $a(1+\alpha.\theta)$, $b(1+\alpha\theta)$ and $c(1+\alpha\theta)$ respectively. So the volume becomes

$$V_2 = a(1+\alpha\theta).b(1+\alpha\theta).c(1+\alpha\theta)$$

$$= a.b.c.\ (1+\alpha\theta)^3 = V_1(1+3\alpha\theta+3\alpha^2\theta^2+\alpha^3\theta^3)$$

$$= V_1(1+3\alpha\theta) \quad .. \quad \text{(iii)}$$

[Neglecting the smaller terms $\alpha^2\theta^2$ and $\alpha^3\theta^3$]

If the coefficient of cubical expansion be γ, then $V_2 = V_1(1+\gamma\theta)$ .. (iv)

From eqns, (iii) and (iv), we get $\gamma = 3\alpha \quad \therefore \quad \alpha = \frac{\beta}{2} = \frac{\gamma}{3}$

[Alternative calculus method :

Let us take a cube of length l. Its area $S = l^2$ and volume $V = l^3$. Differentiating with respect to temperature θ, $\frac{dS}{d\theta} = 2l.\frac{dl}{d\theta}$ and $\frac{dV}{d\theta} = 3l^2.\frac{dl}{d\theta}$

Now, $\beta = \frac{1}{S}.\frac{dS}{d\theta} = \frac{1}{l^2}.\ 2l.\frac{dl}{d\theta} = 2 \times \frac{1}{l}.\frac{dl}{d\theta} = 2\alpha \quad \left(\therefore \quad \alpha = \frac{1}{l}.\frac{dl}{d\theta}\right)$

Again $\gamma = \frac{1}{V}.\frac{dV}{d\theta} = \frac{1}{l^3}.3.l^2\frac{dl}{d\theta} = 3.\frac{1}{l}.\frac{dl}{d\theta} = 3\alpha.$]

3.8. Determination of coefficient of linear expansion of a solid :

In the laboratory, the coefficient of linear expansion of a solid may be determined by Pullinger's apparatus. Fig. 3.9 shows the apparatus.

Description : There is metal cylinder *J* in which a rod *AB* of the material whose co-efficient of linear expansion is to be measured, is inserted. The lower end *B* of the rod rests on a glass slab or a marble slab and the upper end *A* is free to expand through a hole in a cork which closes the mouth of the cylinder. There are two openings—one almost at the top of the cylinder and the other at the bottom of it through which steam can enter and pass out. There are two other openings through which two thermometers T_1 and T_2 are introduced. *G* is a glass plate with a hole at its centre. A spherometer is placed on the glass plate *G* with its central leg just touching the top of the rod *AB* through the hole on the glass plate.

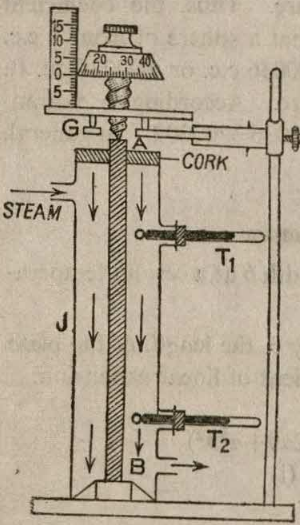


Fig. 3.9

Procedure : Determine the length (*l*) of the rod *AB*. Usually a rod of length 1 metre is taken. Note its initital temperature (t_1) with a thermometer. Place the rod in its position in the cylinder *J* and allow the central leg of the spherometer to just touch the top of the rod. Take the reading of the spherometer. Now by turning the screw of the spherometer, raise the central leg so that the rod *AB* may have sufficient space for expansion. Pass steam through the cylinder. The thermometers T_1 and T_2 will show a gradual increase in temperature. When the thermometers become steady, read the final temperature (t_2). If the thermometers register slightly different temperatures, their average value is to be taken. Screw down the central leg of the spherometer again so as to just touch the top of the rod *AB*. The difference of the two readings of the spherometer will give the increase in length of the rod *AB*. Let it be *x*. Then, we know,

$$\alpha = \frac{\text{Increase in length}}{\text{original length} \times \text{rise of temp.}} = \frac{x}{l(t_2 - t_1)}$$

All the quantities on the *R.H.S.* of the above equation being known, α can be found out.

3.9. Practical applications of the expansion of the solids :

Expansion of solids has many practical applications in the fields of engineering and technology. In our daily life, also, we make use of the expansion of the solids, in some cases to our advantage and in some other to our disadvantage.

Disadvantages : (i) In laying railway tracks, gaps are left between successive lengths of rail to make rooms for the expansion. Sun rays as well as grinding between the wheels of the train and the rails produce much heat. If the gap is not left, the expansion of the rails due to heating may lead to buckling of the track. Rails are joined together by two metal plates —one on each side of the rail, known as '*fish-plate*'—with the help of four bolts. The bolt holes are slotted to allow free movement at the rail joints. But tram lines are welded together to form long, continuous lengths which are bedded in the road having blocks of granite stones and concrete on either side of the lines. This protects the tram lines against considerable rise of temperature. Consequently the lines do not buckle when heated.

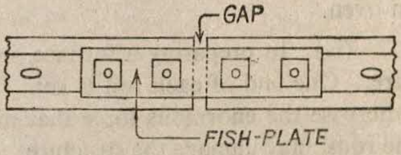


Fig. 3.10

Example : *Railway lines are laid with gaps to allow for expansion. If each piece of rail is 66 ft. long, how much gap is to be left for a temperature difference of 10°C to 67·3°C?* α *for steel* $=11\times10^{-6}$ *per °C.*

Ans. Expansion of 66 ft. long railway line due to a temperature difference of $(67{\cdot}3-10)=57{\cdot}3°C$ is to be first determined.

$$\text{We know, } \alpha=\frac{\text{Increase in length}}{\text{Original length}\times\text{rise of temp.}}$$

So, increase in length = Original length × rise of temp. × α

$$=66\times57{\cdot}3\times11\times10^{-6}\text{ ft.}={\cdot}041\text{ ft.}\;=0{\cdot}5\text{ inch. (nearly)}$$

Hence about 0·5 inch gap is to be left between each two pieces of rail.

(ii) Allowance has also to be provided for the expansion of iron structure of bridges. Various methods are used to overcome the difficulty, a common one being to have one end only of the structure rigidly fixed while the other to rest on rollers (Fig. 3.11). When expansion or contraction takes place, this end can move freely over the rollers.

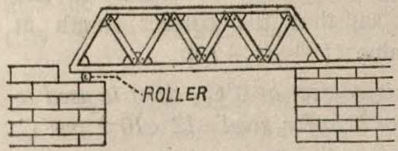


Fig. 3.11

Example : *Assuming that the highest summer temperature is 45°C and the lowest winter temperature is* $-15°C$, *what allowance must be provided for expansion in one of the 1700 ft. steel span of a bridge?* (α *for steel* $=12\times10^{-6}$ *per °C*)

Ans. We know, increase in length = Original length × rise of temp. × coeffⁿ

$$=1700\times[45-(-15)]\times{\cdot}000012$$
$$=1700\times60\times{\cdot}000012=1{\cdot}22\text{ ft.}$$

So, 1·22 ft. gap is to be left for expansion.

(iii) A thick-walled glass vessel often cracks when hot water is poured in it. The inside and the outside walls of a thick-walled vessel do not get equal quantities of heat because glass is not a good thermal conductor and therefore, the walls undergo unequal expansions. It produces stress on the material of the vessel and it cracks. For this reason, sufficient care is taken while manufacturing glass vessels, chimneys, etc.

A piece of stone, also, cracks due to the same reason. Sometimes, pieces of stones mixed with coal, crack with loud report, when the coal is put in an oven.

(iv) In preparing a furnace, iron rods are inserted into a brick-built structure. One end of each rod is kept free to expand due to the heat of the furnace, otherwise the enormous force that may be brought into play by the expansion of the rods, may damage the structure.

(v) Metallic scales, used for the measurement of lengths, are to be corrected for expansion. Such scales can measure lengths correctly at a temperature at which the graduations are made. At all other temperatures, necessary corrections for contraction or expansion of the scale graduations are to be made.

Suppose the scale graduations of a given scale are correct at temperature t_1°C. If the temperature increases to t_2°, say, the distance between any two divisions of the scale increases. Its reading is therefore, too low [fig. 3.12]. Suppose, the correct length of a rod at t_1°C is 1 cm. When the temperature increases to t_2°C, the length should be $(1+x)$ cm but the scale will read it 1 cm. as before. That is, the scale reading is x cm. less than the actual length. Similarly, if the temperature falls to t_2°C, the scale reading will be too high. [Fig. 3.12]

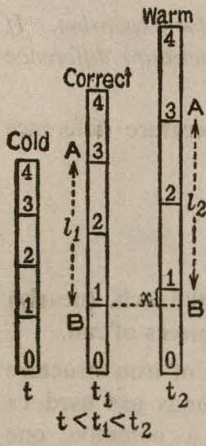


Fig. 3.12

Let the scale reading of the distance between two marks A and B at temperature t_1 is l_1 cm. At a higher temperature t_2°C, the actual distance$=l_1\{1+\alpha(t_2-t_1)\}$. But according to the scale graduation, the distance will still be l_1 cm. So, we can say that the correct length at temperature t_2°C$=$scale value$\times\{1+\alpha(t_2-t_1)\}$.

Example : *A metre scale made of steel is correct at* 0°C. *If it is used to measure distance at 15°C, what will be the error? α for steel$=12\times10^{-6}$ per°C.*

Ans. At 15°C, there will be some expansion in length of the scale. We know, correct length at 15°C$=$scale value$\times\{1+\alpha(t_2-t_1)\}=100\times\{1+12\times10^{-6}\times 15\}=(100+0{\cdot}018)$ cm $=100{\cdot}018$ cm.

So, at 15°C, a length shown as 1 metre or 100 cm. by the scale, is actually 100·018 cm. long.

(vi) The coefficient of linear expansion of glass is not equal to those of other metals. For this reason, a metal wire can not be fused into a glass rod. Unequal expansion or contraction of glass and metals, will leave a gap and the fusion will not be air tight. But because platinum expands almost equally with glass, a platinum wire can easily be fused in a glass tube without any fear of a crack.

Example : *In order to make connection to the carbon anode of a transmitting valve, it is required to thread a copper wire, 2mm in diameter at 20°C, through a smaller hole in the carbon block. If the process can just be carried out by immersing both specimens in dry ice at$-$80°C, calculate the diameter of the hole at 20°C. The coefficients of linear expansion of copper and carbon are 17×10^{-6} and 5×10^{-6}/°C.*

Ans. Let $(d)_{20}$ be the diameter of the hole at 20°C.

Now, $(d)_{-80}=(d)_{20}\{1+5\times10^{-6}(-80-20)\}$

$=(d)_{20}\{1-5\times10^{-6}\times100\}=0{\cdot}9995\,(d)_{20}$

Again, the diameter of the copper wire at $-80°C$,

$=0{\cdot}2\{1+17\times10^{-6}(-80-20)\}=0{\cdot}2\{1-17\times10^{-6}\times100\}=0{\cdot}19966$ cm.

$\therefore\ 0{\cdot}9995.(d)_{20}=0{\cdot}19966$

or $(d)_{20}=\dfrac{0{\cdot}19966}{0{\cdot}9995}=0{\cdot}19976$ cm$=1{\cdot}998$ mm.

(vii) While installing overhead wires of a power transmission system or of telegraph or telephone system, allowance is provided for its expansion or contraction. The wire is so installed between two posts that it can sag or tighten according as it expands or contracts in summer and winter.

Advantages :

(i) You must have heard the process of riveting together the steel plates and girders used in ship building and other constructional works. The plates to be riveted are placed one above the other and a hole is drilled through them. The rivets are first made hot. This softens them so that they are easily burred into a head by hammer after they are inserted into the holes. Their contraction on cooling serves to hold the plates tightly together.

(ii) Expansion and contraction of iron rods are used to straighten the walls of a building which has buckled on the outside. Iron rods are inserted into the walls and fixed tightly with iron plates and screws. The rods are then heated and the screws are tightened more. Afterwards, when the rods become cold, they try to contract lengthwise. The contractile force straightens the curved walls.

(iii) The wheels of hand-carts or bullock-carts are fitted with steel tyres, which have to be renewed from time to time owing to wear. To ensure a tight fit, the diameter of the tyre is kept slightly smaller than that of the wheel. The steel tyre is at first heated uniformly. The resulting expansion enables the tyre to be easily slipped over the wheel. When the tyre cools, its contraction makes a tight fit.

Example : *The diameter of an iron tyre is 99·8 cm. At what temperature will it fit on a wheel whose diameter is 100 cm. and initial temperature is 15°C? With what force per unit area will the tyre press against the wheel when cold?* ($\alpha=1{\cdot}2\times10^{-5}$ *per °C ; Young's modulus of iron* $Y=2\times10^{12}$ *dynes/cm*^{2})

Ans. The circumference of the tyre$=(\pi\times99{\cdot}8)$ cm.

" " " " wheel$=(\pi\times100)$ cm.

So, to fit the tyre on the wheel, the necessary expansion of the circumference of the tyre$=\pi(100-99{\cdot}8)=\pi\times0{\cdot}2$

We know, increase in length=Original length × rise in temp. × coeff^n

or, $\pi\times0{\cdot}2=99{\cdot}8\pi\times(t-15)\times1{\cdot}2\times10^{-5}$

$\therefore\ t-15=\dfrac{0{\cdot}2}{99{\cdot}8\times1{\cdot}2\times10^{-5}}=167$ (approx) or, $t=182°C$

So, the tyre is to be heated to 182°C.

When cold, the diameter of the tyre will decrease from 100 cm to 99·8 cm. It may be supposed that some external force has reduced the diameter by (100 − 99·8) = 0·2 cm. The tyre will press against the wheel with this force.

$$\text{Now,} \quad Y = \frac{\text{Force applied per unit area}}{\text{change of length / original length}}$$

$$\therefore \quad \text{force applied / unit area} = \frac{0{\cdot}2}{100} \times Y = \frac{0{\cdot}2}{100} \times 2 \times 10^{12} = 4 \times 10^{9} \text{ dynes/cm}^2$$

(iv) When the glass stopper of a bottle becomes very tight we generally heat the bottle to loosen it. When the mouth of the bottle is heated, its neck expands slightly but glass being a bad conductor of heat, the stopper receives no heat from the bottle and does not expand. Consequently, the stopper becomes loose.

3.10. Differential expansion :

Different materials expand differently. The differential expansion of various metals has many practical uses. Suppose, two rods AB and CD of different metals, having lengths l_1 and l_2 respectively at temperature t_1 are rigidly connected at A and C [Fig. 3.13]. If l_1' and l_2' be their lengths respectively at a higher temperature $t_2°$, then $l_1' = l_1\{1 + \alpha_1(t_2 - t_1)\}$ and $l_2' = l_2\{1 + \alpha_2(t_2 - t_1)\}$ where α_1 and α_2 are the coefficients of linear expansion of the metals.

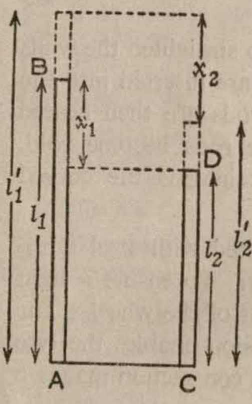


Fig. 3.13

Now, the distance between the ends B and D,

$$x_2 = l_1' - l_2' = l_1 - l_2 + (t_2 - t_1)\ (l_1\alpha_1 - l_2\alpha_2)$$

$$\text{or} \quad x_2 = x_1 + (t_2 - t_1)\ (l_1\alpha_1 - l_2\alpha_2) \quad \dots \quad \dots \quad \text{(i)}$$

Selecting lengths and metals in a proper way, the distance BD may be made to vary with temperature in any one of the following ways.

(a) If the bar CD be made of invar (expansion of invar is negligibly small) the end D will not move with changes of temperature. Ignoring α_2 in eqn. (i) we can write, $x_2 = x_1 + l_1\alpha_1(t_2 - t_1)$.

Thus the short distance BD expands by the same amount as the long distance AB. Consequently, the relative expansion of BD with temperature is much greater than that of a bar of length x_1.

(b) If the bar AB be made of invar, the end B will not move with changes of temperature. Consequently, the distance BD shrinks rapidly as the temperature rises. This principle is applied in a thermostat which is used to maintain refrigerators at constant temperature.

(c) By a suitable choice of lengths and metals, $(l_1\alpha_1 - l_2\alpha_2)$ may be made equal to zero. In that case, we get, $\dfrac{l_1}{l_2} = \dfrac{\alpha_2}{\alpha_1}$ or $l_1\alpha_1 = l_2\alpha_2$

From eqn. (i) it is seen that in this condition $x_2=x_1$ *i.e.* the distance BD does not change with temperature. This is the principle of a compensated pendulum.

Example : *Two bars of brass and steel, standing side by side, have one end of each rigidly attached to the other. The other ends are free to expand. The steel bar is 2 metre long. What should be the length of the brass bar so that the distance between the free ends of the bars remains the same at all temperatures ?* α *for brass*$=2\times10^{-5}/^{\circ}C$ *and* α *for steel*$=1{\cdot}2\times10^{-5}/^{\circ}C$.

Ans. The distance between the free ends of the bars remains the same at all temperatures if $l_1\alpha_1=l_2\alpha_2$ or $2\times100\times1{\cdot}2\times10^{-5}=l_2\times2\times10^{-5}$

$$\therefore\ l=120\text{ cm}=1{\cdot}2\text{ metre.}$$

3.11. Compensated pendulum :

Pendulum clocks keep time by the oscillations of its bob which is generally hung by metallic rods. In summer and winter the length of the rod *i.e.*, the length of the pendulum changes. Since the time period of a pendulum depends on its length, the pendulum clocks will not be able to keep correct time owing to the expansion or the contraction of the pendulum length when temperature variations occur. In summer, the length increases due to rise in temperature and the clock goes slow. In winter on the other hand, the length contracts due to fall of temperature and the clock goes fast. To keep correct time, pendulum clocks should have arrangement for compensating the change of length due to temperature change. Such pendulums are known as *compensated pendulums*.

The table of co-efficients of linear expansion shows that the coefficient of linear expansion of brass is nearly one and half times that of iron. Hence, a brass rod 1 ft. long and an iron rod 1½ ft. long will expand equally for equal rise of temperature. Or, we may say, a brass rod 2 ft. long and an iron rod 3 ft. long will have equal expansion for equal rise of temperature. So, if an iron rod of length 3 ft. and a brass rod of length 2 ft. are placed side by side with one end riveted together, the distance between the other ends of the rods will be always 1 ft. whatever may be the change of temperature [Fig. 3.14]. This is the underlying principle of construction of a compensated pendulum where the distance between the point of suspension to the centre of gravity of the bob is to be kept constant. This distance is known as the effective length of the pendulum.

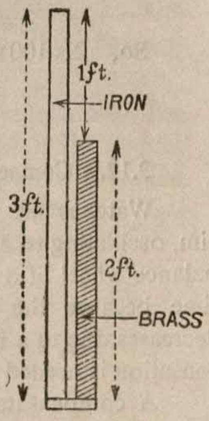


Fig. 3.14

Harrison's grid-iron pendulum : It is a compensated pendulum [Fig 3.15]. Its principle depends upon the differential expansion of a set of brass and iron rods. The co-efficients of linear expansion of brass and iron are approximately in the ratio of 3 : 2. In the grid-iron pendulum, arrangements have been made such that three lengths of iron expand so as to lower the bob, while two lengths of brass expand so as to raise it. The net result is that the position of the bob remains the same whatever may be the temperature-change. In fig. 3.15, the black lines represent the iron rods which expand upward and the other lines brass rods, which expand downward.

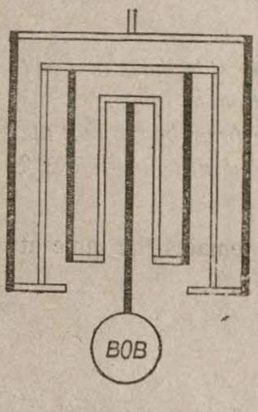


Fig. 3.15

If the average length of each iron rod be l_1 and that of a brass rod l_2, then we can write,

$3l_1\alpha_1.t=2l_2.\alpha_2.t$ or $\frac{l_1}{l_2}=\frac{2\alpha_2}{3\alpha_1}$

The problem of compensation has, to a great extent, been minimised by the discovery of '*invar*'—an alloy of nickel (36%) and steel (64%), because its co-efficient of expansion is negligibly small (·0000009). So, a pendulum, made of invar, will have its length almost unaltered due to seasonal variation of temperature.

Example : *A compensated pendulum has 3 iron rods and 2 brass rods. Each iron rod is, on the average, 100 cm. long and its co-efficient of expansion is* $1{\cdot}2\times10^{-5}$. *If the coefficient of expansion of brass be* $1{\cdot}9\times10^{-5}$, *what is the average length of each brass rod?*

Ans. The total expansion of two iron rods
=total expansion of one brass rod.

Now, the expansion of two iron rods for a rise of t°C
$=2\times100\times{\cdot}000012\times t$

and, the expansion of one brass rod for the same rise of temp.
$=l\times{\cdot}000019\times t$ [l=average length of each brass rod]

So, $2\times100\times{\cdot}000012\times t=l\times{\cdot}000019\times t$

$$\text{or,}\quad l=\frac{2\times100\times{\cdot}000012}{{\cdot}000019}=126{\cdot}3\text{ cm.}$$

3.12. Compensated balance wheel of a watch :

Watches are controlled by a balance. It is a small wheel with a heavy rim, oscillating to and fro under the action of a hairspring. This is called the balance wheel of a watch. An increase in temperature causes the watch to loose time because the diameter of the wheel increases. Again, if the diameter decreases due to a fall of temperature, the watch gains time and goes fast. Compensation is needed to enable the balance wheel to keep correct time.

A compensated balance wheel is generally made by dividing its rim into three portions, each portion being made of a bi-metallic strip [Fig. 3.16]. The strip has a more expansible metal like brass on the outside and stainless steel on the inside. One end of each portion is connected to a rod and the other end carries a small heavy screw. When the temperature rises, the strip bends inwards, compensating the outward expansion of the rods which tend to increase the diameter of the wheel. Consequently the screws

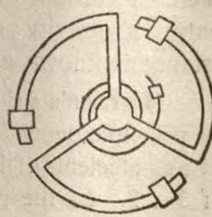


Fig. 3.16

remain at the same distance from the centre of the wheel and keep the time-period unaltered.

3.13. Graham's mercurial pendulum :

It is a compensated pendulum devised by Graham. The pendulum originally consisted of a cast-iron cylinder *A* to which is screwed an iron rod *B*. The cylinder *A* contains mercury. Now-a-days, a glass cylinder is taken in place of cast-iron cylinder. Fig. 3.17 shows a form of Graham's mercurial pendulum.

When temperature rises, the iron rod expands downwards while the mercury expands upwards. The C.G. of mercury is thereby raised. By suitably adjusting the quantity of mercury in the cylinder *A*, the rise of its C.G may be made to counter balance the extension of the rod *B*, so that the length between the centre of suspension and the centre of oscillation remains unchanged. When the temperature falls, the iron rod *B* contracts in length but at the same time, the volume of mercury also shrinks, lowering the C.G. of the mercury. The two being equal, the effective length of the pendulum remains the same.

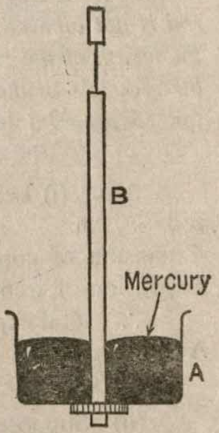


Fig. 3.17

3.14. Thermal stress :

If a rod, with two ends rigidly fixed, be heated or cooled, its tendency to increase or decrease its length will be prevented. As a result a large amount of tensile stress or compressive stress will develop inside the rod. This stress is called *thermal stress.*

Suppose the length and cross-section of a rod are l and A respectively. The coefficient of linear expansion of the material of the rod is α. If the rod be heated through t°C, its linear expansion$=l.\alpha.t$. If the rod returns to its initial temperature, the rod will try to contract to its original length. Suppose the contraction is prevented by applying a force F along the length of the rod. From Hooke's law, we can write,

$$Y=\frac{\text{longitudinal stress}}{\text{,, strain}}=\frac{\text{Applied force}}{\text{cross-section}}\Big/\frac{\text{increase in length}}{\text{original length}}$$

$$=\frac{F/A}{l.\alpha.t/l}=\frac{F}{A.\alpha.t} \quad \therefore \quad F=Y.A.\alpha.t$$

Hence the thermal stress on the rod$=\frac{F}{A}=Y.\alpha.t.$

Examples : (1) *The ends of a steel bar are kept rigidly fixed so that there is no scope for linear expansion. The temperature of the bar is 0°C. Find the thermal stress developed in the bar when it is heated to 30°C. Y for steel*$=2{\cdot}1\times10^{12}$. *dynes/sq. cm. and* $\alpha=1{\cdot}1\times10^{-5}$/°C.

Ans. Thermal stress $=Y.\alpha.t.$; in the present case, $Y=2{\cdot}1\times10^{12}$ dynes/sq. cm. ; $\alpha=1{\cdot}1\times10^{-5}/°C$; $t=(30-0)=30°C$

$\therefore$ Thermal stress $=2{\cdot}1\times10^{12}\times1{\cdot}1\times10^{-5}\times30=6{\cdot}93\times10^{8}$ dynes/sq.cm.

(2) *A composite rod is made by joining a copper rod, end to end, with a second rod of a different material but of the same cross section. At 25°C, the composite rod is 1 metre in length, of which the length of the copper rod is 30 cm. At 125°C, the length of the composite rod increase by 1·91 mm. When the composite rod is not allowed to expand by holding it between two rigid walls, it is found that the length of the two constituents do not change with the rise of temperature. Find the Young's modulus and the coefficient of linear expansion of the second rod. Y for copper*$=1{\cdot}3\times10^{11}$ *newton*/m^2; $\alpha=1{\cdot}7\times10^{-5}$ *per °C.*

[*I.I.T. 1979, Jt. Entrance 1984*]

Ans. (i) Length of the copper rod at $25°C=30$ cm. and that of the second rod $=70$ cm.

Expansion of copper rod $=30\times1{\cdot}7\times10^{-5}\times(125-25)=5{\cdot}1\times10^{-2}$ cm.

Expansion of second rod $=70\times\alpha\times100=7000\,\alpha$ cm.

$\therefore$ total expansion $=(5{\cdot}1\times10^{-2}+7000\alpha)$ cm.

According to the question $5{\cdot}1\times10^{-2}+7000\alpha=0{\cdot}191$

or $7000\alpha=0{\cdot}191-0{\cdot}051=0{\cdot}14$ $\therefore$ $\alpha=2\times10^{-5}/°C$

(ii) Suppose l is the compression in length of the copper rod and F the force acting on the rod. Then, $Y=\frac{F}{a}\cdot\frac{L}{l}$ where, L=original length and a=cross sectional area. $\therefore$ $l=\frac{F}{a}\cdot\frac{L}{Y}$; for copper rod $l=30\times1{\cdot}7\times10^{-5}\times100$

$$\therefore\ 30\times1{\cdot}7\times10^{-5}\times100=\frac{F}{a}\times\frac{30}{1{\cdot}3\times10^{11}}$$

Similarly for second rod, $70\times2\times10^{-5}\times100=\frac{F}{a}\times\frac{70}{Y}$

Dividing, $\dfrac{30\times1{\cdot}7}{70\times2}=\dfrac{30}{70}\times\dfrac{Y}{1{\cdot}3\times10^{11}}$ or, $Y=\dfrac{1{\cdot}7\times1{\cdot}3\times10^{11}}{2}=1{\cdot}11\times10^{11}$ newton/m^2

3.15. Volume expansion of hollow vessel :

A hollow vessel has cubical expansion same as a solid vessel of same volume and material. Take a solid rectangular block of metal and a hollow block of same metal having equal length, breadth and height. Heat them equally. Both will have same expansion in length, breadth and height and hence same expansion in volume. So, the same volume expansion equation or coefficient of expansion which were used in the case of cubical expansion of solid substance may also be used in the case of hollow ones.

Example : *A hollow cylindrical vessel is heated to a range of temperature. Show that the increase in volume is the same as it would be for a solid cylinder of the same dimensions, composed of same substance and heated through the same range of temperatures.*

Ans. Let the radius and the length of the hollow cylinder be r_1 and l_1 at $t_1°$. Its volume $V_1=\pi r_1^2 l_1$. If the temperature be raised to $t_2°$, the new radius $r_2=r_1\,[1+\alpha(t_2-t_1)]$ and new length $l_2=l_1[1+\alpha(t_2-t_1)]$.

So, at $t_2°$, the new volume $V_2=\pi r_2^2 l_2=\pi r^2_1\{1+\alpha(t_2-t_1)\}^2\times l_1\{1+\alpha(t_2-t_1)\}$

$=\pi r_1^2 l_1\{1+\alpha(t_2-t_1)\}^3=\pi r_1^2 l_1\{1+3\alpha(t_2-t_1)\}=V_1\{1+\gamma(t_2-t_1)\}$ $[\because\ \gamma=3\alpha]$

For the volume expansion of a solid cylinder, we have $V_2=V_1\{1+\gamma(t_1-t_2)\}$. Hence, the two expansions are equal.

3.16. Change of density of a solid due to change of temperature :

When a solid substance is heated, it expands in volume but its mass remains unchanged. Hence, the density of the substance will decrease.

Let V_0 and D_0 be the volume and density of a solid substance at 0°. Let V_t and D_t be the corresponding quantities at a higher temperature $t°$.

Since, the mass of the body remains unchanged, we have,

$$V_0\times D_0=V_t\times D_t \quad \text{or} \quad \frac{D_0}{D_t}=\frac{V_t}{V_0}$$

But according to the art. 3.6 we know, $V_t=V_0(1+\gamma t)$

[γ=coeff. of volume expansion of the solid]

$$\text{So,}\ \frac{D_0}{D_t}=\frac{V_0(1+\gamma t)}{V_0}=1+\gamma t \quad \therefore\quad D_0=D_t(1+\gamma.t)$$

Generally, if the density of a solid substance be D_1 and D_2 at temperatures t_1 and t_2 respectively $(t_2>t_1)$, then, $D_1=D_2\{1+\gamma(t_2-t_1)\}$

From the above relation, we get $\gamma=\dfrac{D_1-D_2}{D_2(t_2-t_1)}$

Finding the densities of a solid at two different temperatures, the coefficient of volume expansion γ can be determined from the above relation.

Examples : (1) *A zinc rod is measured by means of a brass scale, correct at 0°C, and is found to be 1·0001 metres long at 10°C. What is the real length of the rod at 0°C?* α *for Zn*$=2{\cdot}9\times10^{-5}$ *and* α *for brass*$=1{\cdot}9\times10^{-5}$.

[*H. S. Exam. 1969*]

Ans. Since the scale is correct at 0°C, every metre mark will extend in length at 10°C. So, at 10°C, the real length of each metre division

$=1(1+{\cdot}000019\times10)$ metre$=1(1+{\cdot}00019)$ metre.

But the scale will show it as 1 metre. *i.e.*, what the scale is showing as 1 metre at 10°C is really $(1+{\cdot}00019)$ metre.

So, the length which is measured as 1·0001 metre at 10°C by the scale, is actually $1{\cdot}0001(1+{\cdot}00019)=1{\cdot}00029$ metre long.

In other words, the correct length of *Zn* rod at 10°C$=1{\cdot}00029$ metres.

Now, suppose l_0=real length of the zinc rod at 0°C. Then, according to the linear expansion of zinc rod, $1{\cdot}00029=l_0(1+{\cdot}000029\times10)=l_0(1+{\cdot}00029)$

$$\therefore\ l_0=\frac{1{\cdot}00029}{1{\cdot}00029}=1\ \text{metre.}$$

(2) *The diameter of a metal sphere is 4·02 cm,; there is a hole of 4·00 cm. diameter in a brass plate. Temperature of both is 30°C. To what temperature*

should the brass plate be raised so that the sphere (still at 30°C) may just pass through the hole ? α *for brass* $=18\times10^{-6}$ *per °C.*

Ans. The required increase in the diameter of the hole

$$=4{\cdot}02-4{\cdot}00=0{\cdot}02 \text{ cm.}$$

Suppose, the required temperature is $t°C$. In this case,

Increase in diameter = Original diameter × coeff^n × temp. rise

$$\text{or, } 0{\cdot}02=4\times18\times10^{-6}\times(t-30)$$

$$\text{or, } t-30=\frac{{\cdot}02}{4\times18\times10^{-6}}=277{\cdot}8 \text{ or, } t=307{\cdot}8°C.$$

(3) *A pendulum clock keeps correct time at 15°C. How many seconds will it go slow at 25°C? The pendulum rod is made of steel whose* $\alpha=11\times10^{-6}/°C$.

Ans. Suppose, the length of the pendulum at 15°C is l_1 and at 25°C l_2. Considering the linear expansion, $l_2=l_1(1+11\times10^{-6}\times10)=l_1(1+11\times10^{-5})$.

Since the pendulum keeps correct time at 15°C, $1=\pi\sqrt{\frac{l_1}{g}}$ .. (i)

Suppose the pendulum completes a beat (*i.e.*, half oscillation) in t sec at 25°C.

Then, $t=\pi\sqrt{\frac{l_2}{g}}=\pi\sqrt{\frac{l_1(1+11\times10^{-5})}{g}}$ .. (ii)

Dividing (ii) by (i), we get $t=\sqrt{1+11\times10^{-5}}=1+\frac{1}{2}\times11\times10^{-5}=1{\cdot}000055$ sec.

In a day there are 86400 sec. So at 25°C, the number of beats executed by the pendulum in a day $=\frac{86400}{1{\cdot}00005}=86395{\cdot}6$ (nearly)

Hence, the pendulum clock will go slow by $(86400-86395{\cdot}6)=4{\cdot}4$ sec. (nearly) in a day at 25°C.

(4) *The volume and mass of an iron ball at 30°C are 1000 c.c. and 7·8 kg respectively. What will be its volume and density at 100°C ?* α *for iron* $=1{\cdot}2\times10^{-5}$ *per°C.*

Ans. If V_{100} be the volume of the ball at 100°C, then we can write,

$$V_{100}=V_{30}[1+\gamma(100-30)]$$

$$\text{or, } V_{100}=1000[1+3\times1{\cdot}2\times10^{-5}\times70] \qquad [\gamma=3\alpha]$$

$$=1000[1+25{\cdot}2\times10^{-4}]$$

$$=1000+2{\cdot}52=1002{\cdot}52 \text{ c.c.}$$

The mass of the ball will not change due to the change of temperature. So, its density at $100°C=\frac{7{\cdot}8\times1000}{1002{\cdot}52}$ gm/c.c. $=7{\cdot}78$gm/c.c.

(5) *A piece of metal weighs 46 gm in air. When it is immersed in a liquid of specific gravity 1·24 at 27°C, it weighs 30 gm. When the temperature of the liquid is raised to 42°C, the metal piece weighs 30·5 gm. Specific gravity of the liquid at 42°C is 1·2. Calculate the coefficient of linear expansion of the metal.* [*I.I.T. 1974*]

Ans. Loss of weight of the metal at $27°C=46-30=16$ gm.

This is equal to the weight of displaced liquid. So, the volume of the displaced liquid $=\frac{16}{1{\cdot}24}=\frac{400}{31}$ c.c. ∴ The volume of the piece at 27°C is $V_{27}=\frac{400}{31}$ c.c.

Hence, the volume of the piece at 42°C, according to the cubical expansion law, $V_{42}=V_{27}\{1\times\gamma(42-27)\}=\frac{400}{31}\{1\times3.\alpha\times15\}$ where, α=coefficient of linear expansion=$\gamma/3$. $\therefore\ V_{42}=\frac{400}{31}(1+45.\alpha)$ .. (i)

But the loss of the weight of the piece at 42°C=46−30·5=15·5 gm.

The volume of the displaced liquid$=\frac{15\cdot5}{1\cdot2}=\frac{155}{12}$ c.c.

$\therefore$ Volume of the metal at 42°C is $V_{42}=\frac{155}{12}$ c.c. .. (ii)

From (i) and (ii), $\frac{400}{31}(1+45.\alpha)=\frac{155}{12}$

or $45.\alpha=\frac{961}{960}-1=\frac{1}{960}$ $\therefore$ $\alpha=2\cdot31\times10^{-5}$

(6) *Three iron rods form an isosceles triangle. Will its base angles change due to a change of temperature?* [*H. S. Exam. 1983*]

Ans. Since the two sides of an isosceles triangle are equal and the rods are made of same material, the increase in length of the rods will be equal *i.e.* the triangle will remain isosceles after the rise of temperature.

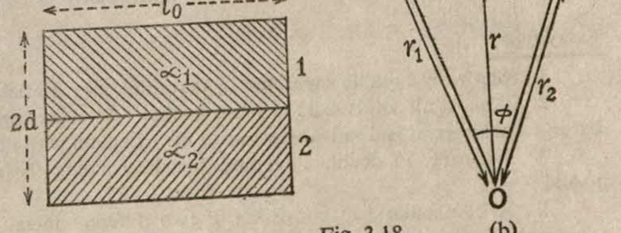
Suppose, the base angle before the change of temperature=θ ; the length of any of the equal rods=a, the length of the base=$2b$, final base angle=θ_1.

Hence, $\cos\theta=\frac{b}{a}$ and $\cos\theta_1=\frac{b(1+\alpha t)}{a(1+\alpha t)}=b/a=\cos\theta$.

$\therefore$ $\theta=\theta_1$ *i.e.* the base angle will remain unaltered.

(7) *Two metal strips, each of length l_0 and breadth d are so riveted together at temperature θ that their edges coincide. The coefficients of linear expansion of the metals are α_1 and α_2 ($\alpha_1>\alpha_2$). When the temperature of the bi-metallic strip is raised to ($\theta+\delta\theta$), it bends in the form of an arc. Find the radius of curvature of the arc.*

Ans. The strips are denoted by the numerals 1 and 2 in the figure. When the temperature is raised from θ to $\theta+\delta\theta$, the bi-metallic strip will bend into an arc because the constituent metals have different expansibility. Suppose, the lengths of the strips at higher temperature are l_1 and l_2 respectively. Then, $l_1=l_0(1+\alpha_1\delta\theta)$ and $l_2=l_0(1+\alpha_2.\delta\theta)$. Suppose the radii of curvature of the strips are r_1 and r_2 respectively. Then $l_1=\phi r_1=l_0(1+\alpha_1\delta\theta)$, [fig. 3.18 (b)] ..(i) and $l_2=\phi r_2=l_0(1+\alpha_2\delta\theta.)$, ... (ii) where ϕ is the angle in radian, at the centre of curvature.



(a) Fig. 3.18 (b)

$$\therefore\ \phi(r_1-r_2)=l_0(\alpha_1-\alpha_2)\delta\theta \quad\text{or}\quad \phi=\frac{l_0(\alpha_1-\alpha_2)\delta\theta}{r_1-r_2}$$

But, $r_1-r_2=2d$; So $\phi=\dfrac{l_0(\alpha_1-\alpha_2)\delta\theta}{2d}$ .. (iii)

Adding eqn. (i) and (ii), $\phi(r_1+r_2)=2l_0+l_0(\alpha_1+\alpha_2)\delta\theta$

$$\therefore\ r_1+r_2=\frac{2l_0+l_0(\alpha_1+\alpha_2)\delta\theta}{\phi}$$

If the mean radius of curvature of the bent strip be r, then,

$r=\dfrac{r_1+r_2}{2}=\dfrac{2l_0+l_0(\alpha_1+\alpha_2)\delta\theta}{2\phi}$. Substituting the value of ϕ from eqn. (iii),

$$r=\frac{l_0\{2+(\alpha_1+\alpha_2)\delta\theta\}}{2l_0(\alpha_1-\alpha_2)\delta\theta}\times 2d=\frac{\{2+(\alpha_1+\alpha_2)\delta\theta\}d}{(\alpha_1-\alpha_2)\delta\theta}$$

(8) *A pressure of 6×10^6 dynes/cm^2 is applied on a solid cube from all sides, compressing the volume of the cube. To what extent should the cube be heated so that it may be restored to its original volume ? Coefficient of volume expansion of the cube$=35{\cdot}7\times10^{-6}/^\circ C$ and its bulk modulus of elasticity$=1{\cdot}03\times10^{11}$ dynes/cm^2.*

Ans. Let the initial volume of the cube be V and v the decrease in volume due to pressure. From the definition of bulk modulus, we have

$$1{\cdot}03\times10^{11}=\frac{\text{Pressure}}{\text{Volume strain}}=\frac{6\times10^6}{v/V} \quad\therefore\quad v=\frac{6\times10^6\times V}{1{\cdot}03\times10^{11}}$$

Again from cubical expansion, we get, increase in volume=Initial volume × coeff^n of volume expansion × diff. in temperature.

or, $v=V\times35{\cdot}7\times10^{-6}\times\theta$ [θ=rise of temperature]

or, $\dfrac{6\times10^6\times V}{1{\cdot}03\times10^{11}}=V\times35{\cdot}7\times10^{-6}\times\theta$

$$\therefore\quad \theta=\frac{6\times10^6\times10^6}{1{\cdot}03\times10^{11}\times35{\cdot}7}=1{\cdot}63^\circ\text{C (nearly)}$$

Exercises

Essay type :

1. Show, by suitable experiments, that different materials expand differently.

2. Almost all solids expand when heated. In some cases, the expansion is advantageous but in some other, it is disadvantageous to us. Explain with illustrations.

3. Describe, in detail, a method for determining the coefficient of linear expansion of a solid.

4. A bi-metallic strip, consisting of two different metal strips riveted together, is used as a control element in the common thermostat. Explain how it works.

5. Prove that the volume coefficient of a solid is three times and the coefficient of surface expansion twice than the coefficient of its linear expansion.

[*H. S. Exam. 1978, '80*]

6. What is a compensated pendulum ? Describe a compensated pendulum you are familiar with.

Short answer type :

7. Define coefficient of linear expansion. Does it depend on the unit of length or of temperature? [*H. S. Exam. 1978, '81*]

8. Answer the following questions :—(i) Why does the glass stopper of a bottle become loose when hot water is poured on its neck ? (ii) Why is a gap left between two rails while laying a railway line ? (iii) What do you understand by the statement that the coefficient of linear expansion of iron is ·000012 ? (vi) Can metallic scales measure lengths correctly at different temperatures ? (v) A platinum wire can easily be fused in a glass rod but a copper wire can not be. Why ?

9. Two plates of identical shape and size—one made of zinc and the other of iron—are placed one over the other and riveted together. What will happen when the plates are heated ?

10. Define the coefficient of volume expansion. What change in the density of copper will be found if a piece of copper, in solid state, is heated to a higher temperature.

11. Establish a mathematical relation between the volumes of solid at a lower and higher temperatures.

12. Explain why a wall clock goes fast in winter and slow in summer.

13. An iron ring is given to you. State whether there will be an increase in (i) the volume (ii) the mass (iii) the density and (iv) the internal diameter of the ring if it is heated.

14. An iron rod connected the opposite sides of a circular iron ring. If the system is equally heated, will the ring remain circular ? [*Jt. Entrance 1985*]

[**Hints** : Yes ; d=diameter ; length of the circumference=πd ∴ length of the rod=πd : $d=\pi$. After heating, the circumference=$\pi d(1+\alpha.\theta)$ and the length of the rod=$d(1+\alpha\theta)$. So ratio=π.]

15. A hollow tube, when heated, expands in length. The wall also thickens due to increase of the thickness of the wall. Will then the tube get narrower ?

[**Hints** : If the diameter of the tube becomes smaller, the internal cross-section of the tube decreases. But due to rise of temperature, the cross-section should increase. So, the tube does not get narrower. Although the tube is hollow, the internal diameter will expand according to expansion laws of its material. As a matter of fact, a hollow tube and a solid tube expand equally on heating.]

16. A metal ball can pass through a metal ring. When the ball is heated, it gets stuck in the ring. What would happen if the ring, rather than the ball, is heated ?

17. Does the change in volume of a body when its temperature is raised depend on whether the body has cavities inside, other things being equal ?

Objective type :

18. Some statements are made below. Write *C* against those which are correct and *W* against those which are incorrect :

(i) When an iron sphere is heated, it undergoes volume expansion as well as surface expansion. —

(ii) On heating a bi-metallic strip, a curvature is set in with the metal having smaller co-efficient of linear expansion on the outside. —

(iii) In laying tram lines, gaps are left between successive lengths of rail to make room for expansion. —

(iv) Glass, being a bad conductor of heat, a thick-walled glass vessel often cracks when hot water is poured in it. —

(v) Measuring scales should be made of a metal whose coefficient of expansion is very small. —

Numerical problems :

19. The temperature of a metal rod, 230 cm. long is increased from 0°C to 100°C and the expansion in length of the rod was 2·75 mm. Calculate the coefficient of linear expansion of the metal. [Ans. $11{\cdot}9\times10^{-6}$]

20. An iron rod and a zinc rod are respectively 25·55 cm. and 25·5 cm. at 0°C. At what temperature will they be exactly equal in length ? α for iron=·00001 and α for zinc=·00003 °C. [Ans. 98·1°C]

21. A rectangular block of copper (8″ × 5″ × 1″) at 0°C is heated to 800°C. Calculate the increase in volume. Coeffⁿ of linear expansion of copper $=0{\cdot}16\times10^{-4}$ per °C. [Ans. 1·536 cu. inches]

22. A railway line is to be constructed by iron rails, each of which is 30 ft. long. If the temperature difference is 50°C, what gap must be left between each two pieces ? [Ans. 0·216 inch]

23. The distance between Allahabad and Delhi is 390 miles. Find the total space that must be left between the rails to allow for a change of temperature from 36°F in winter to 117°F in summer. Coefficient of linear expansion of iron=·000012 per °C. [Ans. 0·2084 miles]

24. There is a gap of 0·5 inch between each two rail-lines of 66 ft. long at 10°C. At what temperature will the gap be filled up ? [*H. S. Exam. 1983*] [Ans. 67·3°C]

25. A hollow tube, made of steel, has length 5 metre and internal cross-section 30 sq. cm. On passing steam through the tube, the temperature rises from 20°C to 100°C. What will be the increase in length and cross-section of the tube ? Coefficient of linear expansion of steel $=11\times10^{-6}$/°C. [Ans. 0·44 cm.; 0·0528 sq. cm.]

26. A scale, made of steel, is correct at 15°C. A certain distance measured with this scale at 30°C, is found to be 2000 ft. Find the error in measurement (α for steel=·000011 per °C). [Ans. 0·33 ft. less]

27. The radius of a wheel is 3 ft. and that of an iron tyre is 2·992 ft. at 0°C. At what temperature will the tyre fit exactly on the wheel ? α for iron $=12\times10^{-6}$°C. [Ans. 222·8°C]

28. An iron sphere should just pass through a circular brass ring. At 20°*C*, the diameter of the sphere is 25 cm. and the internal diameter of the ring is 24·9 cm. At what temperature, will the sphere just pass through the ring when both are heated together ? α for iron $=1{\cdot}2\times10^{-5}$ and α for brass $=2\times10^{-5}$ per °C. [Ans. 525·1°C]

29. Two metal bars *A* and *B* differ in length by 25 cm. whatever might be the change in temperature. If their coefficients of linear expansion are $1{\cdot}28\times10^{-5}$ and $1{\cdot}92\times10^{-5}$ per °C respectively, calculate the lengths of *A* and *B* at 0°C. [Ans. *A*=75 cm.; *B*=50 cm.]

30. An iron and a copper rod differ in length by 2 cm at 50°C as well as at 450°C. Find the lengths of the rods at 0°C if α for iron $=12\times10^{-6}$/°C and that of copper $=17\times10^{-6}$/°C. [*Jt. Entrance 1984*] [Ans. 6·8 cm ; 4·8 cm]

31. A length of 4000 ft. is measured by a steel tape at 10°C. If the tape is correct at 15°C, how much error will come in the measurement ? α for steel $=11{\cdot}9\times10^{-4}$ per °C. [Ans. 2·86 inches]

32. A steel tape is correct at 0°C. The length of a brass rod measured by the tape at 30°C is found to be 4·5 metre. What is the length of the rod at 0°C ? α for steel $=11\times10^{-6}$ per °C and for brass $=19\times10^{-6}$ per °C. [Ans. 4·449 metre]

33. The two ends of a steel rod of cross-section 2 sq. cm. are rigidly fixed and its temperature is raised from 0°*C* to 20°*C*. Calculate the thermal stress and the total force developed along the length of the rod. α for steel $=1{\cdot}1\times10^{-5}$/°C and $Y=2{\cdot}2\times10^{12}$ dynes/sq. cm.

[Ans. $4{\cdot}62\times10^{8}$ dynes/sq. cm.; $9{\cdot}24\times10^{8}$ dynes]

34. The diameter of a wheel is 50 cm. A steel tyre of internal diameter 49·8 cm. is put on the wheel. What force per unit area will the tyre exert on the wheel when it cools ? *Y* for steel $=2{\cdot}1\times10^{12}$ dynes/cm.^{2} [Ans. $8{\cdot}4\times10^{9}$ dynes]

35. An isosceles triangle is made by three pieces of iron wire. The base of the triangle is 6 cm. and the height is 4 cm. The temperature of the triangle is raised from 0°C to 20°C. Show that there will be no change in the base angles. α for iron $=12\times10^{-6}$/°C.

36. The base *BC* of an equilateral triangle is made of an aluminium wire and the other two sides *AB* and *AC* are made of copper. A wire of iron connects the vertex A to the mid-point of the base *BC*. If for a small rise of temperature, the arms of the triangle do not buckle, find the co-efficient of linear expansion of iron. α for Cu $=16\times10^{-6}$ and for aluminium $=26\times10^{-6}$°C. [Ans. $12{\cdot}7\times10^{-6}$/°C]

37. A second's pendulum keeps correct time at 20°C. It goes slow by 30 seconds in a day at a time. What was the temperature at that time ? α for the material of the pendulum rod $=18\times10^{-6}$/°C. [Ans. 58·6°C]

38. A steel tape is correct at 68°F. The length of a brass rod measured by it at 50°C is found to be 1·5 metre. Find out the true length of the rod at 50°C. Linear expansion coefficient of steel $=11{\cdot}2\times10^{-6}/°C$. [*H. S. Exam. 1978*] [Ans. 1·5005 metre (nearly)]

39. An aluminium piston moves through a steel cylinder. At 20°C, the internal diameter of the cylinder is 10 cm. and there is a gap of 0·05 mm. everywhere between the piston and the wall of the cylinder. At what temperature will the piston just fit in the cylinder? α for aluminium $=1{\cdot}6\times10^{-5}$ and for steel $=1{\cdot}2\times10^{-6}/°C$. [Ans. 87·6°C]

40. A steel wire of length 8 metre and diameter 4 mm. is tied between two rigid pillars. The wire is taut. Calculate the increase of tension when the temperature of the wire is reduced by 10°C. α for steel $=1{\cdot}2\times10^{-5}$ per °C and $Y=2\times10^{11}$ dynes/cm.^{2} [Ans. 3×10^6 dynes]

41. Two plates of breadth 20 cm. each—one of brass and the other of steel, are placed one over the other and are riveted so that their edges coincide. The bi-metallic strip buckles in the form of an arc when its temperature is increased by 50°*C*. What is the radius of curvature of the arc? α for brass $=19\times10^{-6}$ and for steel $=11\times10^{-6}$°C. [Ans. 10^5 cm. (nearly)]

Harder problems :

42. The cross-sectional area of a steel rod, 25 cm. long, is 0·8 sq. cm. What stretching force will be necessary to elongate the rod through same extent as will be done by raising the temperature of the rod by 10°C? Y for steel $=20\times10^{11}$ dynes/cm^2 and $\alpha=10^{-5}/°C$.
[*I.I.T. 1971*] [Ans. 16×10^7 dynes]

43. The diameter of an iron ball is 56 cm. When the ball is allowed to slip through a circular hole on a brass plate at 30°C it is found that the diameter of the ball is 0·01 mm. more than the diameter of the hole. At what temperature will the ball just slip through the hole? α for iron $=12\times10^{-6}/°B$ and for brass $=19\times10^{-6}/°C$ [Ans. 53·8°C (nearly)]

44. Three rods form an equilateral triangle at 0°C. Two of the rods are of same material and the third is of invar (whose thermal expansion may be neglected). If after heating to the steam point, the angle between the identical rods be $\left(\frac{\pi}{3}-\theta\right)$. prove that the coefficient of linear expansion of the material of the rods is $\frac{\sqrt{3}.\theta}{200}/°C$. [*Joint Entrance 1976*]

[**Hints :** l=length of the rod ; α=coeff^n of linear expansion. Now, $l_{100}=l(1+100.\alpha)$. Dropping a perpendicular from the vertex of the triangle to the base, we have,

$$\frac{\sin\left(\frac{\pi}{3}-\theta\right)/2}{l/2}=\frac{\sin 90°}{l_{100}}=\frac{1}{l(1+100.\alpha)}\quad\therefore\quad\sin\left(\frac{\pi}{6}-\frac{\theta}{2}\right)=\frac{1}{2(1+100.\alpha)}$$

$$\text{or}\ \sin\frac{\pi}{6}\cos\frac{\theta}{2}-\sin\frac{\theta}{2}.\cos\frac{\pi}{6}=\frac{1}{2(1+100\alpha)}\ \text{or}\ \frac{1}{2}-\frac{\theta}{2}.\frac{\sqrt{3}}{2}=\frac{1}{2(1+100.\alpha)}$$

$$\text{or}\quad 1-\frac{\sqrt{3}.\theta}{2}=\frac{1}{1+100.\alpha}\quad\therefore\quad\alpha=\frac{\sqrt{3}.\theta}{200}$$]

45. Two rods of different materials having the same area of cross-section A, are placed end to end between two massive walls as shown in fig. 3.19. The first rod has a length l_1; coefficient of linear expansion α_1 and Young's modulus Y_1. The corresponding quantities for the second rod are l_2, α_2 and Y_2. The temperature of both the rods are now raised by θ°. (i) Find the force with which the rods act on each other (at the higher temperature) in terms of the given quantities (ii) Also the length of the rods at higher temperature. [*I.I.T. 1975*]

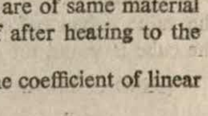


Fig. 3.19

[Ans. (i) $A\theta Y_1Y_2\,(\alpha_1l_1+\alpha_2l_2)/(l_1Y_2+l_2Y_1)$ (ii) $l_1\left\{1+\alpha_1\theta-\frac{Y_2\theta(\alpha_1l_1+\alpha_2l_2)}{l_1Y_2+l_2Y_1}\right\}$

and $l_2\left\{1+\alpha_2\theta-\frac{Y_1\theta(\alpha_1l_1+\alpha_2l_2)}{l_1Y_2+l_2Y_1}\right\}$

46. Two rods of length l_2 and coefficient of linear expansion α_2 are connected freely to a third rod l_1 of coefficient of linear expansion α_1 to form an isosceles triangle. The arrangement is supported on a knife edge at the middle point of l_1 which is horizontal. What relation must exist between l_1 and l_2 so that the apex of the isosceles triangle is to remain at a constant distance from the knife edge as the temperature changes ? [Ans. $l_1 : l_2 = \sqrt{\alpha_1} : 2\sqrt{\alpha_2}$]

47. Two copper and one aluminium rods of same length are joined together at their ends to form an equilateral triangle ABC. Another rod AP of unknown material connects the vertex A to the mid-point P of the base rod which is aluminium rod. For a small rise in temperature of the system, there is no tendency of its sides to buckle. Find the coefficient of linear expansion of the rod AP. α for Cu$=16\times10^{-6}$ and for Al$=26\times10^{-6}$ per°C.

[Ans. $12{\cdot}7\times10^{-6}/°C$]

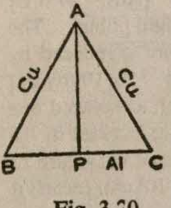


Fig. 3.20

48. A clock with iron pendulum keeps correct time at 20°C. How much will it lose or gain if the temperature changes to 40°C. Coefficient of cubical expansion of iron$=3{\cdot}6\times10^{-5}/°C$.

[*I.I.T. 1977*] [Ans. lose 10·3 sec in a day]

49. Show that a pendulum clock goes fast or slow by an amount equal to half the change of length per unit effective length per second due to variation of temperature.

50. A 'thermal tap' used in certain apparatus consists of silica rod which fits tightly inside an aluminium tube whose internal diameter is 8 mm. at 0°C. When the temperature is raised, the fit is no longer exact. Calculate what change in temperature is necessary to produce a channel whose cross-section is equal to that of a tube of 1 mm. internal diameter ? α for silica$=8\times10^{-6}/°C$ and for aluminium$=26\times10^{-6}/°C$. [Ans. 161°C]

51. A uniform pressure P is exterted on all sides of a solid cube at temperature t°C. By what amount should the temperature of the cube be raised in order to bring its volume back to the value it had before the pressure was applied ? Given coefficient of volume expansion of the cube is γ and the bulk modulus of elasticity is K. [*I.I.T. 1978*] [Ans. $t+P/(K-P)\gamma$]

[**Hints** : Initial volume$=V$; volume after the application of pressure$=V_1$.

Then, $K=\dfrac{P}{(V-V_1)/V}=\dfrac{PV}{V-V_1}$ $\therefore$ $\dfrac{V-V_1}{V}=\dfrac{P}{K}$ or $\dfrac{V}{V_1}=\dfrac{K}{K-P}$. Suppose t_1°C is the temperature at which the original volume is restored. Then from the cubical expansion, we have

$$\gamma=\frac{V-V_1}{V_1(t_1-t)}=\left(\frac{V}{V_1}-1\right)\times\frac{1}{(t_1-t)}=\left(\frac{K}{K-P}-1\right)\times\frac{1}{(t_1-t)}$$

$$\therefore\ t_1-t=\frac{P}{(K-P)\gamma}\ \Big]$$

52. The area A of a rectangular plate is ab. Its coefficient of linear expansion is α. After a temperature rise ΔT, side a is larger by Δa and side b is larger by Δb. Show that if we reject $\Delta a\times\Delta b$, then $\Delta A=2.\alpha A.\Delta T$.

4

EXPANSION OF LIQUIDS

4.1. Expansion of liquids :

Liquids, like solids, expand when heated. As different solids expand differently, different liquids also have different thermal expansions. It may be demonstrated by taking several fairly large glass bulbs with glass stems and filling them to a short distance above the bulb with various liquids [Fig. 4.1].

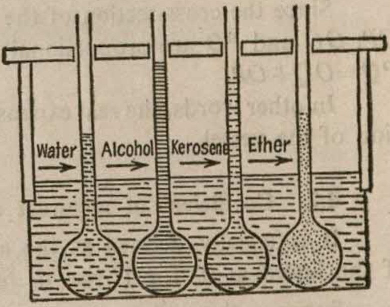


Fig. 4.1

In order to make a fair comparison, the bulbs and the stems must all be of the same size. The bulbs are now immersed in cold water taken in a metal trough and left until they attain a steady temperature. A little extra liquid should now be added, where necessary, to bring the meniscuses of the liquid columns to the same level. This ensures that the volumes of all liquids are equal and that they are at the same temperature. The bath is now heated and well stirred. When the bulbs and their contents have acquired the new temperature of the bath, it will be seen that the liquid levels have attained different heights [Fig. 4.1.]. Thus, for a given rise of temperature, equal volumes of different liquids show different thermal expansions.

Apparent and real expansions of liquid :

Expansion of liquids poses some problems bacause of the fact that liquids have no definite shape of their own; they take up the shape of the vessels in which they are kept. Further, liquids have no linear or surface expansion. Liquids only expand in volume. Furthermore, when a liquid contained in a vessel is heated, the vessel also expands along with the liquid. So, the expansion of a liquid is to be studied with reference to the expansion of its container.

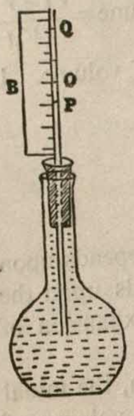


Fig. 4.2

Experiment : Take a glass flask with long and narrow neck. A narrow glass tube is inserted in the vessel through a hole in its stopper. A scale *B* is attached to the side of the glass tube. Fill up the flask with coloured water. Suppose, the level of water reaches the mark *O* of the scale. Place the flask in warm water kept in another vessel. It will be seen that the level of water in the glass tube first comes down to the mark *P* and then goes up past the mark *O*, to the point *Q* [Fig. 4.2]. Why does it happen so?

When the flask is put in warm water, the vessel expands in volume but the heat can not pass on to the liquid inside because glass is a bad conductor of heat. The water, therefore, occupies the additional volume made available by the expansion of the vessel and the level comes down to P. But soon the water in the flask expands receiving heat from water outside, and since liquids expand more than solids, the level of water passes the mark O and reaches the point Q. In the above experiment, the actual volume expansion of water is from P to Q and the volume expansion of glass is from O to P. Although glass is not a good conductor of heat, yet the water in the flask receives heat pretty quickly and because the volume expansion of solid is very small, it appears to us as if water has expanded from O to Q.

For this reason, the volume expansion from O to Q is referred to as *the apparent expansion of volume* of the liquid and the expansion from P to Q as *the real expansion of volume* of the liquid.

Since the cross-section of the glass tube is uniform throughout, the volumes OP, OQ and PQ are proportional to their lengths. From fig. 4.2, it is clear that $PQ=OQ+OP$.

In other words, the real expansion of the liquid = apparent expansion + expansion of the vessel.

4.2. Coefficient of apparent expansion of a liquid :

It is usually defined as the apparent increase in volume per unit volume of the liquid at 0° for 1° rise in temperature.

Suppose the volume of a certain mass of liquid at 0° is V_0. When the temperature is $t°$, let the apparent volume of the liquid be V_t.

So, for $t°$ rise in temperature, the apparent expansion of liquid $=V_t-V_0$

or ,, ,, ,, ,, ,, ,, ,, ,, ,, per unit volume $=\dfrac{V_t-V_0}{V_0}$

∴ ,, 1° ,, ,, ,, ,, ,, ,, per unit volume $=\dfrac{V_t-V_0}{V_0 t}$

This is known as the coefficient of apparent expansion of volume. If it be expressed by γ', then

$$\gamma'=\frac{V_t-V_0}{V_0 t}=\frac{\text{apparent expansion of volume}}{\text{volume at }0°\times\text{rise in temp.}}$$

$$\text{or, } V_t-V_0=V_0\gamma' t \quad \therefore \quad V_t=V_0\{1+\gamma'.t\}$$

It is to be remembered that γ' is not a constant quantity. It depends upon the material of the vessel in which the liquid is kept. It also depends upon the scale of temperature used. If γ' be the coefficient of apparent expansion of a liquid in celsius scale, it will be $\frac{5}{9}\gamma'$ in Fahrenheit scale.

Almost all liquids expand very little in volume. For this reason, the initial temperature may be different from 0°. In other words, if V_1 be the volume of a certain mass of a liquid at t_1° and at a higher temperature t_2°, the volume changes to V_2, then from the above equation, we may write, $V_2=V_1\{1+\gamma'(t_2-t_1)\}$

4.3. Coefficient of real expansion of liquid :

It is usually defined as the real increase in volume per unit volume at 0° for 1° rise of temperature.

Suppose, the volume of a certain mass of a liquid at 0° is V_0 and at a higher temperature t°, the volume actually changes to V_t. Then,

For, t° rise of temperature, the real expansion of volume$=V_t-V_0$

or, ,, ,, ,, ,, ,, ,, ,, ,, per unit volume$=\dfrac{V_t-V_0}{V_0}$

,, 1° ,, ,, ,, ,, ,, ,, ,, per unit volume$=\dfrac{V_t-V_0}{V_0.t}$

This is known as the coefficient of real expansion of the liquid. If it be expressed by γ, then,

$$\gamma=\frac{V_t-V_0}{V_0.t}=\frac{\text{real expansion of volume}}{\text{volume at }0^\circ\times\text{rise of temp.}}$$

It is to be noted that γ does not depend upon the container but depends upon the scale of temperature chosen. In fahrenheit scale the value of γ is $\frac{5}{9}$ times the value in the celsius scale.

Like apparent expansion, the initial temperature in finding out the real coefficient, may be different from 0°. For example, if V_1 be the volume of a certain mass of liquid at t_1° and V_2 its volume (real) at a higher temperature t_2, then, $V_2=V_1\{1+\gamma(t_2-t_1)\}$.

4.4. Relation between the coefficients of apparent and real expansion :

Suppose, γ=coefficient of real expansion.

$\gamma'=$,, ,, apparent expansion.

$\gamma_g=$,, ,, volume expansion of the material of the container.

Also, suppose, the volume of the flask upto the mark O at 0°C, is V_0 [Fig. 4.2]. So, the volume of the water in the flask at 0°C is also V_0. Now, let the temperature be increased to t°C. If the cross-section of the glass tube be S, then,

the expansion of the internal volume of the container$=OP\times S$

,, apparent expansion of volume of the liquid$=OQ\times S$

,, real ,, ,, ,, ,, ,, ,, $=PQ\times S$

From the definitions of the volume coefficients, we know,

$$\gamma_g=\frac{\text{Increase in volume of the container}}{\text{Initial volume of the container}\times\text{rise of temp.}}=\frac{OP\times S}{V_0.t}$$

$$\gamma'=\frac{\text{Apparent increase in volume of the liquid}}{\text{Volume of the liquid at }0^\circ\times\text{rise of temp.}}=\frac{OQ\times S}{V_0.t}$$

$$\gamma=\frac{\text{Real increase in volume of the liquid}}{\text{Volume of the liquid at }0^\circ\times\text{rise of temp.}}=\frac{PQ\times S}{V_0.t}$$

Now, $\gamma'+\gamma_g=\dfrac{S}{V_0.t}\{OP+OQ\}=\dfrac{PQ\times S}{V_0.t}=\gamma.$

i.e. coefficient of apparent expansion of the liquid+coefficient of cubical expansion of the container=coefficient of real expansion of the liquid.

Examples : (1) *The coefficient of apparent expansion of a liquid when a glass container is used, is* $15{\cdot}5\times10^{-5}$ *and when a copper container is used, is* $13{\cdot}2\times10^{-5}$; *if the coefficient of linear expansion of copper is* $1{\cdot}7\times10^{-5}$, *what is it for glass ?*

Ans. The coefficient of real expansion of the liquid$=15{\cdot}5\times10^{-5}+3\alpha$, where α is the coefficient of linear expansion of glass.

Again for the copper container, the coeff^n of real expansion of the liquid$=$ $13{\cdot}2\times10^{-5}+3\times1{\cdot}7\times10^{-5}=18{\cdot}3\times10^{-5}$ $\therefore$ $15{\cdot}5\times10^{-5}+3.\alpha=18{\cdot}3\times10^{-5}$

or, $3\alpha=2{\cdot}8\times10^{-5}$ or, $\alpha=0{\cdot}93\times10^{-5}$

(2) *A long glass tube of uniform capillary bore contains a thread of mercury 1 metre long at 0°C. When the temperature is raised to 100°C, the thread of mercury is found to be 16·5 mm. longer. If the coefficient of absolute expansion of mercury be ·000182, calculate the coefficient of linear expansion of glass.*

[*H. S. (Comp.) 1960*]

Ans. Suppose, the cross-section of the tube at 0°C and 100°C be respectively, A_0 and A_{100} sq. cm. Volume of mercury thread at 0°C $(V_0)=100\,A_0$ c.c. and Volume of mercury thread at 100°C $(V_{100})=101{\cdot}65A_{100}$ c.c.

Now, $A_{100}=A_0(1+100.\beta)$ where β is the coeff^n of surface expansion of glass.

or, $A_{100}=A_0(1+200.\alpha)$,, α ,, ,, ,, ,, linear expansion of glass.

Now, coeff^n of absolute expansion of mercury$=\cdot000182$

$$\therefore\ \cdot000182=\frac{101{\cdot}65A_{100}-100.A_0}{100A_0\times100}=\frac{101{\cdot}65A_0(1+200.\alpha)-100A_0}{100A_0\times100}$$

$$=\frac{101{\cdot}65(1+200.\alpha)-100}{10^4}\quad\therefore\ \alpha=8{\cdot}4\times10^{-6}$$

(3) *The coefficient of expansion of mercury is* $\frac{1}{5550}$. *If the bulb of a mercury thermometer is 1 c.c. and the section of the bore of the tube 0·001 sq. cm., find the position of the mercury at 100°C, if it just fills the bulb at 0°C. Neglect the expansion of the glass.*

Ans. The increase in volume of 1 c.c. of mercury

$$=\text{Original vol}^m\times\text{coeff.}\times\text{diff. in temp.}=1\times\frac{1}{5550}\times100=\frac{2}{111}\text{ c.c.}$$

If the mass of mercury, on expansion, occupies x cm. more of the bore, then, $x\times\dfrac{1}{1000}=\dfrac{2}{111}$ $\therefore$ $x=\dfrac{2000}{111}=18{\cdot}02$ cm. (nearly)

So, the mercury thread will reach the mark 18·02 cm. away from the bulb.

(4) *If a flask is made of a material having co-efficient of volume expansion equal to* 27×10^{-6} *per °C and* $\frac{3}{20}$ *of its volume is occupied by mercury (coeff*^{n} *of absolute expansion*$=180\times10^{-6}$ *per °C), show that the volume of the remaining space will not change with the change of temperature.* [*H. S. Exam. 1963*]

Ans. Suppose, the internal volume of the flask at 0°C$=V_0$. Hence, according to the question, the volume of mercury in the flask at 0°C$=\frac{3}{20}V_0$.

Now, the volume of the remaining space will not change with the change of temperature if the volume expansion of glass is equal to the absolute expansion of mercury. Now, the cubical expansion of glass for t°C rise of temperature $=$Original volume$\times$coeff$^n\times t=V_0\times27\times10^{-6}\times t$.

And, the absolute expansion of mercury for t°C rise of temperature

$$=\frac{3V_0}{20}\times180\times10^{-6}\times t=V_0\times27\times10^{-6}\times t$$

These two expansions are equal. Hence, the volume of the remaining space will not change due to change of temperature.

[N.B. By simply looking at the two coefficients in the above example, it may be said that the volume of the remaining space in the flask will not change.; for the coeff^n of expansion of glass is $\frac{3}{20}$ times that of mercury and $\frac{3}{20}$ th of the volume of the flask is occupied by mercury.]

(5) *The bulb of a thermometer contains 0·45 c.c. of mercury. What must be the cross-section of the bore of the thermometer tube in order that the degree graduations be 2 mm. apart. Coefficient of apparent expansion of mercury in glass is* $1{\cdot}55\times10^{-4}$.

Ans. Let α sq. mm.$=$cross-section of the bore. If the degree graduations are 2 mm. apart, then for 1°C rise in temperature, the apparent expansion of mercury$=2\alpha$ cu. mm.

Now, for 1°C rise of temperature, the apparent expansion of mercury$=0{\cdot}45\times1000\times1{\cdot}55\times10^{-4}$ cu. mm.$=0{\cdot}45\times0{\cdot}155$ cu. mm. [0·45 c.c.$=0{\cdot}45\times1000$ cu. mm.]

$$\therefore\ 2\alpha=0{\cdot}45\times0{\cdot}155\ ;\ \text{or, } \alpha=\frac{0{\cdot}45\times0{\cdot}155}{2}=0{\cdot}0349 \text{ sq. mm.}$$

4.5. Determination of co-efficient of apparent expansion of a liquid :

(A) By the dilatometer or volume thermometer :

Fig. 4.3 shows a dilatometer. It consists of a long glass bulb provided with a narrow, long glass tube about 20—30 cm. long. The glass tube has a volume scale engraved on its body. From the scale, we can ascertain the volume of the liquid filling up the bulb and a part of the steam.

Suppose, the liquid under experiment fills up the bulb and a part of the stem upto the mark P when placed in ice *i.e.* the volume of the liquid at 0°C is obtained from the scale. Let it be V_0. Then place the dilatometer in a vessel containing water, the temperature of which is raised to t°C. Suppose, the liquid expands and reaches the mark Q. Let its volume be V_t.

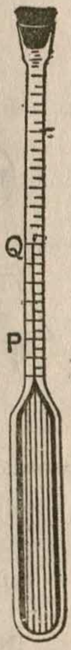


Fig. 4.3

So, $V_t - V_0$=the apparent expansion of volume of the liquid

$$\therefore\ \gamma' = \frac{V_t - V_0}{V_0 . t}$$

(B) By weight thermometer : The weight thermometer consists of a glass bulb with a narrow, bent neck (Fig. 4.4). Weigh it empty and let it be m_1 gm. Now insert the bent neck in the liquid under experiment and heat the bulb a little. The air in the bulb will expand and will go out bubbling through the liquid. If the bulb is now cooled, some liquid will enter the bulb. In this way, the bulb is to be alternately heated and cooled until it is completely filled up by the liquid. Allow the thermometer to return to the room-temperature and then weigh it. Let the weight be m_2 gm. Note the room temperature which is, suppose t_1°C. Again place the thermometer in some warm water whose temperature, suppose is fixed at t_2°C. Liquid, in the bulb will expand and will come out of the stem slowly. When the expulsion of liquid ceases, the thermometer is taken out of the warm bath and is allowed to cool. When it attains the room-temperature again, it is weighed and let it be m_3 gm.

Calculations :

$m_2 - m_1 = M_1$ (say)=mass of liquid filling up the thermometer at t_1°C,

$m_3 - m_1 = M_2$ (say)=mass of liquid filling up the thermometer at t_2°C

So, $M_1 - M_2$=mass of liquid expelled.

Neglecting the expansion of glass, it may be said that the volume of M_1 gm. of liquid at t_1°C=volume of M_2 gm. of liquid at t_2°C.

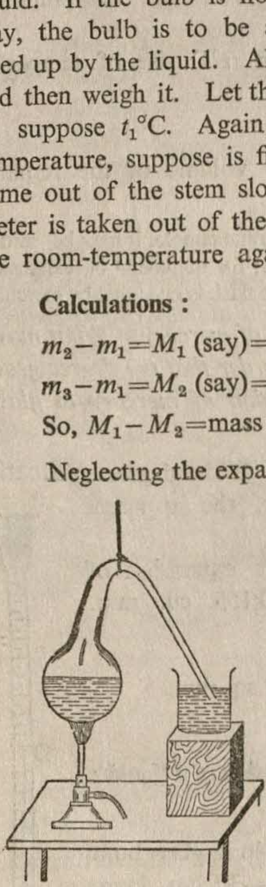


Fig. 4.4

Now, the volume of M_1 gm. of liquid at $t_1°\text{C} = \frac{M_1}{D}$ [D=density of the liquid at t_1°C]

$\therefore\ \frac{M_1}{D}$=volume of M_2 gm. of liquid at t_2°C.

Now, we say that if the temperature of M_2 gm. of liquid be raised from t_1°C to t_2°C, its volume will increase from $\frac{M_2}{D}$ to $\frac{M_1}{D}$. In other words, the coefficient of apparent expansion $\gamma' = \dfrac{\frac{M_1}{D} - \frac{M_2}{D}}{\frac{M_2}{D} \times (t_2 - t_1)} = \dfrac{M_1 - M_2}{M_2(t_2 - t_1)}$

$$i.e.\ \gamma' = \frac{\text{Mass of liquid expelled}}{\text{Mass of liquid left at } t_2°\text{C} \times \text{rise of temp.}}$$

[N.B. (i) The apparatus is called '*weight*' thermometer because the coefficient of apparent expansion of a liquid is determined by noting the weight of liquid instead of volume, by this apparatus. Further, if the coeff^n is known, an unknown temperature can also be ascertained by it and hence it is called '*thermometer*'.]

(ii) Instead of a weight thermometer, a specific gravity bottle may be used.]

Examples : (1) *A weight thermometer contains 300 gm. of mercury at 0°C. When it is placed in boiling water, 4·54 gm. of mercury were expelled. What is the coefficient of apparent expansion of mercury ?*

Ans. Mass of expelled mercury=4·54 gm.

Mass of the mercury left=300−4·54=295·46 gm.

Temperature of boiling water=100°C

We know, $\gamma' = \frac{\text{mass of expelled mercury}}{\text{mass of mercury left} \times \text{temp. rise}} = \frac{4{\cdot}54}{295{\cdot}46 \times 100} = 15 \times 10^{-5}$

(2) *A weight thermometer contains 82 gm. of liquid at 10°C and on heating it to 85°C, 2 gm. of liquid were expelled. Find the co-efficient of absolute expansion of the liquid if the coefficient of linear expansion of glass is 9×10^{-6}.*

Ans. Mass of expelled liquid=2 gm. ; mass of liquid left=82−2=80 gm. ; difference of temp.=85−10=75°C.

If γ' be the coeff^n of apparent expansion of the liquid, then,

$$\gamma' = \frac{\text{mass of expelled liquid}}{\text{mass of liquid left} \times \text{diff. of temp.}} = \frac{2}{80 \times 75} = {\cdot}00033$$

Now, the coeff^n of cubical expansion of glass$=3 \times$coeff^n of linear expansion
$=3 \times {\cdot}000009 = {\cdot}000027$

Again, we know, the absolute expansion of the liquid

=apparent exp^n of the liquid+cubical exp^n of the glass

$= {\cdot}00033 + {\cdot}000027 = {\cdot}00036$

(C) **By hydrostatic method :** This method is based on the principle of Archimedes.

A glass bulb, about 2 inches in diameter is weighted with some metallic shots till it can sink in the liquid whose coefficient of apparent expansion is to be determined. The bulb is suspended by means of a fine thread from the beam of a balance and weighed in air. Let this weight be m_1 gm. It is then suspended in the liquid under test at room temperature t_1°C (say) as shown in the fig. 4.5 and weighed. Let this weight be m_2 gm.

Next, keeping the bulb immersed in the liquid, its temperature is raised to t_2°C. The bulb is again weighed in this condition. Let the weight be m_3 gm.

According to Archimedes principle,

$m_1 - m_2 = M_1$ (say)=the weight of the liquid displaced by the bulb at t_1°C.

$m_1 - m_3 = M_2$ (say)=the weight of the liquid displaced by the bulb at t_2°C.

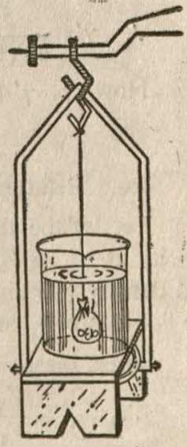


Fig. 4·5

Neglecting the expansion of the bulb, we can say that the volumes of the displaced liquid at t_1 and t_2°C are equal. Let this volume be V. Then

$V=\dfrac{M_1}{\rho_1}=\dfrac{M_2}{\rho_2}$ where ρ_1 and ρ_2 are the densities of the liquid at t_1°C and t_2°C respectively. But, $\rho_1=\rho_2\{1+\gamma'(t_2-t_1)\}$

$$\therefore \quad \frac{M_1}{\rho_2\{1+\gamma'(t_2-t_1)\}}=\frac{M_2}{\rho_2} \quad \text{or} \quad \frac{M_1}{M_2}=1+\gamma'(t_2-t_1)$$

$$\therefore \quad \gamma'=\frac{M_1-M_2}{M_2(t_2-t_1)}$$

Examples : (1) *A piece of glass weighs 90 gm. in air and 49·6 gm. in a certain liquid at 12°C. At 97°C, its apparent weight in the same liquid is 51·9 gm. If the coefficient of cubical expansion of glass is* $2{\cdot}4\times10^{-5}$, *find the coefficient of absolute expansion of the liquid.*

Ans. Let us find the coefficient of apparent expansion of the liquid first,

M_1=mass of the liquid displaced at 12°C=90−49·6=40·4 gm.

and M_2=mass of liquid displaced at 97°C=90−51·9=38·1 gm.

We know, $\gamma'=\dfrac{M_1-M_2}{M_2(t_2-t_1)}=\dfrac{40{\cdot}4-38{\cdot}1}{38{\cdot}1\times(97-12)}=\dfrac{2{\cdot}3}{38{\cdot}1\times85}=0{\cdot}71\times10^{-5}$

Since $\gamma=\gamma'+\gamma_g$, we have $\gamma=71\times10^{-5}+2{\cdot}4\times10^{-5}=73{\cdot}4\times10^{-5}$.

(2) *A metallic bob weighs 50 gm. in air. If it is immersed in a liquid at a temperature of 20°C, it weighs 45 gm. When the temperature of the liquid is raised to 100°C, it weighs 45·1 gm. Calculate the coefficient of cubical expansion of liquid assuming the linear expansion of the metal to be* $12\times10^{-6}/°C$. [*I.I.T. 1973*]

Ans. Let us first find the coefficient of apparent expansion of the liquid. We know, $\gamma'=\dfrac{M_1-M_2}{M_2(t_2-t_1)}$ where M_1=mass of liquid displaced at 25°C=50−45=5 gm.

and M_2=mass of liquid displaced at 100°C=50−45·1=4·9 gm.

$$\therefore \quad \gamma'=\frac{5-4{\cdot}9}{4{\cdot}9(100-25)}=\frac{0{\cdot}1}{4{\cdot}9\times75}={\cdot}00027$$

Now $\gamma=\gamma'+\gamma_g=\gamma'+3.\alpha={\cdot}00027+3\times12\times10^{-6}=27\times10^{-5}+3{\cdot}6\times10^{-5}$
$=30{\cdot}6\times10^{-5}$

4.6. Relation between density and coefficient of real expansion of liquid :

Let 'm' be the mass of a certain quantity of a liquid. Let its volume and density at t_1° be V_1 and D_1 respectively. If the liquid be now heated, its volume and density will change. Suppose, at temperature t_2°, the density and volume of the liquid are D_2 and V_2 respectively ($t_2>t_1$°).

Since, mass=volume×density, we have $m=V_1D_1=V_2D_2$

$\therefore \quad \dfrac{D_1}{D_2}=\dfrac{V_2}{V_1}=\dfrac{V_1\{1+\gamma(t_2-t_1)\}}{V_1}=1+\gamma(t_2-t_1)$ [γ=coeff^n of real expansion of the liquid] $\quad\therefore \quad D_1=D_2\{1+\gamma(t_2-t_1)\}$

If the initial temperature and density be 0° and D_0 respectively, then the density D_t of the liquid at $t°$, is given by

$$D_0=D_t\{1+\gamma.t\}$$

[N.B. The relation between the density and the coeffⁿ of cubical expansion of a solid is the same as that of a liquid. If D_1 and D_2 be the densities of a solid at temperatures $t_1°$ and $t_2°$ respectively, then $D_1=D_2[1+\gamma(t_2-t_1)]$ where γ is the co-effⁿ of cubical expansion of the solid.]

Examples : (1) *If the density of a liquid at 0°C be 8·9 gm/c.c., what will be its density at 20°C ? Coefficient of real expansion of liquid=·000051.*

Ans. We know, $D_0=D_t\{1+\gamma.t\}$
Here $D_0=8{\cdot}9$ gm/c.c. ; $t=20°$C ; $D_t=$?
We have, $8{\cdot}9=D_t\{1+{\cdot}000051\times20\}$

$$\therefore\ D_t=\frac{8{\cdot}9}{1+{\cdot}000051\times20}=\frac{8{\cdot}9}{1{\cdot}00102}=8{\cdot}89 \text{ gm/c.c.}$$

(2) *1 c.c. of water weighs 0·999874 gm. at 0°C and 1 gm. at 4°C. Find the mean coeffⁿ of absolute expansion of water between 0°C and 4°C.*

Ans. Here, $D_0=0{\cdot}999874$ gm/c.c. and $D_t=1$ gm./c.c. ; $t=4°$C
We know, $D_0=D_t\{1+\gamma.t\}$ or, $0{\cdot}999874=1(1+\gamma\times4)$

$$\text{or,}\quad \gamma=\frac{0{\cdot}999874-1}{4}=-\frac{{\cdot}000126}{4}=-{\cdot}0000315.$$

(3) *A vessel is filled completely with 500 gm. of water and 1000 gm. of mercury. When 21,200 calories of heat are given to it, water of mass 3·52 gm. overflows. Calculate the coefficient of volume expansion of mercury. Expansion of the vessel may be neglected. Coefficient of volume expansion of water=1·5×10⁻⁴ per °C ; Density of mercury=13·6 gm/c.c. ; Density of water=1 gm/c.c. ; Sp. heat of mercury=0·03.* [*I.I.T. 1976*]

Ans. Let θ be the rise of temperature. Heat absorbed by the liquids= $(m_1S_1+m_2S_2)\theta=(500\times1+1000\times0{\cdot}03)\theta$ cal$=530\times\theta$ cal.

$$\therefore\ 21{,}200=530\times\theta \qquad \therefore\ \theta=40°\text{C}$$

Volume expansion of the liquids=volume of water overflown $=\frac{3{\cdot}52}{1}=3{\cdot}52$ c.c.

But the volume expansion of the liquids$=V_1.\gamma_1.\theta+V_2.\gamma_2.\theta$

$$=(500\times1{\cdot}5\times10^{-4}+\frac{1000}{13{\cdot}6}\times\gamma_2)\times40$$

$$\therefore\quad 3{\cdot}52=500\times1{\cdot}5\times10^{-4}\times40+\frac{1000\times40}{13{\cdot}6}\times\gamma_2=3+\frac{40000}{13{\cdot}6}\times\gamma_2$$

$$\therefore\ \gamma_2=\frac{0{\cdot}52\times13{\cdot}6}{40000}=1{\cdot}768\times10^{-4}/°\text{C}.$$

(4) *Aniline is a liquid which does not mix with water ; when a small quantity of it is poured into a beaker of water at 20°C, it sinks to the bottom, the densities of two liquids at 20°C being 1·021 gm/c.c. and 0·998-gm/c.c. respectively.*

To what temperature must the beaker and its contents be uniformly heated so that the aniline will form a globule which just floats in water ? Coefficient of cubical expansion of aniline=85×10^{-5} *and of water=*$45\times10^{-5}/°C$.

Ans. As the density of aniline at 20°C is greater than the density of water at the same temperature, aniline sinks in water. The temperature at which their densities will be equal is the required temperature when the aniline forms a globule and floats in water. Let t°C be the required temperature.

If D_t be the density of aniline at t°C, then $D_{20}=D_t\{1+\gamma_a(t-20)\}$

$$\text{or,}\quad D_t=\frac{D_{20}}{1+\gamma_a(t-20)}=\frac{1{\cdot}021}{1+85\times10^{-5}(t-20)}$$

Similarly, D_t' be the density of water at t°C, then

$$D_{20}'=D_t'\{1+\gamma_w(t-20)\}\quad\text{or}\quad D_t'=\frac{D_{20}'}{1+\gamma_w(t-20)}=\frac{0{\cdot}998}{1+45\times10^{-5}(t-20)}$$

$$\text{For floatation, } D_t=D_t' \text{ or, } \frac{1{\cdot}021}{1+85\times10^{-5}(t-20)}=\frac{0{\cdot}998}{1+45\times10^{-5}(t-20)}$$

$$\text{or,}\quad 1{\cdot}021+1{\cdot}021\times45\times10^{-5}(t-20)=0{\cdot}998+85\times{\cdot}998\times10^{-5}(t-20)$$

$$\text{or,}\quad {\cdot}023=10^{-5}(t-20)(84{\cdot}83-45{\cdot}94)=10^{-5}(t-20)\times38{\cdot}89$$

$$\therefore\quad t-20=\frac{2300}{38{\cdot}89}=59\quad\text{or}\quad t=79°\text{C}.$$

4.7. Apparent weight of a body immersed in a liquid at different temperature :

Let W=weight of a body in air

W_1= ,, ,, the body fully immersed in a liquid at temperature t_1°C.

∴ Apparent loss of weight of the body$=W-W_1=m_1$ (say). At t_1°, if the volume of the body be V_1 and density of the liquid be ρ_1 then $m_1=V_1\,\rho_1$ .. (i)

Now, suppose the temperature of the liquid is raised to $t_2°(t_2>t_1)$. If the weight of the body in immersed condition be W_2, then the apparent loss of weight$=W-W_2=m_2$ (say). If V_2 be the volume of the body and ρ_2 the density of liquid at t_2°, then $m_2=V_2.\rho_2$.

If γ_l be the cubical expansion (real) of the liquid and γ_s that of the solid, then

$$V_2=V_1\{1+\gamma_s(t_2-t_1)\}\ \text{and,}\ \ \rho_1=\rho_2\{1+\gamma_l(t_2-t_1)\}$$

$$\because\ m_2=V_1\{1+\gamma_s(t_2-t_1)\}.\rho_1\{1-\gamma_l(t_2-t_1)\}$$
$$=V_1\rho_1\{1-(\gamma_l-\gamma_s)(t_2-t_1)\}$$
$$=m_1\{1-(\gamma_l-\gamma_s)(t_2-t_1)\}\quad\text{[From eq}^n\text{. (i)]}$$

$\gamma_l>\gamma_s$ and $t_2>t_1$; so $m_1>m_2$ *i.e.* $W_1>W_2$. Hence, we may say, that the apparent weight of a body immersed in a liquid increases as the temperature of the liquid is raised.

Example : *A metal piece weighing 100 gm. in air is found to weigh 55·4 gm. in a certain liquid at 20°C. At 95°C, its apparent weight in the same liquid is 57·9 gm. Find the coefficient of real expansion of liquid. Given coefficient of cubical expansion of metal=*$27\times10^{-6}/°C$.

Ans. Here $m_1=100-55{\cdot}4=44{\cdot}6$ gm.; $m_2=100-57{\cdot}9=42{\cdot}1$ gm.; $\gamma_s=27\times10^{-6}$; $(t_2-t_1)=(95-20)=75°C$

We know, $m_2=m_1\{1-(\gamma_t-\gamma_s)(t_2-t_1)\}$

or, $42{\cdot}1=44{\cdot}6\{1-(\gamma_l-27\times10^{-6})\times75\}$

or, $\dfrac{42{\cdot}1}{44{\cdot}6}=1-(\gamma_l-27\times10^{-6})\times75$ or $\gamma_l-27\times10^{-6}=74{\cdot}7\times10^{-5}$

$\therefore\ \gamma_l=74{\cdot}7\times10^{-5}+2{\cdot}7\times10^{-5}=77{\cdot}4\times10^{-5}$

4.8. Determination of coefficient of real expansion of liquid by Dulong and Petit's method :

The apparatus consists of a glass tube bent in the form of a rectangle as shown in the fig. 4.6. This arms *AB* and *CD* are vertical while the arm *BD* is horizontal. The portions of the tube after *A* and *C* have approached each other and then have gone up vertically remaining close to each other. Two scales (*S* and *S*) are fixed alongside these vertical portions of the tubes. The arms *AB* and *CD* are enclosed in two jacket tubes, whose mouths are closed by corks. Through one jacket, steam is passed and through the other ice-cold water is circulated. As a result, liquid column in the arm *AB* will be hot and that in the arm *CD* will be cold. For this difference in temperature, the density of liquid in the arm *AB* will be less than that of the liquid in the other arm, and there will be a level-difference between the liquid columns when equilibrium is established. The height of the less dense liquid column will be greater than the height of the other column. To prevent conduction of heat along the arm *BD*, it is wrapped up by wet blotting paper.

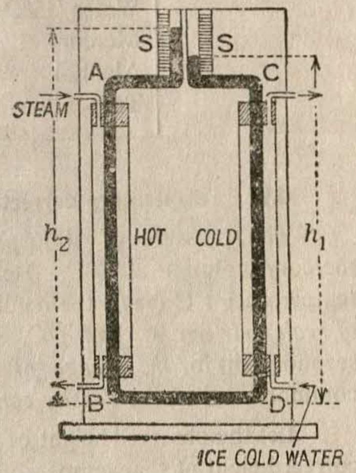


Fig. 4.6

Suppose, when equilibrium is established the height of cold column from the horizontal tube *BD* is h_1 and that of the hot column is h_2. If the temperatures of cold and hot columns are $t_1°$ and $t_2°$ respectively, then we know,

$$\frac{\text{the height of the cold column of liquid } (h_1)}{\text{,, ,, ,, hot ,, ,, ,, } (h_2)}=\frac{\text{Density of hot liquid } (D_2)}{\text{,, ,, cold ,, } (D_1)}$$

We have seen before, $D_1=D_2\{1+\gamma(t_2-t_1)\}$ [see art. 4.6]

$$\therefore\ \frac{h_1}{h_2}=\frac{D_2}{D_2\{1+\gamma(t_2-t_1)\}}=\frac{1}{1+\gamma(t_2-t_1)}$$

or, $h_1+\gamma(t_2-t_1)h_1=h_2$ $\quad\therefore\ \gamma=\dfrac{h_2-h_1}{h_1(t_2-t_1)}$

i.e., the coff^n of real expansion$=\dfrac{\text{Diff. of level of the liquid columns}}{\text{height of cold column}\times\text{diff. in temp.}}$

Example : *A column of mercury 76·35 cm. long at 100°C balances another column of length 75 cm. at 0°C. Calculate the coefficient of real expansion of mercury.* [*H. S. Exam. 1979*]

Ans. We know, the coeff^n of real expansion

$$=\frac{\text{Diff. in level of the liquid columns}}{\text{height of cold column}\times\text{temp. diff.}}$$

$$=\frac{76{\cdot}35-75}{75\times(100-0)}=\frac{1{\cdot}35}{75\times100}=1{\cdot}8\times10^{-4}$$

Table of coeff^{n}s of real expansion of some liquids

Liquids	per °C	per °F
Water (15°C – 100°C)	·00037	·0002
Mercury	·00018	·0001
Alcohol	·0011	·00061
Turpentine	·00105	·00054

4.9. Temperature correction of barometer reading :

The pressure of atmosphere is usually expressed in terms of the height of mercury column at 0°C. Hence if a barometric height is obtained at any temperature t°C (say), the reading should be corrected firstly (i) *for the expansion of scale between* 0° *and* t°C, which commonly called *the scale correction* and secondly (ii) for *the change of density of mercury from* 0°C *to* t°C, which is commonly called the *density correction.*

Let the observed height of the barometer at t°C be h cm. and let H cm. be the correct height at that temperature after scale correction. At t°C, each centimetre of the scale becomes $(1+\alpha.t)$ cm. where α=coefficient of linear expansion of the metal of the scale.

$$\therefore\ H=h(1+\alpha.t) \quad .. \quad \text{(i)}$$

Now this height is to be corrected for density correction of mercury. Let,

H_0=the corrected height

ρ_0=density of mercury at 0°C

ρ= ,, ,, ,, ,, t°C

γ=coefficient of absolute expansion of mercury.

We know, $\rho_0=\rho(1+\gamma.t)$

Since, the column of height H_0 has the same weight per unit area as the column of height H, we have, $\rho.H=\rho_0.H_0=\rho.H_0(1+\gamma.t)$

$$\therefore\ H_0=\frac{H}{1+\gamma.t}=H(1-\gamma.t)$$

Hence considering both the correction, we have

$$H_0=h(1+\alpha t)(1-\gamma.t) \qquad \text{[From eq}^n\text{ (i)]}$$
$$=h(1+\alpha t-\gamma.t-\alpha.\gamma.t^2)$$
$$=h\{1-t(\gamma-\alpha)\} \qquad \text{[Neglecting } \alpha.\gamma.t^2\text{]}$$

Example : *At 15°C, a barometer reads 74·9 cm. The barometer scale, made of brass, is correct at 0°C. Find (i) the height of the column at 15°C and (ii) the height of a column at 0°C which can exert an equal pressure. Coefficient of cubical expansion of brass*$=5{\cdot}4\times10^{-5}/°C$ *and that of mercury*$=1{\cdot}81\times10^{-4}/°C$.

Ans. (i) Coefficient of linear expansion of brass$=\frac{1}{3}\times5{\cdot}4\times10^{-5}$ $=1{\cdot}8\times10^{-5}/°C$.

The required height, $H=74{\cdot}9(1+1{\cdot}8\times10^{-5}\times15)=74{\cdot}92$ cm.

(ii) If H_0 be the height at 0°C, then,

$$H_0=h\{-t(\gamma-\alpha)\}=74{\cdot}9\{1-(18{\cdot}1-5{\cdot}4)\times10^{-5}\times15\}$$
$$=74{\cdot}9\{1-12{\cdot}7\times10^{-5}\times15)\}=74{\cdot}76\text{ cm.}$$

4.10. Anomalous expansion of water :

Most of the liquids expand in volume when heated and contract when cooled. But water is an exception to this general rule. Starting from its freezing temperature of 0°C, and being slowly heated, water contracts until it reaches a temperature of 4°C, and then begins to expand. On the other hand, if a certain mass of water is cooled from a temperature higher than 4°C, it will contract in volume until the temperature of 4°C is reached and then it will commence to expand. The volume expansion of water between 0°C and 4°C is, therefore, quite different from the volume expansion of other liquids. This peculiarity is referred to as the *anomalous behaviour of water*.

From the above discussion it follows that at about 4°C, water reaches its smallest volume. This means that at the aforesaid temperature it has its maximum density.

4.11. Experimental study of anomalous behaviour of water :

The changes of volume described above may be demonstrated by a dilatometer shown in fig. 4.3.

Fill $\frac{1}{7}$th of the volume of the bulb by clean and dry mercury. The coefficient of cubical expansion of glass being about one-seventh that of mercury, any increase in the volume of the glass bulb when it is heated will be compensated by an equal expansion of mercury inside it. The volume of the remaining space will, therefore, be constant as the temperature changes. Hence, if this remaining space is filled with water, any expansion of this water when the temperature is raised will represent the *real* and not the apparent expansion of water.

The dilatometer is placed in melting ice and left until the water level in the stem becomes steady. Note the volume of water from the scale. The dilatometer is then taken out and allowed to warm up slowly while readings of temperature and level of water are taken at regular intervals. In this way, raise temperature of the dilatometer to about 10°C. It will be seen that between 0°C and 4°C the level progressively falls showing a contraction of volume but after this, the level begins to rise.

In fig. 4.7 a graph is drawn between volume of 1 gm. of water and temperature, which clearly shows that volume decreases progressively from 0°C to 4°C and then it increases. It therefore, demonstrates that the volume of a certain quantity of water is minimum at about 4°C and hence the density is maximum. The graph also shows another important feature. The portion of the graph near about 4°C is almost horizontal. This means that slight variation of temperature near about 4°C, will not cause appreciable variation of the density of water. This is why the *density of water at 4°C is taken as unity.*

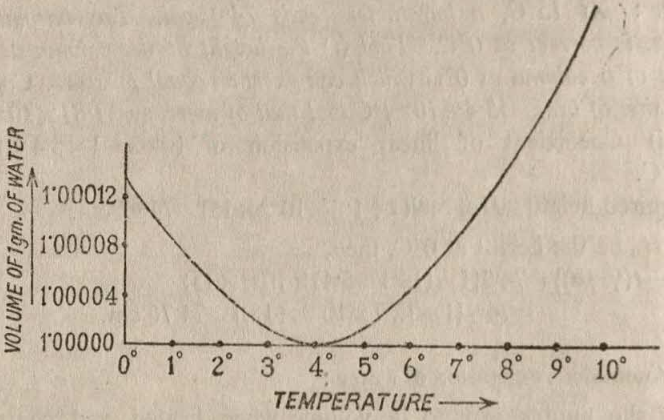


Fig. 4.7

4.12. Hope's experiment to demonstrate the maximum density of water at 4°C:

The apparatus consists of a glass or metal cylinder filled with water and fitted with thermometers near its top and bottom [Fig. 4.8]. Round the middle of the cylinder is a trough containing a freezing mixture of ice and salt, which produces a temperature of about −20°C. There is an exit tube for extracting water obtained by the melting of ice.

Starting with all the water in the vessel at about 10°C, the readings of both the thermometers are taken at regular intervals. It will be noticed that the lower thermometer falls rapidly but the upper thermometer remains stationary. The water near about the freezing mixture cools, contracts and sinks because of its greater density. A circulation of water is thus set up in the lower half of the cylinder and that is why the lower thermometer registers a falling temperature. The water in the upper half, not being disturbed so far, will have its temperature uncharged.

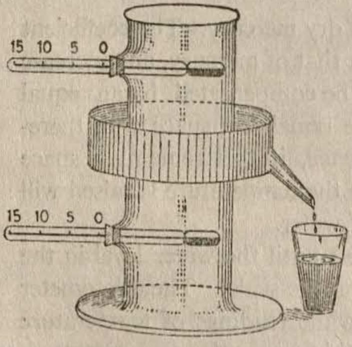


Fig. 4.8

But as soon as the lower thermometer shows a temperature of 4°C ; it becomes stationary. Now the upper thermometer starts showing a fall of temperature.

The reason is that water in the lower half attains maximum density at 4°C and further cooling of water near the centre below 4°C will cause it to expand and the water becomes less dense. It, therefore, floats up and denser water from the upper half comes down. A circulation of water is now set up in the upper half of the cylinder. Eventually the upper thermometer reaches 0°C, while the lower one still indicates 4°C. This experiment, therefore, proves that water attains maximum density at about 4°C.

The variation of temperature with time as registered by the thermometers may be shown graphically as in fig. 4.9. Both the thermometers start at 10°C but the lower thermometer shows a rapid fall of temperature upto 4°C when it becomes almost constant. During this time, the upper thermometer reading is almost constant but it shows a rapid fall when the lower one reads 4°C. There after the fall of temperature shown by the upper thermometer continues till it becomes 0°C.

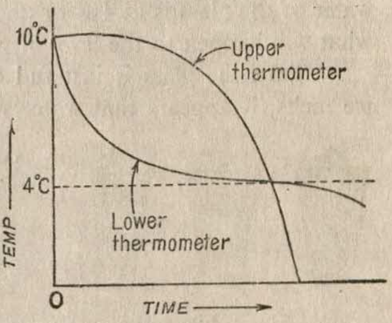


Fig. 4.9

[N.B. (i) When the temperature of water near the freezing mixture goes below 4°C, the mass of water does not either go up or move below. The water at the bottom of the cylinder, being heavier than the water at the middle, remains at the bottom while the water at the top, being at room temperature is lighter and therefore, remains at the top. When some water in the neighbourhood of the freezing mixture, is cooled below 0°C, some ice is formed which floats up and cools down the water at the top. The water there rapidly attains a temperature of 0°C.

(ii) The contraction in volume of water from 0°C to 4°C is explained by supposing that the molecules of water form clusters like H_4O_2, H_6O_3. At first, the contraction due to the formation of these clusters more than offsets the expansion due to the rise of temperature ; but above 4°C, the expansion prevails. As a result, water is found to expand in volume, like all other substances, above 4°C. The metal antimony, however, behaves like water.]

4.13. Consequence of anomalous expansion of water :

The anomalous behaviour of water has an important bearing on the preservation of marine life during very cold weather.

What happens when water of a pond cools towards the freezing point can be realised from the Hope's experiment described earlier. The surface water next to the cold air cools first. Having cooled and contracted, it has a greater density than the water below and it sinks. This process continues until all the water reaches 4°C. Now, as the surface water cools below 4°C, it expands. Becoming less dense, it floats. In due course, ice may form on the top of the water but the lower layers of water remain at 4°C. The lower layers can lose heat only by conduction. Shallow water is thus liable to freeze solid but in deeper water, there will always be layers of water beneath the ice where marine creatures can live [Fig. 4.10].

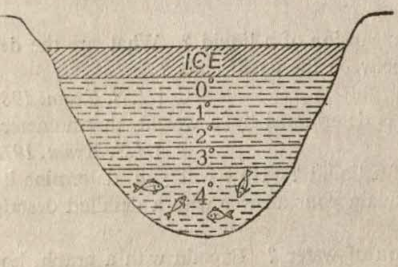


Fig. 4.10

4.14. A problem in connection with the volume of water :

Owing to the anomalous expansion of water, an interesting problem arises, concerning the volume of water. The problem is as follows :

A glass is just full of water and a piece of ice is floating on it. What will happen to the level of water when all ice melts and the water is at 0°C ? If the water of the glass is at 4°C or above 4°C initially and then a piece of ice is floated, what will happen to the level of water when the ice melts completely ?

Since the glass is just full of water and more water is produced when the ice melts, it appears that water will overflow in the first case when the ice melts completely. But that is not the case. The level of water will remain unchanged. The reason is that 11 c.c. of water at 0°C will produce 12 c.c. of ice at 0°C and ice floats with $\frac{11}{12}$th of its volume immersed in water. Hence, ice, on melting, produces as much volume of water as it displaces when it floats on water. So, the level of water will remain the same (see second fig.).

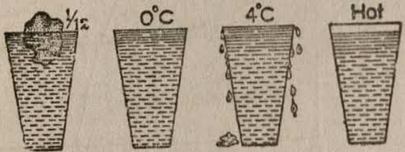


Fig. 4.11

If the initial temperature of water be 4°C, ice will melt receiving heat from water and the temperature of the mixture will be below 4°C. In this case also, the volume of water available by the melting of ice is same as the volume displaced by it but since the temperature of the mixture is below 4°C, there will be expansion in volume of the water which, will, therefore, overflow (see third fig.).

If the initial temperature of water be higher than 4°C, the temperature will fall to a certain extent due to melting of ice but since it is still above 4°C, there will be contraction in volume, although the melting of ice produces same volume of water as it displaces when it floats. Consequently the level of water in the glass will go down a little (see fourth fig.).

Exercises

Essay type :

1. What do you mean by apparent and real expansion of a liquid ? What are the definitions of their coeffcients ? What is the relation between the coefficients ? *[H. S. Exam. 1982]*

2. Which coefficient of expansion of a liquid is determined by a weight thermometer ? Describe the method in detail. *[H.S. Exam. 1978]*

3. What is the coefficient of real expansion of a liquid ? How will you determine it ?

4. What does Hope's experiment prove ? Explain your answer with a detailed description of the experiment.

5. What do you mean by anomalous expansion of water ? Explain with a graph, how the volume of a fixed mass of water changes with the change of temperature between 1°*C* to 20°*C* ?

6. How would you demonstrate the anomalous expansion of water ? *[H.S. Exam. 1979]*

7. What is the difference between real and apparent expansion of a liquid ? Establish the relationship between the coefficients of those expansion. Discuss the effect of anomalous expansion of water on marine life. *[H.S. Exam. 1980]*

Short answer type :

8. A uniform tube is connected to a thin copper bulb. Water fills the bulb and a part of the tube. How will the level of water in the tube change when the bulb is heated (i) quickly and (ii) slowly. Give reasons for your answer.

9. What is a weight thermometer ? Why is the instrument named so ?

10. The density of water is maximum at 4°C. Explain the statement fully. What difference will you notice if mercury and water are heated from 0°C to a higher temperature ?

11. What will happen if water is used instead of mercury in preparing a thermometer designed to read temperatures between 2°C to 20°C ?

12. Answer the questions : (i) How does water at the bottom of a lake remain in liquid state although the upper layers have frozen into ice ? (ii) How does fish live in frozen water ? [*H.S. Exam. 1978*]

13. The top of a lake is frozen. Air in contact with the top is at −15°C. What do you expect the maximum temperature of water (i) in contact with the lower surface of ice (ii) at the bottom of the lake ?

14. A big lump of ice with a piece of lead frozen into it floats in water. What will be the change of level of water (i) when the ice melts completely (ii) when the piece of lead is replaced by a piece of cork of equal volume ?

15. Placing a piece of ice in a beaker, water is poured until the beaker is full to the brim. What change in the level of water will be seen when the ice has melted completely if (i) the water is at 0°C (ii) the water is at 4°C (iii) hot water is taken.

16. A block of wood is floating on water at 0°C with a certain volume V above water-level. The temperature of water is slowly raised from 0°C to 20°C. How will the volume V change [*I.I.T. 1974*] with the rise in temperature ?

[**Hints :** Density of water increases from 0°C to 4°C and exerts greater upthrust. To counter balance it, greater portion on the block will go under the water. So, the volume V floating above the water level will decrease upto 4°C and then the volume V will increase as the temperature is slowly raised from 4°C to 20°C.]

17. A piece of ice is floating in beaker completely filled with a liquid of density 1·5 gm/c.c. [*I.I.T. 1980*] Will there be any overflow of liquid when the ice melts ?

Objective type :

18. Put a tick mark against the correct answer in the following :

(*a*) An observation made on the expansion of a liquid contained in a bulb gives real expansion/apparent expansion/anomalous expansion of the liquid.

(*b*) When the bulb of a thermometer is introduced suddenly in a hot-water bath, the reading first falls and then rises/first rises and then falls/first rises and then remains steady.

(*c*) A piece of ice floats in water at 4°C kept in a beaker. When the ice melts completely, the water level in the beaker comes down/water overflows the beaker/water level remains unchanged.

(*d*) When water is heated from 0°C to a higher temperature, the volume of water is found to be maximum/minimum/unchanged at 4°C.

(*e*) A metal ball immersed in alcohol weighs w_1 at 0°C and w_2 at 50°C. The coefficient of cubical expansion of the metal is less than that of alcohol. Assuming that the density of the metal is large compared to that of alcohol, it can be shown that (i) $w_1 > w_2$ (ii) $w_1 < w_2$ [*I.I.T. 1980*] (iii) $w_1 = w_2$.

Numerical problems :

19. A weight thermometer contains 24 gm. of mercury at 0°C. When heated to 100°C, it contains 23·662 gm. What is the coefficient of apparent expansion of mercury ? If the linear expansion of the container be $6{\cdot}63 \times 10^{-6}$, find the coefficient of absolute expansion of mercury. [Ans. 16×10^{-5} ; $17{\cdot}99 \times 10^{-5}$]

20. A weight thermometer requires when filled completely at 15°C, 45 gm. of alcohol. How much alcohol will be expelled when it is heated to 33°C ? Coefficient of apparent expansion of alcohol$=0{\cdot}00121$. [Ans. 0·96 gm. (nearly)]

21. A piece of glass weighs 48 gm. in air and 32·54 gm. in water at 4°C and 32·76 gm. in water at 60°C. Given the coefficient of linear expansion of glass to be 8×10^{-6}, find the coefficient of cubical expansion for water between 4°C and 60°C. [Ans. $2{\cdot}76\times10^{-4}$]

22. A mercury thermometer, when completely immersed in boiling water, reads 100°C. When the thermometer in held is such a way that the stem from 0°C onwards is at an average temperature of 10°C, the thermometer reads 98·6°C. What is the coefficient of apparent expansion of mercury ? [Ans. $1{\cdot}6\times10^{-4}$]

23. A piece of glass tubing, internal area of cross-section 0·005 sq. cm. has a bulb of 12 c.c. capacity at one end. The bulb is completely filled at 15°C with a liquid whose coefficient of apparent expansion in glass is $0{\cdot}52\times10^{-3}$ per °C. How far will the liquid rise in the tube when the temperature of the bulb is raised to 45°C ? [Ans. 37·44 cm.]

24. The cross-section of the capillary tube of a mercury thermometer is constant at A_0 and V_0 is the volume of the bulb at 0°C. If the mercury just fills the bulb at 0°C, show that the length of the mercury column in the capillary at t°C is given by $l=\frac{V_0}{A_0}(\gamma-3\alpha)t$ when $\gamma=$coeff^n of expansion of mercury and $\alpha=$linear coeff^n of expansion of glass.

25. The capacity of a thermometer bulb is 1 c.c.. It is desired to make the degree graduations on the scale 5 mm. apart. What must be the cross-section of the thermometer tube ? The coefficient of apparent expansion of mercury in glass is $1{\cdot}6\times10^{-4}$ per °C.
[Ans. 0·032 sq. mm.]

26. The capacity of a glass flask is 630 c.c.. What should be the volume of mercury to be put in the glass flask so that the volume of the remaining portion of the flask remains the same at all temperatures ? Co-efficient of real expansion of mercury$=1{\cdot}8\times10^{-4}$/°C and the coefficient of cubical expansion of glass$=2{\cdot}5\times10^{-5}$/°C. [Ans. 87·5 c.c.]

27. What fraction of the inner volume of a glass vessel should be kept filled with mercury so that the volume of the remaining inner space will be same at all temperatures ? Coefficients of volume expansion of glass and mercury are respectively $2{\cdot}4\times10^{-5}$/°C and $1{\cdot}8\times10^{-4}$/°C.
[*H. S. Exam. 1982*] [Ans. $\frac{2}{15}$]

28. 340 gm. of mercury is required to fill completely a bulb at 0°C. Some steel balls are kept in the empty bulb and 255 gm. of mercury is put in the bulb to fill the remaining space at 0°C. When the bulb is heated to 100°C, 4.8 gm. of mercury is expelled. Find the coefficient of linear expansion of steel. Coefficient of cubical expansion of mercury$=180\times10^{-6}$/°C.
[Ans. 12×10^{-6}/°C]

29. A glass tube of uniform diameter is attached with the bulb of a dilatometer made of glass. Filling the bulb and a part of the tube at 0°C, there is 150 gm. of mercury in it. If the temperature is raised to 100°C, how far along the tube will the mercury thread advance ? The cross-section of the tube at 0°C$=8$ sq. cm. and density of mercury$=13{\cdot}6$ gm./c.c. α for glass$=1{\cdot}1\times10^{-5}$ and the coefficient of absolute expansion of mercury$=1{\cdot}82\times10^{-4}$/°C.
[Ans. 20·5 cm.]

30. The density of mercury at 0°C is 13·6 gm./c.c. and at 100°C, it is 13·35 gm./c.c.. Find the coefficient of real expansion of mercury. [Ans. $1{\cdot}87\times10^{-4}$/°C]

31. The brass scale of a barometer is correct at 0°C. At 30°C, the barometer gives a reading of 76 cm. What will be the correct reading at 0°C ? α for brass$=18\times10^{-6}$/°C and the coefficient of cubical expansion of mercury$=18\times10^{-5}$/°C. [Ans. 75·61 cm.]

Harder problems :

32. Mercury is poured in a graduated glass tube at 0°C. The temperature of the tube is also 0°C and the mercury column touches 100 mark. How much should its temperature be raised so that it may touch 101·65 mark ? Coefficient of expansion of mercury$=18\times10^{-5}$ and the coefficient of linear expansion of glass$=8\times10^{-6}$/°C. [Ans. 100°C]

33. A sphere of diameter 7 cm. and mass 266·5 gm. floats in a liquid kept in a vessel. When the temperature of the liquid is increased, the sphere floats just immersed in the liquid at 35°C. If the density of the liquid at 0°C is 1·527 gm/c.c., find the coefficient of cubical expansion of liquid. Expansion of the sphere may be neglected. [*I.I.T. 1962*] [Ans. $8{\cdot}08\times10^{-4}$ per °C]

34. W_0=loss of weight of a solid when it is immersed in a liquid at 0°C, W_t=loss of weight of the body in the same liquid at t°C. Prove that $W_0=W_t\{1+(\gamma l-\gamma_g)t\}$, where γl=coefficient of absolute expansion of the liquid and γ_g=the coefficient of cubical expansion of the solid.

35. A mercury-in-glass thermometer contains 0·4 c.c. of mercury at 0°C. The capillary bore has a diameter of 0·5 mm. What would be the length of the scale of temperature if it has to read a range of temperature from 0°C to 100°C ? Coefficient of apparent expansion of mercury in glass$=1{\cdot}7\times10^{-4}$/°C. [*Jt. Entrance 1973*] [Ans. 0·043 cm. per °C]

36. A vessel contains 8·45 gm of water at 0°C, the remainder of the vessel being filled with paraffin. When water freezes into ice at 0°C, 0·62 gm. of paraffin is expelled from the vessel. Sp. gravity of paraffin at 20°C is 0·8 and its coefficient of expansion is $0{\cdot}9\times10^{-3}$/°C. Find the sp. gravity of ice. [Ans. 0·917]

37. A weight thermometer contains m gm. of a liquid at 0°C. Show that at t°C, $m\gamma_1 t(1-\gamma_1 t)$ gm. of liquid will be expelled from the thermometer. γ_1=coefficient of apparent expansion of the liquid.

38. Two thermometers A and B are made of same kind of glass and contain the same liquid. The bulbs of both the thermometers are spherical. The internal diameter of the bulb in A is 7·5 mm. and the radius of cross-section of the tube is 1·25 mm., the corresponding figures in B being 6·2 mm. and 0·9 mm. Compare appoximately the length of a dgree in A with that in B. [*Jt. Entrance 1970*] [Ans. 0·8 (nearly)]

39. A sinker weigh w_0 has an apparent weigh w_1 when weighed in a liquid at temperature t_1 and w_2 when weighed in the same liquid at temperature t_2. The coefficient of cubical expansion of the material of the sinker is β. What is the coefficient of volume expansion of the liquid ?

[*I.I.T. 1978*] $\left[\text{Ans. } \dfrac{w_1-w_2}{(w_0-w_1)\times(t_1-t_2)}+\beta\right]$

40. Cubical expansivities of benzene and wood are $1{\cdot}2\times10^{-3}$/°C and $1{\cdot}5\times10^{-4}$/°C, their densities at 0°C are 9×10^2 kg/metre^3 and $8{\cdot}8\times10^2$ kg/metre^3 respectively. At what temperature will wood just sink in benzene? [*Jt. Entrance 1984*] [Ans. 21·7°C]

[*Hints* : Wood will just sin K when its density equals the density of benzene. See worked out example 4, page 400]

41. A body weighs m_1 gm. in a liquid of density d_1 and m_2 gm. in a liquid of density d_2. What would be its weight in a liquid of density d_3 ? [*Jt. Entrance 1980*]

$\left[\text{Ans. } \dfrac{m_1(d_2-d_3)-m_2(d_1-d_3)}{(d_1-d_2)}\right]$

42. A cubical block made of aluminium is fully immersed in glycerine. The length, breadth and height of the block are 10 cm. each. The temperature of glycerine is 20°C and density 1·26 gm/c.c. Find the change in apparent weight of the block if the temperature is raised to 100°C ? Coefficient of expansion of glycerine$=500\times10^{-6}$/°C and the coefficient of cubical expansion of aluminium $=25\times10^{-6}$/°C. [Ans. 47 gm. wt.]

43. At temperature t° a barometer reading is h. If the temperature of the barometric liquid be raised by Δt at the same atmospheric pressure the height of the liquid increases by Δh. If the coefficient of expansion of the liquid be γ, show that $\Delta h/\Delta t=\gamma.h$.

44. A closed glass bulb contains some mercury leaving an empty space of 1 c.c. at 0°C. On raising the temperature to 40°C, the volume of the air space is 0·98 c.c. What is the volume of the mercury if the coefficient of linear expansion of glass$=7{\cdot}2\times10^{-6}$ and the coefficient of cubical expansion of mercury$=1{\cdot}81\times10^{-1}$/°C ? [*Lond. Univ.*] [Ans. 3·27 c.c.]

5

EXPANSION OF GASES

5.1. Introduction :

On being heated, gases expand like solids and liquids. Like liquids, gases have only volume expansion and no linear or superficial (in area) expansion. But the striking features of thermal expansion of gases are that they expand much more than solids and liquids and that all gases expand equally for equal rise of temperature. The following simple experiment illustrates these features.

Experiment : Take a thin-walled glass flask and pour some coloured water in it. Insert a thin glass tube through the hole in the stopper so that it reaches almost the bottom of the flask. Besides water, the flask contains air. If the upper part of the flask be now wrapped up by the palms of two hands, coloured water will be found to rise along the narrow glass tube. The air in the flask receives heat from the palms and in trying to expand in volume, exerts pressure on the surface of water which, therefore, rises along the narrow tube.

Now, take two identical flasks of above description and pour equal volumes of coloured water in them so that both may contain equal volumes of gases. Suppose, one flask contains air and the other any other gas—say, hydrogen. Now place the flasks in warm water kept in a wide vessel. It will be noticed that coloured water has risen to the same extent in both the glass tubes. This clearly shows that all gases expand equally for equal rise of temperature. This is not the case with solids and liquids.

5.2. Effects of pressure and temperature on the expansion of gas : Gas laws :

Expansion of a gas is very much influenced by both pressure and temperature. In the case of solids or liquids, the expansion or contraction in volume due to decrease or increase of pressure is so small that it may be easily ignored. But in the case of gases, the volume is found to alter appreciably due to variation of pressure, even if the temperature is kept constant. Similarly, if the temperature is varied, keeping the pressure constant, gases undergo considerable change in volume. Hence to make a full study of expansion of gases, temperature and pressure are to be taken into account. The laws guiding the change of volume and pressure of a gas due to changes of temperature and pressure are known as gas laws.

(A) **Boyle's law** : The law states that *the volume of a fixed mass of gas is inversely proportional to the pressure provided the temperature remains constant.*

Suppose V is the volume of a fixed mass of gas and P its pressure. Then according to Boyle's law, $V \propto \frac{1}{P}$ provided the temperature is constant, or, $V.P = \text{constant}$.

Thus, if V_1, V_2, V_3 etc. be the volumes of a fixed mass of gas when the pressure are P_1, P_2, P_3 etc. respectively, then $V_1P_1=V_2P_2=V_3P_3=\ldots$

(B) **Charles' law** : The law states that *the volume of a fixed mass of gas at constant pressure expands or contracts by* $\frac{1}{273}$ *of its volume at 0°C per degree celsius rise or fall in temperature.*

Suppose, the volume of a fixed mass of gas at 0°C is V_0. Then according to Charles' law.

the volume of the gas at $1°\text{C}=V_0+V_0\cdot\frac{1}{273}$

or, " " " " " " $2°\text{C}=V_0+V_0\cdot\frac{2}{273}$

or, " " " " " " $t°\text{C}=V_0+V_0\cdot\frac{t}{273}$

Now, if the volume of the gas at t°C, be called V_t, then,

$$V_t=V_0+V_0\cdot\frac{t}{273}=V_0\left(1+\frac{t}{273}\right)$$

Similarly, if the temperature be lowered to t°C, below 0°C then the volume of the gas, $V_t=V_0\left(1-\frac{t}{273}\right)$.

(C) **Regnault's Pressure law** : The law states that *the pressure of a fixed mass of gas at constant volume increases or decreases by* $\frac{1}{273}$ *of its pressure at 0°C per degree celsius rise or fall in temperature.*

Suppose, the pressure of a fixed mass of gas at 0°C is P_0. If P_t be the pressure of the gas due to an increase or decrease of temperature t°C, then according to the law, $P_t=P_0\left(1\pm\frac{t}{273}\right)$.

5.3. Experimental verification of Boyle's law :

Fig. 5.1 illustrates an apparatus necessary for verifying Boyle's law experimentally for pressures both above and below atmospheric. It consists of a glass tube B, closed at one end, connected by a length of rubber tubing to another glass tube A, open at both ends. The tubes are fitted against a wooden frame by the side of a scale. The tube A can be moved up and down along the scale. Some air is entrapped in the tube B by a mercury column which fills the entire rubber tubing and parts of the glass tubes A and B. Since, the tube B is of uniform bore, the length of air column in the tube B will be proportional to the volume of the mass of air.

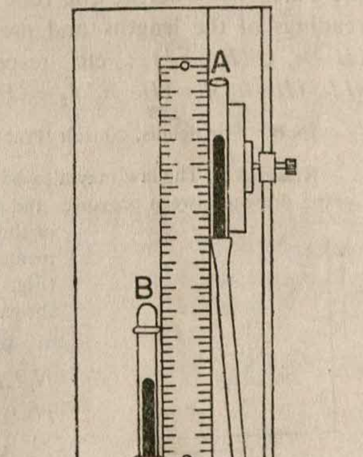


Fig. 5.1

Procedure : Bring the tube A to such a position that the level of mercury in the two tubes are the same. In this position, the pressure of air in the tube B is equal to the atmospheric pressure. Find the length of the air column from the scale. Suppose the length is l and the pressure H (as obtained from a barometer).

Now raise the tube A slowly. The mercury level in the tube A will be higher than that in the tube B. Suppose, the difference of levels is h_1 [Fig. 5.2(a)]. The pressure of air in the tube B is, therefore, $=H+h_1$. Now, find the length occupied by the air column. Suppose it is l_1. A series of such readings are taken by raising the tube A slowly. If the subsequent readings of length and pressure are l_2, l_3 . . . etc. and $(H+h_2)$, $(H+h_3)$ . . . etc. respectively it will be seen that $H.l=(H+h_1)l_1$ $=(H+h_2)l_2=$ . . . etc. This verifies the law for pressures above atmospheric.

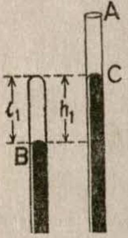


Fig. 5.2(a)

To verify the law for pressures below atmospheric, the tube A is to be lowered such that the level of mercury in it is below the level in the tube B. If, at any such position, the level-difference is h_1' [Fig. 5.2(b)], the pressure of air $=H-h_1'$. Let the length of the air column in this position be l_1'. A series of such readings are taken by lowering the tube A slowly. If the subsequent readings of the lengths and pressures are l_2', l_3' . . . etc. and $(H-h_2')$, $(H-h_3')$ . . . etc. respectively, it will be seen that $H.l=(H-h_1')l'=(H-h_2')l_2'=(H-h_3')l_3'$ . . . etc.

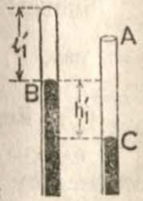


Fig. 5.2(b)

[N.B. For details, consult 'Practical Physics' by the author.]

Remarks : The law may also be verified graphically. If the law is true *i.e.*, $PV=$constant, a graph drawn between pressures and volumes will be a rectangular hyperbola. If the readings of the volume and the pressure as obtained in the above experiment are plotted, a rectangular hyperbola will be obtained (Fig. 5.3). Such $P-V$ graphs are known as *isothermals* of the gas.

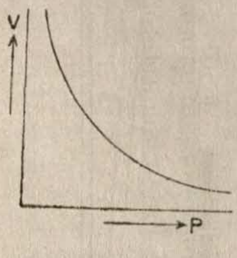


Fig. 5.3

Examples : (1) *What will be the volume at N.T.P. of 10 litres of air at 0°C and 10 atmospheric pressure ?*

Ans. Here, temperature is constant. So Boyle's law may be applied. We know,

$P_1V_1=P_2V_2$ *i.e.* $10\times10=1\times V_2$ $\therefore$ $V_2=100$ litres.

(2) *The air in a bulb of 31·4 c.c. capacity is compressed into **a** narrow tube 5 cm. long and 1 mm. diameter and the pressure of air in the narrow tube is found to be 4 cm. of mercury. What was the pressure of air in the bulb ?*

Ans. Suppose, the pressure of air in the bulb $=H$ cm. of mercury. Now, the volume of the narrow tube $=\pi r^2\times l=3{\cdot}14\times({\cdot}05)^2\times5$ c.c.

We know, $P_1V_1=P_2V_2$

Here, $P_1=H$; $V_1=31{\cdot}4$ c.c. ; $P_2=4$ cm. of mercury, $V_2=3{\cdot}14\times({\cdot}05)^2\times5$ c.c.

Hence, $H\times 31{\cdot}4=3{\cdot}14\times(\cdot 05)^2\times 5\times 4$

or, $H=\dfrac{3{\cdot}14\times(\cdot 05)^2\times 5\times 4}{31{\cdot}4}=\cdot 005$ cm. of Hg.

(3) *An air bubble of diameter 1 mm. is formed at a depth of 238 ft. of water in a lake. What will be its diameter when it reaches the surface? Assume the temperature to be constant. (Height of water barometer = 34 ft.)*

[*H. S. Exam. 1965*]

Ans. The pressure of the bubble at a depth 238 ft. $=238+34=272$ ft. water column. Suppose, the volume of the bubble at the bottom $=V_1$ and that at the surface $=V_2$. Since the temperature is unchanged, $V_1\times 272=V_2\times 34$.

Or, $\dfrac{V_2}{V_1}=\dfrac{272}{34}=8$. But $V_1=\frac{4}{3}\pi(\text{radius})^3=\frac{4}{3}\pi\times(\cdot 05)^3$ and $V_2=\frac{4}{3}\pi r^3$ where $r=$ radius of the bubble at the surface.

$\therefore\quad \dfrac{\frac{4}{3}\pi(r)^3}{\frac{4}{3}\pi(\cdot 05)^3}=8$ or, $\dfrac{r}{\cdot 05}=2\quad\therefore\quad r=2\times\cdot 05=0{\cdot}1$ cm. $=1$ mm.

Hence, the diameter of the bubble at the surface of the lake $=2$ mm.

(4) *The volume of an air-bubble increases five-fold in rising from the bottom of a lake to the surface. If the barometric height be 30 inches, find the depth of the lake. Density of mercury = 13·6 gm/c.c.*

Ans. The barometric pressure when expressed by the column of water $=30\times 13{\cdot}6$ inches. If the depth of the lake be h inches, then the pressure at the bottom = atmospheric pressure + pressure of h inches of water column $=(30\times 13{\cdot}6+h)$ inches of water column.

If V be the volume of the bubble at the bottom, we can write, according to Boyle's law, $(30\times 13{\cdot}6+h)V=5V\times 30\times 13{\cdot}6$

or, $408+h=2040\quad\therefore\quad h=1632$ inches $=136$ ft.

(5) *A vertical cylinder of total length 100 cm. is closed at the lower end and is fitted with a movable frictionless gas-tight disc at the other end. An ideal gas is trapped under the disc. Initially the height of the gas column is 90 cm., when the disc is in equilibrium between the gas and the atmosphere. Mercury is then slowly poured on top of the disc and it just starts overflowing when the disc has descended through 32 cm. Find the atmospheric pressure. Assume the temperature of the gas to remain constant and neglect the thickness and weight of the disc.* [*I.I.T. 1971*]

Ans. At the beginning, the gas pressure = atmospheric pressure P, because the disc is in equilibrium under these two pressures. Volume of the gas $=90\ \alpha$ c.c. where $\alpha=$ cross-section of the cylinder.

When mercury is poured and the disc descends through 32 cm. the volume of the gas entrapped $=(90-32)\alpha=58.\alpha$ c.c. The pressure of this gas $=P+$ pressure

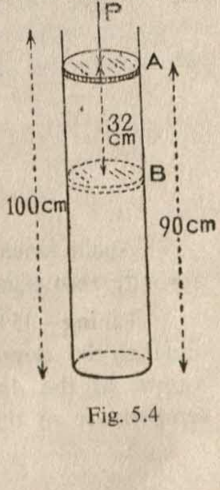


Fig. 5.4

due to (32+10) or 42 cm. of mercury. As the temperature of the gas remains constant, we can write, $P\times 90\alpha=(P+42)\times 58\alpha$.

$$\text{or,}\quad 32\,P=42\times 58 \quad\therefore\quad P=\frac{42\times 58}{32}=76{\cdot}1 \text{ cm. of Hg.}$$

5.4. Absolute scale of temperature :

From Charles' law, we know, that for every degree that the temperature is lowered, the volume of a fixed mass of gas decreases by $\frac{1}{273}$ of its value at 0°C and suggests that—provided the gas does not first liquefy or solidify—if the temperature were lowered 273° below the ice point, the gas would no longer occupy any space at all !

Similarly, from Pressure law, we know that for every degree that the temperature is lowered, the pressure of a fixed mass of gas decreases by the same fraction as above and it means that if the temperature could be lowered to 273° below 0°C, a gas would cease to exert any pressure.

Both laws, therefore, point to the existence of the lowest possible temperature of −273°C where a gas would no longer exert pressure on its container or take up any space. This temperature is called the *absolute zero*. In the laboratory, physicists have been able to come within a few thousandths of a degree of this point.

For all theoretical purposes, scientists measure temperatures upward from the absolute zero as a starting point, using the celsius degree as the unit. The scheme is called the *absolute* or *Kelvin scale* and is written as 'degree Kelvin' (°K). To change from a celsius reading to kelvin, simply add 273. For example, a temperature of t°C is equivalent to $(t+273)$°K.

Thus, the freezing point of ice *i.e.* 0°C is, according to absolute scale 273°K and the boiling point of water (*i.e.* 100°C) 373°K.

Celsius scale (°*C*)	Kelvin scale (°*K*)
−273°	0°
0°	273°
100°	373°
t°	$(273+t)$°

Again since 0°K=−273°C=−459·4° F, according to Fahrenheit scale, the absolute zero is equivalent to −459·4°F.

Taking −459·4° as zero if degree graduations in absolute scale be made equal to the degree graduations in Fahrenheit scale, the scale, so obtained is known as the *Absolute Fahrenheit scale*. After the name of Rankine, the temperature in this scale is written as 'degree Rankine' (°R).

So, $-459{\cdot}4°\text{F}=0°\text{R}$ or $0°\text{F}=459{\cdot}4°\text{R}$

Fahrenheit scale (°F)	Rankine scale (°R)
−459·4°	0°
0°	459·4°
212°	671·4°
$t°$	$(459{\cdot}4+t)°$

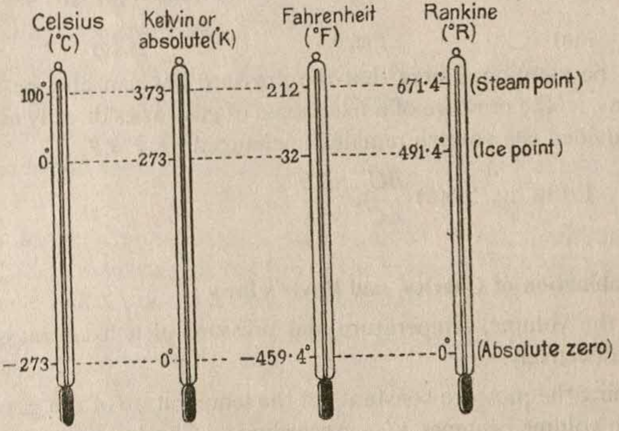


Fig. 5.5

Fig. 5.5 shows comparative readings on four different scales of thermometer.

General relation being $\dfrac{\text{C}-0}{100}=\dfrac{\text{F}-32}{180}=\dfrac{\text{K}-273}{100}=\dfrac{\text{R}-491{\cdot}4}{180}$

[N.B. In Celsius and Fahrenheit scales, zero marks have been fixed arbitrarily having no scientific reasoning behind them. But in absolute scale, zero mark has been fixed with sufficient reasoning because all gases cease to occupy any space or to exert any pressure at that temperature. Any other temperature lower than that is inconceivable. Further the scale, being independent of the nature of gas has justified its name *viz.* 'absolute scale'.]

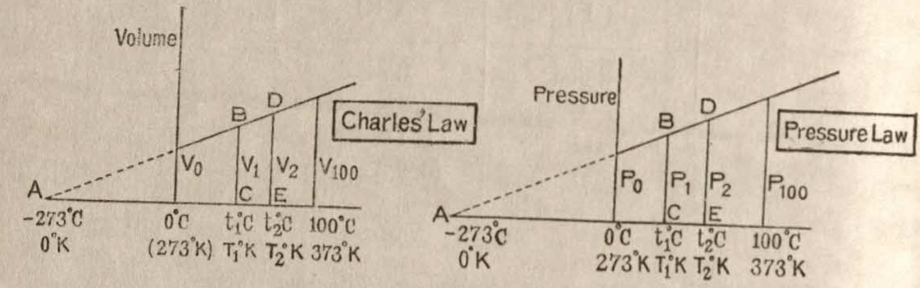
5.5. Alternative form of Charles' law and pressure law :

From Charles' law, we know,

$$V=V_0\left(1+\frac{t}{273}\right)=V_0\left(\frac{273+t}{273}\right)=\frac{V_0T}{273}=\frac{V_0T}{T_0}\text{ or, }\frac{V}{T}=\frac{V_0}{T_0}\text{ i.e. }V\propto T$$

In fig. 5.6(a) $\dfrac{BC}{AC}=\dfrac{DE}{AE}=\cdot\cdot$

i.e. the volume of a fixed mass of gas is directly proportional to its absolute temperature provided the pressure remains unchanged. This may be taken as an alternative statement of Charles' law.



(a) Fig. 5.6 (b)

It should be mentioned here that the pressure law can also be alternatively stated in this way : the pressure of a fixed mass of gas varies directly as its absolute temperature provided the volume remains unchanged *i.e.* $P \propto T$.

$$\text{From fig. 5.6(a) } \frac{BC}{AC} = \frac{DE}{AE} = \ldots$$

5.6. Combination of Charles' and Boyle's laws :

Suppose, the volume, temperature and pressure of a fixed mass of gas are V_1, T_1 and P_1 respectively.

(1) Keeping the pressure constant, let the temperature of the gas be changed to T_2 so that the volume becomes V'. According to Charles' law,

$$\frac{V'}{T_2} = \frac{V_1}{T_1} \quad \text{or} \quad V' = V_1 \frac{T_2}{T_1} \quad \ldots \quad \ldots \quad \text{(i)}$$

(2) Now, keeping the temperature fixed at T_2, the pressure is changed to P_2 so that the volume changes from V' to V_2. According to Boyle's law, we have

$$P_2V_2 = P_1V' = P_1 . \frac{V_1 . T_2}{T_1} \quad \text{[From eq}^n\text{. (i)] or, } \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}.$$

The volume, pressure and temperature of a certain mass of gas are jointly related to one another by this relation.

[**N.B.** Gases, in general, do not obey Boyle's or Charles' laws at all temperatures and pressures. Those which obey, are called *ideal* or *perfect gases* which do not exist in reality.]

Examples : (1) *A quantity of air occupies 1000 c.c. at 20°C and 760 mm. pressure. At what temperature will it occupy 1400 c.c. at 750 mm. pressure ?*

Ans. Here,

$V_1 = 1000$ *c.c.*	$V_2 = 1400$ *c.c.*
$P_1 = 760$ *mm.*	$P_2 = 750$ *mm.*
$T_1 = (20 + 273)$°K	$T_2 = ?$

$$\text{We know, } \frac{V_1P_1}{T_1} = \frac{V_2P_2}{T_2} \quad \text{or,} \quad \frac{1000 \times 760}{273 + 20} = \frac{1400 \times 750}{T_2}$$

$$\text{or, } T_2=\frac{1400\times750\times293}{1000\times760}=404{\cdot}8\ ^\circ\text{K}$$

So, in celsius scale, $t_2=404{\cdot}8-273=131{\cdot}8^\circ$C

(2) *The volume and pressure of 1 litre of a gas at 10°C are doubled by applying heat. Calculate the consequent temperature.*

Ans. Suppose, the initial pressure of the gas $=P$; its volume $=1$ litre and temperature $=10+273=283^\circ$K. Final pressure of the gas $=2P$ and volume $=$ 2 litres; $T_2=?$

$$\text{We know, } \frac{V_1P_1}{T_1}=\frac{V_2P_2}{T_2}; \quad \text{Here, } \frac{1\times P}{283}=\frac{2\times2P}{T_2}$$

$$\text{or, } T_2=4\times283=1132^\circ\text{K.}$$

So, in celsius scale, $t=1132-273=859^\circ$C.

(3) *An air bubble of volume 14 c.c. rose from the bottom to the surface of a lake 34 metre deep. The temperature at the bottom was 7°C and at the surface, the temperature and pressure were respectively 27°C and 75 cm. of mercury. Find the volume of the bubble when it rose to the surface of the lake. Density of mercury $=13{\cdot}6$ gm/c.c.*

Ans. Atmospheric pressure, expressed in terms of water column is $75\times13{\cdot}6=1020$ cm. So, the pressure at the bottom of the lake $=(1020+3400)$ cm $=4420$ cm of water. Boyle's and Charles' laws combined, give us

$$\frac{4420\times14}{273+7}=\frac{1020\times V}{273+27} \quad \text{or} \quad \frac{4420\times14}{280}=\frac{1020\times V}{300}$$

$$\therefore \quad V=\frac{4420\times14\times300}{1020\times280}=65 \text{ c.c.}$$

(4) *A gas is collected over water in a 100 c.c. tube and measures 72·8 c.c.; the temperature and pressure of the atmosphere at the time are 25°C and 74·39 cm. of mercury respectively. Calculate the volume of dry gas at N.T.P. The vapour pressure of water at 25°C is 23·45 mm. of mercury.*

Ans. Since the gas is collected over water, there will be water vapour mixed with dry air in the tube. So, the pressure of dry air $=$ air pressure $-$ pressure of water vapour $=(74{\cdot}39-2{\cdot}345)=72{\cdot}045$ cm. of Hg.

Now, $P_1=72{\cdot}045$ cm. $\quad P_2=76$ cm.
$V_1=72{\cdot}8$ c.c. $\quad V_2=?$
$T_1=25+273=298^\circ$C $\quad T_2=273^\circ$K.

$$\text{We know, } \frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2} \quad \text{or,} \quad \frac{72{\cdot}045\times72{\cdot}8}{298}=\frac{76\times V_2}{273}$$

$$\because \quad V_2=\frac{72{\cdot}045\times72{\cdot}8\times273}{298\times76}=63{\cdot}22 \text{ c.c.}$$

(5) *Two vessels of volumes V_1 and V_2 are filled with a gas and pressures in them are P_1 and P_2 respectively. The vessels are then connected by a narrow tube with a stop cock. What will be the final pressure of the gas if the stop cock is opened. Assume the temperature to remain unchanged.*

Ans. Let the pressure of the second vessel be changed from P_2 to P_1 without change of temperature so that the volume of the gas changes from V_2 to V'. Applying Boyle's law to the second vessel, we have $P_2V_2=P_1V'$ or $V'=\frac{P_2V_2}{P_1}$

Now, the volume V_1 at pressure P_1+volume V_2 at pressure P_2

$$=\text{Volume } V_1 \text{ at pressure } P_1+\text{volume} \frac{P_2V_2}{P_1} \text{ at pressure } P_1$$

$$=\text{Volume } \left(V_1+\frac{P_2V_2}{P_1}\right) \text{at pressure } P_1$$

When the stop-cock is opened, let the common pressure be P. The total volume of the gas is evidently $=V_1+V_2$. Again applying Boyle's law, we get,

$$P(V_1+V_2)=\left(V_1+\frac{P_2V_2}{P_1}\right)\times P_1=P_1V_1+P_2V_2$$

$$\therefore \quad P=\frac{P_1V_1+P_2V_2}{V_1+V_2}$$

5.7. Dalton's law of partial pressure :

In example 5 above we see that after the stop-cock is opened the final pressure in both the vessels is $P=\frac{P_1V_1+P_2V_2}{V_1+V_2}=P_1 \frac{V_1}{V_1+V}+P_2 \frac{V_2}{V_1+V_2}$ .. (i)

Now, suppose the air in the first vessel, having volume V_1 and pressure P_1 is allowed to expand and occupy both the vessels. Its pressure, in that case, is suppose p_1. Then, $p_1(V_1+V_2)=P_1V_1$ or $p_1=P_1.\frac{V_1}{V_1+V_2}$

Similarly, $p_2.=P_2.\frac{V_2}{V_1+V_2}$ is the pressure p_2 which the gas in the second vessel would have, if it expanded to occupy both the vessels.

From eqn. (i) we get $P=p_1+p_2$; thus, *the total pressure of the mixture is the sum of the pressures which the individual gases exert, when they have expanded to fill the vessels containing the mixture.*

The pressure exerted by an individual gas in a mixture is called its *partial pressure* ; it is the pressure which would be observed if the gas alone were allowed to occupy the volume of the mixture at the same temperature. We may, therefore, say that *the pressure of the mixture of gases is the sum of the partial pressures of its constituents.* This statement was first made by Dalton in 1801, and is called *Dalton's law of partial pressure.*

Example : *Two vessels of volumes 5 and 3 litres contain air at pressures of 3 and 7 atmospheres respectively. What will be the resultant pressure when they are connected together by a small tube? Assume the temperature to remain constant throughout.* [*I.I.T. 1970*]

Ans. Let p be the final pressure of the mixture.

Then, $p(V_1+V_2)=(P_1V_1+P_2V_2)$ or $p\times8=5\times3+3\times7=36$

$\therefore \quad p=\frac{36}{8}=4{\cdot}5$ atmosphere.

5.8. The ideal gas equation :

Combining Boyle's and Charles' laws, we have seen in the art. 5.6 that for a perfect gas $\frac{PV}{T}$=constant. If the constant be taken as R, then we can write, $PV=RT$.

The above equation connecting the volume, the pressure and the temperature of an ideal gas is called the *ideal gas equation or equation of state.*

Molar gas constant and ordinary gas constant :

The constant R in the ideal gas equation is very significant. If one gramme-molecule (or one mole) of the gas is considered, the costant is called the *universal gas constant or molar gas constant* because its value is constant for all gases, It is generally denoted by R_0.

On the other hand, if n gm-molecules of the gas are considered, the above equation can be written as $PV=nR_0T=KT$; $[K=nR_0]$

Here, the constant K is dependent on the mass of the gas taken.

If m be the mass of n gm-mol. of the gas whose molecular wt. is M, then $n=m/M$ $\therefore$ the gas constant $K=nR_0=\frac{mR_0}{M}$. .. (i)

So, for m *gm* of the gas, the gas equation becomes, $P.V=\frac{mR_0T}{M}$.

Further, if R be the gas constant per unit mass and M the molecular mass, then from eqn. (i) $R=\frac{R_0}{M}$. It is called the ordinary gas constant.

It is needless to mention that the value of R is different for different gases. (see art. 5.10)

5.9. Magnitude of universal gas constant :

In the case of one gm-molecule of a gas, the ideal gas equation $PV=R_0T$ is applicable at all temperatures and pressures. In other words, $R_0=\frac{PV}{T}=\frac{P_0V_0}{T_0}$.

Here V_0=volume of one gm-molecule of the gas at *N.T.P.*
$=22{\cdot}4$ litres$=22400$ c.c. (according to Avogadro hypothesis)
P_0=normal pressure (*i.e.* 76 cm. of Hg. pressure)
$=76\times13{\cdot}59\times980=1{\cdot}013\times10^6$ dynes/sq. cm.
T_0=normal temp. (*i.e.* $0°C$)$=(0+273)=273°$K

$$\therefore \quad R_0=\frac{1{\cdot}013\times10^6\times22400}{273}=8{\cdot}31\times10^7 \text{ ergs/°K}$$

5.10. Magnitude of ordinary gas constant :

When 1 gm of a gas at N.T.P. is taken, the gas constant is called the ordinary gas constant and is denoted by the letter R. Its value is different for different gases because the volumes occupied by 1 gm of different gases at N.T.P. are not the same.

(i) *For hydrogen* : For m gm of any gas, the gas constant$=\frac{m.R_0}{M}$. So, for 1 gm of hydrogen the gas constant $R=\frac{R_0}{M}$. Since molecular weight of hydrogen $M=2$, $R=\frac{8 \cdot 31\times 10^7}{2}=4 \cdot 15\times 10^7$ ergs/gm-°K.

(ii) *For Oxygen* : Molecular weight of oxygen$=32$. So, gas constant for 1 gm of oxygen $R=\frac{R_0}{32}=\frac{8 \cdot 31\times 10^7}{32}=0 \cdot 26\times 10^7$ ergs/gm-°K.

(iii) *For carbon-dioxide* : Molecular weight of $CO_2=44$, So, gas constant for 1 gm of CO_2 is $R=\frac{R_0}{44}=\frac{8 \cdot 31\times 10^7}{44}=0 \cdot 189\times 10^7$ erg/gm-°K.

Examples : (1) *The density of hydrogen at N.T.P. is* $8 \cdot 96\times 10^{-2}$ *gm/litre. Find the value of the gas constant for 100 gm. of the gas.*

Ans. If R_0 be the gas constant for 1 gm-molecule of a gas, then for m gm., the gas-constant $K=\frac{mR_0}{M}$ where M is the molecular wt. of the gas. Now, the molecular weight of hydrogen$=2$ gm. Since $8 \cdot 96\times 10^{-2}$ gm. of H_2 at *N.T.P.* occupies 1 litre, 2 gm. of H_2 at *N.T.P.* occupy$=\frac{2}{8 \cdot 96\times 10^{-2}}=22 \cdot 32$ litres

$=22 \cdot 32\times 10^3$ c.c.

So, $V_0=22 \cdot 32\times 10^3$ c.c. ; $P_0=1 \cdot 013\times 10^6$ dynes/sq. cm. ; $T_0=273°K$

$$\therefore\ R_0=\frac{P_0V_0}{T_0}=\frac{1 \cdot 013\times 10^6\times 22 \cdot 32\times 10^3}{273}=8 \cdot 28\times 10^7 \text{ ergs/°K (nearly)}$$

$$\text{Now, } K=\frac{mR_0}{M}=\frac{100\times 8 \cdot 28\times 10^7}{2}=4 \cdot 14\times 10^9 \text{ erg/°K. (nearly)}$$

[Alternatively, for 1 gm of H_2 $R=4 \cdot 15\times 10^7$ ergs/gm-°K. So for 100 gm the gas constant $=4 \cdot 15\times 10^7\times 100=4 \cdot 15\times 10^9$ erg/°K]

(2) *A cylinder contains oxygen at 27°C and* $2 \cdot 5\times 10^6$ *dyne/cm*^{2} *pressure. The volume of the cylinder is* 10^4 *c.c. When some oxygen is let out of the cylinder, the pressure becomes* $1 \cdot 3\times 10^6$ *dyne/cm*^{2}. *What is the mass of oxygen let out ? Molecular weight of oxygen=32.*

Ans. For n mole of gas, we know, $PV=n\ R_0T$ or $n=\frac{PV}{R_0T}$. If there be n_1 mole of oxygen in the cylinder at the beginning, then $n_1=\frac{P_1V}{R_0T}$. After some gas is let out, suppose there are n_2 mole of gas in the cylinder. Then $n_2=\frac{P_2V}{R_0T}$.

$\because$ The no. of moles of gas expelled $=n_1-n_2=\frac{(P_1-P_2).V}{R_0T}$

$$=\frac{(2 \cdot 5-1 \cdot 3)\times 10^6\times 1\times 10^4}{8 \cdot 3\times 10^7\times (273+27)}=\frac{1 \cdot 2\times 10^{10}}{8 \cdot 3\times 10^7\times 300}=0 \cdot 48$$

Since mass of 1 mole of oxygen $=32$ gm., the mass of oxygen expelled $=0{\cdot}48\times32=15{\cdot}36$ gm.

(3) *Two gas cylinders with volumes of 100 c.c. and 1000 c.c. respectively are connected by a tube of negligible volume and contain air at a pressure of 100 cm. of mercury. If the temperature of both the cylinders is originally 0°C, how much air will pass through the connecting tube when the temperature of the smaller is raised to 100°C ? Express your answer in 0°C and 76 cm. of mercury pressure.*

Ans. Some air will pass from hot cylinder to the cold one but the total mass of the gas will remain unchanged. If R be the gas constant per unit mass, then for m gm of mass, we can write, $PV=m.R.T$ or $m=\frac{PV}{R.T}$.

In the first case the total mass of the gas in the cylinders $=\frac{PV}{RT}=\frac{100\times1100}{R\times273}$

Afterwards the total mass $=\frac{p\times100}{R(273+100)}+\frac{p\times1000}{R\times273}$ [$p=$ final common pressure]

Since total mass remains unchanged,

$$\frac{p\times100}{R(273+100)}+\frac{p\times1000}{R\times273}=\frac{100\times1100}{R\times273} \text{ or, } p\left(\frac{100}{373}+\frac{1000}{273}\right)=\frac{100\times1100}{273}$$

$$\therefore\quad p=\frac{1100\times373}{4003}=102{\cdot}5 \text{ cm of } Hg$$

Now, the initial pressure of gas in the smaller cylinder at 0°C and 76 cm. pressure $=100\times\frac{100}{76}=131{\cdot}6$ c.c. At 0°C and 76 cm. pressure, the final volume of the gas in the smaller cylinder $=100\times\frac{102{\cdot}5}{373}\times\frac{273}{76}=98{\cdot}7$ c.c.

$\therefore$ The volume of the gas passing through the tube at 0°C and 76 cm of pressure $=131{\cdot}6-98{\cdot}7=32{\cdot}9$ c.c.

5.11. Relation between the pressure, temperature and density of a gas :

Suppose, the mass of a certain quantity of gas is 'm' and its volume and density at T_1°K are V_1 and D_1 respectively. If the temperature changes to T_2°K, its volume and density will change but its mass will remain unchanged.

Hence, $V_1D_1=m=V_2D_2$ or, $V_1=\frac{m}{D_1}$ and $V_2=\frac{m}{D_2}$.

If the pressures of the gas at those temperatures be P_1 and P_2 respectively, then from ideal gas equation, we have,

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2} \text{ or, } \frac{P_1m}{D_1T_1}=\frac{P_2.m}{D_2.T_2} \text{ or, } \frac{P_1}{D_1T_1}=\frac{P_2}{D_2T_2} \text{ i.e. } \frac{P}{DT}=\text{constant.}$$

(i) Keeping the pressure unchanged, if the temperature be varied, then, $D_1T_1=D_2T_2$.

(ii) Keeping the temperature unchanged, if the pressure be varied, then, $P_1D_2=P_2D_1$.

Examples : (1) *A litre of dry air at N.T.P. weighs 1·293 gm. What would be the weight of 3 litres at 100°C and a pressure of 3 atmospheres ?*

Ans. The density of dry air at *N.T.P.* (D_1 say)$=1\cdot293$ gm./litre.
Its density at 100°C and 3 atmosphere pressure$=D_2$ (say)
Here, $P_1=1$ atmos ; $T_1=273°$K ; $D_1=1\cdot293$ gm/litre.
and $P_2=3$ atmos ; $T_2=273+100=373°K$; $D_2=$?

We know, $\frac{P_1}{D_1T_1}=\frac{P_2}{D_2T_2}$ or, $D_2=\frac{P_2D_1T_1}{P_1T_2}=\frac{3\times1\cdot293\times273}{1\times373}=2\cdot84$ gm/litre

∴ mass of 3 litres of dry air$=3\times2\cdot84$ gm$=8\cdot52$ gm.

(2) *A glass bulb of volume 1 litre is heated from 0°C to 100°C at sea-level with its mouth open. Find the mass of air expelled from the bulb. Density of air at N.T.P.$=129\times10^{-5}$ gm/c.c.*

Ans. The pressure of the bulb remains constant as its mouth is open to atmosphere. In this case,

$$\frac{V_1}{T_1}=\frac{V_2}{T_2} \text{ or, } \frac{1}{273}=\frac{V_2}{273+100} \text{ or } V_2=\frac{373}{273}=\left(1+\frac{100}{273}\right) \text{ litre.}$$

So, the volume of expelled air$=\frac{100}{273}$ litre$=\frac{10^5}{273}$ c.c.

As the pressure remains unchanged, we have $D_1T_1=D_2T_2$

$$\therefore\quad D_2=\frac{D_1T_1}{T_2}=\frac{129\times10^{-5}\times273}{273+100}=\frac{129\times10^{-5}\times273}{373}$$

∴ the mass of expelled air$=\frac{10^5}{273}\times\frac{129\times10^{-5}\times273}{373}=\frac{129}{373}$ gm$=0\cdot346$ gm.

(3) *A glass vessel contains air at 60°C. To what temperature must it be heated to expel $\frac{1}{3}$ of the air, pressure remaining constant.*

Ans. If V be the volume of the gas at 60°C, then the volume at higher temperature should be$=\frac{3V}{2}$; because if $\frac{1}{3}$ of the volume is expelled, the volume of expelled air$=\frac{V}{2}$; hence the volume of the remaining air $=\frac{3V}{2}-\frac{V}{2}=V.$ Since the pressure is kept constant, $\frac{V}{273+60}=\frac{\frac{3}{2}V}{273+t}$ [$t°C=$reqd. temp.]

or, $273+t=333\times\frac{3}{2}=499\cdot5$; ∴ $t=499\cdot5-273=226\cdot5°$C.

[*Alternatively :* Since pressure remains constant, we have

$$\frac{V_1}{V_2}=\frac{T_1}{T_2}=\frac{273+60}{T_2}=\frac{333}{T_2}. \text{ So, } \frac{V_2-V_1}{V_2}=\frac{T_2-333}{T_2}$$

According to the question, $\frac{V_2-V_1}{V_2}=\frac{1}{3}$ ∴ $\frac{1}{3}=\frac{T_2-333}{T_2}$ or, $T_2,=499\cdot5°$K]

(4) *In manufacturing electric vacuum tubes, a tube is sealed at temperature 27°C and pressure $1\cdot2\times10^{-6}$ cm. The volume of the tube is 100 c.c. Calculate the number of gas molecules left in the tube. Avogadro number i.e. the number of molecules in 22·4 litre of any gas at N.T.P.$=6\cdot02\times10^{23}$.* [*Jt. Entrance 1967*]

Ans. Let the number of gas molecules left in the tube be n.

The density of the gas at N.T.P. $=\dfrac{6{\cdot}02\times10^{23}}{22{\cdot}4\times10^{3}}$

We know, $\dfrac{P}{D.T}=\dfrac{P_1}{D_1T_1}$ Here, $P=76$ cm. ; $D=\dfrac{6{\cdot}02\times10^{23}}{22{\cdot}4\times10^{3}}$; $T=273°$K.

$$P_1=1{\cdot}2\times10^{-6}\text{ cm. } D_1=\frac{n}{100}\ ;\ T_1=273+27=300°\text{K.}$$

Hence, $\dfrac{76\times22{\cdot}4\times10^{3}}{6{\cdot}02\times10^{23}\times273}=\dfrac{1{\cdot}2\times10^{-6}\times100}{n\times300}$

or, $n=\dfrac{1{\cdot}2\times10^{-6}\times100\times6{\cdot}02\times10^{23}\times273}{300\times76\times22{\cdot}4\times10^{3}}=3{\cdot}86\times10^{13}$

5.12. Coefficient of expansion of gases :

It has already been mentioned that the expansibility or the compressibility of gases is much more pronounced than those of solids and liquids. Consequently, the volume or pressure of a certain mass of gas can easily be varied by slight variation of its temperature. For this reason, gases are supposed to have two co-efficients of expansion *viz*, (i) *Volume co-efficient* giving the change of volume due to change of temperature when the pressure is kept constant and (ii) *Pressure co-efficient* giving the change of pressure due to change of temperature when the volume is kept constant.

(i) **Volume co-efficient (γ_p) :**

Definition : *The volume co-efficient of expansion of a gas at constant pressure is the fraction of its volume at 0°C by which the volume of a fixed mass of gas expands per degree celsius rise in temperature.* The value of this co-efficient is same for all gases.

Suppose, a fixed mass of gas occupies volumes V_0 and V_t at 0°C and t°C respectively. Hence, rise of temperature $=t-0=t$°C and increase in volume $=V_t-V_0$.

$\therefore$ For 1°C rise in temperature, the increase in volume $=\dfrac{V_t-V_0}{t}$

and ,, ,, ,, ,, ,, ,, ,, per unit volume $=\dfrac{V_t-V_0}{V_0.t}$

$$\text{Hence, volume coeff}^{n}\ (\gamma_p)=\frac{V_t-V_0}{V_0.t}$$

(ii) **Pressure co-efficient (γ_v) :**

Definition : *The pressure co-efficient of expansion of a gas at constant volume is defined as the fraction of its pressure at 0°C, by which the pressure of a fixed mass of gas increases per degree celsius rise in temperature.*

As before, suppose, the pressures of a fixed mass of gas at 0°C and t°C are respectively P_0 and P_t.

Here, rise of temperature $=t-0=t$°C and increase in pressure $=P_t-P_0$.

$\therefore$ For 1°C rise in temperature, the increase in pressure $=\dfrac{P_t-P_0}{t}$

and ,, ,, ,, ,, ,, ,, ,, ,,

per unit pressure $=\dfrac{P_t-P_0}{P_0.t}$

$$\therefore \text{ Pressure coeff}^n\ (\gamma_v)=\frac{P_t-P_0}{P_0 t}.$$

5.13. Reason for taking initial volume at 0°C in calculating the volume co-efficient of a gas :

In the case of a gas, the initial volume is always taken at 0°*C* in calculating the volume coefficient or volume expansion. In the case of solid or liquid, it may be mentioned here, that the initial volume may be taken at any other temperature. The reason is that the coefficient of cubical expansion of solids and liquids is very small while that of gases is sufficiently large ($\frac{1}{273}$). For example, if the volumes of a piece of solid or of some liquid at t_1°C and t_2°C be V_1 and V_2 respectively, then we can write without much error,

$V_2=V_1\{1+\gamma(t_2-t_1)\}$ [γ=coeff^n of cubical expansion of solid or liquid]

But in the case of a gas, we can not write in the above way ; we should write $V_1=V_0\{1+\gamma t_1\}$ and $V_2=V_0\{1+\gamma t_2\}$. [γ=coeff^n of cubical expansion of a gas.]

Let us take one concrete example. Suppose, the volume of a certain mass of gas at 0°C is 273 c.c. Then at 100°C and 120°C, the volumes of the gas, according to its expansion law, will be,

$$V_{100}=V_0\left(1+\frac{1}{273}\times 100\right)=273\left(1+\frac{100}{273}\right)=373 \text{ c.c.}$$

$$\text{and } V_{120}=V_0\left(1+\frac{120}{273}\right)=273\left(1+\frac{120}{273}\right)=393 \text{ c.c.}$$

Let us now see what becomes the value of V_{120} taking V_{100} as the initial volume.

$$V_{120}=V_{100}\left\{1+\frac{1}{273}\left(120-100\right)\right\}=373\left\{1+\frac{20}{273}\right\}=400{\cdot}3 \text{ c.c.}$$

The volume obtained is much more than the volume (393 c.c.) available according to the Charles' law. So this method is erroneous. *It is, therefore, to be borne in mind that in the case of gases, the initial volume must be taken at 0°C.*

5.14. Relation between the co-efficients of expansion of a perfect gas :

Suppose, the volume of a fixed mass of gas increases from V_0 to V_t due to an increase of temperature from 0°C to t°C, when the pressure is kept constant. From the definition of volume co-efficient, we have,

$$\gamma_p=\frac{V_t-V_0}{V_0.t} \quad \text{or} \quad V_t=V_0+V_0.\gamma_p.t=V_0(1+\gamma_p.t)\ldots.\text{(i)}$$

Now, suppose, the temperature is kept constant at t°C, but the pressure is changed from P_0 to such a value P_t that the volume of the gas V_t decreases to original value V_0. Applying Boyle's law, we get, $P_t.V_0=P_0.V_t$....(ii)

Combining eq^n. (i) and (ii), we have,

$P_t.V_0=P_0V_0(1+\gamma_p.t)$ or, $P_t=P_0(1+\gamma_p.t)$....(iii)

If, on the other hand, the temperature of the gas is increased from 0°C to t°C, keeping the volume of the gas constant, then from the definition of the pressure coefficient, we may write, $\gamma_v=\dfrac{P_t-P_0}{P_0.t}$ or $P_t=P_0+P_0.\gamma_v.t=P_0(1+\gamma_v.t)$....(iv)

Comparing eq^n. (iii) and (iv), we have, $\gamma_p=\gamma_v$.

i.e., the two coefficients of expansion of a perfect gas are equal. In this connection, it may be mentioned that the value of the coeff^n is $\frac{1}{273}$ or 0·00366.

It is worthwhile to note that solids and liquids have different coefficients of expansion but all gases have same coefficient of expansion. It is an important distinction between the properties of gases and those of liquids and solids.

5.15. Experimental determination of the two coefficients of expansion of gas :

For the determination of volume coeff^n (γ_p) of a gas, Regnaults' constant pressure thermometer and for the determination of pressure coeff^n (γ_v) Jolly's constant volume thermometer apparatus are necessary.

(A) Regnault's constant pressure thermometer and determination of volume coefficient :

Description : A quantity of dry air is contained in a glass bulb A, which forms part of a U-tube graduated to read volume in c.c. This bulb and the U-tube are partially filled by sulphuric acid which can be drained out, if necessary, by opening the stop-cock D (Fig. 5.7). The temperature of the air is controlled by a water-bath which may be heated by passing in steam through a bent copper tube MN. A stirrer (S) is provided to stir the water and a thermometer T to read the temperature of air.

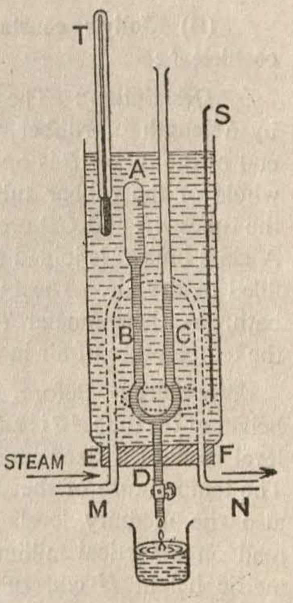


Fig. 5.7

Procedure : By draining out the acid or by pouring in a little more through the open end of the U-tube, the acid-levels in the two arms are to be made equal. The enclosed air will then be at atmospheric pressure. Having noted the volume of the air and the temperature, steam is allowed to pass through the copper tube MN. The water is then continuously stirred and its temperature raised in steps of about 10°C at a time.

At each step, suitable quantities of acid are to be drained off slowly in order to equalise the acid-levels in the two arms. This will ensure that the pressure of the enclosed air is kept constant and is equal to the atmospheric pressure. Having done this, the volume of the air and its temperature are recorded after allowing sufficient time for the air to attain the temperature of the bath in each case.

From the results obtained, a graph of volume against temperature is plotted (Fig. 5.8). The graph will be a straight line. In the figure, *AB* is the straight line. If the straight line is produced backwards it cuts the volume axis at a point which gives the volume which the air would have occupied at 0°C. Suppose it is V_0. Now, take any point *P* on the graph and draw a perpendicular *PN* on the temperature axis. *ON* represents a temperature and *PN*, the corresponding volume of the air.

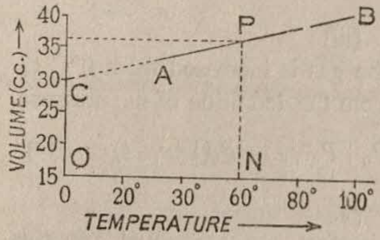


Fig. 5.8

From the fig. 5.8 it is seen that $OC=V_0=30$ c.c. ; $ON=t=60°C$; $PN=V_t=36{\cdot}6$ c.c.

$$\text{From art. } 5{\cdot}12,\ \gamma_p=\frac{V_t-V_0}{V_0.t}=\frac{36{\cdot}6-30}{60\times 30}=\frac{6{\cdot}6}{1800}=\frac{1}{273}.$$

[N.B. This experiment may also be taken as a verification of Charles' law.]

(B) Jolly's constant volume thermometer and determination of pressure coefficient :

Description : The apparatus consists of two glass tubes *AB* and *CD* joined by a length of rubber tubing (Fig. 5.9). One end of the tube *AB* is open to atmosphere. The whole of the rubber tubing and some parts of the tubes *AB* and *CD* are filled up with mercury. A glass bulb *E* is joined to the tube *CD* and it is filled by dry air. The bulb is placed in a water-bath. A thermometer *T* is provided to record the temperature of air in the bulb.

Procedure : Before heating the water, the height of the tube *AB* is adjusted until the mercury level reaches a fixed mark *F* on the tube *CD*. The temperature of the water-bath is noted and also the mercury levels in the two tubes are read on a vertical millimetre scale. The barometric height *H* cm. of mercury is read from a Fortin's barometer. If the difference in the mercury levels of the two tubes be h_1 cm., the pressure of the air in the bulb *E* is given by $H \pm h$ according as the level in the tube *AB* is above or below *F*.

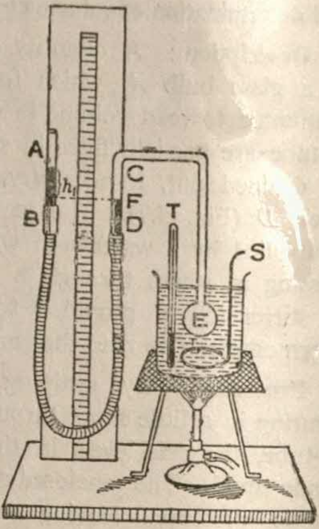


Fig. 5.9

A series of such readings are taken, as the temperature of the water-bath is raised by about 10°*C* intervals. Each time the mercury level in the tube *CD* is carefully adjusted to the mark *F* to ensure that the volume of the air remains constant. The water is stirred well and sufficient time is allowed for the air to attain the temperature of the bath before the readings are taken.

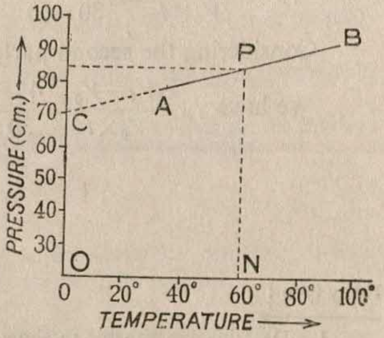


Fig. 5.10

From the results obtained, a graph of pressures against temperature is plotted as in the previous experiment. The graph will be a straight line (Fig. 5.10). If the straight line is produced backwards, it cuts the pressure axis at a point *C* which gives the pressure that the air will exert at 0°C. Suppose it is P_0. Now, take any point *P* on the graph and draw a perpendicular *PN* on the temperature axis. *ON* represents a temperature *t* and *NP* the corresponding pressure P_t of the air. From the figure it is seen that,

$$OC = P_0 = 70 \text{ cm. } ; ON = t^\circ C = 60^\circ C \text{ and } PN = P_t = 85{\cdot}4 \text{ cm.}$$

From art. 5.12, $\gamma_v = \dfrac{P_t - P_0}{P_0 . t} = \dfrac{85{\cdot}4 - 70}{70 \times 60} = \dfrac{15{\cdot}4}{4200} = \dfrac{1}{273}$.

It may be pointed out here that with this apparatus, an unknown temperature may be determined, provided the volume coefficient or the pressure coefficient is known. For these reasons, the instruments are called thermometers. As a matter of fact, use of gas thermometers as standard thermometers is due to these instruments.

Examples : (1) *A certain mass of gas in a constant volume air thermometer is at the temperature of melting ice under a pressure of 75 cm. of mercury. When the temperature is raised to that of the steam from water boiling under standard pressure, the pressure on the gas is 102·4 cm. When the bulb of the thermometer is placed in a furnace, the pressure on the gas is 256·4 cm. Calculate the temperature of the furnace.*

Ans. Temperature of melting ice=0°C ; temperature of water boiling under standard pressure=100°C. Considering the first part of the problem, we have, $P_0 = 75$ cm. ; $P_{100} = 102{\cdot}4$ cm ; $t = = 100^\circ$C.

Now, $\gamma_v = \dfrac{P_{100} - P_0}{P_0 . t} = \dfrac{102{\cdot}4 - 75}{75 \times 100} = {\cdot}00365$

Considering the second part, we have, $P_0 = 75$ cm., $P_t = 256{\cdot}4$ cm., $\gamma_v = {\cdot}00365$, $t = ?$

We have, $\gamma_v = \dfrac{P_t - P_0}{P_0 . t} = \dfrac{256{\cdot}4 - 75}{75 \times t}$ or $t = \dfrac{181{\cdot}4}{75 \times {\cdot}00365} = 663^\circ$C (nearly)

(2) *In a constant pressure air thermometer, volume of air in the bulb is found to be 30 c.c. at 0°C and 36·6 c.c. at 60°C. When the bulb is introduced in the vapour of a boiling liquid, the volume of air is found to be 45 c.c. Calculate the boiling point of the liquid.*

Ans. From the first part, we get $V_0=30$ c.c., $V_{60}=36{\cdot}6$ c.c., $t=60°C$;

Now, $\gamma_p=\dfrac{V_{60}-V_0}{V_0\times t}=\dfrac{36{\cdot}6-30}{30\times 60}={\cdot}00366.$

Considering the second part, $V_0=30$ c.c. ; $V_t=45$ c.c., $\gamma_p={\cdot}00366$; $t=$?

We have $\gamma_p=\dfrac{V_t-V_0}{V_0\times t}=\dfrac{45-30}{30\times t}$ $\therefore\ t=\dfrac{15}{30\times{\cdot}00366}=136{\cdot}6°C$ (nearly)

Exercises

Essay type :

1. Describe experiments to show that gases expand when heated. What are gas laws ? What is the experimental verification of Charles' law?

2. State Boyle's law. How would you verify it experimentally?

3. State Dalton's law of partial pressure. How will you establish the law?

4. What are the coefficients of expansion of a gas? What are their definitions? What is their relation?

5. Describe Regnault's constant pressure air thermometer. How can you determine the volume coefficient of a gas?

Short answer type :

6. What is absolute scale of temperature? How can you get the scale from Charles' law? What is the significance of the scale being called 'absolute'?

7. Find the values of zero degree in the absolute scale in terms of Celsius and Fahrenheit degrees.

8. Establish a relation between the pressure, volume and temperature of a certain mass of gas.

9. The volume V versus temperature T for a certain amount of a perfect gas at two pressures P_1 and P_2 are as shown in fig. 5.11. It follows from the graph that P_1 is greater than P_2. Do you agree ? [*I.I.T. 1982*]

10. What is molar gas constant? Is it same for all gases? What is its magnitude? Is the gas constant for 1 gm. of every gas the same ?

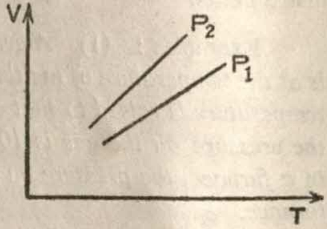


Fig. 5.11

11. Why are temperature and pressure mentioned while determining the volume expansion of a gas?

12. The pressure coefficient of a gas is $\frac{1}{273}$ °K^{-1}. Explain the statement.

13. Keeping pressure constant, if the temperature of a certain mass of gas increases, will its volume decrease or increase ?

14. (i) In what respect does the expansion of a gas differ from that of a liquid or a solid?
(ii) If hot air rises, why is it cooler at the top of a mountain than near sea level?

15. A liquid has an apparent coefficient of expansion but gases have none. Why?

16. Why do we always take the initial volume and pressure at 0°C while we define coefficient of expansion of a gas in contrast to the cases of solid and liquid?

17. Given samples of 1 c.c. of hydrogen and 1 c.c. of oxygen both at N.T.P. Which sample has a larger number of molecules? [*I.I.T. 1979*]

[**Hints :** According to Avogadro's hypothesis, they contain equal number of moleules.]

Objective type :

18. Put a tick mark against the correct answer in the following :—

(i) When equal volumes of different gases are heated equally, they expand differently/equally/proportionate to their densities.

(ii) Pressure remaining constant, the density of a gas varies directly/inversely/equally with its temperature in absolute scale.

(iii) Temperature remaining constant, the density of a gas varies inversely/directly/equally with its pressure.

(iv) To double the volume of a given mass of an ideal gas at 27°C, keeping the pressure constant, one must raise the temperature in degree centigrade to 54°/270°/327°/600°.

Numerical problems:

19. The volume of a gas is doubled by changing its temperature at constant pressure. The initial temperature of the gas was 13°C. Calculate the final temperature. [Ans. 299°C]

20. A volume of gas at S.T.P. is heated so that its pressure and volume are doubled. What will be its final temperature? [*H. S. Exam. 1980*] [Ans. 819°C]

21. At what depth in a lake will a bubble of air have one-half the volume it will have on reaching the surface? The height of the barometer at the time is 76 cm. of mercury and density of mercury 13·6 gm/c.c. [Ans. 1033·6 cm.]

22. The volume of an air bubble increases two-fold in rising *h* metres of depth in a sea. If the barometric height at that time is 750 mm. and the densities of mercury and sea-water are 13·58 and 1·05 gm/c.c. respectively, find the value of '*h*'? [*H. S. Exam. 1961*] [Ans. 9·7 metre]

23. A certain quantity of gas at 15°C and at a particular pressure is heated to double its volume, pressure remaining the same. What is the final temperature? [Ans. 303°C]

24. A vessel contains 1000 litres of gas at 0°C and 740 mm. of mercury pressure. If the temperature be increased to 27°C, what would be the pressure of the gas, volume supposed to be constant? [Ans. 813·1 mm.]

25. The volume occupied by a certain mass of gas at 27°C and 740 mm. of mercury pressure is 400 c.c. If the temperature be changed to 0°C and pressure to 760 mm. of mercury, what would be the volume of the gas? [Ans. 354·4 c.c.]

26. A cylinder contains a perfect gas at a temperature of 20°C and a pressure of 15 atmosphere in a volume of 100 litres. A piston is lowered into the cylinder decreasing the volume occupied by the gas to 80 litres and raising the temperature to 25°C. What is the gas pressure now? [Ans. 19 atmos]

27. 100 c.c. of gas is collected over water at 20°C and 760 mm. pressure. The space occupied by the gas is saturated with water vapour. What will be the volume of the gas at N.T.P.? The saturation vapour pressure at 20°C=17·4 mm. [Ans. 91·04 c.c.]

28. A thin walled glass bulb was sealed at 27°C. The pressure inside the bulb is one atmosphere. The bulb can withstand maximum internal pressure of 95 cm. of mercury. At what temperature will the bulb burst? [Ans. 102°C]

29. Three litres of air at 0°C and a pressure of 1 atmosphere are heated in such a way as to keep its volume unchanged. Calculate the temperature of the air when its pressure becomes 5 atmospheres. The temperature is now kept constant and the pressure reduced to 1 atmosphere. What will be the resulting volume? If, finally this last pressure is maintained, calculate the temperature for which the volume will be reduced to 2 litres? [Ans. 1092°C ; 15 litre ; −91°C]

[**Hints** : First apply pressure law *i.e.* $P \propto T$; then apply Boyle's law *i.e.* $V \propto \frac{1}{P}$ and finally apply Charles' law.]

30. A motor car tyre has a volume of 15 litres and contains air at 2·5 times atmospheric pressure when the temperature is 17°C. What will be the pressure inside the tyre in lb/inch^2 when the temperature rises to 37°C and the volume of the tyre increases to 15·5 litres? 1 atmospheric pressure=15 lb/sq. inch. [*Jt. Entrance 1962*] [Ans. 38·8 lb/sq. inch (nearly)]

31. A glass vessel contains air at 42°C. To what temperature must it be heated to expel 1/4th of the air, pressure remaining constant? [Ans. 147°C]

32. The volume of a certain mass of gas expands from 1 litre to 1·128 litre, when the temperature is changed from 0°C to 35°C, pressure remaining constant. Calculate the value of absolute zero from these data. [Ans. −273°C]

33. A balloon, on the surface of the earth contains 1000 cu. ft. of helium gas at 14·7 lb/sq. inch pressure and 27°C temperature. If the balloon ascends 10 miles above the surface and enters stratosphere, where the pressure is 1·5 lb/sq. inch and temperature is −55°C, what will be the volume of the gas inside the balloon? [Ans. 7121·3 cu.ft.]

34. A capillary glass tube with fine bore has both the ends sealed and a small mercury pellet inside. The pellet divides the tube into two parts whose lengths are in the ratio of 3:1. If the initial temperature of the whole arrangement is 0°C and if it is heated afterwards to 273°C what will be the change in the air-pressure inside the tube? [Ans. Pressure in each part doubled]

35. A vessel of volume 5 litres contains gas at one atmospheric pressure. What will be the pressure inside vessel when it is heated to 100°C with mouth closed? What fraction of the gas will be expelled if the above heating is done with its mouth open? [Ans. 1·37 atmos; $\frac{100}{373}$]

36. Two glass bulbs of equal volume are joined by a narrow tube and are filled with a gas at N.T.P. When one bulb is kept in melting ice and the other in a hot bath, the new pressure is 877·6 mm. of mercury. Calculate the temperature of the bath. [Ans. 97°C]

[**Hints** : If R be the gas constant per unit mass, then for m gm of the gas $PV=m.R.T.$ or $m=\frac{PV}{RT}$; at first both the bulbs contain same mass of gas. If m be the mass of gas in each bulb, $m=\frac{76V}{R\times 273}$ $\therefore$ Total mass$=2m=\frac{152V}{R\times 273}$. Afterwards gas from hot bulb will flow to the cold bulb till the pressure is same everywhere, which is 877·6 mm or 87·76 cm. If m_1 be the mass of gas left in the hot bulb$=m_1=\frac{87{\cdot}76\times V}{R(273+t)}$ where t°C=temperature of hot bulb and that in the cold bulb$=\frac{87{\cdot}76\times V}{273}$.]

37. Two glass bulbs of equal volume are connected by a narrow tube and are filled with a gas at 0°C and a pressure of 76 cm. of mercury. One of the bulbs is then placed in a water bath maintained at 62°C. What is the new value of the pressure inside the bulbs ? The volume of the connecting tube is negligible. [*I.I.T. 1985*] [Ans. 83·74 cm of *Hg*]

38. A cylinder closed at both ends is divided into two parts by a thermally insulated piston. Both halves of the cylinder contain equal masses of gas at a temperature of 27°C and a pressure of 1 atmosphere. What distance from the centre of the cylinder will the piston move if the gas in one half is heated to 77°C. The initial length of each half of the cylinder is 26 cm. [Ans. 2 cm.]

39. An empty bottle is corked when the air inside it is at 10°C, and the barometer reads 75 cm. of *Hg*. If the cork blows out when the pressure inside the bottle exceeds the atmospheric pressure by 100 cm. of *Hg*, calculate the temperature at which the bottle must be heated to blow the cork. Assume that the volume of the bottle remains constant when heated. [Ans. 387·3°C]

40. An open glass bottle of 0·5 litre capacity contains air at atmospheric pressure of 76 cm. of *Hg* and at temperature 27°C. It is heated uniformly to 167°C allowing air to escape and then corked. What is the new pressure of air inside the flask if it is cooled to 27°C ?

[Ans. 51·82 cm.]

41. The volume of a certain mass of gas at constant pressure increases from 1 litre to 1·128 litre due to an increase of temperature from 0°C to 35°C. From these data, find the value of absolute zero. [*H. S. Exam. 1973*] [Ans. −273°C]

42. At constant pressures, 5 litres of gas at 0°C increases by 640 c.c. when the temperature is increased to 35°C. Find the value of absolute zero in celsius scale.

[*H. S. Exam. 1965*] [Ans. −273°C]

Harder problems :

43. Two identical barometer tubes are standing side by side. In one tube, the mercury column has just reached the top of the tube while in the other it is 1 cm. below the top because of the presence of some dry air over it. When the temperature of air is raised by 63°C, the mercury column is depressed 1 mm. more. What was the initial temperature of air ? [Ans. 27°C]

44. In the middle of a narrow horizontal tube sealed at both ends is a column of mercury 12 cm. in length. Both halves of the tube contain air under a pressure of 76 cm. of mercury. By what distance will the mercury column be displaced if the tube is held vertically ? The length of the tube is 100 cm. [Ans. 3·5 cm.]

45. A glass capillary tube, sealed at both ends, is 100 cm. long. It lies horizontally with the middle 10 cm. occupied by mercury. The two ends of the tube (which are equal in length) contain air at 27°C and at a pressure of 76 cm. of mercury. Keeping the tube in a horizontal position, the air-column at one end is now kept at 0°C and the other end is maintained at 127°C. Calculate the length of the air column which is at 0°C and also its pressure. Neglect the change in volume of mercury and glass. [*I.I.T. 1975*] [Ans. 36·5 cm. ; 85·2 cm. *Hg*]

[**Hints :** After change of temperature, the pressures of air change to a new value but it will be equal in both the columns. Let l=length of the air column at 127°*C* ; the length of the other column=$(90-l)$cm. Now $\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$ or $\frac{p\times l\times\alpha}{400}=\frac{p\times(90-l)\times\alpha}{273}$ $\therefore$ $l=53\cdot5$ cm.

$\therefore$ Length of the air column at $0°\text{C}=90-53\cdot5=36\cdot5$ cm.

Again, $\frac{P_1V_1}{T_1}=\frac{P_0V_0}{T_0}$ or $\frac{p\times 53\cdot5\times\alpha}{400}=\frac{76\times45\times\alpha}{300}$ $\therefore$ $p=85\cdot2$ cm.]

46. An air bubble gradually rises from a depth of 10 metre in a lake where the temperature is 10°C to the surface where the temperature is 25°C Compare the radii of the bubble in the two places. Atmospheric pressure=75 cm. of *Hg* and density of mercury=13·6 gm/c.c. [Ans. 1·275]

47. An air bubble starts rising from the bottom of a lake. Its diameter is 3·6 mm. at the bottom and 4 mm. at the surface. The depth of the lake is 2·5 metre and the temperature at the surface is 40°C. What is the temperature at the bottom of the lake ? Assume that the variation of density of water with depth is negligible. Atmospheic pressure is 76 cm. of *Hg* and $g=980$ cm/s^2. [*I.I.T. 1966*] [Ans. 10·2°C]

[**Hints :** Apply $\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$; $P_1=(76\times13\cdot6+250)$ cm. $V_1=\frac{4\pi}{3}(0\cdot18)^3$ c.c.

$P_2=(76\times13\cdot6)$ cm. ; $V_2=\frac{4\pi}{3}(0\cdot2)^3$ c.c. ; $T_1=$? $T_2=(273+40)$°K.]

48. A balloon may carry a total load of 175 kg. when the temperature and pressure are normal. What load will the balloon carry on rising to a height where the barometric pressure is 50 cm. of *Hg* and the temperature is −10°C ? Assume that the envelope of the balloon maintains a constant volume. [Ans. 119·5 kg]

49. A bulb *A* of volume 1 litre is connected to a second bulb of volume 400 c.c. and the whole is initially full of air at N.T.P. If the bulb *A* is now heated to 100°C find (i) the new pressure in the bulbs (ii) the mass of air transferred from one bulb to the other. Density of air at N.T.P.= 1·29 gm/litre. [Ans. (i) 93·99 cm. (ii) 0·12 gm.]

50. Two glass bulbs of volume 3 litres and 1 litre respectively are connected by a capillary tube. Air at a pressure of 76 cm. at 30°C is contained in the apparatus which is then hermetically sealed. If the 3 litre bulb be immersed in steam at 100°C, the other remaining at 0°C, what would be the pressure in the bulbs ? Neglect the increase in volume of the 3 litre bulb. [*Jt. Entrance 1981*] [Ans. 88·4 cm.]

51. A mercury column of length 15 cm. encloses an air column of length 10 cm. at 27°C in a capillary tube when the tube with its open end upwards is held vertically. Now the tube is inverted and the length of air column becomes 15 cm. What is the temperature of air in the tube when it encloses air column of length 20 cm. in its inverted position ? [Ans. 127°C]

52. The density of a gas at 0°C and 760 mm. of *Hg* pressure is 1·293 gm/litre. Some of the gas is enclosed in a cylinder of volume 2·5 litre at temperature 17°C and pressure 780 mm. of *Hg*. What is the mass of the gas enclosed ? [Ans. 3·14 gm.]

53. The apparent weight of a piece of silver when counterpoised by brass weights in a delicate balance is found to be 25 gm when the air temperature is 32°C and barometric pressure 75·5 cm. of mercury. What is the actual mass of the piece ? Density of silver=10·5 gm/c.c. and that of brass=8·4 gm/c.c., both at 32°C. [Ans. 24·99 gm.]

54. The pressure in a constant volume gas thermometer immersed in melting ice is 60 cm. and when immersed in boiling water it is 80 cm. The thermometer is now placed in another liquid which is boiling and the pressure recorded is 90 cm. Find the boiling point of the liquid.

[Ans. 150°C]

6

CHANGE OF STATE

6.1. Introduction :

Matter, we know, can exist in three states *viz.* solid, liquid and gas. When a substance changes from one state to another, say, from solid to liquid or from liquid to gas etc. it is called a change of state.

From Solid to Liquid

6.2. Melting and Solidification :

Suppose, a lump of ice is at a temperature of −10°C. If we heat it, its temperature will rise until at 0°C, the ice begins to melt and then its temperature will remain constant at 0°C, although heat is supplied to the lump. This state of affairs will continue till the lump melts completely. After that, of course, the temperature of water will rise, if the heating is continued.

If a time-temperature plot be made for the solid, the graph will be like that shown in fig. 6.1(a). The initial temperature of the solid is represented by the point *A*. As the body is heated, its temperature rises but it remains in the solid state. This is represented by the portion *AB* of the curve. At the temperature (θ) corresponding to the point *B*, the solid starts melting and then the temperature remains constant. This is represented by the horizotal portion *BC* of the curve. θ, therefore, gives the melting point of the solid. At the point *C*, the melting is complete and the whole mass is converted into liquid. The portion *CD* of the curve represents the rising temperature of the liquid obtained from the melting of the solid mass. During the interval between the points *B* and *C i.e.*, during the process of melting, we have a mixture of solid and liquid.

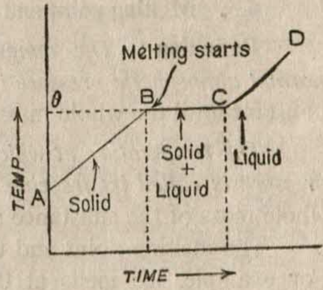


Fig. 6.1(a)

In the same way, if a quantity of pure water be gradually cooled, its temperature will fall until at 0°C, the water begins to freeze and then its temperature will remain constant at 0°C, although heat is being extracted from the mass of water. This state of affairs will continue till the whole mass of water freezes into ice. After that, of course the temperature of ice will fall, if the cooling is continued.

If, again, a time-temperature plot be made for the liquid, the graph will be like that shown in fig. 6.1(b). The initial temperature of the liquid is represented by the point *A*. As the liquid is cooled, its temperature falls but it remains in the liquid state. This is represented by the portion *AB* of the curve. At the temperature (θ) corresponding to the point *B*, the liquid starts solidifying and then the temperature remains constant. This is represented by the horizontal portion *BC* of the curve. θ, therefore, represents the freezing or the solidification point of the liquid. At the point *C*, the solidification is complete and the whole mass of liquid is converted into solid. The portion *CD* of the curve represents the fall of the temperature of the solid obtained from the solidification of liquid. During the interval between the points *B* and *C* *i.e.*, during the process of solidification, we have a mixture of liquid and solid.

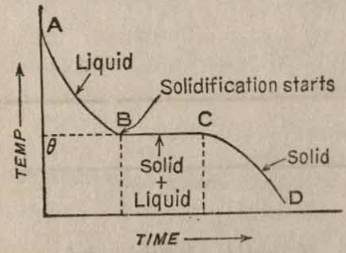


Fig. 6.1(b)

From the above description, it may be said that the temperature of a solid rises when heated until at a particular temperature, the solid begins to melt when its temperature remains constant, although heat is being supplied to it. This phenomenon is known as *melting* of solids.

Similarly, the temperature of a liquid falls when cooled until at a particular temperature, the liquid begins to solidify. Then its temperature becomes constant, although heat is being extracted from it. This phenomenon is known as *solidification* of liquids.

6.3. Melting point and freezing point of a substance :

Definition : *The temperature at which a substance begins to melt under normal atmospheric pressure is called its melting point.* This temperature remains constant until the whole mass of the substance melts.

The temperature at which a liquid begins to solidify under normal atmospheric pressure is called its freezing point. The temperature remains constant until the whole mass of the substance solidifies.

The melting point and the freezing point of almost all substances are equal. For example, ice melts at 0°C under normal atmospheric pressure and water solidifies at the same temperature. But some non-crystalline substances like, grease, butter, glass, wax etc.—have no sharp melting or freezing point. They pass into a viscous condition before melting. For example, butter melts somewhere between 28°C and 33°C and freezes at some temperature between 23°C and 20°C.

It is, however, important to note that the melting point or freezing point of a substance is not constant. It depends upon pressure.

6.4. Supercooling :

It has already been mentioned that a liquid freezes to the solid state at a given temperature. But if the cooling of the given mass of liquid be carried on *very slowly*, the temperature may be brought down below the freezing point without the liquid being frozen. This condition, however, is not a stable one because a slight disturbance will cause immediate solidification of the whole liquid and the temperature will quickly rise to its normal freezing point. This phenomenon is known as *supercooling.*

The time-temperature plot of supercooling of a solution of hypo (sodium thiosulphate) has been shown in fig. 6.1(c). The initial temperature of the solution is nearly 80°C. The temperature of the solution falls as it is cooled. The fall of temperature continues beyond the freezing point (about 50°C) without the liquid being frozen. This is represented by the portion *AB* of the curve. The temperature corresponding to the point *B* is much lower than that corresponding to *C*, the freezing point of the solution. In this condition, the liquid is said to be supercooled. If at this moment a crystal of solid hypo is dropped into the solution, the temperature at once rises to the freezing point (shown by dotted line) and the solution starts to freeze. While the hypo is freezing, its temperature remains constant at the freezing point which is represented by the horizontal portion *CD* of the curve. When all the hypo has frozen, the temperature starts to fall again. The portion *DE* of the curve represents the fall of temperature of the frozen hypo.

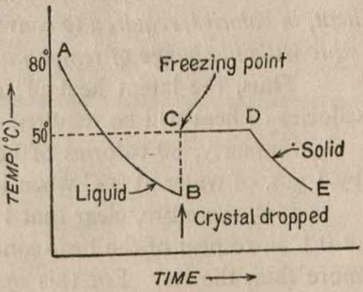


Fig. 6.1(c)

6.5. Latent heat :

When a substance is heated, its temperature generally changes. By observing such change of temperature with the help of a thermometer, we realise that the body is absorbing heat. Such heat is known as '*sensible heat.*' But we have already seen that a piece of ice at 0°*C*, when heated, does not show any change of temperature but melts into water and the temperature remains at 0°*C* till the whole piece melts although heat is being continually added to it. In this way, whenever a solid changes into a liquid, it absorbs a quantity of heat which does not manifest itself by a rise of temperature of the substance, but remains latent in the substance. For this reason, it is called *latent heat.*

Instead of increasing the temperature of a solid, the latent heat breaks up the geometrical configuration of the molecules and converts the solid into its liquid state.

In the same way, when a liquid freezes into a solid, it gives up some quantity of heat which is not detectable by a thermometer. It is also called latent heat.

In general, we may say that when a change of state takes place, a body absorbs or gives up some quantity of heat which does not indicate its presence by producing a rise or a fall of temperature, but remains 'hidden' in the body. This heat is known as latent heat.

Latent heat, in general, may be of four different types, *viz.* (i) *latent heat of fusion* (ii) *latent heat of solidification* (iii) *latent heat of vaporisation* (iv) *latent heat of condensation.*

6.6. Latent heat of fusion :

Definition : The latent heat of fusion of a substance is the quantity of heat required to convert unit mass of the substance form the solid to the liquid state without change of temperature.

In the C.G.S. system, *the latent heat of fusion of a substance is the quantity of heat, in calories, required to convert 1 gm. of the substance from the solid to the liquid state without change of temperature.*

Thus, the latent heat of fusion of ice is 80 calories/gm. It means that 80 calories of heat will be required to convert 1 gm. of ice at 0°C to water at 0°C.

Similarly, 80 calories of heat will be given up (latent heat of solidification) by 1 gm. of water at 0°C when it completely solidifies into ice at 0°C.

It is, therefore, clear that 1 gm. of ice at 0°C is different from 1 gm. of water at 0°C in respect of the heat-content because the water contains 80 calories of heat more than the ice. For this reason, some water kept at 0°C, will remain in the liquid state and will freeze only when 80 calories of heat are extracted from each gramme of the water.

Latent heat of fusion or solidification in F.P.S. system :

In the F.P.S. system, the latent heat of fusion of ice may be obtained in the following way :

Since 1 lb.$=453 \cdot 6$ gm. and 1 Btu.$=252$ calories, the latent heat of fusion of ice in the F.P.S. system$=\frac{80 \times 453 \cdot 6}{252}=80 \times \frac{9}{5}=144$ Btu/lb.

[N.B. It is worth-while to note that the conversion factor $\frac{9}{5}$ is equal to the ratio of a degree celsius to a degree fahrenheit. The value of latent heat, therefore, does not depend on the unit of mass but depends on the scale of temperature. For example, latent heat of fusion of tin is 14 cal/gm. In F.P.S. system, it is $14 \times \frac{9}{5}=25 \cdot 2$ Btu/lb.]

6.7. Determination of latent heat of fusion of ice : (a) By the method of mixture :

A copper calorimeter and a stirrer are first weighed empty. The stirrer should have a wire-gauge net fitted at one end. Then with the calorimeter about $\frac{2}{3}$ full of water warmed to a temperature about 5°*C* above room temperature the weighing is repeated.

A thermometer is inserted in the water. Some pieces of ice are carefully dried on a blotting paper. The water is then stirred, the temperature noted and immediately the dry ice is added. Stirring continuously, sufficient ice is added to bring the final temperature of the mixture to about 5°*C* below room temperature. The final steady temperature is noted and the calorimeter with its contents reweighed.

Calculations : Let the latent heat of fusion of ice$=L$ cal/gm.

Wt. of the empty calorimeter+stirrer$=m_1$ gm.

" " " calorimeter+water$=m_2$ gm.

" " " calorimeter+water+water obtained by melting of ice$=m_3$ gm.

Initial temperature of calorimeter and its contents $=t_1$°C.

Final „ „ „ „ „ „ $=t_2$°C.

Sp. heat of the material of the calorimeter $=S$

Wt. of water taken $=m_2-m_1=m$ (say) gm.

„ „ ice „ $=m_3-m_2=M$ (say) gm.

Ice will take heat from the calorimeter and its contents at first and it will melt into water at 0°C which afterwards will warm up from 0°C to t_2°C.

Now, heat necessary to melt the ice

$=$mass of ice × latent heat of fusion $=M.L$ cal.

and heat necessary to raise the temperature of melted ice from 0°C to t_2°C $=$ mass of ice × rise of temp. $=M(t_2-0)=M.t_2$ cal.

So, the total heat gained by ice $=M.L+Mt_2$ cal.

Now, heat given up by the calorimeter $=$ mass × sp. heat × fall in temp. $=m_1\times S\times(t_1-t_2)$ cal. and heat given up by the water $=$ mass × fall in temp. $=m\times(t_1-t_2)$ cal.

So, the total heat given up $=m_1S(t_1-t_2)+m(t_1-t_2)$ cal.

Since, heat gained $=$ heat lost.

$$\therefore\quad ML+M.t_2=m_1S(t_1-t_2)+m(t_1-t_2)$$

$$=(m_1S+m)(t_1-t_2)$$

$$\text{or,}\quad ML=(m_1S+m)(t_1-t_2)-M.t_2.$$

$$\text{or,}\quad L=\frac{(m_1S+m)(t_1-t_2)}{M}-t_2$$

If $W=$water equivalent of the calorimeter, then $W=m_1S$;

In this case, $L=\dfrac{(t_1-t_2)(W+m)}{M}-t_2$

Sources of errors and precautions :

(1) Ice taken should be dry ; if there be any water with the ice, the water will absorb some heat which can not be accounted for.

(2) Ice should not be allowed to float in water, as the exposed portion of the floating ice may absorb some heat from outside. This is why ice should be always pressed down in water by wire-gauge stirrer.

(3) Addition of too much ice should be avoided . Otherwise there might be a chance of condensation of water vapour present in the atmosphere over the surface of the calorimeter. This will involve error.

(4) Since the final temperature is lower than the room-temperature, absorption of heat by the calorimeter and its contents from outside due to radiation may take place. This can be avoided either by making radiation correction or by initially raising the temperature of water in the calorimeter 4° or 5°C above the room-temperature.

(b) By Black's ice-calorimeter :

Black's ice-calorimeter consists of a big block of ice *A* with a hole cut out as shown in the fig. 6.2. The hole can be covered by another block of ice *B* which serves as the lid.

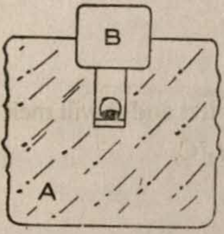


Fig. 6.2

We take a lump of solid of known mass, say *m* gm. and allow it to be heated in a steam-heater or in any other bath of constant temperature say, *t*°C. Then the lump of the solid is suddenly introduced in the cavity of the ice-calorimeter after removing the lid *B*. The lid is then quickly replaced. After a while, the lid is removed and the melted ice is sucked up into a pipette and the quantity of water so obtained is weighed. Let the mass of water obtained by melting of ice be *M* gm.

Calculation : If *S* be the specific heat of the solid, the heat given out by it $=m.S.t$ cal. The heat absorbed by *M* gm of ice to melt into water$=M.L$ cal.

Since heat lost=heat gained.

$$m.s.t=M.L \quad \text{or,} \quad L=\frac{m.s.t}{M}.$$

This type of ice-calorimeter, however, does not give accurate result for :

(i) All the water that is obtained by melting of ice cannot be completely drawn out.

(ii) Some ice may melt in the cavity due to heat supplied from external sources other than the hot body.

Table of latent heat of fusion of some substances

Substance	Latent heat	Substance	Latent heat
Ice	80 cal.	Silver	21·07 cal.
Lead	5·86 ,,	Tin	14·0 ,,
Zinc	28·13 ,,	Mercury	2·82 ,,
Paraffin	35·1 ,,		

Examples : (1) *A copper calorimeter weighs 112·5 gm. and with certain amount of water it weighs 187·5 gm. The temperature of water is 30°C. When a few pieces of ice are dropped into the water, the temperature falls to 24·5°C. When the calorimeter is re-weighed it is found to be 192 gm. If the sp. heat of copper be 0·1, calculate the latent heat of fusion of ice.*

Ans. Suppose, the latent heat of fusion of ice$=L$ cal.

Wt. of water taken$=187{\cdot}5-112{\cdot}5=75$ gm.

,, ,, ice ,, $=192-187{\cdot}5=4{\cdot}5$ gm.

Heat necessary for melting of ice=mass of ice × latent heat$=4{\cdot}5\ L$ cal.

,, ,, to raise the temperature of melted ice from 0°C to 24·5°C =mass of water × rise of temp.$=4{\cdot}5\times(24{\cdot}5-0)=4{\cdot}5\times24{\cdot}5=110{\cdot}25$ cal.

$\therefore$ Total heat gained $=4{\cdot}5L+110{\cdot}25$ cal.

Heat lost by the calorimeter = its mass × its sp. heat × fall of temp.

$=112{\cdot}5\times0{\cdot}1\times(30-24{\cdot}5)=61{\cdot}87$ cal.

Heat lost by the water = its mass × fall of temp.

$=75\times(30-24{\cdot}5)=412{\cdot}5$ cal.

$\therefore$ Total heat lost $=412{\cdot}5+61{\cdot}87=474{\cdot}37$ cal.

Since heat lost = heat gained,

we have, $4{\cdot}5L+110{\cdot}25=474{\cdot}37$ or $4{\cdot}5L=364{\cdot}12$

$$\therefore \quad L=\frac{364{\cdot}12}{4{\cdot}5}=80{\cdot}9 \text{ cal.}$$

(2) *A copper calorimeter of mass 70·5 gm contains 0·1 kg of water at 19·5°C. Napthalene (melting point=79·9°C) is melted in a test-tube, cooled to 80°C and then poured into the calorimeter. If the highest temperature reached by the water is 28·7°C and the final mass of the calorimeter and its contents is 188·3 gm, calculate the latent heat of fusion of napthalene. Specific heat of copper 400 and of napthalene 1300 J. kg*$^{-1}$*K*$^{-1}$.

Ans. Mass of napthalene taken $=0{\cdot}1883-(0{\cdot}0705+0{\cdot}1)=0{\cdot}0178$ kg

Heat lost by napthalene $=0{\cdot}0178\times L+0{\cdot}0178\times1300\times(79{\cdot}9-28{\cdot}7)$

$=0{\cdot}0178\times L+0{\cdot}0178\times1300\times51{\cdot}2$

$=0{\cdot}0178\times L+1184{\cdot}77$ joule

Heat gained by water and calorimeter $=0{\cdot}1\times4200\times(28{\cdot}7-19{\cdot}5)+0{\cdot}0705\times400\times(28{\cdot}7-19{\cdot}5)$

$=0{\cdot}1\times4200\times9{\cdot}2+0{\cdot}0705\times400\times9{\cdot}2$

$=4123{\cdot}44$ joule

$\therefore \quad 0{\cdot}0178\times L+1184{\cdot}77=4123{\cdot}44$

or, $0{\cdot}0178\times L=2938{\cdot}67 \quad \therefore \quad L=1{\cdot}65\times10^5 J \text{ kg}^{-1}$

[*N.B.* Note carefully the units of different quantities]

(3) *5 gm. of ice at −10°C are mixed with 20 gm. of water at 39°C. Will all ice melt? If so, what is the final temperature of the mixture? Sp. heat of ice=0·5 and latent heat of fusion of ice=80 cal.*

Ans. To melt, the piece of ice must have its temperature raised from −10°C to 0°C at first and then taking 80 calories of heat per gramme, it will melt. If all the heat necessary is available from hot water, the ice will melt.

Heat necessary for the first step. = mass of ice × its sp. heat × rise in temp.

$=5\times0{\cdot}5\times[0-(-10)]=5\times0{\cdot}5\times10=25$ cal.

Heat necessary for the 2nd. step $=5\times80=400$ cal.

So, total heat necessary $=400+25=425$ cal.

The heat lost by 20 gm. of hot water in coming down from 39°C to 0°C $=20\times(39-0)=20\times39=780$ cal.

Since the heat lost is more than the heat necessary to melt the ice, it is clear that all ice will melt and the excess heat will raise the temperature of melted ice.

Suppose, the final temperature of the mixture $=t$°C.

So, heat lost by hot water when its temperature falls from 39°C to t°C $=20\times(39-t)=780-20\times t$ cal.

Heat gained by ice to raise its temperature from -10°C to $0^\circ\text{C}=25$ cal (see above). Heat necessary to melt the ice$=5\times 80=400$ cal.

Heat necessary to raise the temperature of melted ice from 0°C to $t^\circ\text{C}=5\times(t-0)=5\times t$ cal. Since, heat gained=heat lost,

we have, $780-20\times t=425+5\times t$ or, $25.\ t=355$ or, $t=\dfrac{355}{25}=14{\cdot}2^\circ$C

(4) *10 gm of ice at -80°C are dropped into some water at 20°C. When all ice melts, the temperature of the mixture is 15°C. After this, 15·6 gm of ice at 0°C are again added with the mixture. After melting, the temperature of the mixture is brought down to 10°C. Find the latent heat of fusion of ice at 0°C. Find also the initial mass of water. Specific heat of water between -80°C to 0°C $=0{\cdot}5$.*

Ans. Let the initial mass of water be m gm. During the first stage, heat given up by m gm. of water$=m(20-15)=5m$ cal. Heat absorbed by ice to raise its temperature from -80°C to $0^\circ\text{C}=10\times 0{\cdot}5\times 80=400$ cal. Heat absorbed by ice to melt$=10\times L$ cal. Heat absorbed by melted water to rise from 0°C to $15^\circ\text{C}=10\times 15$ cal$=150$ cal.

$\therefore$ Total heat absorbed$=400+10L+150=550+10\times L$ .. .. (i)

During the second stage, total mass of water$=(m+10)$ gm. and its temperature is 15°C. Heat given up by water in falling from 15°C to $10^\circ\text{C}=(m+10)\times 5$ cal.

Again, heat absorbed by 15·6 gm of ice to melt$=15{\cdot}6\times L$ cal. Heat absorbed by melted water to rise from 0°C to $10^\circ\text{C}=15{\cdot}6\times 10$ cal.

$\therefore$ Total heat absorbed$=(15{\cdot}6\times L+15{\cdot}6\times 10)$ cal.

So, $(m+10)\times 5=15{\cdot}6\times L+15{\cdot}6\times 10$

or $5m=15{\cdot}6\times L+106$ .. (ii)

From eq^{n}s. (i) and (ii), $10\times L+550=15{\cdot}6\times L+106$

or $5{\cdot}6\times L=444$ or $L=79{\cdot}3$ cal/gm

Putting this value in eq^n. (i), $5m=550+10\times 79{\cdot}3$ or $m=268{\cdot}6$ gm.

(5) *Densities of water and ice at 0°C are 1 gm/c.c. and 0·916 gm/c.c. respectively. A piece of metal, weighing 10 gm is heated to 90°C and is then dropped into a mixture of water and ice. Some ice melts and the volume of the mixture contracts by 0·1 c.c. without any change of temperature. Find the sp. heat of the metal, if the latent heat of fusion of ice=80 cal/gm.* [*I.I.T.* *1964*]

Ans. It is clear that the contraction of volume is due to melting of ice. Now, volume of 1 gm of ice at $0^\circ\text{C}=\dfrac{1}{0{\cdot}916}$ c.c. and that of water of 1 gm mass$=1$c.c.

So, contraction of volume due to metling of 1 gm of ice$=\dfrac{1}{0{\cdot}916}-1$ $=\dfrac{0{\cdot}084}{0{\cdot}916}$ c.c.. Hence mass of ice melted for 0·1 c.c. of contraction

$$=\frac{0{\cdot}916}{0{\cdot}084}\times 0{\cdot}1=\frac{91{\cdot}6}{84}\text{ gm.}$$

Heat necessary for melting $\frac{91\cdot6}{84}$ gm of ice $=\frac{91\cdot6}{84}\times80$ cal. This heat comes from the hot metal piece. If S be the sp. heat of metal, then $10\times S\times100$ $=\frac{91\cdot6}{84}\times80$ or $S=0\cdot087$ (nearly).

(6) *10 gm of a substance was taken in the solid state at -10°C. 64 calories were required to heat it to 2°C (still in the solid state) and 880 and 900 calories were required to heat it to the liquid state at 1°C and 3°C respectively. Assuming that the specific heat of the material in the solid and liquid state has values S_1 and S_2 $(S_1\neq S_2)$ respectively, find their values. Show that the latent heat of fusion L is related to the melting point temperature t_m by $L=79+0\cdot2\,t_m$.* [*Jt. Entrance 1982*]

Ans. Heat absorbed at the first stage $=$ 64 cal; at the second stage $=880-64$ $=816$ cal and at the third stage $=900-880=20$ cal.

In the first stage, we have $64=10\times S_1\times[-2-(-10)]=80\times S_1$ $\therefore$ $S_1=0\cdot8$

In the third stage, $20=10\times S_2\times(3-1)=20\times S_2$ $\therefore$ $S_2=1$.

In the second stage,
$$816=10\times0\cdot8\times(t_m+2)+10\times L+10\times1(1-t_m)$$
$$=8t_m+16+10L+10-10t_m$$
$$\therefore\ L=79+0\cdot2\times t_m$$

6.8. Change of volume during melting and solidification :

In general, when solids change into liquids, there is an expansion in volume and when liquids solidify, there is a contraction in volume. For example, when molten paraffin wax is allowed to solidify in a test tube, a deep cleft will be produced due to contraction in volume (Fig. 6.3). But water, cast-iron, brass, bismuth, antimony and some other materials are exception to this general rule. When they melt, they contract in volume ; when they solidify, they expand in volume. 11 c.c. of water at 0°C, for example, solidify to form 12 c.c. of ice *i.e.* there is about 9% increase in volume. In the same way, cast iron increases by about 7% in volume when it solidifies.

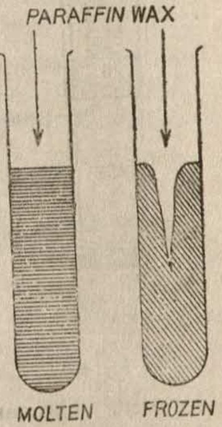


Fig. 6.3

Advantage and disadvantage of change of volume : In cold countries, when water solidifies into ice, the consequent expansion in volume causes much disadvantage. Frequently water pipes are found to burst due to enormous force brought about by the expansion of volume when water in the pipe freezes into ice. *The hot water-pipes are, however, very prone to get burst than cold water-pipes.* The reason is, in hot water all the dissolved air is driven out while in cold water some air remains dissolved in it. When cold water solidifies, the dissolved air is expelled and the expansion of volume is to some extent compensated by the expelled air. Consequently, the force on the pipe is not sufficient enough to break it. But in hot water, such thing cannot happen and the hot water-pipes are liable to burst frequently.

For the same reason, rocks and stones in extreme cold countries are found to develop gaping fissures and moist ground swells up during frost.

On the other hand, the expansion of volume when iron and brass change from liquid to solid state is advantageously applied to some useful purposes. In the process of casting, molten metal is run into a mould and the expansion of the solidifying mass forces the metal into every corner of the mould and a casting with sharp edge is obtained. Types for printing press are also made in the same process.

6.9. Effects of pressure on melting point :

The relations between the pressure and the melting point are as follows :

(1) Substances which contract in volume on melting have their melting points lowered by pressure. Thus, the melting point of ice is lowered by about 0·007°C per atmosphere increase in pressure. The reason is simple. The increased pressure helps the substance in contracting the volume and hence its melting point is lowered.

(2) Substances which expand in volume on melting have their melting points raised by the application of pressure. Wax, for example, has its melting point raised by about 0·04°C per atmosphere increase in pressure. Here also, the reason is simple. Increased pressure prevents the substance from expanding and hence raises its melting point.

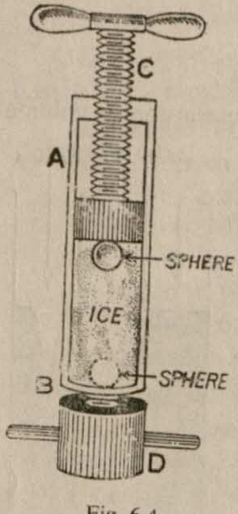


Fig. 6.4

Expt. : AB is a cylinder made of wrought iron. Its lower end can be closed or opened by a screw-plug D. A piston C provided with a handle can work inside the cylinder. Fill half of the cylinder by water and freeze the water into ice by freezing mixture. Place a metallic ball on the ice and exert pressure on the ball with the help of the piston by screwing it down after having surrounded the cylinder with ice. If, now the screw-plug D is removed, it will be found that the metallic ball has come down but ice above remains solid as before (Fig. 6.4). How can this happen ? When pressure is exerted by the piston, the melting point of ice is lowered. The surrounding temperature being 0°C, the ice in contact with the ball melts and the ball goes down. As soon as the pressure is relieved due to the ball going down, the water again solidifies into ice. In this way, the ball gradually cuts its way down and the water above freezes into ice. This experiment was devised by Mousson.

[**N.B.** The relation connecting change in pressure and the consequent change in melting point, is known as Clausius-Clapeyron equation. It is : $\frac{dP}{dT}=\frac{L}{T(V_2-V_1)}$, where dP=change in pressure ; dT=corresponding change in melting point ; L=latent heat of fusion, V_2, V_1=volumes of 1 gm of the substance in solid and liquid state respectively.]

6.10. Regelation :

You perhaps know that two pieces of ice pressed together form a single lump. During hailstorm, you must have made a big snow ball pressing together several pieces of small hail. How does it happen ?

When two pieces of ice are pressed, the melting point at their junction is lowered below 0°C. Consequently, ice at their junction melts into water. But when the pressure is withdrawn, the melting point at the junction is raised and the water immediately solidifies joining the two pieces together.

Definition : Melting of ice by the application of pressure and resolidification of molten ice by the withdrawal of pressure are known as *Regelation*.

The following laboratory experiment devised by Bottomley illustrates the phenomenon of regelation in a striking manner.

A block of ice rests on two supports and a thin copper wire with a heavy weight is hung over it. After some time, it will be seen that the wire cuts right through the block and falls on the floor leaving the ice still in a solid block (Fig. 6.5). The reason is as follows :

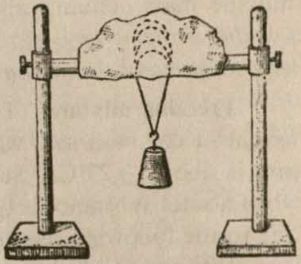


Fig. 6.5

The pressure of the wire lowers the melting point of the ice in contact with it and so the ice melts and flows up above the wire. The latent heat required for the melting comes from the copper wire and the surrounding air. If the surrounding air is cold, this thing will not happen. As soon as the water passes above the wire, it is no longer under pressure and therefore, resolidifies. In so doing it gives out latent heat which is conducted by the wire to provide heat for further melting of the ice below. In this way, the wire cuts right through the block but the block is not split into two.

From the above discussion, it is clear that to make the experiment a success, the wire must be thin and made of a material having high thermal conductivity. A thin cotton thread, for example, will not pass through at all.

When the wheels of a carriage pass over snow, ice melts due to pressure of the wheels. When the wheels pass over, water formed on the wheel solidifies due to regelation. For this reason, the wheels are found to be covered with snow. Skating is possible on snow due to the formation of a thin layer of water below the skates due to increase of pressure. The layer acts as a lubricant.

6.11. Freezing point of a solution :

When a substance is dissolved in a liquid, it is found that the solution has a freezing point lower than the freezing point of the solvent. Thus, the freezing point of the solution of common salt in water is —2°C, *i.e.*, 2°C below the freezing point of water.

But when a solution freezes, pure solvent separates out in crystalline form and the solution becomes more and more concentrated. This fact is applied in preparing salt from sea-water in cold countries. Sea-water condenses at about

$-2°C$. As more and more sea-water condenses, pure water separates out in the form of ice and the percentage of salt in the sea-water increases. Afterwards, the sea-water is evaporated and the salt is separated.

In cold countries, the radiator tubes of automobiles burst due to condensation of water. To prevent it, glycol or glycerine is mixed with water which lowers the freezing point of water.

When a solution of common salt in water is gradually cooled, ice separates out at about 3° to 4°C below 0°C. The concentration of the remaining part of the solution, therefore grows stronger. If it is cooled further, more and more ice separates out until at $-23°C$, whole of the remaining solution solidifies at a time, and the mass contains about 23·6% salt. *This temperature at which a solution solidifies without any separation between solvent and solute is known as eutectic temperature and the mixture is known as eutectic mixture.*

Freezing mixture : Three parts by weight of powdered ice and one part by weight of common salt, when mixed together, produce a mixture whose temperature is about $-23°C$. Such mixture is known as freezing mixtures. Generally when a solid substance is dissolved in a liquid, the temperature of the mixture falls due to the following reason.

When a solid changes into liquid, it requires some latent heat. The solid generally gets the latent heat from the solvent ; consequently the temperature of the mixture falls. In a mixture of ice and common salt, each gramme of ice takes up 80 calories of heat from the water adhering to ice and common salt. As a result, the temperature of the mixture falls appreciably.

In the same way, a mixture of equal quantities of water and ammonium nitrate produces a temperature of $-15°C$. A mixture made of 4 parts by weight of crystalline calcium chloride and three parts by weight of powdered ice, gives a temperature of about $-50°C$. Such low temperature mixtures have wide use in scientific works of physics, chemistry etc.

Freezing mixtures are used for various purposes. Perishable materials, covered by freezing mixtures, retain their freshness for some time. For this reason, fish is preserved in freezing mixtures. Freezing mixtures are also used to prepare '*Kulpi Baraf*'.

6.12. Laws of fusion :

Various facts so far discussed in connection with melting and solidification may be listed in the form of some laws, usually known as the *laws of fusion*. They are as follows :—

(i) Under a given pressure, a substance (solid) melts at particular temperature which remains constant as long as the mass of the substance melts. This temperature is called the melting point of the particular substance.

(ii) Substances like ice, cast-iron etc. which contract in volume on melting have their melting points lowered by pressure. On the other hand, substances like wax etc. which expand in volume on melting, have their melting points raised by the application of pressure.

(iii) The melting point of an alloy made of two or more metals, is lower than those of the constituent metals. Thus, an alloy of 5 parts of tin and 1 of lead (solder) melts at 194°C whereas the melting points of tin and lead are 232°C and 327°C respectively. *Rose's metal* is an alloy of tin, lead and bismuth. It melts at 94·5°C but none of the constituent metals *i.e.* tin, lead and bismuth melt at a temperature lower than 210°C. For this reason, alloys are used as fire extinguisher and safety arrangements like fuses etc.

(iv) A solution has a freezing point lower than that of a solvent.

(v) Every substance has a definite latent heat of fusion or solidification.

From Liquid to Vapour

6.13. Vapour and vaporisation :

Vapour means the aeriform state of a liquid and vaporisation means the process by which a liquid passes to its corresponding vapour. It has been mentioned earlier that a liquid requires some quantity of heat to pass to its vapour state, which remains latent in the vapour. It is known as the latent heat of vaporisation.

In this connection, it may be mentioned that gas and vapour are not the same thing, though ordinarily, we do not make any difference between them. It has been found experimentally that a vapour given out by a liquid can not be condensed back to the liquid state by exerting pressure if the vapour is kept at any temperature. For condensation, the vapour must be kept at a particular temperature or below it, which is known as the *critical temperature*. If a vapour is below its critical temperature, it should be called a vapour. If it is above the critical temperature, it should be called a gas.

Further vapours come out of liquids at all temperatures but gases are produced by chemical process.

6.14. Different ways of vaporisation :

There are three ways in which a liquid can pass into its vapour state, *viz*, (i) *evaporation*, (ii) *boiling* and (iii) *sublimation*.

(i) **Evaporation** : Evaporation is a slow and gradual conversion of a liquid to its corresponding vapour. It takes place at the surface of the liquid and at any temperature. The drying up of ponds, lakes etc. in summer, the disappearance of water left in an uncovered vessel, drying up of wet clothes etc. are due to evaporation.

(ii) **Boiling** : It is a rapid process of evaporation of a liquid. The whole mass of liquid takes part in the process of boiling which occurs at a particular temperature depending upon the superincumbent pressure. The temperature of the liquid during boiling remains constant till the whole mass of the liquid boils away.

(iii) **Sublimation** : It is a direct conversion of a solid to its corresponding vapour without passing through the intermediate liquid state. Camphor, napthalene etc. are found to sublime even at the ordinary temperature.

6.15. Difference between evaporation and boiling :

The following are the differences between evaporation and boiling :

(a) Boiling is a rapid process while evaporation is a slow one.

(b) Boiling takes place throughout the mass of the liquid but evaporation takes place from the surface of the liquid.

(c) During boiling, the whole mass of the liquid is agitated but during evaporation no such agitation is caused in the liquid.

(d) Boiling takes place at a particular temperature under normal atmospheric pressure, but evaporation takes place at all temperatures.

6.16. Factors influencing rate of evaporation :

The following factors influence the rate of evaporation of a liquid :

(a) **Dryness of air** : The more the air is dry, the greater is the rate of evaporation. For this reason, wet clothes dry up sooner in winter than in rainy season.

(b) **Atmospheric pressure** : With the increase of atmospheric pressure, the rate of evaporation decreases. Evaporation takes place very quickly in vacuum where the atmospheric pressure is almost nil.

During evaporation, some of the high-speed molecules leave the liquid surface and get mixed up with air above. As a result ,the kinetic energy of the remaining liquid decreases and the temperature falls. Some of the liquid molecules leaving the surface may re-enter the liquid due to collision against the air-molecules above. So, less the air-molecules present above the liquid surface, *i.e.* less the atmospheric pressure, greater will be the rate of evaporation.

(c) **Surface area of the liquid** : An extended surface of the liquid helps to quicken the evaporation. For this reason, hot tea or milk when poured in a saucer becomes cold quickly.

(d) **Nature of liquid** : The more volatile a liquid, the quicker will be the rate of evaporation. Spirit, ether, alcohol, petrol etc. evaporate quickly because they are all volatile liquids.

(e) **Renewal of air** : Renewal of air in contact with the surface of the liquid facilitates evaporation. If there be air-current, the vapour coming out of the liquid may not remain in contact with the liquid for a long time. This makes the air above the liquid surface dry and accelerates evaporation. This is why fanning dries up wet clothes quickly.

6.17. Cold caused by evaporation :

It has been mentioned earlier that a liquid requires some latent heat to change from liquid to vapour. If this latent heat is not supplied from some external source, the liquid will derive the heat from its own body or from its surroundings. As a result, the liquid or the surroundings will become cold. Many such cases of cooling produced by evaporation may be cited.

(1) If a little methylated spirit or *eau-de-cologne* is spilt on the hand it evaporates rapidly and the hand feels very cold. To change from liquid to vapour, the spirit requires latent heat. This it obtains from the hand, which thus loses heat and cools. For the same reason, during high fever doctors prescribe eau-de-cologne bandage or simply water-bandage on the forehead. Water from the bandage takes heat from the body and gets evaporated. As a result, the temperature of the body comes down.

(2) During summer, when body perspires, fanning gives a comfortable feeling. Fanning facilitates evaporation of perspiration and the body becomes cold giving up heat necessary for evaporation.

It is relevant to mention here that perspiration is the body's natural method of maintaining a constant temperature. When perspiring heavily after exercise it is unwise to stand about in a draught, lest there should be overcooling due to evaporation. The resulting chill may lower the body resistance to infection.

Dogs, who do not perspire from the skin, hang out their tongues during hot weather in order to get a cooling effect.

(3) In summer days, drinking water is kept in an earthenware pitcher because such pitchers keep the water cool. Water always percolates through the pores of the pitcher and continually evaporates taking latent heat from the pitcher which becomes cold. The water, in the pitcher, also becomes cold. Metal vessels or glass vessels cannot make the water so cold, because these vessels do not possess pores and therefore, water cannot percolate out and get evaporated. Old earthenware pitchers, whose pores have been choked by dust, sand etc., cannot keep the water so cold as a new one.

(4) In summer, rooms are kept cold by hanging '*Khaus Khaus*' soaked in water on windows. Water evaporates, taking latent heat from '*Khaus Khaus*' which, therefore, loses heat and becomes cold. Air passing into the room through cold '*Khaus Khaus*' is also made cool.

(5) It is unwise to sit in a draught with wet clothes on. The reason is simple. Water evaporates quickly in a draught taking latent heat from the body. As a result, the person may catch cold.

(6) In summer, a person feels uncomfortable in a packed hall or auditorium. Perspiration from his body cannot evaporate. As soon as he comes out of the hall, he feels a sudden cooling sensation. Perspirations evaporate quickly taking heat from his body and he feels comfortable.

The working of an ice-making machine is based on the cooling produced by evaporation. In this machine, liquid ammonia is quickly evaporated and the rapid change from the liquid to the vapour state requires large amount of latent heat. Water supplies this heat and is frozen into ice.

Refrigerators also work on the same principle. Perishable food-stuffs like fish, meat, egg, fruits etc. retain their freshness for a pretty long time, when kept in the cool atmosphere of a refrigerator.

6.18. Boiling of liquid :

Take some water in a glass flask and close the mouth of the flask by a rubber stopper. Insert a thermometer *T* and a bent glass tube *E* into the flask through two holes in the stopper. The bulb of the thermometer should be a little above the water surface (Fig. 6.6). Now heat the flask by means of a burner.

When the water is heated a little, some vapour rises from the surface and the dissolved air comes out as bubbles and gets deposited on the wall of the flask. The mercury column in the thermometer is found to rise gradually. When, in this way, the mercury thread touches 70°C/80°C mark, small bubbles form at the bottom of the flask. Rising upwards these bubbles come in contact with comparatively cold water and break. At this time, a simmering sound is produced. At last, when the thermometer reads 98°C/99°C, big bubbles come up from the bottom and reaching the surface, they burst, producing a violent agitation in the whole mass of the liquid. Profuse steam now comes out of the tube *E* and the thermometer shows a steady temperature. We now say that the liquid is boiling.

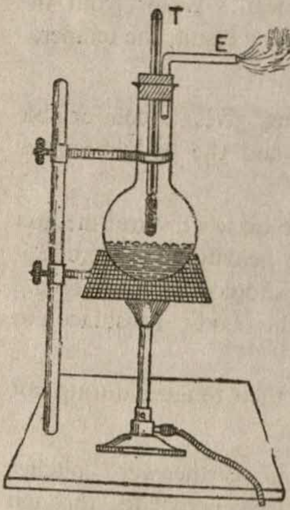


Fig. 6.6

[**Note** : The account of boiling as given above is rather crude because we did not consider the effect of surface tension. Consideration of surface-tension shows that a bubble can be formed in a liquid only if there is an excess pressure inside it. The smaller the size of the bubble, greater is this internal excess pressure. Mathematical calculations show that a bubble can never start from zero radius, because it would require an infinite vapour pressure to do so which is not possible. Bubbles actually form on roughnesses in the vessel or specks of solid suspended in the liquid. Very clean water in a very smooth beaker may not boil until it is well above 100°C ; if bubbles then form somehow, they grow and burst violently. This is known as '*bumping*' of liquid. To prevent bumping, pieces of broken glass or pipe-clay are put in the liquid because they offer fine and sharp points for the bubbles to form on.]

6.19. Definition of boiling point of a liquid :

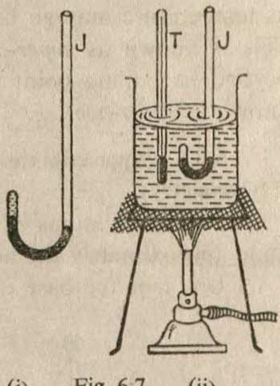
The temperature at which a liquid boils under normal atmospheric pressure is called its normal boiling point. For example, the normal boiling point of pure water is 100°C. The boiling point remains constant during the whole period of boiling and depends upon the superincumbent pressure.

6.20. At the boiling point, the vapour pressure of a liquid is equal to the superincumbent pressure :

Take a narrow glass *U*-tube (*J*) with one end closed and the other open. Fill the tube completely with clean and dry mercury. During filling care should be taken so that no air bubbles stick in the tube. Then, at the top of mercury in the open limb, some distilled water, free from dissolved air, is poured. Closing the open end by thumb, the tube is inverted. Water, being lighter than mercury, will be collected at the closed end of the tube [fig. 6.7(i)].

Now by introducing a thin solid rod, some mercury is removed from the longer limb of the *U* tube so that the mercury level in the longer limb is lower than the level in the closed limb. At the top of mercury column in the closed limb, some water together with water-vapour exists [fig. 6.7(i)].

Next place the tube *J* in a beaker containing water and a thermometer *T* as shown in fig. 6.7(ii). The water in the beaker is heated by a burner. A gradual rise of mercury meniscus in the longer limb of the tube *J* will be noticed. This indicates that with the rise of temperature the vapour pressure of water increases. When the water in the beaker boils, the mercury meniscuses in the two limbs stand at the same level. This shows that at the boiling point, the vapour-pressure of water is equal to the atmospheric pressure *i.e.* the superincumbent pressure. It is to be noted that water taken in the tube *J* should be in sufficient quantity so that even during boiling, some water is left in the closed limb of the tube *J*.



(i) Fig. 6.7 (ii)

Explanation : At the time of boiling, big bubbles are formed at the bottom of the water and they move up through the mass of water to the surface. As long as the bubble remains inside the water, the vapour pressure in the bubble should, at least, be equal to the surrounding pressure ; otherwise the bubble will disappear at the greater surrounding pressure. For this reason, bubbles formed below disappear on the way of upward journey when the temperature of water is low. The pressure just below the surface of water may be taken to be equal to atmospheric pressure. So, if the pressure inside the bubble is less than the atmospheric pressure, bubbles cannot come up to the surface and the liquid cannot boil. The temperature at which the saturated vapour pressure of water becomes equal to the outside atmospheric pressure, therefore, makes the mass of water boil.

6.21. Similarity between fusion and boiling :

The following similarities are observed between fusion and boiling :

(1) In both the operations, there is a change of state of matter which takes place at constant temperature. In the case of fusion, the temperature is called '*melting temperature*' and in the case of boiling, the '*boiling temperature*'.

(ii) Some heat is absorbed in both the processes.

(iii) During the change of state in both the processes, a change of volume takes place.

(iv) Both the melting temperature and the boiling temperature are dependent on superincumbent pressure.

(v) The freezing point of a solution is lower than the freezing point of the pure solvent but the boiling point of a solution is higher than that of the pure solvent. In both the cases, the difference depends on the amount of solvent present in the solution.

(vi) Under suitable arrangement, a pure liquid can be slowly cooled so that its temperature may go below its freezing point without the liquid having frozen. This is known as *super-cooling*. Similarly, a pure liquid can be slowly heated beyond its boiling point without the liquid having turned into vapour. This is known as *super-heating*.

6.22. Approximate determination of height by observation of boiling points (Hypsometry) :

The observations of boiling points at any two stations may be used to determine approximately the height between the stations.

Let A be the base of a hill and B its top [Fig. 6.8]. Boiling point of water at the lower station A is observed by means of a hypsometer. In the same way the boiling point of water is measured at the higher station B. From the boiling point observations, the atmospheric pressures at the two stations may be calculated from the fact that normal boiling point of water is 100°C and that for each degree change in the boiling point of water, pressure changes by 27 mm. Atmospheric temperatures at both the stations are noted. The difference of atmospheric pressures as calculated from boiling point observations is equal to the weight of the column of atmosphere of unit sectional area and length equal to the height between the two stations A and B.

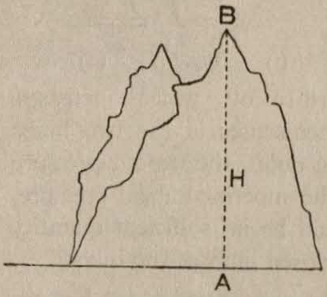


Fig. 6.8

Let h cm be the difference in atmospheric pressures at the two stations. The corresponding pressure is given by

$$P = h.\rho.g = h \times 13{\cdot}6 \times g \text{ dynes/cm}^2. \quad [\rho = \text{density of mercury}]$$

Let H cm be the height between the stations A and B and d the mean-density of air between the stations. Then the weight of the column of atmosphere of unit sectional area $= Hdg$ dynes/cm^2.

$$\therefore \quad H.d.g = h \times 13{\cdot}6 \times g \quad \text{or} \quad H = \frac{h \times 13{\cdot}6}{d}.$$

Example : *The boiling point of water at a lower station is 100°C and that at the upper station is 96°C, the temperature of air at the two stations being 14°C and 10°C respectively. Density of air at N.T.P. is* $1{\cdot}293 \times 10^{-3}$ *gm/c.c. Find the height between the stations.*

Ans. Pressure at the lower station $=760$ mm of Hg. At the upper station, the boiling point of water being 4°C less, the atmospheric pressure there

$$= 760 - 27 \times 4 = 652 \text{ mm.}$$

$$\therefore \quad h = 760 - 652 = 108 \text{ mm} = 10{\cdot}8 \text{ cm. of mercury}$$

Hence $P = 10{\cdot}8 \times 13{\cdot}6 \times g$ dynes/cm^2

Now, mean atmospheric pressure between the stations $= \dfrac{760+652}{2} = 706$ mm.

and the mean temperature $= \dfrac{14+10}{2} = 12$°C

From $\frac{d.T}{P}=\frac{d_0.T_0}{P_0}$, we have $d=\frac{d_0.T_0}{P_0}\times\frac{P}{T}=\frac{1{\cdot}293\times10^{-4}\times273}{760}\times\frac{706}{285}$

$\therefore\quad H=\frac{10{\cdot}8\times13{\cdot}6\times760\times285}{1{\cdot}293\times10^{-4}\times273\times706}$ cm. $=12{\cdot}77$ km. (nearly)

6.23. Latent heat of vaporisation :

If we take some water and heat it, its temperature rises. This rise of temperature can be recorded by a thermometer. When the temperature of water becomes 100°C, the water boils. Once the water starts boiling, the temperature remains constant at 100°C, although heat is being steadily absorbed by the water from the burner. This heat, which is going into the water, but is not increasing the temperature of water, is the energy needed to convert the water from the liquid state to the vapour state. This heat remains latent in the steam and is known as the latent heat of vaporisation.

Definition : *The latent heat of vaporisation of a liquid is the quantity of heat required to change unit mass of the liquid at its boiling point to the vapour state without change of temperature.*

For example, latent heat of steam is 537 calories per gramme. This means that 1 gm. of water at 100°C will require 537 calories of heat for conversion into steam at 100°C. Conversely, when 1 gm. of steam condenses to water at 100°C, it will give up 537 calories of heat.

So, we see that in respect of heat-content, 1 gm. of water at 100°C is different from 1 gm of steam at 100°C. The latter contains 537 calories of heat more than the former. For this reason, steam at 100°C is a good heating agent and has many practical applications. In cold countries, rooms of a building are kept warm by the circulation of steam. As steam contains more heat than water, it will cause severe burn than water at 100°C. In F.P.S. system the latent heat of steam

$$=\frac{537\times453{\cdot}6}{252}=537\times\frac{9}{5}=966{\cdot}6 \text{ Btu. per lb.}$$

6.24. Method for determining the latent heat of steam :

The apparatus required is shown in the fig. 6.9. It consists of a copper boiler *A* which produces steam. Steam passes along the rubber tubing *B* to a steam-trap. Any water droplets formed by condensation of the steam in the rubber tube are caught in the trap and cannot enter the calorimeter *C*. To reduce the loss of heat, the calorimeter is well lagged. A thermometer *T* and a stirrer *S* are inserted into the calorimeter. To prevent direct passage of heat from the boiler to the calorimeter, an asbestos partition *P* is placed between them.

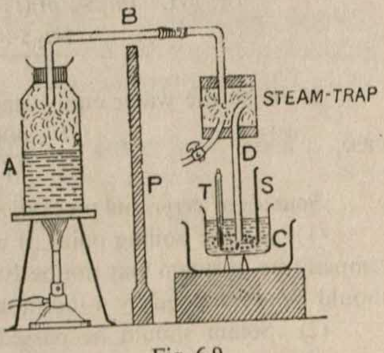


Fig. 6.9

Procedure : The calorimeter and the stirrer are first weighed empty and then when the calorimeter is about three-quarters full of cold water. The boiler is started in good time so that the apparatus has time to warm up and produce a steady flow of dry steam. The calorimeter is brought up to the steam supply and its temperature noted immediately before inserting the tube D below the surface of water. During the passage of steam the water is continually stirred to ensure thorough mixing. When the temperature has risen 4°C/5°C above the room temperature, the calorimeter is removed from the steam and the final maximum steady temperature is noted. The calorimeter is then weighed.

Calculation : Suppose, the weight of the empty calorimeter $= m_1$ gm.

" " " calorimeter + water $= m_2$ gm.

" " " calorimeter + water + condensed steam $= m_3$ gm.

Initial temperature of the calorimeter $= t_1$°C

Final steady temperature of the mixture $= t_2$°C

So, heat of the material of the calorimeter $= S$

Latent heat of steam $= L$

Now, weight of water taken $= (m_2 - m_1) = m$ *gm.* (say)

and " " condensed steam $= (m_3 - m_2) = M$ gm. (say)

Here, steam will lose heat in two stages—first, each gram of steam gives out L calories of heat in condensing to form water at 100°C; and afterwards, this water cools from 100°C to the final steady temperature t_2°C. The calorimeter and the cold water will absorb this heat and its temperature will rise from t_1°C to t_2°C.

Now, heat lost by steam in condensing to water

$$= \text{mass of the steam} \times \text{latent heat} = M.L. \text{ calories}$$

Heat lost by condensed steam $= M(100 - t_2)$ cal.

So, the total heat lost $= ML + M(100 - t_2)$ cal.

Again, heat gained by the calorimeter + stirrer $= m_1S(t_2 - t_1)$ cal.

and " " " " water $= m(t_2 - t_1)$ cal.

So, the total heat gained $= m_1S(t_2 - t_1) + m(t_2 - t_1)$

$$= (m_1S + m)(t_2 - t_1) \text{ cal.}$$

Since, heat lost = heat gained,

we have $ML + M(100 - t_2) = (m_1S + m)(t_2 - t_1)$

or, $ML = (m_1S + m)(t_2 - t_1) - M(100 - t_2)$

$$\therefore \quad L = \frac{(m_1S + m)(t_2 - t_1)}{M} - (100 - t_2)$$

If W be the water equivalent of the calorimeter, then $m_1S = W$ and in that case,

$$L = \frac{(W + m)(t_2 - t_1)}{M} - (100 - t_2)$$

Sources of error and precautions :

(1) As the boiling point of water is dependent on atmospheric pressure, the temperature of steam may not be 100°C. For this reason, the temperature of steam should be ascertained by a thermometer.

(2) Steam should be passed slowly into the calorimeter, otherwise water may spill out.

(3) Supply of steam in the steam-trap should be slow, otherwise some water may enter along with steam through the delivery tube, and make the result erroneous.

(4) The heat given up by the condensation of steam may partly be lost due to convection and radiation. To prevent it, the calorimeter should be placed in Regnault's apparatus.

(5) To avoid radiation loss, the calorimeter and its contents should be cooled 4° or 5°C below the room temperature before passing steam and the amount of steam sent into the calorimeter should be so adjusted that the final temperature of the mixture may be 4° or 5°C higher than the room temperature.

6.25. Graphical representation of the steps by which ice is heated through fusion and vaporisation :

For the sake of simplicity, let us consider the case of 1 gm of ice at -10°C (say). Before fusion, certain quantity of heat should be supplied to raise the temperature of ice from -10°C to 0°C. Taking sp. heat of ice equal to 0·5, heat required for the above purpose $=1\times0{\cdot}5\times10=5$ calories. In the calori-temperature curve [fig. 6.9(a)], the absorption of heat at this stage is represented by *OA*. Application of heat during this stage does not change the temperature of ice but fusion of ice commences. About 80 calories of heat (latent heat of fusion of ice) will be necessary for complete fusion of 1 gm of ice. This is represented by the horizontal part *AB* of the curve. When fusion is complete, further application of heat will raise the temperature of water as obtained from the melting of ice. Temperature rise of water will continue with the application of heat, till the temperature becomes 100°C. The amount of heat absorbed during this process by 1 gm of water $=1\times1\times100=100$ calories and this is represented by the curve *BC*. Further application of heat at this stage again, will not cause any rise in temperature but water will be converted into vapour. For complete conversion of 1 gm of water at 100°C into vapour at 100°C, the amount of heat absorbed $=537$ calories and this is represented by yet another horizontal portion *CD*. If the steam is contained in a vessel under constant pressure, further application of heat will raise its temperature.

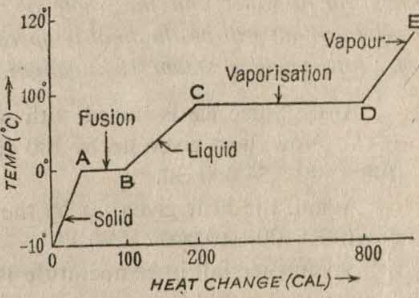


Fig. 6.9(a)

If the process be just reversed, the abstraction of heat at different stages may be shown simply by retracing the temperature-calorie curve.

Table for latent heat of vaporisation of some liquids

Liquids	Latent heat
Water	537 calories
Alcohol	362 ,,
Ether	90 ,,
Turpentine	70 ,,

Examples : (1) *A vessel of water equivalent 100 gm., contains 500 gm of water at 40°C. Steam at 100°C is allowed to condense in the vessel. What quantity of steam is required to raise the water to its boiling point ? Latent heat of steam is 537 cal/gm.*

Ans. Let's 'm' gm of steam be required.

Now, heat necessary to raise temperature of the vessel together with its contents from 40°C to 100°C (boiling point of water) $=(100+500)(100-40)=600 \times 60$ cal.

This heat will be supplied by steam in condensing to water at 100°C. 1 gm. of steam gives up 537 cal. in condensing to water at 100°C.

$\therefore$ 600×60 cal. of heat will be available from $\frac{600 \times 60}{537}=67$ gm (nearly) of steam. So, 67 gm of steam will be required.

(2) *100 gm. of steam are passed through a mixture of 1 kilogram of ice and 1 kilogram of water and the whole of the steam is condensed. How much ice will melt ? What will be the final temperature ? Latent heat of fusion of ice = 80 cal/gm ; latent heat of steam = 540 cal/gm.*

Ans. Since ice is mixed with water, the initial temperature of the mixture = 0°C. Now, heat given up by 100 gm. of steam at 100°C in condensing to water $=100 \times 540=54{,}000$ cal.

Again, the heat given up by the condensed water in coming from 100°C to 0°C $=100 \times 100=10{,}000$ cal.

No further fall in temperature is possible because the initial temperature of ice-cold water is 0°C *i.e.* no more heat will be available from the steam.

Hence, the total heat given up by steam $=54{,}000+10{,}000=64{,}000$ cal.

But the heat necessary to melt 1 kg. of ice $=1000 \times 80=80{,}000$ cal.

So, it is clear that all ice will not melt. Since, some ice is left, the final temperature of the mixture is also 0°C. Now, we know,

80 calories of heat are necessary to melt 1 gm. of ice.

$\therefore$ 64,000 calories of heat will melt $\dfrac{64{,}000}{80}=800$ gm. of ice.

(3) *By passing steam at 100°C through a mixture of ice and water, it is found that 2 gm of steam are necessary for complete melting of the ice and raising the temperature of the vessel to 4°C. How much ice was in the vessel ? Water-equivalent of the vessel and water = 60 gm ; latent heat of ice = 80 cal/gm ; latent heat of steam = 540 cal/gm.*

Ans. Heat given up by 2 gm of steam in condensing to water at 100°C and then falling to 4°C $=2 \times 540+2(100-4)=1272$ cal.

Heat absorbed by the vessel and water $=60 \times 4=240$ cal.

Heat necessary for melting ice $=m \times 80$ cal. [m = mass of ice]

Heat absorbed by melted water in raising its temperature from 0°C to 4°C $=4m$ cal.

So, total heat absorbed $=240+80m+4m=(240+84m)$ cal.

$$\therefore \quad 240+84m=1272 \quad \text{or} \quad m=12{\cdot}3 \text{ gm (nearly).}$$

(4) *If 64,800 calories of heat are withdrawn from 100 gm. of steam at 100°C, what will be the result ? Latent heat of steam = 540 cal/gm. and latent heat of fusion of ice = 80 cal/gm.*

Ans. Due to extraction of heat, steam will first condense into water at 100°C and for this purpose, the amount of heat withdrawn $= 100 \times 540 = 54,000$ cal. Then, the condensed water will fall in temperature. The amount of heat extracted $= (64,800 - 54,000) = 10,800$ cal.

Now, heat given up by water in coming down to $0°C = 100 \times 100 = 10,000$ cal. So, the remaining amount of heat extracted $= (10,800 - 10,000) = 800$ cal. which will form ice out of water.

We know, 80 calories of heat are to be extracted for forming 1 gm. of ice. Hence 10 gm. of ice will be formed by the withdrawal of 800 cal.

So, the result is that 10 gm of ice will be mixed up with 90 gm of water at 0°C.

(5) *There is some water in a vessel. The vessel is rapidly evacuated causing intense evaporation of water. The consequent cooling helped some water to freeze. What fraction of the initial mass of water can be frozen into ice by this process. Latent heat of ice = 80 cal/gm and latent heat of evaporation = 540 cal/gm.*

Ans. Let m gm be the initial mass of water ; m_1 gm = the mass of water frozen and m_2 gm = mass of water evaporated. Then $m = m_1 + m_2$ .. (i)

Heat given out by m_1 gm of water in freezing into ice $= m_1 \times 80$ cal . This heat will evaporate m_2 gm of water. For this purpose, the amount of heat necessary $= 540 \times m_2$ cal.

$$\therefore \quad 540 \times m_2 = 80 \times m_1 \quad \text{or} \quad m_2 = \frac{80}{540} . m_1$$

Putting this value in eq^n. (i), $m = m_1 + \frac{80}{540} . m_1 = \frac{620}{540} . m_1$

$$\therefore \quad \frac{m_1}{m} = \frac{540}{620} = \frac{27}{31}$$

(6) *A metal container is partly filled with 5 gm of water and sealed. The container is then dipped in liquid oxygen at −180°C for some time and then transferred quickly to copper calorimeter containing water whose total thermal capacity is 100 cal. The water in the calorimeter cools from 20° to 10°C. Assuming no loss of heat find the mean specific heat of ice between −180°C to 10°C. The water equivalent of the metal container is 1 gm. Latent heat of fusion of ice = 80 cal.*

Ans. When the metal container is dipped in liquid oxygen, the water in the container at once freezes into ice at −180°C.

Now, heat gained by the container = water equivalent × rise of temp. = $1 \times [10 - (-180)] = 190$ cal.

Heat gained by the water in the container $= 5 \times S \times 180 + 5 \times 80 + 5 \times 10$
$= (900S + 450)$ cal.
[S = mean sp. heat of ice]

$\therefore$ Total heat gained $= 190 + 900S + 450 = 640 + 900S$ cal.

Heat lost by the calorimeter and water $= 100 \times (20 - 10) = 1000$ cal.

Now, heat gained = heat lost. $\therefore$ $640 + 900S = 1000$. or $S = 0{\cdot}4$

(7) *A mixture of 250 gm of water and 200 gm of ice at 0°C is kept in a calorimeter which has a water-equivalent of 50 gm. If 200 gm of steam at 100°C is passed through this mixture, calculate the final temperature and weight of the contents of the calorimeter. Latent heat of ice = 80 cal/gm and latent heat of steam = 540 cal/gm.*
[*I.I.T. 1974*]

Ans. Heat given out by 200 gm of steam in condensing to water at 100°C = $200 \times 540 = 108{,}000$ calories.

Heat absorbed by the calorimeter and its contents to rise to 100°C

$$= (250+50)\times 100 + 200\times 80 + 200\times 100$$
$$= 300\times 100 + 200\times 80 + 200\times 100 = 66{,}000 \text{ cal.}$$

As heat absorbed is less than the heat given out, a part of the steam will condense and the temperature of the mixture will be 100°C. Let m gm of steam condense.

Heat given out by m gm of steam to condense into water at 100°C $= m\times 540$ cal $\therefore$ $m\times 540 = 66{,}000$ or $m = 122{\cdot}2$ gm.

$\therefore$ Wt. of the contents of the calorimeter $= 250 + 200 + 122{\cdot}2 = 572{\cdot}2$ gm.

6.26. Effects of pressure on boiling point :

The boiling point of a liquid, as has been mentioned earlier, depends upon the pressure under which the liquid is allowed to boil. If the superincumbent pressure is reduced, the liquid boils at a lower temperature. On the other hand, if the pressure is increased, the liquid boils at a higher temperature. Following experiments illustrate these facts.

(1) **Lowering of boiling point due to reduction of pressure ; Franklin's experiment :** Take some water in a round-bottomed flask and boil it. Steam will push out all air from the flask. Having closed the mouth of the flask with a cork, insert a thermometer through a hole and keep the flask inverted on a stand as shown in fig. 6.10. Remove the source of heat. The water will now stop boiling and the space above water will be filled up by water vapour. Now pour cold water on the flask. After some time, water will be found to boil again, although the thermometer shows a temperature lower than 100°C. How is it possible ?

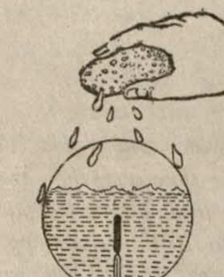


Fig. 6.10

Due to pouring of cold water, some water-vapour in the flask condenses into water and the pressure over the water surface is reduced. This lowers the boiling point of water and the then temperature of water being equal to the lowered boiling point, the liquid starts boiling again.

Applications : Lowering of boiling point due to reduction of pressure is applied in many industries like preparing hydrogen peroxide, crystals of sugar from sugar solution etc.

(2) **Raising of boiling point due to increase of pressure : Regnault's experiment :** The experimental arrangement is shown in the fig. 6.11. It consists of a

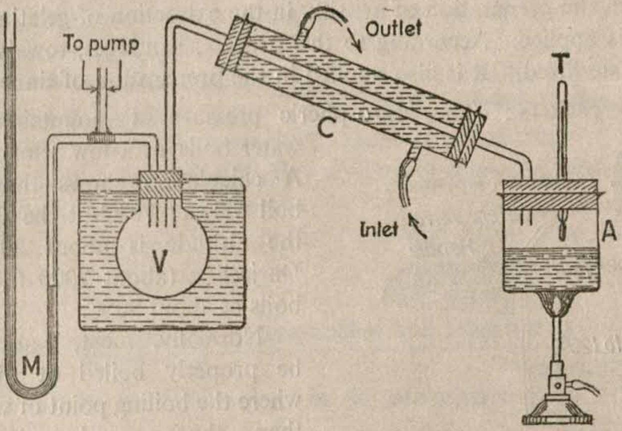


Fig. 6.11

round-bottomed copper vessel *V* connected to a boiler *A* through a condenser *C*. There is arrangement for water-circulation through the condenser tube. The liquid under test is taken in the boiler *A*, into which a thermometer (*T*) is inserted. The vessel *V* is kept in a water-bath so that the temperature within it remains constant. The vessel is connected to a compression pump and a manometer *M*.

Procedure : Keeping the air pressure in the vessel *V* equal to atmospheric pressure, heat the boiler *A*. The liquid vapour entering into the condenser will liquefy again and will return to the boiler *A*. Consequently, there will be no variation of pressure on the liquid surface; the pressure will remain equal to the atmospheric. After some time, the liquid starts boiling; the corresponding temperature given by the thermometer *T* is the normal boiling point of the liquid.

Now, by working the compression pump, increase the pressure of air in the vessel *V*. Hence, the pressure on the liquid surface in the boiler *A* increases. If now, the liquid is made to boil by supplying heat, it will be seen that the thermometer records a boiling temperature higher than the normal boiling point. In this way, if the pressure of air be increased further by working the compression pump, the boiling point of the liquid will also be raised further.

Lowering of boiling point due to reduction of pressure can also be demonstrated by this arrangement. For this, the compression pump is to be replaced by an air-exhaust pump. This pump will exhaust air from the vessel *V* and the pressure on the liquid surface will be reduced. Under this circumstance, the liquid will be found to boil at a temperature lower than its normal boiling point.

Experiments show that for every change of 27 mm. of air pressure there is a change of about 1°*C* in the normal boiling point (100°C) of water.

[*Note* : Change of boiling point due to change of pressure is given by Clasius-Clapeyron equation : $\frac{dP}{dT}=\frac{L}{T(V_1-V_2)}$ where dP=change of pressure ; dT=consequent change of boiling point, V_1, V_2=volumes occupied by 1 gm. of the substance in gaseous and liquid state respectively.]

Application : Raising of boiling point of a liquid due to increase of pressure has several applications. In the manufacture of paper pulp from saw-dust and caustic soda, in the preparation of art silk, in the extraction of gelatin from bones, etc. this fact is applied. According to this process, bandages, towels etc. used in hospitals, are sterilized. It is also applied in the preparation of tinned food.

Pressure cookers : The atmospheric pressure at mountains being low, water boils at a low temperature there. A calculation shows that water will boil only at 70°C at the Everest where the altitude is about 29,000 ft. At Darjeeling (about 7000 ft. high) water boils at about 90°C.

Normally, meat, eggs etc. cannot be properly boiled at high altitudes where the boiling point of water is lower than the normal value. *Pressure cookers* are used in such cases to raise the pressure and therefore to the right boiling temperature of water for proper cooking.

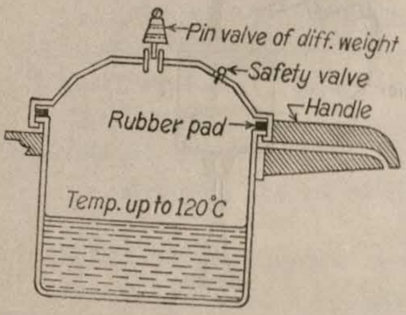


Fig. 6.12

Fig. 6.12 shows a modern pressure cooker. It consists of a stout metallic container fitted with a lid having a rubber sealing ring. Steam is allowed to escape through a loaded pin valve which can be set to blow at various desired pressures. By this means it is possible to cook at temperatures upto 120°C or more, with a corresponding saving of time and fuel. If, for some reason or other, the pressure inside the cooker exceeds the safe limit, a safety valve opens and relieves the excess pressure. This cooker is also known as '*Papin's digester*' as it was designed by a Frenchman named Denys Papin.

6.27. Factors influencing the boiling point of a liquid :

The following are the factors which influence the boiling point of a liquid :

(i) **Pressure on the liquid** : The boiling point of a liquid depends upon the superincumbent pressure. With the increase of pressure, the boiling point rises and vice versa. It has been mentioned earlier that the normal boiling point of water (100°C) increases or decreases by about 1°C for 27 mm. increase or decrease of atmospheric pressure.

(ii) **Pressure of impurities in the liquid** : Presence of impurities in the liquid raises its boiling point. For example, the boiling point of water (100°C) is raised by about 9°C when common salt is dissolved in it. For this reason, in determining the boiling point of water, the thermometer bulb is never immersed in the liquid but kept in contact with the vapour coming out of the liquid. The temperature of the vapour remains same as the normal boiling point of the liquid.

(iii) **Nature of the liquid** : The normal boiling point of a liquid depends upon the nature of the liquid. Different liquids have different normal boiling points.

6.28. Laws of ebullition :

The facts so far discussed in connection with the ebullition or boiling of a liquid may be summarised in the following way in the form of laws, usually known as the *laws of ebullition.*

(i) Every liquid has a normal boiling point which is the temperature at which the liquid boils under normal atmospheric pressure. The boiling point remains constant as the whole mass of the liquid is boiled away.

(ii) The boiling point of a liquid increases or decreases with the increase or decrease of the superincumbent pressure.

(iii) The boiling point of a solution is always higher than that of the pure solvent.

Water-vapour in atmosphere and Hygrometry

6.29. Presence of water-vapour in the atmosphere :

Owing to continual evaporation of water from the sea and other water surfaces, the atmosphere always contains water-vapour. The amount of water vapour which the air holds varies almost everyday. In rainy season, the air is 'wet' *i.e.*, it contains too much water-vapour while in winter, the air becomes 'dry'.

Various natural phenomena like cloud, fog, rain occur due to the presence of water-vapour in the atmosphere. *Hygrometry is considered as that branch of science which deals with the condition of atmosphere with regard to the aqueous vapour present in it.* For a fruitful study of this subject, knowledge of saturated and unsaturated vapours is necessary. In the following article, we shall briefly discuss about it.

6.30. Saturated and unsaturated vapour :

Suppose we have some liquid in a vessel whose mouth is closed by an air-tight stopper. Owing to evaporation, the space above the liquid begins to fill with vapour. The vapour molecules move about in all directions and exert pressure when they bounce off the walls of the vessel. They also strike the liquid surface and many re-enter it. Finally, a state of equilibrium is reached when the rate at which molecules leave the liquid equals the rate at which others return to it. Under these conditions, the space above the liquid is said to be *saturated* with vapour and the pressure exerted is called the *saturation vapour pressure* (abbreviated as S.V.P.). The following experiment is very illustrative.

Definition : A vapour is said to be saturated if it is in a state of equilibrium with its own liquid.

Experiment : Take two barometer tubes *A* and *B*. The mercury columns in the tubes will be of same height because both of them indicate the same atmospheric pressure. Take some water in a pipette and insert the bent end of the pipette in the tube *B*. Blowing at the other end, allow some water to enter into the tube. Water being lighter than mercury, it will at once, go up in the Torricellian vacuum, where the water will be evaporated due to very low pressure existing there. Since this vapour exerts pressure, the mercury column will be depressed a little. In this way, if water be allowed to enter into the tube slowly, the mercury column will also be gradually depressed till some water will be found to accumulate on the top of the mercury column (Fig. 6.13). After that, the height of the mercury column will not change which shows that the vapour pressure is changing no more. The space above the mercury column then, is saturated with aqueous vapour and the pressure it exerts is called the *saturated vapour pressure* or *aqueous tension*. The difference of the heights of the mercury columns in the tubes *A* and *B* gives the saturated vapour pressure.

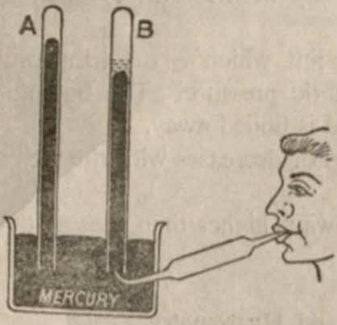


Fig. 6.13

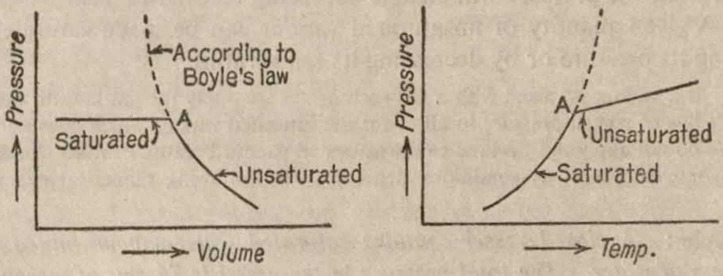
For the above reason, the saturated vapour pressure of a liquid is generally expressed by the heights of mercury column. Thus, the vapour pressure of a liquid at 25°C is 30 mm. means that at 25°C saturated vapour pressure of the liquid is equal to the pressure exerted by 30 mm. of mercury column.

Before water accumulates at the top of the mercury column in the tube *B*, the vapour present in the Torricellian vacuum is called unsaturated vapour and the pressure exerted by such vapour is known as *unsaturated vapour pressure.*

Applicability of gas laws :

What change in the saturation vapour pressure occurs when the volume of the saturated vapour changes ? Does it follow Boyle's law ? To study it, suppose we raise the mercury reservoir (Fig. 6.13) a little. The volume of the saturated vapour above the mercury column in the tube *B*, will decrease. But the difference in the heights of the mercury columns in the two tubes will be found to remain the same ; only the amount of water standing on top of mercury column will increase slightly due to condensation of some vapour. So, decrease in the volume of the saturated vapour produces no change in its pressure ; it causes some of the vapour to condense into water. Similarly, if the mercury reservoir be lowered a little, the volume of the saturated vapour will increase but it causes no change in the difference of height between the mercury columns. This means that increase or decrease in volume of saturated vapour does not produce any change in its pressure ; it causes some of the water to evaporate or some vapour to condense into water. So, the conclusion is that *saturated vapour does not obey Boyle's law.*

If, now, the temperature of the Torricellian vacuum in the tube *B*, be increased, more water will be evaporated showing that the capacity of the space to hold water-vapour has increased. In other words, the saturation vapour pressure has also increased but not according to Charle's law. The increase is more than what the law allows. Fig. 6.14(a) shows the variation of pressure and volume of saturated and unsaturated vapour. As long as the vapour is unsaturated, the pressure-volume variation is according to Boyle's law but as soon as the vapour becomes saturated, the pressure remains constant although the volume is changed. Then the graph becomes a straight line parallel to the volume axis [Fig. 6.14(a)]. Fig. 6.14(b) shows graphically the temperature-pressure variation. With the rise of temperature, more and more liquid is evaporated and the pressure increases. When the point *A* is reached, the whole of the liquid is evaporated. With further rise of temperature, the vapour becomes unsaturated and it rises more sedately. The dotted line shows how the pressure would have increased had the vapour been saturated still.



(a) Fig. 6.14 (b)

From the above experiments, we come to the following important conclusions :

(i) Vapour is capable of exerting pressure.

(ii) A closed space has a maximum limit of holding vapour and when the limit is reached, the vapour is called saturated vapour and its pressure saturated vapour pressure.

(iii) The capacity for holding vapour increases with the increase of temperature of the space and hence the pressure of the vapour also increases.

6.31. Difference between saturated and unsaturated vapours :

(i) A vapour in contact with the liquid in a closed space at a particular temperature is said to be a saturated one and it exerts a pressure which is maximum for the vapour to exert at the given temperature. This pressure is called the saturation vapour pressure at that temperature.

When a closed space contains vapour but no liquid, the space may be either just saturated with the vapour or unsaturated.

A space is said to be just saturated when partial condensation of vapour to corresponding liquid takes place when the volume of the space is slightly diminished. The pressure of this vapour will be the same as the saturation vapour pressure.

(ii) When the vapour is unsaturated, change of volume of the space holding the vapour will produce change in pressure according to Boyle's law. But in the case of saturated vapour, the change of volume will not cause any change of pressure but there may be fresh evaporation or condensation according as the volume of the space is increased or decreased.

(iii) If a space is saturated with vapour, then with the increase of temperatures, saturation vapour pressure increases. But this increase of pressure will not be according to Charles' law, although the volume of the space is kept constant. In the case of unsaturated vapour, if the temperature is changed, keeping the volume constant, the pressure will change according to Charles' law.

(iv) A given quantity of unsaturated vapour can be made saturated either by increasing its pressure or by decreasing its temperature.

[Note : If a vapour is mixed with a gas such as air, we apply the gas laws to the *air* after using Dalton's law of partial pressure to allow for the saturated vapour which does not obey the gas laws. We do not apply the gas laws to the saturated vapour because its mass changes due to condensation or evaporation as conditions demand. The following numerical example illustrates it.]

Example : *A closed vessel contains saturated water-vapour mixed with air and an excess of water. The total pressure in the vessel is 76 cm. of mercury when the temperature is 27°C ; what will be the total pressure when the temperature has been raised to 100°C. Saturation vapour pressure of water at 27°C=2·8 cm. of mercury.*

Ans. Gas laws are applicable to air only. According to Dalton's law of partial pressure, the pressure of dry air at 27°C=Total pressure−the saturation vapour pressure=(76−2·8)=73·2 cm. of Hg.

Now, from gas laws, we know, that pressure of a gas is directly proportional to its absolute temperature, when volume remains unchanged ; *i.e.*,

$$\frac{P_1}{T_1}=\frac{P_2}{T_2} \quad \text{or} \quad \frac{73{\cdot}2}{273+27}=\frac{P_2}{273+100} \quad \text{or} \quad \frac{73{\cdot}2}{300}=\frac{P_2}{373}$$

$$\therefore \quad P_2=\frac{73{\cdot}2\times 373}{300}=91{\cdot}01 \text{ cm. of Hg}$$

Now, the saturation vapour pressure of water at 100°C=76 cm. of Hg.

∴ Total pressure in the vessel at 100°C=91·01+76=167·01 cm.of Hg.

6.32. Triple point :

The triple point may be defined as that point representing a given temperature and pressure at which the three phases of a substance, solid, liquid and vapour co-exist simultaneously in equilibrium with each other without any change in their proportions. For a given substance there is only one triple point. Thus triple point of water represents a pressure of 4·6 mm of mercury and a temperature of 0·01°C ; for carbon-dioxide triple point presents a pressure of 3800 mm. of mercury and a temperature of −56·6°C.

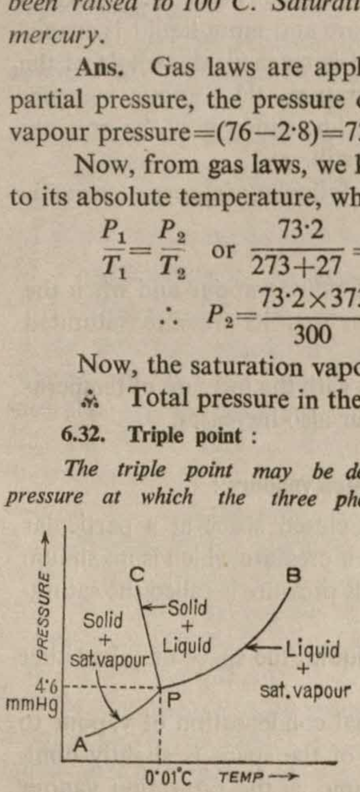


Fig. 6.15

Explanation : Consider the case of water. The curve *AP* represents the change of the saturated vapour pressure of ice with temperature. Ice and water-vapour are in thermal equilibrium at any point on this curve. The portion *PB* represents the variation of saturated vapour pressure of water with temperature [Fig. 6.15]. At any point on it, water is in thermal equilibrium with its vapour. The part *PC* of the curve gives the relation between the freezing point of ice with pressure. At any point on this portion of the curve, ice and water are in equilibrium. The three curves meet at the point *P*, whose temperature and pressure values are 0·01°C and 4·6 mm. of Hg respectively. At this temperature and pressure only, ice, water and water-vapour can co-exist simultaneously. If either the temperature or the pressure is altered, at least, one phase of the substance vanishes. The point *P* is, therefore, called the triple point for water.

6.33. Dew point :

Condensation of water-vapour present in the atmosphere is the reason for the formation of *dews*. Generally, the atmosphere is not saturated with the water-vapour present in it, but if, for some reason, the atmosphere is cooled, it may become saturated. During night, the surface of the earth becomes cold due to radiation of heat etc. and the mass of air in contact with the surface also becomes cold. It then contracts in volume due to which its capacity for holding vapour diminishes. Gradually, the cold earth surface cools the air in its vicinity to a temperature below that for which the water-vapour present is sufficient to saturate the air. Excess vapour then condenses in the form of droplets. These are known as dews and the highest temperature at which this occurs is called the dew point.

Definition : *The dew point is defined as the temperature at which the water-vapour present in the air is just sufficient to saturate it.*

Alternatively, it may be said that when the temperature of the air reaches dew point, the air becomes saturated with the water-vapour present in it. It goes without saying that when the atmosphere is perfectly dry *i.e.*, the atmosphere does not contain any water-vapour, there will be no formation of dew.

The dew-point at a place is 12°C—this statement, therefore means that the air at that place will be saturated with water-vapour present in it at 12°C.

6.34. Humidity and relative humidity :

The term 'humidity' signifies the dampness of the atmosphere due to its holding of water-vapour.

The absolute humidity of the air is defined as the mass of water-vapour present per cubic metre. However, for practical purposes, relative humidity (R.H.) is more important than absolute humidity.

Definition : *It is the ratio of the mass of water-vapour present in a given volume of air at a particular temperature to the mass of water-vapour required to saturate the same volume of air at the same temperature.* So,

$$\text{R.H.} = \frac{\text{Mass of water-vapour in a given volume of air}}{\text{,, \quad ,, \quad ,, \quad required to saturate the same volume of air at the air-temperature}}$$

Since, the mass of water-vapour is proportional to its pressure, the above ralation may be written as

$$\text{R.H.} = \frac{\text{Pressure of water-vapour in the air}}{\text{Saturated vapour pressure at the air-temperature}}$$

Further we know that the mass of water-vapour present in the air will saturate it at the dew point. The original vapour pressure in the atmosphere is therefore equal to the saturation vapour pressure at the dew point. The above ratio may alternatively be written as,

$$\text{R.H.}=\frac{\text{Saturation vapour pressure of water at the dew point}}{\text{,, \quad ,, \quad ,, \quad ,, \quad ,, at the original air-temperature}}$$

Relative humidity is usually expressed in percentage. Any one of the above ratios multiplied by 100, will give us the relative humidity in percentage.

The saturated vapour-pressure at the dew point being f and that at the room temperature being F, $\text{R.H.}=\frac{f}{F}\times 100\%$.

'The relative humidity of air is 60%' — this statement means (i) the vapour pressure at the room-temperature is 60/100 or 3/5 part of the saturated vapour pressure at the same temperature; (ii) the water vapour present in a certain volume of air in the room is only 60% of the water vapour needed to saturate the air at the room-temperature.

6.35. Effect of relative humidity on daily life :

The feeling of comfort or uneasiness due to the atmosphere being 'dry' or 'wet' does not depend only on the water-vapour present in the air. It also depends upon the temperature. *The main factor which controls the sensation of comfort or uneasiness is the relative humidity.* Following examples will clarify the matter.

(a) Two rooms having same temperature but of different relative humidity will cause a difference in the feeling of comfort. The room having greater relative humidity will cause greater discomfort because evaporation of perspiration from the body becomes slow in that room due to the presence of greater amount of moisture in the air. Quick evaporation cools down the body and produces pleasant sensation.

How will the relative humidity of a room change if the temperature of the room is raised ? Due to rise of temperature, relative humidity will be lowered. At a higher temperature, a greater amount of water-vapour will be necessary to saturate the air in the room. The denominator of the ratio given in the definition of the relative humidity, is thereby increased while the numerator remains the same. As a result, the relative humidity is reduced.

How will the relative humidity change if some water is sprinkled in the room? Due to spraying of water, the mass of water-vapour present in the air increases. As the temperature remains unaltered, the mass of water-vapour required to saturate the air remains the same. Hence, the numerator of the ratio giving the relative humidity increases while the denominator remains unchanged. As a result, the relative humidity increases.

(b) Wet clothes are found to dry up quicker in winter than in rainy season although the temperature in winter season is less than that in rainy season. In winter, the atmosphere is less humid which facilitates the rate of evaporation. In rainy season the atmosphere is about hundred per cent humid which hinders evaporation. For this reason, wet clothes are found to dry more quickly in winter than in rainy season.

(c) The uncomfortable feeling due to greater humidity may well be illustrated by mentioning that a hot day in Puri causes greater discomfort than an equally hot day in Delhi, because the atmosphere of Puri is more humid due to proximity of the sea than the atmosphere of Delhi.

(d) In winter season, skin, lips etc. of many persons get parched due to low humidity. Atmosphere absorbs water from the comparatively soft part of uncovered portion of the body. Lips are therefore, very prone to get parched in winter. But a coating of glycerine prevents parching. Glycerine, being hygroscopic, draws water-vapour from the atmosphere and prevents atmosphere from absorbing water from the lips.

Importance of relative humidity : For various reasons, information about relative humidity is necessary. It has been found that we perspire and feel uncomfortable if the relative humidity exceeds 50—60%. High percentage of relative humidity heralds rain. For this reason, the Meteorological office keeps a continuous record of the change of relative humidity of the atmosphere and announces it through newspaper and radio. In certain manufacturing processes, like cotton fibres, damp climate is conducive. For this reason, cotton mills are generally located at damp climate. Some bacteria thrive in moist atmosphere, for which the department of Health and Hygiene keeps a note of the humid condition of the atmosphere. As a measure of safety, a pilot of an aircraft always avoids a region of humid air.

On the other hand, the atmosphere of a factory where electrical or electronic apparatus and instruments are manufactured or assembled must be kept free from humidity. In the same way, places where food articles, tobacco, wood etc. are stored, must be dry. In all these places, special arrangements are made to reduce the humidity of the air.

6.36. Hygrometers :

The instrument by which we can determine the dew point and hence the relative humidity at any time is called a hygrometer. There are many types of hygrometers of which Daniell and Regnault's hygrometer are very important.

(a) Daniell hygrometer : It is called a dew point hygrometer because with the help of this instrument, we directly determine the dew point and therefrom we measure relative humidity.

It consists of a bulb *A*, at the lower part of which there is a gilt band. This bulb is connected to another bulb *B* by a tube *C* as shown is fig 6.16(a). The bulb *B* is surrounded by a piece of muslin. The bulb *A* contains some ether and the rest of the apparatus contains

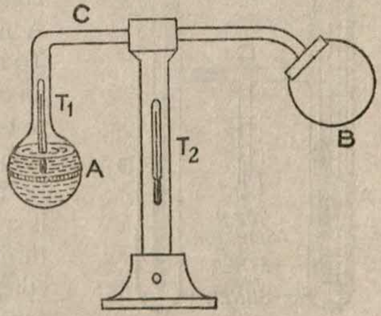


Fig. 6.16(a)

nothing but ether vapour. Presence of air will make the arrangement inefficient. A thermometer T_1 is kept inside the bulb *A*, dipping into the ether and another thermometer T_2 is mounted on the stand of the instrument to read the room temperature.

On pouring ether on muslin wrapping the bulb *B*, rapid evaporation of ether takes place causing a marked cooling of the bulb *B*. Consequently the vapour in *B* condenses, causing evaporation of ether in *A*. The bulb *A* is then slowly cooled due to evaporation of ether in it so that the temperature of gilt band falls slowly until moisture present in the surrounding air deposits on the gilt band as dew. The temperature at which dew appears is recorded from the thermometer T_1. Pouring of ether on *B* is then discontinued. Evaporation ceases and the gilt band warms up. The temperature at which dew just starts disappearing is noted from the thermometer T_1. Mean of these two temperatures gives the dew point. Next, by consulting Regnault's table, aqueous tensions at the dew point and also at the room-temperature are found out. From these, the relative humidity can be calculated.

Disadvantages :

(1) Glass being a bad conductor of heat, the thickness of the glass between the liquid and the gilt band makes some difference of temperature outside and inside the bulb.

(2) Ether cannot be stirred. Consequently, uniformity of temperature within the mass of ether cannot be ensured.

(3) It is impossible to regulate the rate of cooling. The outside air surrounding the apparatus also becomes cooled, thus introducing an error in recording the room temperature from the thermometer T_2.

(4) It is difficult to note the exact temperatures of appearance and disappearance of dew.

(b) **Regnault's hygrometer** : It consists of two thick-walled glass test-tubes *A* and *B* joined by a narrow tube *C* [Fig. 6.16(b)]. The lower portions of the test-tubes are removed and are replaced by thin silver caps. T_1 and T_2 are two thermometers. The test-tube *A* contains some ether but the test-tube *B* is empty. A narrow bent tube *D* is inserted into the ether. To a branch tube connected to the tube *C*, is joined an aspirator. The part the tube *C* connected to the test-tube *B* is completely closed so that air cannot enter into the test-tube *B*.

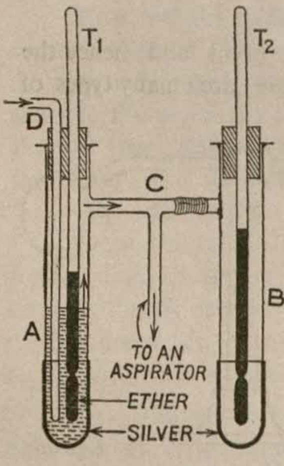


Fig. 6.16(b)

Now, as water comes out of the aspirator (which is a reservoir of water with a tap) air is drawn in through the bent tube *D*, producing bubbles in the ether. It causes rapid evaporation of ether and a consequent cooling. The silver cap which is in contact with the liquid is also cooled. Ultimately the temperature of the silver cap is so lowered that the air surrounding the cap is cooled to its dew point and the water-vapour is deposited as dews on the silver cap. This can be easily detected by noting the loss of lustre of the silver cap of *A* as compared with that of *B*. Note the temperature indicated by the thermometer T_1 at this moment. Now close the aspirator. The test-tube *A* will gradually warm up and the dews will disappear. The temperature indicated by the thermometer T_1 when the dews begin to disappear is to be noted. The average of these two temperatures will give the dew point at that time. There being no ether in the test-tube *B*, its silver cap retains its lustre always and offers a good contrast for detecting the loss of lustre of the silver cap of the tube *A*. Further, the thermometer T_2 gives the room-temperature.

Suppose the dew-point is t°C and the room-temperature as given by the thermometer T_2 is T°C. Saturation vapour pressure at t°C and T°C are to be ascertained from the saturation vapour pressure table prepared by Regnault. Suppose the pressures are f and F respectively.

$$\text{Then, relative humidity} = \frac{\text{Saturation vapour pressure at } t°\text{C}}{\text{,, \quad ,, \quad ,, \quad ,, } T°\text{C}} \times 100\% = \frac{f}{F} \times 100\%$$

Advantages :

(1) Due to bubbling of air through the ether, the entire mass is automatically stirred ensuring uniform temperature throughout which is not the case with Daniell hygrometer.

(2) Silver being a good conductor of heat, the silver cap readily assumes the temperature of ether. The thermometer, the liquid, the silver cap and the air around the cap will all be at the same temperature which is not always the case with Daniell hygrometer.

(3) Rate of cooling can be controlled by adjusting the outflow of water from the aspirator.

(4) By comparing the brightness of two silver thimbles, the appearance and the disappearance of dew can be more correctly noted.

Saturation vapour pressure table

(Regnault's table)

[Pressure is given in mm. of mercury and temperature in celsius scale]

Temp.	Pressure	Temp.	Pressure	Temp.	Pressure
0°	4·6	11°	9·8	22°	19·6
1°	4·9	12°	10·4	23°	20·6
2°	5·3	13°	11·1	24°	22·2
3°	5·7	14°	11·9	25°	23·5
4°	6·1	15°	12·7	26°	25·0
5°	6·5	16°	13·5	27°	26·5
6°	7·0	17°	14·4	28°	28·1
7°	7·5	18°	15·3	29°	29·9
8°	8·0	19°	16·3	30°	31·5
9°	8·5	20°	17·4	35°	41·8
10°	9·1	21°	18·5	40°	54·9

6.37. Mass of wet air :

Suppose, we are to find the mass of 1 litre of wet air which is at a temperature of t°C and a pressure of P cm. of mercury. Also suppose, the aqueous tension at that temperature is f cm. of mercury. Then, the pressure of dry air$=(P-f)$ cm. of Hg.

If ρ_m=density of wet air at t°C ; ρ_d=density of dry air at t°C and $(P-f)$ cm. pressure and ρ_w=density of water-vapour at t°C and f cm. pressure, then

$$\rho_m=\rho_d+\rho_w.$$

Again, we know, the density of dry air at N.T.P.$=1{\cdot}293\times10^{-3}$ gm/c.c. and the density of aqueous vapour$=0{\cdot}622\times$density of dry air$=0{\cdot}622\times1{\cdot}293\times10^{-3}$ gm/c.c. So, from gas law $\dfrac{P_1}{\rho_1T_1}=\dfrac{P_2}{\rho_2T_2}$ we get,

$$\frac{P-f}{\rho_d\times(273+t)}=\frac{76}{273\times1{\cdot}293\times10^{-3}} \quad \therefore \quad \rho_d=\frac{P-f}{76}\times\frac{273}{273+t}\times1{\cdot}293\times10^{-3}\text{ gm/c.c.}$$

Similarly, for aqueous vapour we can write that at t°C and f cm. pressure, the density of aqueous vapour is given by

$$\rho_w=\frac{f}{76}\times\frac{273}{273+t}\times0{\cdot}622\times1{\cdot}293\times10^{-3}\text{ gm/c.c.}$$

Now, $\rho_m=\rho_d\times\rho_w$

$$=\frac{P-f}{76}\times\frac{273}{273+t}\times1{\cdot}293\times10^{-3}+\frac{f}{76}\times\frac{273}{273+t}\times0{\cdot}622\times1{\cdot}293\times10^{-3}$$

$$=1{\cdot}293\times10^{-3}\times\frac{273}{273+t}\left[\frac{(P-f)+0{\cdot}622f}{76}\right]$$

$$=1{\cdot}293\times10^{-3}\times\frac{273}{273+t}\times\frac{(P-0{\cdot}378f)}{76}\text{ gm/c.c.}$$

$$=1{\cdot}293\times\frac{273}{273+t}\times\frac{(P-0{\cdot}378f)}{76}\text{ gm/litre}$$

Hence, the mass of 1 litre of moist air at t°C and P cm. of mercury pressure

$$=\text{volume}\times\text{density}=1\times1{\cdot}293\times\frac{273}{273+t}\times\frac{(P-0{\cdot}378f)}{76}\text{ gm.}$$

Examples : (1) *On a certain day, the temperature was found to be 14°C and the dew point 8°C. Saturation vapour pressure at 8°C and 14°C are respectively 8 mm. and 12 mm. of Hg. Calculate the relative humidity on that day.*

Ans. We know, relative humidity

$$=\frac{\text{Sat. vapour pressure at the dew point}}{\text{,, ,, ,, ,, ,, air temp.}}\times100\%$$

$$=\frac{\text{Sat. vapour pressure at 8°C}}{\text{,, ,, ,, ,, 14°C}}\times100\%=\frac{8}{12}\times100\%=66{\cdot}6\%$$

(2) *On a certain day, the dew point and the room temperature were 12°C and 25°C respectively. If the saturation vapour pressure at 12°C is 10·4 mm., what is the pressure of the vapour present in the air ?*

Ans. From the definition of dew point we know that water-vapour present in the air at the room temperature saturates the air at the dew point. That is, vapour pressure at room-temperature becomes the saturated vapour pressure at dew point. Since the dew point is 12°C and the S.V.P. at that temperature is 10·4 mm., the pressure of water-vapour at 25°C will be equal to 10·4 mm.

(3) *The dew point and the temperature on a certain day were respectively 12°C and 16·5°C. The saturation vapour pressure at 12°C, 16°C and 17°C are respectively 1·046 cm., 1·364 cm. and 1·442 cm. What is the relative humidity on that day ?*

Ans. S.V.P. at 16°C=1·364 cm.

„ „ 17°C=1·442 „

So, the increase of S.V.P. due to increase of 1°C temperature

$$=1{\cdot}442-1{\cdot}364=\cdot078 \text{ cm.}$$

„ „ „ „ „ „ „ increase of 0·5°C temp.

$$=\cdot078\times0{\cdot}5=\cdot0390 \text{ cm.}$$

Hence, S.V.P. at 16·5°C=1·364+·039=1·403 cm.

$$\therefore \text{ R.H.}=\frac{\text{S.V.P. at dew point}}{\text{,, ,, room temp.}}\times100\%=\frac{1{\cdot}046}{1{\cdot}403}\times100\%=74{\cdot}5\%$$

(4) *The relative humidity in a closed room at 20°C is 45%. If the temperature increases to 30°C, what will be the relative humidity ? Saturated vapour pressure at 20°C and 30°C are 17·4 and 31·5 mm. of mercury respectively.*

Ans. Since the saturated vapour pressure at 20°C is 17·4 mm. and the relative humidity at that temperature is 45%, the pressure of water-vapour in the room at 20°C$=\frac{45}{100}\times17{\cdot}4=7{\cdot}83$ mm.$=P_1$ (say).

Let the pressure exerted by the water-vapour in the room at 30°C be P_2. As unsaturated vapour obeys gas laws, we have,

$$\frac{P_1}{P_2}=\frac{T_1}{T_2} \text{ or } \frac{7{\cdot}83}{P_2}=\frac{273+20}{273+30}=\frac{293}{303} \quad \therefore \ P_2=8{\cdot}1 \text{ mm (nearly)}$$

$$\therefore \text{ R.H. at } 30^\circ=\frac{8{\cdot}1}{31{\cdot}5}\times100=25{\cdot}7\%$$

(5) *Air at 30°C and 90% humidity is drawn into an air-conditioning unit and cooled to 20°C, the relative humidity being reduced to 50%. How many grams of water-vapour must be removed by the air-conditioner from a cubic metre of air ? Neglect the change in volume of air. Given density of saturated water-vapour in air at 30°C is 30 gm/m³ and at 20°C 17 gm/m³.* [*Jt. Entrance 1972*]

Ans. As the initial humidity is 90%, we can write,

$$\frac{90}{100}=\frac{\text{mass of water vapour present in 1 cu metre of air at 30°C}}{\text{,, ,, ,, required to saturate 1 cu. metre of air at 30°C}}$$

$\therefore$ mass of water vapour present in 1 cu. metre of air at 30°C

$$=\frac{90}{100}\times30=27 \text{ gm.}$$

Final humidity being 50% we can similarly write, mass of water vapour present in 1 cu. metre of air at 20°C$=\frac{50}{100}\times 17=8{\cdot}5$ gm.

$\therefore$ Mass of water vapour removed$=27-8{\cdot}5=18{\cdot}5$ gm.

(6) *What is the dew point when at 20°C the relative humidity is 50%. The S.V.P. at 20°, 10°C and 9°C are respectively 17·5, 9·2 and 8·6 mm. of mercury.*

Ans. We know, $\text{R.H.}=\frac{\text{S.V.P. at dew point}}{\text{,, ,, } 20^\circ\text{C}}$

$$\text{or} \quad \frac{50}{100}=\frac{p}{17{\cdot}5}\ [p=\text{S.V.P. at dew point}]$$

$$\text{or} \quad p=\frac{50\times 17{\cdot}5}{100}=8{\cdot}75 \text{ mm.}$$

Due to difference of 1°C temperature around 9°C, the difference of S.V.P. $=(9{\cdot}2-8{\cdot}6)=0{\cdot}6$ mm.

$\therefore$ $(8{\cdot}75-8{\cdot}6)=0{\cdot}15$ mm. diff. of S.V.P. will be due to $\frac{0{\cdot}15}{0{\cdot}6}=0{\cdot}25^\circ$C

$\therefore$ Dew point$=9+0{\cdot}25=9{\cdot}25^\circ$C.

6.38. Condensation of water-vapour present in atmosphere :

Condensation of water-vapour present in atmosphere, under various conditions, gives rise to many natural phenomena like dews, fog, clouds etc.

Formation of dews, fog and mist :

The earth radiates heat at night and becomes cold. Although the radiated heat passes through the atmosphere, yet the atmosphere is not heated thereby. But the air in contact with the surface of earth gets cooled as the surface becomes cold. In this way, when the temperature of air reaches the dew point, a slight cooling will cause the water-vapour present in the air to condense as small drops of water on grass, leaves etc. These are called dews. In autumn, copious dews are found on grass and leaves in the early morning. The following factors are conducive to copious deposition of dews :

(i) **Clear cloudless sky** : The surface of the earth becomes cold quickly due to radiation in a clear, cloudless night. The radiant heat does not come back to the surface of the earth after reflection by the cloud. For this reason, a clear cloudless sky is helpful for copious deposition of dew.

(ii) **Absence of wind** : If there be little or no wind, a certain amount of air will stay longer in contact with a cold body. This helps the air to arrive at the dew point quickly, leading to the deposition of dew.

(iii) **Presence of water vapour in air** : If the initial humidity of air is high, a little cooling will cause deposition of dew.

(iv) **A good radiator but a bad conductor** : A good radiator but a bad conduct or quickly radiates heat and becomes cold. The air in contact with such substances can, therefore, attain dew point very quickly. It is desirable that those substance should be as near to the surface of the earth as possible. Otherwise, cold air, being heavy, will go down and hot air from above will occupy the position. This will produce an air current which will prevent deposition of dew. For this reason, dews are formed on the grass and small plants and not on the leaves of large trees.

If the temperature of a wide region of atmosphere falls below the dew point, the water-vapour present in the atmosphere condenses as small droplets on dust particles, smoke particles, coal dusts etc. and remain floating in the air. This is called *fog*. Generally fog is formed if the temperature of wet ground is higher than the temperature of atmosphere. Fog is very often seen in winter season. *Mist* is another form of fog, formed over a sheet of water. Fog disappears in the noon because the higher temperature of the noon time causes evaporation of the droplets and the atmosphere becomes unsaturated.

Clouds and rains : When air containing water-vapour becomes light for various reasons, it goes high up where, due to low pressure, it expands in volume and gets cooled. Due to gradual cooling in this way, the temperature of air, at one stage, goes below the dew point and the water-vapour present in the air, condenses as small droplets of water on the suspended dust particles. The mass of floating droplets is called *cloud.* There is, therefore, no significant difference between fog and cloud, except that fog is formed at a lower level while cloud is formed at a higher level.

When the water drops of the floating cloud coalesce and form bigger drops, they fall down and if in the way, they meet any dry and warm layer of air, they again get evaporated and move up. If on the other hand, they pass through a wet layer of air, they grow bigger in size instead of being evaporated and drop down on the surface of the earth as *rain.*

The tiny water drops in a cloud always pass through a series of formation and disintegration. Sometimes, a few tiny drops coalesce to form a bigger drop, sometimes a bigger drop disintegrates into smaller droplets. Whenever a drop disintegrates, a separation of its electrical charges takes place. This is, perhaps, a reason of the presence of lightning during a thunderstorm, which is always attended with a heavy shower.

Snow and hail : During severe winter the water-vapour present in the atmosphere, solidifies and remains floating in the air or falls on the surface of the earth in the form of a drizzle. This is known as *snow-fall.* Snow-fall takes place frequently in polar regions and sometimes in hilly countries in severe winter.

During rainfall, if the drops come in contact with a very cold layer of air, the drops solidify and fall on the ground as pieces of ice. This is called *hailstorm.*

Hoar-frost : Hoar-frost is deposited instead of dew if the temperature of the slowly-cooling atmosphere reaches the freezing point of water before saturation occurs. The process of formation of hoar-frost and of snow does not consist in the freezing of dew but the direct change of state from aqueous vapour into the solid form.

Exercises

Essay type :

1. 'An enormous force is developed when water solidifies'—illustrate it with a suitable experiment. What relation does it bear with the crack found in rocks ?

2. What is the effect of pressure on melting point ? Explain with illustration.

3. What are the factors which determine the rate of evaporation ? What is the difference between a gas and a vapour ? *[H. S. Exam. 1980]*

4. What are evaporation and boiling ? What are the factors which influence the rate of evaporation ?

5. What is boiling point ? What is its relation with the pressure over the surface of boiling liquid ? Explain your answer with suitable experiments. *[cf. H. S. Exam. 1982]*

6. "A liquid can be made to boil at a much lower temperature than its boiling point".—describe an experiment in support of this statement.

7. Describe an experiment in each case of the effect of (i) increased pressure (ii) reduced pressure, on the boiling point of a liquid . Mention some of the applications of each case.

8. What are the factors on which the boiling point of a liquid depends ? What are the laws of boiling ? 'At the boiling point of a liquid, the vapour pressure is equal to superincumbent pressure.' Prove it by an experiment.

9. Describe a method for determining the latent heat of fusion of ice. *[H. S. Exam 1980]*

10. Describe a method of determining the latent heat of steam. Why is steam-trap used in the method ?

11. What is a hygrometer ? What does it determine ? Describe the construction and explain the action of Regnault's hygrometer.

12. What does humidity of air mean ? Describe an apparatus which determines the humidity of air. Draw a neat sketch of the apparatus you describe.

13. Define 'dew point' and 'relative humidity' and describe a method of determining the dewpoint. *[H. S. Exam. 1979]*

14. Mention the differences between a saturated and an unsaturated vapour. Describe a simple arrangement for measuring S.V.P. at room temperature.

15. Explain with suitable diagrams how the pressure of (a) a gas and (b) a vapour change with the change (i) of volume at constant temperature and (ii) of temperature at constant volume.

Short answer type :

16. What do you mean by fusion and solidification ? What is meant by saying that the melting point of platinum is 1755°C ? Are melting point and freezing point of a substance always identical ?

17. Explain 'regelation'. Explain how a loaded copper wire cuts through a piece of ice without dividing the piece into two halves. State with reasons what will happen if (i) the copper wire be replaced by an ordinary thread and (ii) temperature of air goes below 0°C. *[H. S. Exam. 1981]*

18. Answer the following questions :—(a) Does every substance exhibit sharp melting point ? (b) In cold countries, hot water pipes are found to burst more frequently than cold water pipes. Why ? (c) Why do cracks occur in rocks ? (d) When common salt is mixed with ice, the temperature of the mixture goes much below 0°C. Why ? (e) What is eutectic temperature ?

19. What are 'sensible heat' and 'latent heat' ? What do you mean by saying that the latent heat of fusion of ice is 80 cal/gm. ? *[H. S. Exam. 1981]*

20. What are boiling and evaporation ? What are their differences ? *[H. S. Exam. 1980]*

21. Answer the following questions :—(a) Why do we feel comfort under an electric fan in summer ? (b) Water kept in an earthen-ware vessel is cold but water kept in a brass vessel is not so cold. Why ? (c) Why is it unwise to sit with wet clothes on ? (d) Why are *Khaus-Khaus* used in doors and windows in summer ? (e) Why do two pieces of ice combine when pressed ? (f) Why is ice at 0°C a better cooling agent than water at 0°C ? (g) Why does steam at 100°C produce more severe burns than water at 100°C ? (h) In determining the boiling point of a liquid, the bulb of a thermometer is not kept immersed in the liquid. Why ? (i) Why is the time required to boil off a certain quantity of water longer than that required to bring the same quantity of water to its boiling point ? (j) Why are some pieces of broken glass put in a vessel while boiling water in the vessel ? (k) What is triple point ? (l) How is it that ice-cream appears cooler to mouth than water (cooled with ice) at 0°C ?

22. (a) A small quantity of water is placed in a hole in a large block of ice. Will the water freeze ? Give reasons for your answer. [*H. S. Exam. 1980*]

(b) When can heat be supplied to a substance without causing a change of temperature ? How can this heat be recovered ?

(c) State why electric fuse wires are usually made of alloys ?

23. Answer the following questions : (a) A flask containing some water is heated until the water boils. After sometime, the mouth of the flask is closed and the flask is immediately removed from the source of heat. On, now, dipping the flask in cold water, it is found that the water in the flask starts boiling again. Why does it happen so ? (b) Why is the process of cooking food different in the top of a mountain from that on the plain ? (c) What is the effect of height on boiling point ? (d) While cooking rice, if a lid is placed on the pan, rice is well-cooked. Why ?

24. A mixture of crushed ice and water is poured into a vessel containing a thermometer. What will be the effect on the reading of the thermometer (a) when more water is poured in (b) when more ice is put in (c) when a handful of salt is stirred in ? Give reasons in each case of your answer.

25. A beaker of water is kept on a table in a room. Can this water be made to boil by passing steam through it at atmospheric pressure ? Give reasons.

26. What is latent heat of vaporisation ? What do you understand by the statement that the latent heat of steam is 537 cal/gm ? What is the difference between water at 100°C and steam at 100°C in respect of the heat content ?

27. 'An astronaut poured water from his thermosflask at 20°C in a beaker on the surface of the moon. Describe in brief, the condition of water.

[**Hints** : There is no atmosphere on the moon. As soon as water is poured, some will be immediately evaporated. The latent heat required for this will be supplied by the rest of the water, which will be so cooled that it will get solidified at once.]

28. Explain the terms :—(i) Dew point and (ii) Relative humidity.

29. Define 'Dew point'. Of what use it is when it has been determined ? What becomes the condition of the atmosphere when its temperature is equal to its dew point ?

What effect will be produced on (i) the dew point and (ii) the relative humidity, when the temperature of a room is increased ?

30. Is it possible for the dew point to go below 0°C ? State your reasons.

[**Hints** : No ; at a temperature below 0°C, dew will be solidified.]

31. Answer the following questions :—(a) Under what condition, will the room temperature be equal to the dew point ? (b) When is dew point not found ? (c) Why is hot but wet air more uncomfortable than hotter but dry air ? (d) Delhi is more comfortable than Puri on an equally hot day. Why ? (e) Dews are found to deposit sometimes on the outer surface and sometimes on the inner surface of a glass pane. Under what conditions such formation of dews may occur ?

32. Answer the following questions :—(i) Wet clothes usually dry up quicker in winter than in rainy season although the temperature in winter is much less. Why ? (ii) Why do dews accumulate on the outer side of a glass vessel when ice water is poured in it ? (iii) The temperature of two rooms is 24°C ; but the relative humidity of one is 80% while that of the other is 60%. Which room will appear more comfortable ? (iv) When air is blown by mouth on a piece of glass in winter morning the glass becomes foggy. Why ?

33. What effect will be produced on (i) the dew point and (ii) the relative humidity, if some water is sprinkled in the room ? [*H. S. Exam. 1979*]

34. Of relative humidity and absolute humidity, which one is necessary for weather forecasting ? [*H. S. Exam. 1980*]

35. A vessel contains some air and saturated vapour. If the volume of the vessel be reduced, will there be any change of pressure ?

36. What are dews ? How are they formed ? Why do dews form more on some susbtances than others ? What are the factors responsible for copious deposition of dews ?

37. Air is blown through some ether in a copper can. Explain (a) the effect on the temperature of the ether (b) the formation of mist on the outside of the can. On another day, repetition of the experiment produced mist at a lower temperature. Explain the reason.

[**Hints** : Mist appears at a lower temperature because relative humidity was low and air contained lesser amount of water-vapour that day.]

38. A solid material is supplied with heat at a constant rate. The temperature of the material is changing with the heat input as shown in fig. 6.17. Study the graph carefully and answer the following questions :

(i) What do the horizontal region AB and CD represent ? (ii) If $CD=2AB$, what do you infer ? (iii) What does the slope DE represent ? (iv) The slope of $OA>$the slope of BC. What does this indicate ? [*I.I.T. 1980*]

[**Hints** : (i) Change of state from solid to liquid and from liquid to vapour. (ii) Heat required for change from liquid to vapour double the heat required for change from solid to liquid. (iii) The rate of increase of temperature of vapour. (iv) Rate of increase of temperature in solid state greater than that at the vapour state.]

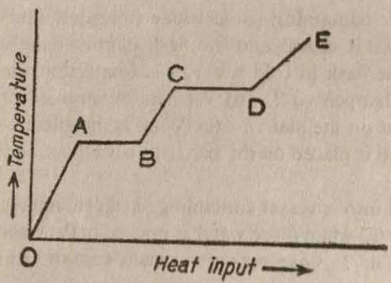


Fig. 6.17

39. In the graph shown in fig. 6.17, a point is taken on the horizontal line AB as well as on the horizontal line CD. What are the states of the material at those two points ?

Objective type :

40. Put a tick mark against the correct answer :—

(i) Does 1 gm. of water at 0°C contain more heat than 1 gm. of ice at 0°C ?
[Ans. (a) yes (b) no]

(ii) Does the heat absorbed or given out by a body during change of state indicate its presence by producing a change of temperature ?
[Ans. (a) yes (b) no]

(iii) Will the boiling point of a liquid be higher at the top of a mountain than at the plains ?
[Ans. (a) yes (b) no]

(iv) Can a gas be liquefied by exerting pressure if its temperature is above the critical temperature ?
[Ans. (a) yes (b) no]

(v) Can the dew point ever go below 0°C ?
[Ans. (a) yes (b) no]

(vi) Two equally hot rooms have different relative humidity ? Will the room with higher relative humidity appear more comfortable than the other ?
[Ans. (a) yes (b) no]

(vii) Will the mass of water-vapour present in the atmosphere saturate it at its dew point ?
[Ans. (a) yes (b) no]

(viii) Does the vapour pressure at room temperature become the saturated vapour pressure at the dew point ?
[Ans. (a) yes (b) no]

(ix) Do cloudy nights help better deposition of dews than cloudless night ?
[Ans. (a) yes (b) no]

Numerical problems :

41. What will be the result of mixing 2 gm. of ice with 4 gm. of water at 45°C ? Latent heat of ice=80 cal./gm. [Ans. All ice will melt and the final temperature=3·3°C]

42. A vessel of water equivalent 5 gm contains 55 gm of water at 20°C. 25 gm of ice at 0°C is dropped into it. What will be the composition of the contents of the calorimeter and the temperature of the final mixture ? Latent heat of ice=80 cal/gm.

[*H. S. Exam. 1982*] [Ans. 70 gm water and 10 gm ice at 0°C]

43. 25 gm of ice at 0°C is mixed with 120 gm of water at 10°C. How much ice will melt ? Latent heat of ice=80 cal per gm. [Ans. 15 gm.]

44. In trying to evaporate 20 gm of water at 0°C quickly, some evaporated and the rest was frozen into ice. If no heat is supplied from external source, how much water evaporated and how much was converted into ice ? Latent heat of vaporisation of water=600 cal/gm and that of fusion of ice=80 cal/gm. [Ans. 18 gm ; 2 gm (nearly)]

[**Hints** : $80m=(20-m)\times 600$]

45. The density of ice at 0°C is 0·916 gm/c.c. and that of water is 1 gm/c.c. A piece of metal weighing 600 gm. is raised to a temperature of 80°C and placed in a mixture of ice and water melting some of the former. The volume of the mixture is found to decrease by 8·4 c.c.. How much ice is melted and what is the specific heat of the metal ? [Ans. 91·6 gm. ; 0·152]

46. 100 gm. of copper nails are heated to 100°C and are then dropped into a calorimeter whose mass is 100 gm. The calorimeter contains 40 gm. of a mixture of ice and water. If the final temperature is 10°C, find the mass of ice in the mixture. Sp. heat of copper=0·09 and latent heat of fusion of ice=80 cal/gm. [*I.I.T. 1962*] [Ans. 4 gm.]

47. Steam coming out of boiling water is passed through a mixture of water and ice for some time. The mass of the mixture is found to increase by 2 gm. How much ice melted ? Latent heat of steam=536 cal/gm and of ice=80 cal/gm. [Ans. 15·9 gm (nearly)]

48. 5 gm. of water at 30°C and 5 gm. of ice at −20°C are mixed together in a calorimeter. Find the final temperature of the mixture. Water equivalent of the calorimeter is negligible. Sp. heat of ice=0·5 ; Latent heat of ice=80 cal/gm. [*I.I.T. 1977*] [Ans. 0°C]

49. An aluminium container of mass 100 gm contains 200 gm of ice at −20°C. Heat is added to the system at the rate of 100 calories per second. What is the temperature of the system after 4 minutes ? Sp. heat of aluminium=0·2 ; Sp. heat of ice=0·5 ; L=80. cal/gm. [*I.I.T. 1973*] [Ans. 25·5°C]

[**Hints** : Heat supplied $=4\times 60\times 100=24{,}000$ cal.
Heat absorbed by container $=100\times 0{\cdot}2\times (t+20)$ cal.
" " " ice $=(200\times {\cdot}5\times 20+200\times 80+200\times t)$ cal.]

50. If 69,600 calories of heat are withdrawn from 100 gm. of steam at 100°C, what will be the result ? Latent heat of steam=540 cal/gm ; latent heat of ice=80 cal/gm. [Ans. 70 gm. of ice mixed with 30 gm. of water at 0°C]

51. After allowing a current of steam at 100°C to pass over a piece of ice at 0°C, the mass of water collected is 225 gm. On weighing the piece of ice, its weight is found to decrease from 850 gm. to 650 gm. Calculate the latent heat of steam. [Ans. 540 cal/gm.]

52. When a piece of metal weighing 48·3 gm at 10·7°C was immersed in a current of steam at 100°C, 0·762 gm of steam was found to condense. Calculate the specific heat of the metal. Assume latent heat of vaporisation of water as 540 cal/gm. [*I.I.T 1963*] [Ans. 0·095]

53. In an industrial process 10 kg of water per hour is to be heated from 20°C to 80°C. To do this, steam at 150°C is passed from a boiler into a copper coil immersed in water. The steam condenses in the coil and is returned to the boiler at 90°C. How many kg of steam are required per hour ? (Sp. heat of steam=1 cal/gm/°C. Latent heat of steam=540 cal/gm.) [*I.I.T. 1972*] [Ans. 1 kg]

[**Hints** : Heat absorbed by water per hour $=10\times (80-20)$ kg-cal.
" given out by steam $=m\times 1\times (150-100)+m\times 540+m\times (100-90)$ kg-cal.
where m=mass of steam in kg.]

54. 7·5 gm. of copper at 27°C were dropped into liquid oxygen at its boiling point (−183°C) and the oxygen evaporated occupied 1·89 litres at 20°C and 750 mm. pressure. Calculate the latent heat of vaporisation of oxygen. Sp. heat of copper=0·08 ; Density of oxygen at N.T.P.=1·429 gm/litre. [Ans. 50·74 cal/gm]

55. In a closed vessel there is some water mixed with water vapour at 27°C. Total pressure of the mixture is $1{\cdot}07\times 10^6$ dynes/cm^2. At this temperature, vapour is just saturated. What will be the pressure in the vessel due to air only at 40°C ? What will be the total pressure in the vessel when the temperature is reduced to 17°C ? S.V.P. at 17°C and 27°C are respectively $1{\cdot}9\times 10^4$ and $3{\cdot}7\times 10^4$ dynes/cm^2. [Ans. $1{\cdot}078\times 10^6$; $1{\cdot}018\times 10^6$ dynes/cm^2]

56. A capillary tube of uniform bore has some air entrapped by a water index. When the atmospheric pressure is 76·25 cm. and the temperature is 20°C, the air-column is 15·6 cm. long. With the tube immersed in water at 50°C, it was 19·1 cm. long, the atmospheric pressure remaining the same. If the S.V.P. of water at 20°C is 17·5 mm., deduce its value at 50°C.

[Ans. 9·17 cm. of Hg]

57. Some ether is gradually introduced into the space above the mercury in a simple barometer tube until a little remains above the surface. The height of the mercury falls from 76 cm. to 36 cm. What is the S.V.P. of ether at room temperature (a) in cm. of Hg (b) in dynes/cm^2. Density of mercury=13·6 gm/c.c. [Ans. (i) 40 cm (ii) $5{\cdot}34\times10^5$ dynes/cm^2]

58. On a certain day, when the temperature of the air was 30°C, the dew point was found to be 15°C. The saturation vapour pressure at those temperatures were respectively 31·5 mm. and 12·7 mm. What was the relative humidity at that time ? [Ans. 68·8%]

59. If the dew point were 20°*C* instead of 15°C in the above problem, will the relative humidity increase or decrease ?

60. The dew point on a particular day was 15°C while the temperature of air was 31°C. If the saturation vapour pressure at 15°C, is 12·3 mm., what was the pressure of the vapour present in the air ? [Ans. 12·3 mm.]

61. On a certain day, the temperature of air is 18·5°C and the dew point is 12°C . Find the relative humidity. The aqueous tensions at 18°C, 19°C and 12°C are 15·16, 15·86 and 10·46 mm. of mercury respectively. [*H. S.* (*Comp*) *1962*] [Ans. 67·4%]

62. The temperature in an enclosure is 17°C and the relative humidity is 50% . If the temperature rises to 27°C, what will be the relative humidity ? The aqueous tensions at 17°C and 27°C are respectively 14·4 and 26·5 mm. of Hg. [Ans. 29·02%]

[**Hints** : See example 4, page 469]

63. Calculate the mass of 1 litre of moist air at 27°C when the barometer reads 753·6 mm of Hg and the dew point is 16·1°C. S.V.P. of water at 16·1°C is 13·6 mm of Hg, density of air at N.T.P.=0·001293 gm./c.c. ; density of saturated water vapour at N.T.P.=0·000808 gm/c.c.

[*I.I.T. 1977*] [Ans. 1·16 gm (nearly)]

[**Hints** : See art. 6.37]

64. What fraction of water vapour present in the air will condense if the air temperature falls to 5°C from 20°C, the initial 60% humidity remaining the same ? S.V.P. at 20°C=17·5 mm. and at 5°C=6·5 mm. [Ans. 0·381]

Harder problems :

65. 25 gm of ice at 0°C are put in 26 gm of water at 70°C kept in a vessel whose thermal capacity is 4 cal. What will be the final temperature of the mixture ? [Ans. 1·8°C]

66. An earthen pitcher loses 1 gm of water per minute due to evaporation. If the water equivalent of the pitcher is 0·5 Kg and the pitcher contains 9·5 Kg of water, calculate the time required for the water in the pitcher to cool to 28°C from its original temperature of 30°C. Neglect radiation effects. Average latent heat of vaporisation of water in this range of temperature=580 cal/gm. [*I.I.T. 1970*] [Ans. 34·4 minutes]

67. 2 gm of steam at 100°C is allowed to pass through 55 gm of water at 10°C, kept in a copper calorimeter of mass 50 gm. The final temperature of the mixture is 30°C. Find the latent heat of vaporisation of water. Sp. heat of copper=0·1. [*Jt. Entrance 1972*] [Ans. 530 cal/gm]

68. A copper calorimeter weighing 100 gm contains 150 gm of water at 30°C. Pieces of ice which have not been dried are dropped in and the final temperature after stirring is 5°C. The weight of the calorimeter and its contents is then 300 gm. How much water was put in with the pieces of ice ? Latent heat of ice=80 cal/gm and sp. heat of copper=0·1. [Ans. 3·125 gm]

[**Hints** : Mass of ice+water=300−(100+150)=50 gm. Let m=mass of ice. ∴ mass of water=$(50-m)$gm. Now, heat lost by water+cal=$100\times0{\cdot}1\times25+150\times25$; Heat gained by ice=$80m+5m=85m$. Heat gained by water sticking with ice=$(50-m)\times5$ cal.]

69. How should 1 kg of water at 5°C be so divided that one part of it when turned into ice at 0°C would by this change of state, gives out a quantity of heat just sufficient to vaporise the other part ? [*Jt. Entrance 1980*] [Ans. 881·96 gm ; 118·04 gm]

70. A calorimeter of water-equivalent 120 gm contains 800 gm of water at 20°C. 300 gm of ice at 0°C are dropped into the water. How much ice will remain unmelted ? How much steam at 100°C should be passed into it to raise its temperature to 20°C ? Latent heat of ice 80 cal/gm and latent heat of steam=540 cal/gm. [Ans. 70 gm of ice ; 48·4 gm of steam]

71. 8 gm of ice at 0°C are dropped into a mixture of oil and water at 20°C and are just melted in cooling the mixture to 0°C. If the sp. heat of the oil is 0·5 and the total weight of the liquid at the end of the experiment is 60 gm, find the quantities of oil and water in the original mixture. Latent heat of fusion of ice=80 cal/gm. [Ans. oil=40 gm., water=12 gm]

72. A sample of steam initially at 110°C and a piece of ice originally at −15°C come to thermal equilibrium with water at a temperature of 40°C at normal atmospheric pressure. Find the ratio of the mass of ice to the mass of steam used. Latent heat of steam and ice are 540 and 80 cal/gm respectively and specific heats of steam and ice are 0·53 and 0·48 respectively.

[Ans. 4·73]

73. The temperature on a certain day was 20°C and the relative humidity of air 60%. If the temperature fell to 5°C, what fraction of water-vapour condensed into water ? S.V.P. at 27°C and 5°C are respectively 17·5 mm and 6·5 mm of mercury. [*Jt. Entrance 1976*] [Ans. 0·31]

74. The measurement of a closed room is 5 metre × 5 metre × 4 metre. Temperature of the room is 27°C. If the relative humidity of air is reduced from 90% to 40% what volume of water-vapour will condense into water ? S.V.P. at 27°C=26·7 mm. of mercury.

[Ans. $1{\cdot}59\times10^6$ c.c.]

[**Hints :** Volume of the room=100 cu. metre=10^8 c.c. ; $T_1=273+27=300$°K

Pressure of water vapour at 90% humidity $p_1=26{\cdot}7\times\frac{9}{10}$ mm and that at 40% humidity $p_2=26{\cdot}7\times\frac{4}{10}$ mm. Decrease of pressure$=(p_1-p_2)=26{\cdot}7\times\frac{5}{10}=13{\cdot}35$ mm$=1{\cdot}335$ cm. Normal pressure $(P_0)=76$ cm and temperature $(T_0)=273$°K

$$\text{Volume of water-vapour condensed}=\frac{(p_1-p_2)\times V}{T_1}\times\frac{T_0}{P_0}=\frac{1{\cdot}335}{300}\times10^6\times\frac{273}{76}=15{\cdot}9\times10^6\text{ c.c.}]$$

75. In a jar there is some gas along with a few drops of water at T°K. The pressure of the jar is 830 mm of mercury. The temperature of the jar is reduced by 1%. S.V.P. at the two temperatures being 30 mm and 25 mm of mercury respectively, find the new pressure of the jar.

[Ans. 817 mm of Hg]

76. A vessel of water is put in a dry sealed room of volume 76m^3 at a temperature of 17°C. The S.V.P. of water at 17°C is 15mm of mercury. How much water will evaporate before the water is in equilibrium ? Assume that the vapour obeys perfect gas laws.

[*I.I.T. 1978*] [Ans. 1·14 litre (nearly)]

77. A closed vessel contains air, a saturated water-vapour, and an excess of water. The total pressure in the vessel is 76 cm of Hg when the temperature is 25°C. What will it be when the temperature has been raised to 100°C ? (S.V.P. of water at 25°C=2·4 cm of Hg)

[Ans. 168·1 cm. of Hg]

78. A mixture of air and saturated alcohol vapour in the presence of liquid alcohol exerts a pressure of 12·8 cm of Hg at 20°C. When the mixture is heated at constant volume to the boiling point of alcohol (*i.e.* 78°C) at standard pressure the vapour remaining saturated, the pressure becomes 86 cm. of Hg. Find the S.V.P. of alcohol at 20°C. [Ans. 44·5 mm of Hg]

79. The temperature of air in a closed room is observed to be 15°C and the dew-point is 8°C. If the temperature falls to 10°C, how will the dew-point be modified ? S.V.P. of water at 7°C=7·49 mm and at 8°C=8·02 mm Hg. [Ans. 0·25°C less]

80. 300 gm of water are evaporated in a room containing 60 cu. metre of dry air at 27°C and 76 cm of Hg. What will be the relative humidity of air in the room. S.V.P. at 27°C=30·8 mm of Hg. [Ans. 16·9% (nearly)]

7

MECHANICAL EQUIVALENT OF HEAT

7.1. Heat is a form of energy :

It is our common experience that when work is done by friction, heat is produced. The different parts of a big machine become heated due to friction unless they are lubricated with a suitable lubricating oil. A fire may be produced by rubbing two pieces of stone. Many such examples of heat produced by friction may be cited.

When pressure is applied on an enclosed gas, some work is done, which, in its turn, produces heat. You may have noticed it while inflating the tyre of a cycle or a football bladder.

When electric current flows through an electric lamp, the lamp produces light as well as heat. This shows that electric energy can be transformed into light energy or heat energy.

We frequently come across the examples of kinetic energy being transformed into heat energy. When a man runs a certain distance, his body becomes heated. When a blacksmith hammers a piece of iron, the piece becomes very hot. All these are illustrations of kinetic energy being transformed into heat energy.

From the examples mentioned above, heat may be supposed to be a form of energy as other forms of energy can be converted into heat and vice versa.

Caloric theory : Long ago, at the beginning of the eighteenth centrury, the foremost thinkers of the day considered heat as a weightless fluid called *caloric*. They were of opinion that the flow of caloric from one body to another meant flow of heat. The body in which caloric enters gets heated while the body from which it comes out becomes cold. As exchange of heat does not make any difference in the weights of the bodies concerned, the early thinkers considered caloric as weightless. The caloric theory gained ground and occupied a prominent place in the study of heat for some time to come.

Count Rumford : In 1798 Count Rumford observed certain facts for which he could not find the caloric theory very satisfactory. Count Rumford was an engineer and was in charge of arsenal at Munich. While preparing a cannon, he noticed that a great deal of heat was produced during the boring of a big brass piece by a drill. According to the caloric theory, this was caused by caloric being flown from air into the metal. As a result, the surrounding air should be cooled down. So Rumford carried out a test experiment in which a lump of metal was immersed in water and drilled. Soon he found that the water was so much heated that it started boiling. Clearly, caloric could not have come from water, since water had gained caloric. So, the caloric theory failed to explain it properly. Count Rumford further observed that more heat was produced if the boring was done by a blunt drill. Further, more the drilling operation continued, the more heat was produced. But according to the caloric theory, a body containing a definite amount of caloric, cannot be an inexhaustible store of heat.

From all these experiments, Rumford expressed doubts about the validity of the caloric theory and was led to believe that heat had some relation with motion.

Sir Humphry Davy : After a couple of years, Sir Humphry Davy, performing another experiment, supported the opinion expressed by Rumford. Rubbing two pieces of ice against each other, Devy found that the heat produced was sufficient to melt the pieces. Performing the experiment in a vacuum, he got the same result. He became sure that heat did not flow to the pieces of ice from the surroundings because there can be no surrounding air in a vacuum. Further, it was known that water had greater thermal capacity than an equal amount of ice. Where did this excess heat come from in the water obtained by the melting of ice? All these questions remained unanswered by the caloric theory.

From some other experimental results, Davy finally came to the conclusion that friction increased the motion of atoms and molecules of a substance and this increased motion was transformed into heat of the substance. In other words, *heat is a form of energy.*

James Prescott Joule and principle of conservation of energy : Generally speaking, the main body of scientific opinion at that time was hostile to the new ideas propounded by Rumford and Davy, but eventually, a series of experiments carried out by James Prescott Joule during the period 1840—50, settled the problem of the nature of heat beyond all reasonable doubt. He argued that if heat was a form of energy it should obey the principle of conservation of energy which says that no energy is lost when one form of energy is converted into the other. So, when mechanical energy is converted into heat energy, each will be proportional to the other. In other words, a definite amount of work is needed to obtain a definite amount of heat. In 1847, Joule succeeded in devising an experiment which showed that there was an exact equivalence between the mechanical work done and the heat produced. After this, the caloric theory was completely rejected and the idea that 'heat is a mode of motion' was universally accepted.

7.2. Internal energy of a body :

From kinetic theory of gases we know that the average kinetic energy of the gas molecules is proportional to its temperature. Besides kinetic energy, the constituent atoms of the molecules have vibrational energy. Moreover, there are mutual attraction and repulsion between the molecules of a substance which endow the molecules with some potential energy. The sum of the kinetic and potential energies of the molecules of a substance *i.e.* the sum of the mechanical energies of the molecules is called the *internal energy of* the body.

For a long time, heat energy was regarded as the total internal energy of a body. At present, according to kinetic theory, heat is regarded as that part of the internal energy which flows from one place to another due to a difference of temperature.

We cannot measure the total internal energy of a body; but that does not create any difficulty because for all practical purposes we require to know the change of internal energy and not the actual internal energy possessed by a body. There are, however, methods for measuring the change of internal energy accurately.

Internal energy is generally denoted by the symbol U. If a mechanical work W (either in the form of compression or expansion) is done on a system whose internal energy is thereby changed from U_1 to U_2, then $W=U_2-U_1$. Internal energy of a system changes during certain thermal processes *i.e.* during change of state, say from liquid to vapour.

7.3. Work done by a gas :

Consider some gas enclosed in a cylinder fitted with a frictionless piston [Fig. 7.1]. The pressure on the piston $=p$. If the piston has a sectional area α, then the force on the piston $P=p.\alpha$. If we allow the piston to move outwards a distance dl, the gas will expand and its pressure will fall. But if the distance moved by the piston be very short, the change in pressure will be very small so that we may consider the pressure to remain constant. The force P is then constant and the work done $dW=$ force × distance $=p.\alpha.dl$.

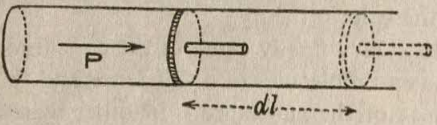


Fig. 7.1

The product $\alpha.dl$. is the increase in volume dV of the gas, so that $dW=p.dV$.

If the volume of the gas changes from V_1 to V_2 under a constant pressure,

then $W=\int_{V_1}^{V_2} p.dV=p.(V_2-V_1)$ *i.e.* work done = pressure × volume change.

If the pressure p is in dynes/cm^2 and the area α is in cm^2, the force P is in dynes and the work done is in ergs.

[Note : If the gas expands isothermally (*i.e.* without change of temperature) we have $p=\frac{R_0T}{V}$ (for 1 mole of the gas)

$$\therefore \quad W=\int_{V_1}^{V_2} \frac{R_0T}{V} \, . \, dV=R_0T\Big[\log_e V\Big]_{V_1}^{V_2}=R_0T.\log_e \frac{V_2}{V_1}\Big]$$

Examples : (1) *100 calories of heat are supplied to a gas enclosed in a cylinder fitted with a piston. The gas expands against a constant pressure of 76 cm. Hg. Calculate the change in volume of the gas, assuming that its internal energy remains unchanged. Density of Hg=13·6 gm./c.c. and J=4·2×10⁷ erg/cal.*

Ans. Work done by the gas, $W=J.H.=4{\cdot}2\times10^7\times100$ erg.

Pressure $p=76$ cm. Hg $=76\times13{\cdot}6\times980=1{\cdot}013\times10^6$ dynes/cm^2

Now, volume change $=\frac{\text{work done}}{\text{pressure}}=\frac{4{\cdot}2\times10^7\times100}{1{\cdot}013\times10^6}=4{\cdot}14\times10^3$ c.c. (nearly)

(2) *Five moles of an ideal gas is compressed isothermally from an initial volume of 5 litres to a final volume of 1 litre. Calculate the work done in the process, if the temperature of the gas is 27°C. Given $R_0=8{\cdot}31$ j/mole °K.* [*I.I.T. 1972*]

Ans. If the volume of a gas changes isothermally, we have for 1 mole of the gas, the work done $W=R_0T\log_e\frac{V_2}{V_1}=2{\cdot}3026\ R_0T\log_{10}\frac{V_2}{V_1}$

$\therefore$ For n moles of the gas, the work done $W=2{\cdot}3026.nR_0T\log_{10}\frac{V_2}{V_1}$

Here, $n=5$; $R_0=8{\cdot}31$ j/mole °K, $T=27+273=300°$R ; $V_2=1$ litre and $V_1=5$ litre.

$$\therefore\ W=2{\cdot}3026\times5\times8{\cdot}31\times300\times\log_{10}(\tfrac{1}{5})$$
$$=2{\cdot}3026\times5\times8{\cdot}31\times300\times(-0{\cdot}6990)$$
$$=-2{\cdot}006\times10^4\ J$$

[**Note** : The negative sign shows that work is done on the system.]

7.4. Internal energy and latent heat :

When 1 gm of water is converted into steam at 100°C and 76 cm Hg pressure, the volume occupied by the steam is found to be 1672 c.c. 1 gm of water, having volume 1 c.c., thus expands by 1671c.c. when turned into steam. During expansion it evidently does work against atmospheric pressure. The total latent heat of vaporisation of water is the sum of (i) the heat equivalent of the above work supplied to water to make it overcome atmospheric pressure as it evaporates and (ii) heat equivalent of the work done in separating the molecules, against their mutual cohesive force. The former is known as *external latent heat*' and the latter '*internal latent heat*'. The internal latent heat is a measure of the increase in potential energy of the molecules of water in the gaseous state over that in the liquid state at the same temperature.

The work done by 1 gm of water in expanding from liquid to vapour state =atmospheric pressure×increase in volume$=p\times dV=76\times13{\cdot}6\times980\times1671$ erg.

$$\therefore\ \text{External latent heat}=\frac{76\times13{\cdot}6\times980\times1671}{J}\text{cal}$$

$$=\frac{76\times13{\cdot}6\times980\times1671}{4{\cdot}2\times10^7}=40{\cdot}3\text{ cal.}$$

This result indicates that the external latent heat is much less than the internal part. Since the total latent heat of vaporisation is nearly 540 cal/gm, the internal latent heat, is therefore$=540-40{\cdot}3=499{\cdot}7$ cal/gm.

As 1 mole of water weighs 18 gm and Avagadro number being $6{\cdot}03\times10^{23}$, the number of molecules in 1 mole of water is $6{\cdot}03\times10^{23}$ which weigh 18 gm. and possess total potential energy$=18\times499{\cdot}7\times4{\cdot}2\times10^7$ erg.

$\therefore$ Potential energy of a molecule of water when changing from liquid state at 100°C to vapour state (steam) at 100°C$=\frac{18\times499{\cdot}7\times4{\cdot}2\times10^7}{6{\cdot}03\times10^{23}}=6{\cdot}26\times10^{-13}$ erg.

7.5. First law of thermodynamics and mechanical equivalent of heat :

The section of the study of heat which deals with the relation between the mechanical energy and heat energy is called *thermodynamics*. Now-a-days the scope of thermodynamics has been widened and it includes the discussion of

7.7. The value of J : (a) Joule's experiment :

Joule's experiment is illustrated in Fig. 7.4. A paddle wheel provided with several *vanes* (P) was caused to rotate inside a copper calorimeter C containing some water. The calorimeter was also provided with several fixed vanes (V) which prevented the water from being carried round bodily.

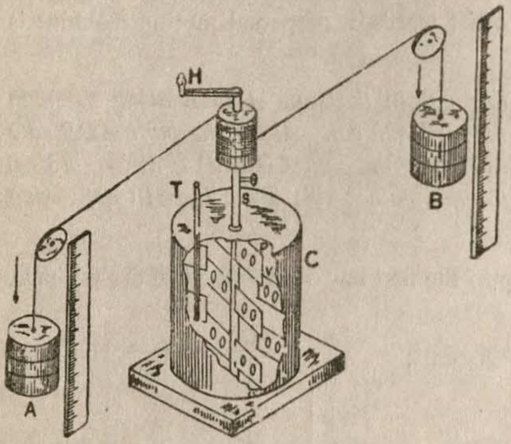


Fig. 7.4

The rotational motion of the paddle was brought about by two identical weights A and B which were allowed to fall several times through a certain fixed height. These weights were connected to strings wound round the spindle of the paddle as shown in the diagram. After each fall, the weights could be wound up again by the handle H without turning the paddle. When the paddle rotated, thorough churning of water took place with the result that considerable resistance was offered to the motion of water by the fixed vanes. The work done against the opposing force was converted into heat which produced a rise in temperature of the water and the calorimeter. A thermometer T was inserted into the calorimeter to record this rise in temperature.

Calculation : Mass of each weight A or $B=M$ gm.

Distance fallen$=h$ cm.

Work done by each weight for each fall$=M.g.h.$ erg

Total work done by the two weights$=2M.g.h.$ erg

If the number of falls be 'n', the total work done$=2n\ M.g.h.$ erg

If m gm be the mass of water, W gm the water-equivalent of the calorimeter and t°C the rise in temperature, then, the heat produced$=(m+W)t$ cal.

$$\therefore\quad J=\frac{\text{Work done}}{\text{Heat produced}}=\frac{2nM.g.h}{(m+W)t}\ \text{erg/cal.}$$

All the quantities on the right hand side of the equation being known, the value of J can be calculated.

[**Remarks** : It has been assumed that the whole potential energy of the weights has been utilised in heating the water. But actually before touching the ground the weights possess some kinetic energy for which some correction is necessary. If v the velocity acquired by each weight just before touching the ground, then the total kinetic energy of the two weights$=2\times\frac{1}{2}Mv^3=Mv^2$.

$\therefore$ the energy utilised$=2Mgh-Mv^2=M(2g.h.-v^2)$

So, the total energy utilised after n falls$=nM\ (2g.h-v^2)$

$$\therefore\quad J=\frac{nM(2g.h-v^2)}{(m+W)t}$$

Example : *In one of the above experiments of Joule, the following data were obtained :—Mass (M) of each weight=13 kg. ; height of each fall=160 cm.; no of falls (n)=20 ; mass of water (m)=6000 gm., water-equivalent of the calorimeter (W)=300 gm; rise in temperature=0·3°C. Find the value of J.*

Ans. Total work done $=2n.M.g.h.=2\times20\times13000\times980\times160$

$=8{\cdot}15\times10^{10}$ ergs.

Heat developed $=(m+W)t=(6000+300)\times0{\cdot}3=1890$ cal.

$$\therefore\quad J=\frac{\text{Work done}}{\text{Heat produced}}=\frac{8{\cdot}15\times10^{10}}{1890}=4{\cdot}31\times10^{7}\text{ ergs/cal}$$

(b) Card board tube and lead shots experiment :

This is a laboratory method for approximate determination of the value of *J*.

Take a card board tube about a metre long provided with corks at each end. Take a certain quantity of lead shots and note its initial temperature. Remove one of the corks. Holding the tube horizonal insert the lead shots into the tube. Close the mouth again with the cork and invert the tube quickly as shown in the fig. 7.5 so that the lead shots drop from one end of the tube to the other. Repeat the inversions quickly for several times. At the end of the operation, note the final temperature of the lead shots.

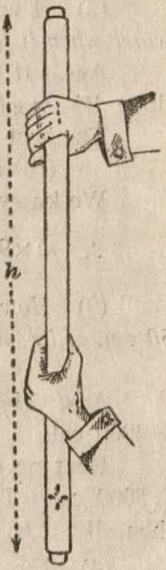


Fig. 7.5

Calculation : Let h be the average height through which the shots fall during each inversion. Let m gm be the mass of the lead shots.

Work done in each fall $=m.g.h$ ergs.

If the total number of inversions be n, then the total work done $=n.m.g.h$ ergs.

If θ_1 and θ_2 be the initial and final temperatures of the lead shots and S the sp. heat of lead, then the heat developed $=mS\ (\theta_2-\theta_1)$ calories.

$$\therefore\quad J=\frac{\text{Work done}}{\text{Heat produced}}=\frac{n.m.g.h.}{mS(\theta_2-\theta_1)}=\frac{n\,g\,h}{S(\theta_2-\theta_1)}\text{ ergs/cal.}$$

Example : *A tube of heat insulating material, closed at both ends, contains 800 gm. of lead shots. The tube is 1 metre long and is held vertically. It is then suddenly inverted so that the lead shots fall vertically to the other end. After 50 such inversions, the temperature of lead shots is found to rise by 3·89°C. Assuming that the whole heat generated remains within the lead shots, calculate the mechanical equivalent of heat. Sp. heat of lead=0·03.* [*H.S. Exam. 1978*]

Ans. Work done in each fall $=mgh=800\times980\times100=8\times98\times10^{5}$ ergs. Work done after 50 fall $=8\times98\times10^{5}\times50=4\times98\times10^{7}$ ergs.
Heat absorbed by the lead shots $=m.s.\theta=800\times0{\cdot}03\times3{\cdot}89$ cal $=24\times3{\cdot}89$ cal.

$$\therefore\quad J=\frac{\text{Work done}}{\text{Heat produced}}=\frac{4\times98\times10^{7}}{24\times3{\cdot}89}=4{\cdot}2\times10^{7}\text{ ergs/cal.}$$

Miscellaneous examples : (1) *A glass tube 1 metre long and closed at the ends, contains 200 gm. of lead shots, the remainder of the tube containing 100 gm. of water. The tube is held vertically and then inverted, causing the shots to fall entire length of the tube. How many times must this action be repeated to produce a temperature rise of 1°C?*

Ans. Suppose, the required number of fall $=n$

The energy spent $=n.m.g.h.=n\times200\times980\times100$ ergs.

$=n\times2\times98\times10^5$ ergs.

Hence, the heat produced $=\frac{W}{J}=\frac{n\times2\times98\times10^5}{4{\cdot}2\times10^7}=\frac{7n}{15}$ cal.

The heat will be absorbed by water. The rise of temperature being 1°C, the amount of heat absorbed $=1000\times1$ cal.

$$\therefore\quad \frac{7n}{15}=1000 \quad\text{or}\quad n=\frac{15{,}000}{7}=2143 \text{ (approx).}$$

(2) *A water-fall is 5000 cm. high. What will be the rise in temperature of water when it falls to the ground?* ($J=4{\cdot}2\times10^7$ *erg/cal*)

Ans. If the mass of water falling at any moment be m gm. then the work done $W=m.g.h.=m\times980\times5000$ ergs.

The heat produced $H=m.s.t.=m\times1\times t$

(t=rise in temperature and sp. heat of water $s=1$)

We know, $W=J.H.$

$$\therefore\quad m\times980\times5000=4{\cdot}2\times10^7\times m\times t \quad\therefore\quad t=\frac{980\times5000}{4{\cdot}2\times10^7}=0{\cdot}116^\circ\text{C (nearly).}$$

(3) *How much work must be done to supply the heat necessary to convert 50 gm. of ice at 0°C into water at 100°C? Latent heat of fusion of ice=80 cal/gm.*

[*H.S. Exam. 1969*]

Ans. Heat necessary to melt 50 gm. of ice at 0°C to water at 0°C $=50\times80$ $=4000$ cal.

Heat necessary to raise the temperature of water to 100°C $=50\times(100-0)$ $=5000$ cal. Total heat $=4000+5000=9000$ cal. If W be the required work, then, $W=J.H.=4{\cdot}2\times10^7\times9000$ erg $=37{\cdot}8\times10^{10}$ ergs.

(4) *The speed of a 42 kilogram meteor was retarded from 20 kilometres per minute to 5 kilometres per minute while passing through earth's atmosphere. Calculate the heat developed, in calories, due to the change in speed.* $J=4{\cdot}2\times10^7$ *ergs/cal.*

[*H. S. Exam. 1964*]

Ans. The initial kinetic energy of the meteor

$$=\tfrac{1}{2}mu^2=\tfrac{1}{2}\times42\times10^3\left(\frac{20\times10^5}{60}\right)^2 \text{ ergs.}$$

The final kinetic energy $=\frac{1}{2}mv^2=\frac{1}{2}\times42\times10^3\times\left(\frac{5\times10^5}{60}\right)^2$ ergs.

$$\therefore\quad \text{Change in kinetic energy}=\tfrac{1}{2}\times42\times10^3\times\frac{10^5\times10^5}{60\times60}\left(400-25\right)$$

$$=\frac{42\times10^{13}\times375}{2\times60\times60} \text{ ergs.}$$

This amount of energy will be converted into heat. If H cal. be the heat developed, $H=\frac{42\times10^{13}\times375}{2\times60\times60\times4{\cdot}2\times10^7}=5{\cdot}2\times10^5$ cal.

(5) *A lead bullet just melts when stopped by an obstacle. Assuming that 25% of the heat is absorbed by the obstacle, find the velocity of the bullet if its initial temperature is 27°C. Melting point of lead=327°C, Sp. heat of lead=0·03, latent heat of fusion of lead=6 cal/gm; J=4·2 joules/cal.* [*I.I.T. 1981*]

Ans. Kinetic energy of the bullet $=\frac{1}{2}mv^2$. Heat produced by the impact $=\frac{1}{2}\frac{mv^2}{J}$ cal. Since 75% of the heat is retained by the bullet, heat absorbed by the bullet $=\frac{1}{2}\frac{mv^2}{J}\times\frac{75}{100}$ cal.

Now, heat required to melt the bullet $=m\times0{\cdot}03\times(327-27)+m\times6=15m$ cal.

$\therefore\ \frac{1}{2}\frac{mv^2}{J}\times\frac{75}{100}=15\ m.$ or $v^2=\frac{15\times2\times100\times4{\cdot}2\times10^7}{75}$ $\therefore\ v=4{\cdot}1\times10^4$ cm/s.

(6) *A lead bullet at 27°C strikes a steel plate with a velocity of 700 metre/s and due to heat produced by the impact, the bullet just melts. Assuming that the heat generated has been shared by the bullet and the plate, what percentage of heat has been absorbed by the steel target? Sp. heat of lead=0·03 and its melting point=327°C; its latent heat of fusion=5·4 cal/gm.* [*Jt. Entrance 1984*]

Ans. Total heat produced $=\frac{1}{2}.\ \frac{mv^2}{J}=\frac{1}{2}\times m\times\frac{(700\times100)^2}{4{\cdot}2\times10^7}=\frac{700m}{12}$ cal.

[m=mass of the bullet]

Heat absorbed by the lead bullet $=m\times0{\cdot}03\times(327-27)+m\times5{\cdot}4=14{\cdot}4m$ cal.

$\therefore$ Percentage of total heat absorbed by the lead bullet $=\frac{14{\cdot}4\ m\times12}{700\ m}\times100=24{\cdot}7\%$

Hence, percentage of total heat absorbed by the steel plate $=(100-24{\cdot}7)=75{\cdot}3\%$.

(7) *From what height should a piece of ice fall so that the heat generated may just melt the piece? Assume that 50% of the energy is confined in the ice. Latent heat of fusion of ice=80 cal/gm.*

Ans. Let the mass of the piece of ice be m gm and the height be h cm. The work done by the piece $=m.g.h.$ erg. Since 50% of the energy is confined in the ice, the energy so confined $=\frac{50}{100}\ m.g.h.=\frac{1}{2}m.g.h.$ erg.

We know, $W=J.H.$ $\therefore$ $\frac{1}{2}m.g.h.=J.H.$ or $H=\frac{m.g.h.}{2J}$

But the heat necessary to melt the piece $=mL=80.m$ cal.

$\therefore\ \frac{mgh}{2J}=80m$ or $h=\frac{80\times2\times J}{g}=\frac{80\times2\times4{\cdot}2\times10^7}{980}$ cm $=68{\cdot}6$ km.

(8) *In an X-ray tube, 10^{18} electrons per second arrive with a speed of 2×10^6 metres/s. at a metal target of mass 200 gm and specific heat 0·12. If the mass of an electron is $9{\cdot}1\times10^{-28}$ gm and assuming 98% of the incident energy, is*

converted into heat, find how long the target will take to rise in temperature by 50°C? $J=4{\cdot}2\times10^7$ ergs/cal.

Ans. Energy spent per second $=n\times\frac{1}{2}mv^2=10^{18}\times\frac{1}{2}\times9{\cdot}1\times10^{-28}\times(2\times10^8)^2$
$=18{\cdot}2\times10^6$ erg

Effective energy converted into heat $W=\frac{98}{100}\times18{\cdot}2\times10^6$ erg.

Heat produced per second $=\frac{W}{J}=\frac{98\times18{\cdot}2\times10^6}{100\times4{\cdot}2\times10^7}$ cal.

Heat absorbed by the target $=m.s.\theta=200\times0{\cdot}12\times50=1200$ cal.

$\therefore$ Required time $=\frac{1200\times100\times4{\cdot}2\times10^7}{98\times18{\cdot}2\times10^6}$ sec $=47$ min. (nearly).

7.8. Specific heat of gases :

Generally, specific heat of a substance is defined as the ratio of the heat supplied to the unit mass of the substance to the rise in its temperature. This definition, although applicable to solids and liquids, is not applicable to gases for the following reasons.

Suppose, we take unit mass of a gas and suddenly compress it by the application of pressure. The temperature of the gass will, thereby, rise although no heat is supplied to it. In this case, the ratio of heat supplied and the rise in temperature of the gas *i.e.* the specific heat of the gas is zero. Again suppose, the unit mass of the gas is suddenly allowed to expand. The temperature of the gas will, now fall and if the fall of temperature is prevented by supplying necessary heat to the gas from some external source, the temperature of the gas may be kept unaltered. In this case, the gas undergoes no change of temperature although it receives heat from external source. Consequently its specific heat *i.e.* the ratio of heat supplied to the rise in temperature becomes infinite. So, we see that according to the above definition, the specific heat of a gas can vary from zero to infinity. This is certainly an absurd thing. The absurdity lies in the fact that when a certain mass of gas is heated, its volume and pressure change along with the change of temperature. In the cases of solids and liquids the changes of pressure and volume are negligibly small. As the behaviour of a gas is determined by its volume, pressure and temperature, due consideration should be given to its volume and pressure, in defining its specific heat. For this reason, gases are supposed to have two specific heats *viz.* (i) *Specific heat at constant volume* and (ii) *Specific heat at constant pressure.*

Definition : *Specific heat of a gas at constant volume* (C_v) *is defined as the quantity of heat required to raise 1 gm. of the gas through 1°C when its volume is kept constant.*

If M be the molecular weight of the gas, then the quantity of heat required to raise M gm. of the gas through 1°C without any change of volume $=MC_v$; it is known as the *gm-molecular specific heat or molar specific heat.* So, the molar specific heat at constant volume $C_v'=MC_v$. For example, molecular weight of hydrogen is 2 and its specific heat at constant volume $C_v=2{\cdot}41$ cal/gm. Hence, the molar specific heat of hydrogen at constant volume $C_v'=2\times2{\cdot}41=4{\cdot}82$ cal/gm. mol.

Definition : *Specific heat of a gas at constant pressure (C_p) is defined as the quantity of heat required to raise 1 gm of the gas through 1°C when its pressure is kept constant.*

As before, the molar specific heat of a gas at constant pressure $C_p' = M.C_p$; the specific heat of hydrogen at constant pressure $C_p = 3{\cdot}4$ cal/gm; hence, its molar specific heat $C_p' = 2 \times 3{\cdot}4 \times 6{\cdot}8$ cal/gm-mol.

[N.B. Specific heats for unit mass *i.e.*, 1 gm of a gas, are some times called principal specific heats.]

7.9. C_p is greater than C_v :

Keeping volume constant, if a certain mass of gas be heated, the kinetic energy of the molecules of the gas increases, thereby increasing the temperature and pressure of the gas. The gas does not perform any external work in this case because the volume of the gas remains unchanged.

But keeping pressure constant, if a certain mass of gas be heated the gas expands in volume. The heat supplied to the gas performs, in this case, two fold purposes. Firstly, it increases the temperature of the gas and secondly, it does some work in expanding the volume of the gas against the external pressure. Now, the heat required to raise the temperature of the gas through a given range *when the volume is kept constant* is evidently equal to that required for the same purpose *when the pressure is kept constant* but in the second case, *some additional heat will be necessary for doing the external work.* So, the heat required to raise the temperature of 1 gm of a gas through 1°C when the pressure is kept constant is greater than that required for the same purpose when the volume is kept unchanged *i.e.*, C_p is greater than C_v.

It can be proved in the following way that $C_p - C_v = \frac{R}{J}$ where R=gas constant for 1 gm. of the gas (ordinary gas constant) and J=Joule's heat equivalent.

Proof : Suppose we take 1 gm. of a gas. If its temperature be increased from T_1 to T_2, keeping its volume constant, the quantity of heat supplied $= C_v(T_2 - T_1)$; where C_v=specific heat of the gas at constant volume. Since no external work is done in this case, according to the first law of thermodynamics, we have, $C_v(T_2 - T_1)$=Increase in internal energy$=dU$............(i)

Now, suppose, the temperature of the gas is increased from T_1 to T_2, keeping its pressure constant. The volume of the gas will change. Suppose the volume changes from V_1 to V_2. Here, the external work done$=p(V_2 - V_1)$ where p= constant pressure of the gas. Since the increase in internal energy of the gas depends upon the rise of temperature which is the same in both the cases, the increase in internal energy in both the cases will be equal. The amount of heat supplied to the gas in the second case$=C_p(T_2 - T_1)$ where C_p=Sp. heat of the gas at constant pressure. According to the first law of thermodynamics,

$$C_p(T_2 - T_1) = dU + p(V_2 - V_1) = C_v(T_2 - T_1) + p(V_2 - V_1) \quad \text{[From eqn. (i)]}$$

$$\text{or, } C_p - C_v = \frac{p(V_2 - V_1)}{T_2 - T_1}$$

Now, from Boyle's law we have, $pV_1=RT_1$ and $pV_2=R.T_2$

$\therefore$ $C_p-C_v=R$. [where R=gas constant for 1 gm of the gas]

If all the quantities are expressed in *heat units*, $C_p-C_v=\dfrac{p(V_2-V_1)}{J(T_2-T_1)}=\dfrac{R}{J}$

[**N.B.** If 1 gram molecule of the gas is taken (instead of 1 gm), then $C_p'-C_v'=R_0$ where R_0=universal gas constant or molar gas constant.]

Further, the values of molar specific heats of a gas depend on the number of atoms in the molecules of the gas. Thus,

for monatomic gas, $C_v'=\frac{3}{2}R_0$; hence $C_p'=R_0+C_v'=\frac{5}{2}R_0$

for diatomic gas, $C_v'=\frac{5}{2}R_0$; hence $C_p'=R_0+C_v'=\frac{7}{2}R_0$

for triatomic gas, $C_v'=3R_0$; hence $C_p'=R_0+C_v'=4\ R_0$ and so on.

7.10. Importance of the ratio of two specific heats of a gas :

For a gas, $\gamma=\dfrac{\text{Sp. heat at constant pressure } (C_p)}{\text{,, ,, ,, ,, volume } (C_v)}$

For a number of reasons, this ratio is important. Firstly, from a knowledge of its value, we can form an idea of the molecular constitution of the gas. For example, its value is 1·66 for monatomic gases like helium, neon etc. For diatomic gases, its value is nearly 1·4 except for chlorine which has got γ slightly less than 1·4. For triatomic gases, γ is nearly equal to 1·3 but for polyatomic gases, γ lies between 1·3 and 1·1. So, the value of γ for a gas gives us an idea as to whether the gas is monatomic, diatomic etc. Secondly, the value of γ is necessary to determine the velocity of sound in a gas. As a matter of fact, the theoretical value obtained by Newton for the velocity of sound in a gas was found to be defective and Laplace, later on, corrected the value with the help of the ratio of the two specific heats of a gas (vide Sound). Thirdly, a knowledge of the above ratio is necessary for a fruitful study of adiabatic processes in a gas (see next articles).

Examples : (1) *Calculate* C_v' *for hydrogen given* $C_p'=6{\cdot}85$ *cal/gm. molecule, density of hydrogen at N. T. P.*$=8{\cdot}99\times10^{-2}$ *gm/litre,* $J=4{\cdot}18\times10^7$ *ergs/cal and gm-molecular weight of hydrogen=2·016.*

Ans. For 1 gm. molecule, $R_0=\dfrac{P_0V_0}{T_0}$

Now, P_0=standard pressure$=76\times13{\cdot}59\times980=1{\cdot}013\times10^6$ dynes/sq.cm.

V_0=volume of 1 gm. molecule of hydrogen at N.T.P.$=\dfrac{2{\cdot}016}{8{\cdot}99\times10^{-2}}$ litre

$=\dfrac{2{\cdot}016\times10^3}{8{\cdot}99\times10^{-2}}$ c.c. ; T_0=standard temperature=273°K.

$$\therefore\ R_0=\frac{1{\cdot}013\times10^6\times2{\cdot}016\times10^3}{8{\cdot}99\times10^{-2}\times273}=8{\cdot}32\times10^7 \text{ ergs.}$$

Now, $C'_p-C'_v=\dfrac{R_0}{J}$. or, $6{\cdot}85-C'_v=\dfrac{8{\cdot}32\times10^7}{4{\cdot}18\times10^7}=1{\cdot}99$

$\therefore$ $C'_v=6{\cdot}85-1{\cdot}99=4{\cdot}86$ cal/gm. mol.

(2) *One litre of hydrogen at 0°C and 760 mm. pressure weighs 0·0896 gm. The specific heats of hydrogen per gram at constant pressure and volume are 3·409 and 2·411 respectively. Calculate the value of J, given* $g=980$ *cm/sec*^{2} *and density of mercury=13·6 gm/c.c.*

Ans. Volume per gram of hydrogen $=\dfrac{1000}{0{\cdot}0896}$ c.c.

Normal pressure $=76\times 13{\cdot}6\times 980$ dynes/sq. cm.
,, temperature $=273$°K.

For 1 gm, the gas constant $R=\dfrac{76\times 13{\cdot}6\times 980\times 1000}{{\cdot}0896\times 273}$; Now, $C_p-C_v=\dfrac{R}{J}$

$$\therefore \quad J=\frac{R}{C_p-C_v}=\frac{76\times 13{\cdot}6\times 980\times 1000}{0{\cdot}0896\times 273\times(3{\cdot}409-2{\cdot}411)}=4{\cdot}15\times 10^7 \text{ ergs/cal.}$$

(3) *5 gm of a gas is heated from 10°C to 100°C. Calculate the* (i) *the heat supplied* (ii) *change in internal energy* (iii) *the external work done by the gas if the gas is heated* (*a*) *at constant volume* (*b*) *at constant pressure. For the gas,* $C_v=0{\cdot}153$ *cal/gm/°C and* $C_p=0{\cdot}217$ *cal/gm/°C.*

Ans. **(a) at constant volume :**

(i) The heat supplied, $Q=mC_v(\theta_2-\theta_1)$ cal. $=5\times 0{\cdot}153(100-10)=68{\cdot}85$ cal.

(ii) & (iii) As the volume remains constant, no external work is done ; hence $dW=0$.

From the 1st law of thermodynamics, $dQ=dw+dU$

$\therefore$ change in internal energy $dU=dQ=68{\cdot}85$ Cal.

(b) at constant pressure :

(i) Heat supplied $Q=mC_p(\theta_2-\theta_1)$ cal. $=5\times 0{\cdot}217(100-10)=97{\cdot}65$ cal.

(ii) As the temperature change in both cases is the same, the change in internal energy which depends on the temperature change, is the same *i.e.* $dU=68{\cdot}85$ cal.

(iii) From, $dQ=dW+dU$, we have $dW=dQ-dU=97{\cdot}65-68{\cdot}85=28{\cdot}8$ cal.

(4) *Calculate the difference in the two sp. heats of 1 gm. of helium, given that molecular weight of helium=4 and gas constant for 1 gm molecule of the gas* $R_0=8{\cdot}40\times 10^7$ *ergs/°K.*

Ans. For 1 gm. molecule of the gas, $C_p'-C_v'=\dfrac{R_0}{J}=\dfrac{8{\cdot}40\times 10^7}{4{\cdot}2\times 10^7}=2.$

Since the molecular weight (M) of helium is 4, the difference in sp. heat for 1 gm. of helium is given by, $C_p-C_v=\dfrac{C_p'-C_v'}{M}=\frac{2}{4}=0{\cdot}5$ cal/gm.

(5) *The density of a gas is* $1{\cdot}775\times 10^{-3}$ *gm/c.c. at 27°C and 76 cm Hg. pressure and its specific heat at constant pressure is* 846×10^4 *erg/gm/°K. Find the ratio of its specific heat at constant pressure to that at constant volume.*

Ans. Let us consider 1 gm of the gas. Its volume $=\dfrac{10^3}{1{\cdot}775}$ c.c.

$$\text{Now, gas constant } R=\frac{P.V}{T}=\frac{76\times13{\cdot}6\times980}{27+273}\times\frac{10^3}{1{\cdot}775}=190{\cdot}2\times10^4\ \text{erg/gm/°K}.$$

But $C_p-C_v=R$ (for 1 gm in work unit) or $846\times10^4-C_v=190{\cdot}2\times10^4$

or $C_v=(846-190{\cdot}2)\times10^4=655{\cdot}8\times10^4$ erg/gm/°K.

$$\therefore\quad \gamma=\frac{C_p}{C_v}=\frac{846\times10^4}{655{\cdot}8\times10^4}=1{\cdot}29\ \text{(nearly)}.$$

7.11. Isothermal process :

Any process where a gas undergoes changes in pressure and volume under thermal communication with the surroundings so as to keep its temperature constant, is called an isothermal process.

If a gas be suddenly compressed, some heat may be produced which may change the temperature of the gas. If, on the other hand, the gas is compressed slowly and the heat that is produced is removed from the gas at once, the temperature of the gas will remain the same although its volume will change. Such change is called isothermal change.

In the same way, if a gas is allowed to expand suddenly, a cooling will be produced due to which the temperature of the gas may fall. But if the expansion takes place slowly, and the cooling is compensated by the supply of requisite amount of heat from some external source, the temperature of the gas will remain unchanged although its volume will change. Such change is also called isothermal change. The first one is called *isothermal compression* and the second one *isothermal expansion.*

For isothermal process, it is necessary, therefore, that the gas should be in thermal communication with the surroundings so that heat may be supplied to or taken away from the gas, as the case may be. Further, the vessel containing the gas should be made of good conductor of heat and should be surrounded by a medium of high thermal capacity. Also, the process of compression or expansion should be carried out slowly so that sufficient time may be available for the supply or the extraction of heat. For this reason, *any slow thermal process is usually regarded as isothermal process.*

It may be mentioned here that the volume and pressure of a certain mass of gas under isothermal condition is related by the equation PV=constant (Boyle's law).

As temperature remains constant in an isothermal change, the internal energy of the system remains the same *i.e.* $dU=0$. So, from the first law of thermodynamics, we get $dQ=dW$. *i.e.* the heat supplied to the gas in an isothermal process is spent entirely in doing external work.

7.12. Adiabatic process :

Any process where a gas undergoes changes in pressure and volume under conditions of thermal insulation *i.e.* when no heat is allowed to enter or leave the gas, is called an adiabatic process.

If the heat produced due to sudden compression of the gas, is not allowed to leave the gas, the temperature of the gas will rise. On the other hand, if the cooling produced due to sudden expansion of the gas, is not compensated by allowing heat to enter into the gas, the temperature of the gas will fall. The first one will be called *adiabatic compression* and the second one *adiabatic expansion.*

So, in an adiabatic change, temperature does not remain constant. It is also clear that for a successful adiabatic process, the vessel containing the gas should be made of an insulator of heat and that the process should be carried out quickly giving no time for the heat to enter or leave the gas. For this reason, *any rapid thermal process is usually regarded as an adiabatic process.*

Under adiabatic condition, the volume and pressure of a gas are related by the equation, $P.V^{\gamma}=$constant, where γ is the ratio of the specific heats of the gas.

Cooling or heating of gas under adiabatic process :

From first law of thermodynamics, we can understand why an adiabatic expansion of a gas causes cooling and an adiabatic compression of the gas causes heating.

Since no heat is supplied from outside to the gas in an adiabatic process, $dQ=0$; in the case of expansion, the work done dW is taken positive since the gas itself does the work. So, from the first law of thermodynamics, it follows that $dW=-dU$ which means that the internal energy of the gas decreases. As the change of internal energy is proportional to the temperature, it follows that an adiabatic expansion of the gas causes a fall in its temperature. For this reason, when a cycle tyre or an automobile tyre suddenly bursts, the air that comes out is cold.

Again, in adiabatic compression, work is done on the gas for which the external work dW is regarded as negative. Hence from the first law, we get $0=dU-dW$ or $dW=dU$; this shows that the internal energy of the gas and hence its temperature increases. For this reason, when a football bladder or a cycle tyre is inflated, the air appears to be hot.

7.13. Adiabatic relations for a perfect gas :

The followings are the relations between the volume, temperature and pressure of a given mass of perfect gas under adiabatic condition :—

(i) **Pressure-volume relation :** If V be the volume and P the pressure of a given mass of gas, then $PV^r=$constant, where r is the ratio of the sp. heat of the gas at constant pressure and that at constant volume.

(ii) **Volume-temperature relation :** If V be the volume and T, the absolute temperature of a given mass of gas, then $T.V^{r-1}=$constant.

(iii) **Pressure-temperature relation :** If P be the pressure and T the absolute temperature of a given mass of gas, then, $T^rP^{1-r}=$constant.

The proof of the above relations is beyond the scope of the book.

7.14. Slopes of isothermal and adiabatic curves :

As we draw isothermal between pressure (P) and volume(V) of a gas at a given temperature, so also we can draw isothermal between P and V under adiabatic condition, known as *adiabatic curves.*

Consider a point M on the $P-V$ curve and suppose the pressure, volume and temperature of a given mass of gas corresponding to the point M be respectively P, V and T (Fig. 7.6). If the gas be now compressed *isothermally*, its volume will decrease and its pressure will increase. Let the point A represent the present volume and pressure of the gas at the given constant temperature.

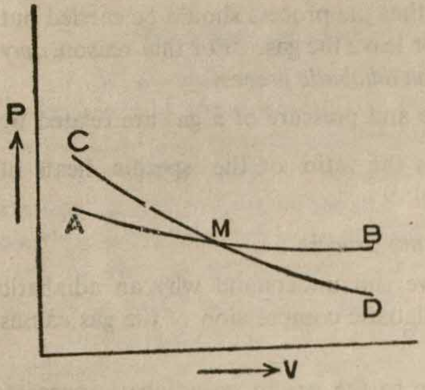


Fig. 7.6

If, on the other hand, the gas is compressed *adiabatically* to the same extent as before, the heat produced will remain confined in the gas which will increase the pressure of the gas more than the previous isothermal case. Let the volume and pressure of the gas in this condition be represented by the point C. Then the curve CMD will be the adiabatic curve for the gas. From the diagram it is clear that *the adiabatic curve is steeper than the isothermal curve.* It can also be proved very easily with the help of calculus in the following way.

For an isothermal, PV=constant. Differentiating, we get $P.dV+V.dP=0$

$\therefore \dfrac{dP}{dV}=-\dfrac{P}{V}$ $\qquad\therefore$ the slope of the isothermal curve$=\dfrac{dP}{dV}=-\dfrac{P}{V}$.

For an adiabatic $P.V^{\gamma}$=constant. Differentiating we get,

$\gamma.P.V.^{\gamma-1}dV+V.^{\gamma}dP=0 \qquad \therefore \dfrac{dP}{dV}=-\gamma.\dfrac{P}{V}$

$\therefore$ The slope of adiabatic curve$=\dfrac{dP}{dV}=-\gamma\dfrac{P}{V}$

or, the slope of the adiabatic$=\gamma\times$the slope of the isothermal. As the value of γ is always greater than 1, the adiabatic curve is steeper than the isothermal.

Examples (1) : *An ideal gas at 17°C has a pressure of 76 cm Hg and is compressed* (i) *isothermally* (ii) *adiabatically until its volume is halved. Calculate in each case the final pressure and temperature of the gas.* $\gamma=1{\cdot}4$; $2^{1\cdot4}=2{\cdot}64$ *and* $2^{0\cdot4}=1{\cdot}32$.

Ans. (i) For isothermal change, $PV=P_1V_1$ or $76\times V=P_1\times\dfrac{V}{2}$

$\therefore\ P_1=152$ cm. Hg.

In isothermal process, temperature does not change. So final temperature=17°C.

(ii) For adiabatic change, (art. 7.13) $P_1V_1^{\gamma}=P.V^{\gamma}$ or $P_1\left(\frac{V}{2}\right)^{\gamma}=76\times V^{\gamma}$

or $P_1=76\times\left(\frac{V}{V/2}\right)^{1\cdot4}=76\times2^{1\cdot4}=76\times2\cdot64=200$ cm Hg (nearly).

Again, for adiabatic process, [art. 7.13 (ii)]

$T.V^{\gamma-1}=T_1V_1^{\gamma-1}$ or $(273+17)V^{(1\cdot4-1)}=T_1\left(\frac{V}{2}\right)^{1\cdot4-1}$

or $290\times V^{0\cdot4}=T_1\left(\frac{V}{2}\right)^{0\cdot4}$ $\therefore$ $T_1=290\times2^{0\cdot4}=290\times1\cdot32=382\cdot8$°K.

$\therefore$ Final temperature$=382\cdot3-273=109\cdot8$°C.

(2) *Calculate the work done when one mole of a perfect gas is compressed adiabatically. The initial pressure and volume of the gas are 10^5 N/m^2 and 6 litres respectively. The final volume of the gas is 2 litres. Molar specific heat of the gas at constant volume is $3R/2$.* [*I.I.T. 1982*]

Ans. In adiabatic compression the work done is given by,

$$W=\frac{1}{\gamma-1}\,(P_2V_2-P_1V_1)\ldots.(\text{i})$$

Now, we know $R=C_p'-C_v'=C_p'-\frac{3}{2}R$ $\therefore$ $C_p'=\frac{5}{2}R.$

Also, $\gamma=\frac{C_p'}{C_v'}=\frac{(5/2)R}{(3/2)R}=\frac{5}{3}.$

Further, for adiabatic compression, $P_1V_1^{\gamma}=P_2V_2^{\gamma}$ or $P_2=P_1\left(\frac{V_1}{V_2}\right)^{\gamma}$

Here, $P_1=10^5N/m^2$; $V_1=6$ litres and $V_2=2$ litres

$$\therefore\ P_2=10^5\times\left(\frac{6}{2}\right)^{5/3}=6\cdot24\times10^5\ N/m^2$$

Substituting the values in eqn. (i), we get

$$W=\frac{1}{\frac{5}{3}-1}\times(6\cdot24\times10^5\times0\cdot002-10^5\times0\cdot006)\text{ joules.}$$
$$=972\text{ joules.}$$

Exercises

Essay type :

1. Give evidence in support of the statement that heat is a form of energy.

2. How should you determine experimentally the value of the mechanical equivalent of heat ?

3. Why do gases possess two specific heats ? Obtain a relation between the two.

4. What are isothermal and adiabatic processes ? What are the conditions necessary for a successful adiabatic process ? Why is a gas heated and cooled respectively when it is adiabatically compressed and expanded ?

Short answer type :

5. What is mechanical equivalent of heat ? What are its values in the F.P.S. and the C.G.S. units ?

6. If the value of J in the C.G.S. system is 4·2 joules/cal. what will it be in the F.P.S. system, given 1 ft. lb. = 1·356 joules and 1 Btu. = 252 cal. ?

7. Cite two common examples illustrating the fact that mechanical work can be converted into heat. What do you mean by the statement that the mechanical equivalent of heat is 4·2 joules/cal. ?

8. State and explain 'First law of thermodynamics'. What is meant by the internal energy of a gas ? [*H. S. Exam. 1978*]

9. Can one distinguish between whether the internal energy of a body was acquired by heat transfer or acquired by performance of work ?

10. Temperature increases with the increase of kinetic energy. But when a piece of iron is thrown with a great force, its kinetic energy increases but the temperature does not. On the other hand, when the piece is hammered, keeping it at rest, its temperature increases. Why is this difference ?

11. Sparks are produced when cutleries are sharpened ? Where does the heat come from ?

12. Distinguish between the specific heat of a gas at constant pressure and the specific heat at constant volume. Define them.

13. What is molar specific heat of a gas ? What is the importance of the ratio of two specific heats of a gas ?

14. State which of the following processes are isothermal and which are adiabatic :— (i) The explosion of a bomb, (ii) Slow leakage of air through a balloon, (iii) Bursting of a motor-car tyre, (iv) The passage of compression and rarefaction through a gaseous medium.

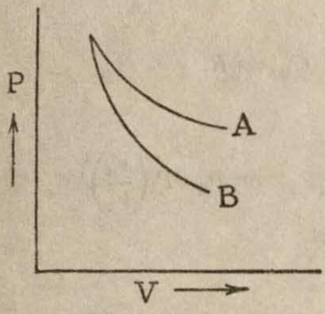


Fig. 7.7

15. The curves A and B in the fig. 7.7 show $P-V$ graphs for an isothermal and an adiabatic process for an ideal gas. The isothermal process is represented by the curve A. Do you agree ? [*I.I.T. 1985*]

16. Distinguish between isothermal and adiabatic conditions. Write down the relations between, (i) volume and pressure (ii) volume and temperature (iii) pressure and temperature under the above two conditions.

17. Does a gas do any work when it expands adiabatically ? If so, what is the source of energy needed to do this work ?

18. One mole of a perfect gas in a cylinder fitted with a piston has a pressure P, volume V and absolute temperature T. If the temperature is increased by 1°K, keeping pressure constant, show that (i) the increase in volume of the gas is $\frac{V}{T}$ (ii) mechanical work done by the gas is $\frac{PV}{T}$ (iii) the heat absorbed by the gas is $\frac{PV}{JT}$.

Objective type :

19. (a) Some statements are made below. Write C against those which are correct and W against those which are incorrect :—

(i) Clausius' interpretation of the first law of thermodynamics is nothing but the expression of the well-known principle of conservation of energy. —

(ii) The specific heat of a gas at constant volume is greater than that a constant pressure.—

(iii) When an automobile tyre suddenly bursts, the air that comes out is cold because it undergoes isothermal expansion. —

(iv) For isothermal process, it is necessary that the substance should be in thermal communication with the surroundings. —

(b) 70 calories of heat are required to raise the temperature of 2 moles of an ideal gas at constant pressure from 30°C to 35°C. The amount of heat required to raise the temperature of the same gas through the same range of the temperature at constant volume is (a) 30 cal (b) 50 cal (c) 70 cal (d) 90 cal. Which is correct? Gas constant $R=2$ cal/mole°K? *[I.I.T. 1985]*

Numerical problems :

20. A cylindrical tube made of some non-conducting material is 15 cm. long. Both the ends of the tube are closed and it contains 500 gm. of lead shots. When the tube is vertical the shots occupy a length of 6 cm. When the tube is suddenly inverted, the lead shots fell from one end to the other. The inversion is repeated quickly for 200 times. The temperature of the shots is found to increase by 1·4°C. Assuming that no heat is lost due to radiation etc, find the mechanical equivalent of heat. Sp. heat of lead=0·03 ; $g=980$ cm/s². [Ans. $4{\cdot}2\times10^{7}$ ergs/cal]

21. A mass of 1 kg. falls to the ground from a height of 1 kilometre. What will be the heat developed if all the energy is transformed into heat ? $J=4{\cdot}2\times10^{7}$ erg/cal.
[H. S. Exam. 1981] [Ans. $2{\cdot}33\times10^{3}$ cal]

22. Two weights, each of mass 13 kg. are allowed to drop 20 times from a height of 160 cm. The work done raises the temperature of 6 kg. of water through 0·3°C. The water is kept in a calorimeter of water equivalent 300 gm. Find the mechanical equivalent of heat.
[Ans. $4{\cdot}3\times10^{7}$ ergs/cal]

23. A copper calorimeter, weighing 500 gm. contains 500 gm. of water at 18°C. The water is churned by a revolving paddle which raises the temperature of water to 20°C. Calculate the work done by the paddle. Assume that the whole work is transformed into heat. Sp. heat of copper=0·1 ; $J=4{\cdot}2$ joules/cal. [Ans. 4620 joules]

24. A piece of lead falls through 200 metres and strike the ground. If all the energy is converted into heat, what will be the rise in temperature of the piece ? Sp. heat of lead=·03.
[Ans. 15·5°C]

25. A metal plate weighing 12·6 kg at 0°C fell from a height of 230 cm. on a piece of ice. If the whole energy is converted into ice, how much ice will melt ? $J=4{\cdot}2\times10^{7}$ erg/cal, $L=80$ cal/gm. $g=980$ cm/s². [Ans. 0·84 gm]

26. An iron ball travelling with a velocity of 50 metres/sec. is suddenly stopped. If the whole energy is converted into heat, what will be the final temperature, the initial temperature of the ball being 25°*C*. Sp. heat of iron=0·1. [Ans. 27·9°C]

27. The heat developed due to the impact of a moving bullet of lead against a target raises the temperature of the bullet by 200°C. Calculate the speed of the bullet. Sp. heat of lead=·03.
[Ans. 224 metres/sec. (nearly)]

28. A bullet of mass 40 gm. moving with a speed of 15 metre/s struck against a body and was brought to rest. If 80% of the kinetic energy is converted into heat, find the amount of heat developed. [Ans. 0·85 cal]

29. A lead bullet collided against a concrete wall with a speed of 0·4 km/s. If ¼th of its kinetic energy is converted into heat, how much will be the rise of temperature of the bullet ? Sp. heat of lead=0·03. [Ans. 161·8°C]

30. A moving lead bullet at 47°*C* struck against an obstacle. The heat produced was just sufficient to melt the bullet. What was the velocity of the bullet ? Melting point of lead=327°C ; Sp. heat=0·03 : Latent heat=6 cal/gm and $J=4{\cdot}2\times10^{7}$ ergs/cal.
[Ans. $3{\cdot}48\times10^{4}$ cm/s]

31. Find the equivalent work, in ergs, of the amount of heat required to raise 1 kilogram of water through 10°C. [Ans. $4{\cdot}2\times10^{11}$ ergs]

32. The rate at which the energy is received by the earth from the sun is 8·4 joules/cm²/min. Calculate the time required to raise a quantity of water weighing 100 gm. from 25°C to its boiling point when contained in a blackened copper vessel of mass 150 gm. and kept at the focus of a concave mirror of aperture 1000 sq. cm., which is facing the sun. [Given $J=4{\cdot}2$ joules/calorie and sp. heat of copper=0·1.] [Ans. 4 mint. 18 sec.]

33. An iron ball having fallen from rest through 30 metres contains kinetic energy sufficient to raise the temperature through 0·°C. What does this give for the mechanical equivalent of heat ? Sp. heat of iron=0·1. *[H. S. Exam. 1961]* [Ans. $4{\cdot}2\times10^{7}$ ergs/cal.]

34. Calculate the difference in temperature between the top and bottom of a waterfall 800 metres high assuming that 90% of the heat generated remains in the water. [*H. S. Exam. 1963*] [Ans. 0·21°C]

35. The height of a waterfall is 500 cm. When the water falls on the ground, what will be its rise of temperature ? $J=4{\cdot}2\times10^7$ ergs/cal. [Ans. 0·0116°C]

36. Sea water is projected horizontally with a speed of 10 metre/s by a hose-pipe and the water fell into a tank 20 metres below the level of the pipe and came to rest. What will be the rise of temperature of water ? Sp. heat of sea-water=0·94 and $J=4{\cdot}2\times10^7$ ergs/cal.

[Hints : Energy of water$=\frac{1}{2}mv^2+mgh$; Heat produced$=m.s.\theta$] [Ans. 0·062°C]

37. Find the work equivalent of heat necessary to convert 5 gm of ice at −5°C into steam. Sp. heat of ice=0·5 ,Latent heat=80 cal/gm ; Latent heat of steam=537 cal/gm ; $J=4{\cdot}2\times10^7$ ergs/cal. [Ans. $1{\cdot}5\times10^4$ joules (nearly)]

38. 1 gm of water when converted into steam at 100°C and 76 cm. *Hg* pressure occupies 1672 c.c. Calculate the gain of potential energy of a molecule of water when changing from liquid to vapour state. Latent heat of vaporisation of water=540 cal/gm. [Ans. $6{\cdot}2\times10^{-13}$ erg]

39. Calculate the value of *J*, given $C_p'=6{\cdot}85$ cal/gm-mol., $C_v'=4{\cdot}87$ cal/gm. mol. and $R_0=8{\cdot}31\times10^7$. [Ans. $4{\cdot}18\times10^7$ ergs/cal.]

40. The temperature of 2 gm. of oxygen is raised from 30°C to 120°C. Find the increase in internal energy when the expansion takes place at constant volume. C_v for $O_2=0{\cdot}155$ cal/gm. [Ans. 27·9 cal]

41. The density of hydrogen gas at N.T.P. is $8{\cdot}96\times10^{-2}$ gm/litre and its sp. heat at constant volume is 2·41 and the ratio of specific heats at constant pressure to that at constant volume is 1·4. Find the value of *J*. [Ans. $4{\cdot}29\times10^7$ ergs/cal]

Harder problems :

42. The speed of a train, weighing 200 ton, is reduced from 40 miles/hr to 30 miles/hr in 30 seconds by applying brakes. If the whole energy is transferred to heat, how much heat will be produced ? $J=778$ ft. lb/Btu. [Ans. 1359 Btu]

43. A lead bullet strikes against a steel armour plate with a velocity of 300 metres/s. If the bullet falls dead after the impact, find the rise in temperature of the bullet assuming that the heat produced is shared equally between the bullet and the target . Sp. heat of lead=0·03 ; $J=4{\cdot}2\times10^7$ ergs/cal. [*I.I.T. 1974*] [Ans. 178·5°C]

44. Two lead balls of mass 0·1 kg and 0·4 kg respectively are moving towards each other with a speed of 1 metre/*s* and 0·1 metre/s. They suffer a head on collision and stick together. The combined mass moves with a velocity of 12 cm/sec. If the temperature of both the balls before collision be the same, find the increase in temperature of the combined mass. $J=4{\cdot}2\times10^7$ erg/cal ; Sp. heat of lead=0·03. [Ans. $0{\cdot}77\times10^{-3}$°C]

45. A big block of ice of mass 10 kg is dragged over a horizontal sheet of ice through a distance of 100 metres, all the ice being at 0°C. Calculate the mass of ice melted. Coefficient of friction between the ice surfaces=·03 and $L=80$ cal/gm.

[Hints : Frictional force$=\mu\times$normal reaction$=\cdot03\times10\times1000\times980$ dynes ; Work done, $W=\cdot03\times10\times1000\times980\times100\times100=3\times98\times10^7$ erg. Heat produced$=\frac{W}{J}=\frac{3\times98\times10^7}{4{\cdot}2\times10^7}=70$cal.

$\therefore$ mass of ice melted$=\frac{70}{80}=0{\cdot}875$ gm]

46. A steel drill making 180 revolutions a minute is used to drill a hole in a block of steel. The mass of the steel block and the drill is 180 gm. If the entire mechanical work is used up in producing heat and the rate of rise in temperature of the block is 0·5°C per second, find (i) the rate of working of the drill in watt (ii) the couple required to drive the drill. Sp. heat of steel =0·1 cal/gm. [*I.I.T 1969*] [Ans. (i) 37·8 Watt (ii) 2·00 Newton-metre]

[Hints : (i) Heat produced/sec$=180\times\cdot1\times\cdot5=9$ cal/s. Watt.$=J.H=9\times4{\cdot}2=37{\cdot}8$ watt.

(ii) $2\pi n\times\tau=9\times4{\cdot}2$ or $2\pi\times\frac{180}{60}\times\tau=9\times4{\cdot}2$]

47. A body of mass 2 kg is being dragged with a uniform velocity of 2*m*/sec on a rough horizontal plane. The coefficient of friction between the body and the surface is 0·2. Calculate the amount of heat generated in 5 sec. [$J=4{\cdot}2$ joule/cal; $g=9{\cdot}8$ m/s^2] [*I.I.T. 1980*] [Ans. 9·33 cal]

48. A 1 kg hammer, moving with a velocity of 50 m/s, strikes a 200 gm iron stake, driving it into ground. If half of the energy goes to heating the iron rod, what will be the rise in temperature ? Sp. heat of iron=0·105. [Ans. 7·1°C]

49. A glass tube 1 metre long, closed at the ends, contains 25 gm of lead shot, the remainder of the tube being filled with 1 litre of water. The tube is held vertically and then inverted, causing the shots to fall the entire length of the tube. How many times the action is to be repeated to produce a rise of temperature of 1°C ? Neglect the heat supplied to the glass. Sp. heat of lead =0·02 ; $J=4{\cdot}2$ joule/cal. [*Jt. Entrance 1981*] [Ans. 17150]

50. A 600 watt electric heater is used to raise the temperature of a certain mass of water from room temperature to 80°C. Alternatively, by passing steam from a boiler into the same initial mass of water at the same initial temperature the same temperature rise is obtained in the same time. If 16 gm. of water were being evaporated every minute in the boiler find the value of the mechanical equivalent of heat. Latent heat of vaporisation of water=515·7 cal/gm.

[Ans. $4{\cdot}36\times10^7$ ergs/cal]

51. A gas is compressed from a volume of 20 litres to 10 litres by a constant pressure of 10^6 dynes per cm^2. Calculate the heat developed. [*Jt. Entrance 1982*] [Ans. $0{\cdot}24\times10^3$ cal]

52. If the ratio of the principal specific heats of a certain gas is 1·4 and its density at N.T.P. is 0·09 gm/c.c., calculate the values of the sp. heat at constant pressure and at constant volume. Normal atmospheric pressure=$1{\cdot}01\times10^6$ dynes/cm^2.

[Ans. $C_p=3{\cdot}43$ cal/gm/°K ; $Cv=2{\cdot}43$ cal/gm/°K]

53. Given that the volume of a gas at N.T.P. is 22·4 litres/mol. and that standard pressure is $1{\cdot}01\times10^6$ dynes/cm^2, calculate a value for the molar gas constant R_0 for the gas and use it to find the difference between the quantities of heat required to raise the temperature of 10 gm. of oxygen from 0°C to 10°C when (a) the pressure (b) the volume of the gas is kept constant. Relative molecular mass of oxygen=32. [Ans. $R_0=8{\cdot}3$ joule/mol/°K; 6·17 cal (nearly)]

[**Hints** : Difference=$10\times(C_p-Cv)\times10=10\times R/J\times10$ where R=gas constant for 1 gm. of oxygen. $R=\frac{R_0}{M}=\frac{R_0}{32}$]

54. A quantity of oxygen is compressed isothermally until its pressure is doubled. It is then allowed to expand adiabatically until its original volume is restored. Find the final pressure in terms of the initial pressure. γ for oxygen=1·4 and $(2)^{1{\cdot}4}=2{\cdot}64$.

[Ans. 0·8 times initial pressure]

8

KINETIC THEORY OF GASES

8.1. Molecular structure of matter :

We see many kinds of substance around us, some of which are solid, some liquid and some are gaseous. From very early times, thinkers were wondering as to what should be the structure of these different kinds of matter. Thanks to the labour, perseverance and intelligence of scientists, we now know that every substance consists of a large number of discrete particles called molecules. A molecule is the smallest particle of a substance which can exist in a free state and possesses all the characteristic properties of the substance. Molecules of a particular substance are all alike but molecules of different substances are different in all respects. As a matter of fact, there are as many molecules in this universe as there are substances. Thus, molecules of water—be it sea-water, spring-water, or riverwater, are all alike but molecules of water are quite different from molecules of any oil. Thousands of molecules present in a piece of iron are all similar to one another but iron molecules are not similar to copper molecules.

This concept of molecular structure of matter is not a new one. In the first century A. D., Lucretius, the Greak Philosopher and the ancient Indian Sage Kanad propagated the same idea.

Each molecule of a substance attracts other molecules towards it and at the same time exerts a repulsive force to keep them at a definite distance from itself. This mutual forces of attraction and repulsion among the molecules of a substance are known as *intermolecular forces*. The existence of intermolecular forces explains satisfactorily the three states of matter. In solids, the intermolecular forces are considerable for which solid substances have definite shape and volume. The molecules in a liquid are farther apart than in a solid and exert less intermolecular forces. They can not easily leave the liquid but are free to move about anywhere within the liquid. This explains why a liquid has a definite volume but no shape of its own. In a gas, the intermolecular forces are very feeble and the gas molecules are free to move about over the entire space available. This is why a gas has neither any shape nor any volume of its own. On this fundamental conception of molecular structure of matter, many physical and chemical properties of the three states of matter have been explained.

Evidences in favour of molecular motion :

Various observations have also revealed that the molecules of a substance are in a state of incessant motion. Of the three different states, the gas molecules have the greatest mobility, next come the liquid molecules, the solid molecules having the least mobility.

(1) The phenomena of *diffusion* and *solution* suggest the molecular structure of matter and agitation of the molecules. If a drop of odorous oil falls in the corner of a room, the smell at once, spreads throughout the

room. Pour some sugar in a glass of water. The sugar crystals will disappear in a short while and a sweet taste will spread throughout the water of the glass. These examples of diffusion and solution show that molecules are always in a state of rapid motion and very quickly move from one place to another.

(2) The phenomena of *evaporation* and *vaporisation* are intimately connected with the motion of the molecules. When a liquid is heated, the molecules of the liquid move more rapidly. Finally, with the rise of temperature, the motion becomes so rapid that some of the molecules escape out of the liquid. The liquid is then said to vaporise.

(3) The familiar phenomenon of *expansibility* of gases clearly show the tendency of the molecules to fly away from each other.

(4) A little over a century ago, an English botanist named Robert Brown first observed a direct effect of the movement of molecules, and from that time, the motion is referred to as '*Brownian motion*'.

8.2. Brownian motion :

Although no one has ever observed directly the random motions of molecules, it is possible to observe in a microscope the resultant recoils of larger particles under their continual bombardment. The effect was first observed by Robert Brown, who noticed the irregular but lifelike motions of small particles (pollen grains) suspended in a liquid. These microscopic particles appear to be continually agitated and make a succession of quick jumps first one way and then another. This motion goes on for ever and shows no sign of abatement or stoppage. This is called Brownian motion. The possible path of a single particle is illustrated in fig. 8.1.

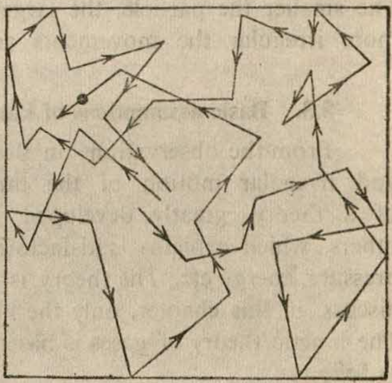


Fig. 8.1

Various scientists investigated into this strange perpetual motion and the information collected by them may be summarised as follows :—

(i) This motion is incessant, ever-lasting and completely random and irregular. In general, two particles do not possess identical motion, not even if they are situated in the same region.

(ii) This motion is independent of the movement of the vessel.

(iii) Less the viscosity of the liquid, the more is the motion.

(iv) The smaller the particle, the more lively and noticeable are its movements.

(v) At same temperature, particles of same weight and size are equally agitated.

(vi) The motion increases with the rise of temperature.

It was, at first, believed that this motion was due to surface tension of the liquid, irregular changes in the temperature of the liquid, chemical action and so on. All these explanations were found inadequate and the phenomenon remained unexplained for a pretty long time. At last, it was realised that movement was due to the motion of thermal agitation of liquid molecules. Einstein succeeded in formulating a quantitative theory for the Brownian movement which was put to experimental verification. Perrin carefully carried out this verification, using colloidal solutions in which fine granules were suspended and was able to verify the haphazard and incessant molecular motion.

However small the suspended particles may be, their size is million times greater than the size of a molecule. Hence, the suspended particles are visible in a microscope, although the molecules are not. Due to thermal agitation, the liquid molecules continually bombard against the particles from all sides. It is easily understood that the forces due to impact of the liquid molecules will almost completely balance if the size of the particle is very large, so that Brownian movement cannot be observed with particles of large size as is the case with a piece of floating wood or cork. But if the particle is sufficiently small, the forces will not balance and a resultant force will act on the particle causing its motion. Since the molecular impacts are all haphazard, the resultant force acting on the particle is also haphazard. Hence the particle will move in a random fashion. The smaller the particle, the larger is the resultant force and consequently, the more irregular the movements are.

8.3. Basic assumptions of kinetic theory of gases :

From the observations on Brownian motion, we are aware of the incessant and irregular motion of the molecules of a substance. This forms the basis of a theory, greatly developed by Maxwell, Boltzmann, Van-der-waals and others, which explains satisfactorily certain physical concepts like temperature, pressure, energy etc. The theory is known as *kinetic theory of matter*. We shall discuss, in this chapter, only the kinetic theory of gases because of its simplicity. The kinetic theory of gases is based on some fundamental assumptions which are as follows :

(i) Every gas is composed of molecules. The molecules of a particular gas are identical but molecules of different gases are different. The molecules are considered to be perfectly elastic so that during collision they do not get deformed.

(ii) Molecules are all point masses, they are of negligible size as compared with their distances apart.

(iii) The molecules are in a state of incessant, random motion, moving in all direction with all possible velocities, ranging from zero to infinity.

(iv) The molecules in their motion collide with one another and with the walls of the containing vessel. Between collisions the molecules move in a straight line with uniform velocity. The distance between two collisions is called the *free path* of the molecule ; the average distance travelled by a molecule between successive collisions is known as the *mean free path*.

(v) The time spent in a collision is negligible as compared with that during which the molecules are moving independently *i.e.* the collisions are instantaneous.

(vi) The collisions are perfectly elastic and no forces of attraction or repulsion act between the molecules and the walls of the container. This means that the energy of the gas is entirely kinetic.

(vii) Though the molecules collide against each other and against the walls of the containing vessel yet in the steady state, the molecular density of the gas *i.e.* the number of molecules per unit volume of the gas remains constant.

It may be mentioned that all the above assumptions are applicable to an ideal gas.

8.4. Pressure of a gas according to kinetic theory :

According to the kinetic theory, the pressure exerted by a gas upon the walls of the containing vessel is due to the incessant bombardment by the rapidly moving gas molecules on the walls. If the temperature of the gas is raised, the molecules move faster and the pressure rises, whereas if the temperature is lowered, they move slower and the pressure decreases. The more gas that is pumped into a vessel of constant volume, the more molecules there are to bombard the walls per second and the greater is the resultant pressure.

In any reasonable large volume of gas, there are numerous molecules (about 3×10^{19} molecules per c.c. at N.T.P.) and at any given instant, some of them are moving in one direction and some in another, some are travelling fast, some slow and a few are momentarily at rest. Under these circumstances, according to the mathematical laws of probability, some average speed can be determined, which if possessed by all the molecules, would correspond to the same temperature and would give rise to the same wall pressure. If this average velocity be C, then, every molecule, when it rebounds from the wall, has velocity $-C$ and a change of momentum $=[mC-(-mC)]=2mC$. According to Newton's laws of motion, this exerts a force on the wall. In this way, the force exerted per unit area of the wall by the molecules of the gas per sec gives the pressure on the wall. It can be proved* that the pressure $P=\frac{1}{3}\rho\times C^2$, where ρ is the density of the gas. This is the expression for the pressure of a gas according to the kinetic theory.

It may be mentioned here that from the above expression of gas pressure, we can establish Boyle's law, Charles' law, ideal gas equation, Avogadro's hypothesis etc. relating to the behaviour of a gas.

8.5. Root mean square velocity or R.M.S. velocity :

The average velocity of the gas molecules mentioned in the previous article, is in fact, the root mean square velocity of the gas molecules. It is abbreviated as R.M.S. velocity. It will be clear from the following description.

Suppose, there are '*n*' number of molecules in a vessel whose velocities at the given pressure and temperature are c_1, c_2, c_3, ... c_n. The average velocity C of the gas molecules is given by, $\overline{C}=\dfrac{c_1+c_2+c_3+\,.\,.+c_n}{n}$

[*The proof is beyond the scope of the book.]

But if the average of the squares of the velocities of the molecules be $\overline{c^2}$, then, $\overline{c^2}=\frac{c_1^2+c_2^2+c_3^2+..+c_n^2}{n}$.

If now the square root of $\overline{c^2}$ is taken, then.

$$C=\sqrt{\overline{c^2}}=\sqrt{\frac{c_1^2+c_2^2+c_3^2+..+c_n^2}{n}}$$

This value of C is called the root mean square velocity. It is to be noted that C and $\overline{C}$ are not equal *i.e. the average velocity is not equal to the r.m.s. velocity.*

Suppose we consider the velocities of six molecules, which are say, 1,2,3,4 ; 5 and 6 units. Their average velocity $\overline{C}=\frac{1+2+3+4+5+6}{6}=\frac{21}{6}=3{\cdot}5$ unit

The square of this average velocity $(\overline{C})^2=(3{\cdot}5)^2=12{\cdot}25$

Again average of their squares $(\overline{C^2})=\frac{(1)^2+(2)^2+(3)^3+(4)^2+(5)^2+(6)^2}{6}$

$$=\frac{91}{6}=15{\cdot}2$$

So, $(\overline{C})^2$ is not same as $(\overline{C^2})$ and hence $\overline{C}$ and C are also not equal.

Example. *Calculate the r.m.s. velocity of air molecules at N.T.P. given density of air=0·00129 gm/c.c.*

Ans. We know, $P=\frac{1}{3}\rho.C^2$ or, $C=\sqrt{\frac{3P}{\rho}}$

Here, $P=76\times13{\cdot}6\times980$ dynes/sq. cm. (normal pressure) and $\rho=0{\cdot}00129$ gm/c.c. $\therefore$ $C=\sqrt{\frac{3\times76\times13{\cdot}6\times980}{{\cdot}00129}}=4{\cdot}85\times10^4$ cm/sec.

8.6. Concept of temperature according to kinetic theory :

Consider one gm-molecule of a gas. Suppose its volume is V at pressure P and temperature T. Its mass, suppose, is M_0. From art. 8.4 we can write, $P=\frac{1}{3}\frac{M_0}{V}.C^2$ or, $PV=\frac{1}{3}M_0C^2$. But for an ideal gas, $PV=R_0T$ $\therefore$ $\frac{1}{3}M_0C^2=R_0T$. or, $\frac{1}{2}M_0C^2=\frac{3}{2}R_0T$. In other words, the *K.E.* of a gm-molecule of a gas$=\frac{3}{2}R_0T$.

If N=Avogadro number, then the number of molecules in a gas-molecule of the gas$=N$. Hence, the average *K.E.* of each molecule$=\frac{3}{2}\frac{R_0T}{N}=\frac{3}{2}K.T$ $[K=\frac{R_0}{N}$ a constant]. This constant is known as *Boltzmann constant*. It is defined as the gas constant per molecule and is a universal constant. Its value is $1{\cdot}37\times10^{-16}$ erg/°K.

Hence the average kinetic energy of a molecule is proportional to the absolute temperature of the gas. Since, the molecules of a perfect gas exert no force of attraction or repulsion among themselves, the molecules have no potential energy—their energy is entirely kinetic and this kinetic energy of the gas determines its temperature. The more the kinetic energy, the higher is the temperature and vice versa. Alternatively it may be said that the temperature is the manifestation of the kinetic energy of the molecules of a gas. This is the kinetic interpretation of temperature of a gas. When the temperature is zero, the kinetic energy of the molecules also becomes zero, *i.e.* the motion of the gas molecules ceases. According to Kinetic theory, this temperature corresponds to the zero degree of the absolute scale.

8.7. Variation of r.m.s. speed :

For a gm-molecule of a gas, we have seen that, $\frac{1}{3}M_0C^2=R_0T$ or $C^2=\frac{3R_0T}{M_0}$.

From this, we can say (i) the *r.m.s.* speed or velocity of the molecules of a *given gas* $\propto\sqrt{T}$ *i.e.* r.m.s. speed increases with the increase of temperture.

(ii) the r.m.s. speed or velocity of the molecules of different gases at the *same temperature* $\propto \sqrt{\frac{1}{M_0}}$ *i.e.* the molecules of gases having higher molecular mass have smaller r.m.s. speeds.

Examples : (1) *At 0°C and 760 cm Hg pressure, the r.m.s. speed of hydrogen molecules is* $1{\cdot}84\times10^5$ *cm/s. What will be the r.m.s. speed when the temperature is raised to 100°C, pressure remaining the same?*

Ans. We know, $C\propto\sqrt{T}$ $\therefore$ $\frac{C_0}{C_{100}}=\sqrt{\frac{T_0}{T_{100}}}$. Now $T_0=0+273=273°K$ and $T_{100}=273+100=373°K$. Hence,

$$\frac{C_0}{C_{100}}=\sqrt{\frac{273}{373}} \text{ or } C_{100}=1{\cdot}84\times10^5\sqrt{\frac{373}{273}}=2{\cdot}15\times10^5 \text{ cm/s.}$$

(2) *Assuming that the velocity of sound in an ideal gas is equal to the r.m.s. speed of the molecules of the gas, show that* $\frac{v_1}{v_2}=\sqrt{\frac{M_2}{M_1}}$ *at constant temperature where* v_1 *is the speed of sound in a gas of molecular weight* M_1 *and* v_2 *that in a gas of molecular weight* M_2.

Ans. Referring to art 8.7, we have seen that the *r.m.s.* speed of the molecules of a perfect gas is given by $C=\sqrt{\frac{3R_0T}{M_0}}$. [$R_0$=molar gas constant and M_0 =molecular wt. of the gas] Since the velocity of sound in an ideal gas has been assumed equal to the *r.m.s.* speed, the velocity of sound is given by $v=\sqrt{\frac{3R_0T}{M_0}}$.

So, for the first gas, $v_1=\sqrt{\frac{3R_0T}{M_1}}$ and for the second, $v_2=\sqrt{\frac{3R_0T}{M_2}}$, since R_0 and T are constants. $\therefore$ $\frac{v_1}{v_2}=\sqrt{\frac{M_2}{M_1}}$.

[N.B. From above it follows that the velocity of sound in a given gas is directly proportional to the square root of its absolute temperature or $v \propto \sqrt{T}$]

8.8. Some important deductions from the kinetic theory of gases :

(i) **Boyle's law** : From the kinetic theory, we know $P=\frac{1}{3}\rho C^2$. Suppose there are n number of gas molecules enclosed in a cubical vessel. If m be the mass of each molecule, then $\rho=\frac{mn}{V}$ [V=volume of the vessel]. Hence $P=\frac{1}{3}\frac{mn}{V}\cdot\ C^2$ or, $PV=\frac{1}{3}mnC^2=\frac{1}{3}M.C^2$ [$M=mn$=mass of a given quantity of gas]

When the temperature of the gas is kept constant, the mean square velocity (C^2) of the gas molecules remains constant. So, temperature remaining constant, for a given quantity of gas, we see PV=constant. This is Boyle's law.

(ii) **The ideal gas equation,** $PV=RT$: Since the average kinetic energy of a molecule is proportional to the absolute temperature of the gas, we have,

$\frac{1}{2}MC^2 \propto T$ or $\frac{1}{2}MC^2=KT$ [K=constant of proportionality]

But we know, $PV=\frac{1}{3}MC^2=\frac{2}{3}\times\frac{1}{2}MC^2=\frac{2}{3}.K.T.=R.T.$ where $R=\frac{2}{3}K$=a constant which depends upon the mass M of the gas. This is known as the ideal gas equation.

(iii) **Charles' law** : From kinetic theory of gas, it has already been established that $PV=RT$. If the pressure (P) is now kept constant, we can say $V \propto T$ *i.e.* the volume of a certain mass of gas is proportional to its absolute temperature when the pressure of the gas remains unaltered. This is Charles' law.

(iv) **Avogadro's hypothesis** : Suppose, we take two different gases, having volume 1 *c.c.* each and temperature and pressure T and P respectively. If the number of molecules per c.c. of the gases be n_1 and n_2 and if the mass of each type of molecules be m_1 and m_2 respectively, then from the kinetic theory of gas, we have $P=\frac{1}{3}m_1n_1c_1{}^2=\frac{1}{3}m_2n_2c_2{}^2$ .. .. (i)

[c_1 and c_2 are the *r.m.s.* values of velocities of the gas molecules]

Again, since the temperature of the gases is equal, their mean kinetic energy is also equal *i.e.* $\frac{1}{2}m_1c_1{}^2=\frac{1}{2}m_2c_2{}^2$ .. .. .. (ii)

From equations (i) and (ii), we have, $n_1=n_2$

That is, the number of molecules per *c.c.* of the gases is equal. In this way, taking any volume and any number of gases, it can be proved that under same condition of temperature and pressure equal volumes of different gases contain equal number of molecules. This is Avogadro's hypothesis.

(v) **Graham's law of diffusion :**

When a gas passes through a porous plug (a pad of cotton wool say) it is said to diffuse. There is a fundamental difference between the diffusion of a gas and the flow of a gas, say through a tube. The flow of a gas through a tube represents motion of the gas *in bulk* but the diffusion of a gas through a porous plug represents the motion of *individual molecules* of the gas. Graham established a law in connection with the diffusion of a gas, which is known as Graham's law of diffusion. The law states that the *rate of diffusion of a gas is inversely proportional to the square root of its density.*

The law can be readily explained by kinetic theory of gases. We know, that the mean kinetic energies of the molecules of different gases at a given temperature $T°K$ are equal since $\frac{1}{2}mC^2=\frac{3}{2}KT$, where K is a universal constant.

Denoting air and hydrogen by the subscripts a and h respectively, we get,

$\frac{1}{2}m_aC_a^2=\frac{1}{2}m_hC_h^2$ or $\frac{C_a^2}{C_h^2}=\frac{m_h}{m_a}$. Now, at a given temperature and pressure, the density ρ of a gas is proportiorial to the mass m of its molecules, since equal volumes contains equal number of molecules according to Avogadro's hypothesis.

$$\therefore \quad \frac{m_a}{m_h}=\frac{\rho_a}{\rho_h} \quad \text{or} \quad \frac{C_a^2}{C_h^2}=\frac{\rho_h}{\rho_a} \quad \text{or} \quad \frac{\sqrt{C_a^2}}{\sqrt{C_h^2}}=\frac{\sqrt{\rho_h}}{\sqrt{\rho_a}}$$

The average speed of the molecules of a gas is roughly equal to but strictly proportional to the square root of its mean squared speed. The above equation therefore, shows that the average molecular speeds are inversely proportional to the square roots of the densities of the gas. Now, rates of diffusion being dependent on molecular speeds, it may be said that the rate of diffusion is inversely propartional to the square roots of the densities. This is Graham's law.

Examples : (1) *A one litre flask contains* 10^{25} *oxygen molecules, each of mass* $2{\cdot}7\times10^{-25}$ *gm. and having r.m.s. velocity* 4×10^4 *cm/sec. Calculate the pressure in the flask.*

Ans. We know $P=\frac{1}{3}\frac{mn}{V}\cdot C^2$

Here, $m=2{\cdot}7\times10^{-25}$ gm; $n=10^{25}$; $C=4\times10^4$ cm/sec and $V=1000$ c.c.

$$\therefore \quad P=\frac{1}{3}\times\frac{2{\cdot}7\times10^{-25}\times10^{25}\times(4\times10^4)^2}{1000}=14{\cdot}4\times10^5 \text{ dynes/sq.cm}$$

(2) *If the root mean square velocity of hydrogen at N.T.P. is 1·84 km/sec., find the r.m.s. velocity of oxygen molecules at N.T.P., given the molecular weight of hydrogen and oxygen are 2 and 32 respectively.*

Ans. We know, $MC^2=3RT=\text{constant}$. $\therefore \; M_HC_H^2=M_OC_O^2$

Now, $M_H=2$, $M_O=32$ and $C_H=1{\cdot}84$ km/sec.

$$\therefore \quad C_O=\sqrt{\frac{M_HC_H^2}{M_O}}=C_H\sqrt{\frac{M_H}{M_O}}=1{\cdot}84\times\sqrt{\frac{2}{32}}=1{\cdot}84\times\frac{1}{4}=0{\cdot}46 \text{ km/sec.}$$

(3) *At what temperature will r.m.s. velocity of nitrogen molecule be double its value at N.T.P., pressure remaining constant?*

Ans. We know, $C=\sqrt{\frac{3P}{\rho}}$. For an ideal gas $PV=RT$ or $P=\frac{RT}{V}$

$$\therefore \quad C=\sqrt{\frac{3RT}{V.\rho}}=\sqrt{\frac{3RT}{M}} \quad [M=\text{mass of the gas}=V.\rho] \quad \textit{i.e.} \quad C\propto\sqrt{T}$$

Now, if at normal temperature (T_0 say), the *r.m.s.* velocity be C then suppose at $T°K$, it is $2C$.

$$\therefore \quad \frac{2C}{C}=\sqrt{\frac{T}{T_0}} \quad \text{or} \quad 4=\frac{T}{T_0} \quad \therefore \quad T=4T_0=4\times 273=1092°K$$

So, the required temperature in Celsius scale, $t=1092-273=819°C$.

(4) *Calculate the kinetic energy per gm. molucule of helium at 27°C, given* $R_0=8{\cdot}3\times 10^7$ ergs/mol/°K.

Ans. We know, the K.E. per gm molecule of a gas$=\frac{3}{2}R_0T$

Here, $R_0=8{\cdot}3\times 10^7$ ergs/mol/°K; $T=273+27=300°K$

$\therefore$ K.E.$=\frac{3}{2}\times 8{\cdot}3\times 10^7\times 300$ erg$=3735\times 10^7$ ergs.$=3735$ joules.

8.9. Degrees of freedom and the law of equipartition of energy :

If a single gas atom is constrained to move along a straight line, it is said to have only one degree of freedom of motion; if it is free to move in a plane, it has two degrees of freedom, for, two co-ordinate axes on the plane are enough to specify its position. It is only when it is free to move in three-dimensional space, that it possesses three degrees of freedom because all the three co-ordinates are now required to specify the position of the atom. These degrees of freedom are known as *degrees of freedom of translational motion.*

Besides translational motion, a gas atom may have a rotational motion about three mutually perpendicular axes through a fixed point. These motions endow the atom three more degrees of freedom, known as *degrees of freedom of rotational motion.* An atom, capable of translatory and rotatory motions simultaneously has, in total, six degrees of freedom—three for translatory and three for rotatory motion. Now consider a diatomic molecule where the atoms A and B are fixed rigidly to each other at a certain distance apart (fig 8.2). Clearly, this system has three degrees of translatory motion along the three mutually perpendicular axes X, Y and Z. The possible rotational motion of the system being only about the axes Y and Z, the molecule has only two degrees of rotational motion. So, a diatomic molecule has, in all, five degrees of freedom.

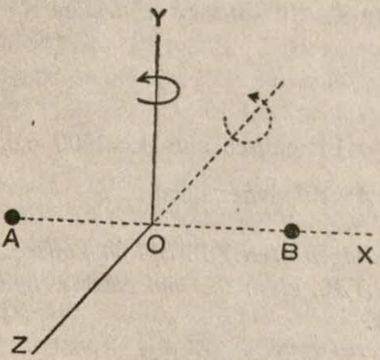


Fig. 8.2

In the same way, if we consider the case of a triatomic molecule, the number of degrees of freedom for it will be found to be 6 and so on.

Principle of equipartition :

In establishing the kinetic theory of gases, we resolve the velocity C of a molecule into three components u, v and ω along the three axes of co-ordinates (or the three degrees of freedom), so that $C^2=u^2+v^2+\omega^2$

Taking average over all the molecules of the gas in the enclosure, we have, $\overline{C^2}=\overline{u^2}+\overline{v^2}+\overline{\omega^2}$ where, C^2 is the mean square velocity of the molecules and $\overline{u^2}$, $\overline{v^2}$ and $\overline{\omega^2}$ are the squares of the average component velocities along the three axes. According to the fundamental assumptions of kinetic theory, the molecular density of the gas remains unaltered. This means that the average velocities of the molecules in the three directions must be same *i.e.* $\overline{u^2}=\overline{v^2}=\overline{\omega^2}$. Correspondingly, the kinetic energies of the molecules in the three directions are also same *i.e.* $\frac{1}{2}m\,\overline{u^2}=\frac{1}{2}m\,\overline{v^2}=\frac{1}{2}m\,\overline{\omega^2}$.

But we know, $\overline{C^2}=\overline{u^2}+\overline{v^2}+\overline{\omega^2}$, so, $\frac{1}{2}m\overline{u^2}=\frac{1}{2}m\overline{v^2}=\frac{1}{2}m\overline{\omega^2}=\frac{1}{3}.\frac{1}{2}m\overline{C^2}$

We have seen that the average *K.E.* per molecule$=\frac{1}{2}m\overline{C^2}=\frac{3}{2}KT$, where K is the Boltzman constant.

$$\therefore\ \frac{1}{2}m\overline{u^2}=\frac{1}{2}m\overline{v^2}=\frac{1}{2}m\overline{\omega^2}=\frac{1}{3}.\frac{3}{2}KT=\frac{1}{2}KT$$

So, average *K.E.* of a molecule per degree of freedom$=\frac{1}{2}KT$. This is known as the law of equipartition of energy and it is true for all degrees of freedom – translational as well as rotational The law may, therefore, be stated as.

The total energy of a dynamical system in thermal equilibrium is partitioned equally among all its degrees of freedom, the energy per molecule per degree freedom being equal to $\frac{1}{2}KT$.

From the above it follows that the energy of a diatomic molecule, which has five degrees of freedom, is $\frac{5}{2}KT$ and that of a triatomic molecule which has six degrees of freedom is $3KT$ etc.

8.10. Ratio of two specific heats of a gas :

We know that a gas molecule has always three degrees of freedom of translational motion but the number of degrees of freedom of rotational motion depends on the number of atoms and their arrangement in the molecule. Let r be the number of degrees of freedom of rotational motion of a polyatomic molecule. The total number of degrees of freedom is, therefore$=3+r=x$ (say)

We have already seen that the average energy per gm-molecule of a gas per degree of freedom at temperature T is $\frac{1}{2}R_0T$. Hence the total energy (E) of a gm-molecule of the gas for all the x degrees of freedom $E=\frac{1}{2}x.R_0T$

Since molar specific heat of a gas at constant volume (C_v') is the quantity of heat required to raise the temperature of 1 gm. molecule of gas through 1 degree at constant volume, it is obviously the differential coefficient of the total heat energy of 1 gm mol of the gas with respect to the temperature T.

$$\text{Hence, } C_v'=\frac{dE}{dT}=\frac{1}{2}x.R_0$$

$$\text{Also, } C_p'-C_v'=R_0 \text{ or } C_p'=R_0+C_v'=R_0+\frac{1}{2}xR_0=R_0\left(1+\frac{x}{2}\right)$$

$$\therefore\ \gamma=\frac{C_p'}{C_v'}=\frac{R_0\left(1+\frac{x}{2}\right)}{\frac{x}{2}.\,R_0}=1+\frac{2}{x}$$

Now, for *mon atomic* gas, $x=3$ $\therefore\ \gamma=1+\frac{2}{3}=1{\cdot}67$
for *di atomic* gas, $x=5$ $\therefore\ \gamma=1+\frac{2}{5}=1{\cdot}4$
for *tri atomic* gas, $x=6$ $\therefore\ \gamma=1+\frac{2}{6}=1{\cdot}33$

All these results agree with the experimental results for different gases, like, helium, oxygen, etc. Thus, the value of γ goes on decreasing with increasing atomicity of the gas but it is always greater than 1 since $C'_p>C'_v$.

Example : *The density of a gas is $1{\cdot}765\times10^{-3}$ gm/c.c. at 27°C and 76 cm. Hg pressure and its specific heat at constant pressure is 846×10^4 erg/gm/°K. Show, from the ratio of two specific heats of the gas that the gas is polyatomic.*

Ans. Let us consider 1 gm. of the gas. Its volume$=\frac{10^3}{1{\cdot}765}$ c.c.

Now, gas constant $R=\frac{P.V}{T}=\frac{76\times13{\cdot}6\times980}{27+273}\times\frac{10^3}{1{\cdot}765}=191{\cdot}3\times10^4$ erg/gm/°K.

But $C_p-C_v=R$ (for 1 gm in work unit)
or, $846\times10^4-C_v=191{\cdot}3\times10^4$ or $C_v=(846-191{\cdot}3)\times10^4=654{\cdot}7\times10^4$ erg/gm/°K

$$\therefore\quad \gamma=\frac{C_p}{C_v}=\frac{846\times10^4}{654{\cdot}7\times10^4}=1{\cdot}29 \text{ (nearly)}$$

The ratio shows that the gas is polyatomic.

8.11. Limitation of the ideal gas equation : The equation which relates the volume, pressure and temperature (in absolute scale) of a perfect gas is referred to as the ideal gas equation. We know that $PV=R.T$ where R is the gas constant, is the above mentioned gas equation which can be easily established from Boyle's and Charles' laws. In other words, we can say that a gas which obeys Boyle's and Charles' laws under all circumstances is a perfect gas. Unfortunately, no real gas is, in this sense, a perfect gas.

Boyle himself found experimentally that ordinary gases obey his law only at high temperature and low pressure (from ·03 atmospheric pressure to about 4 atmospheric pressures). In 1827 Despretz, extending the range upto 20 atmospheres, compressed equal volumes of different gases and found that at least some of the gases did not obey Boyle's law strictly. In 1847, Regnault, after a series of experiments on the behaviour of ordinary gases towards Boyle's law upto pressures of 30 atmospheres and within a temperature range of 0°*C* to 100°*C* found that real gases do not behave according to the Boyle's law under all circumstances.

There are mainly two reasons for the deviations of the real gases from the ideal gas equation. From the equation $PV=RT$, it is easily understood that if the pressure of the gas be increased keeping its temperature constant, the volume of the gas diminishes until at a very high pressure, the volume of the gas practically reduces to zero. But actually this can never happen because the gas molecules have always a certain volume of their own which can never reduce to zero. Secondly, it has been assumed that the molecules of a perfect gas do not exert any attractive force on one another. But in a real gas, the molecules do exert a force of attraction—though very small—on one another, especially when the molecules come very close to one another under high pressure. Due to this force of attraction, there is a slight contraction in the volume of the gas and at high pressure, the volume occupied by the gas is less than that occupied by an ideal gas.

Remembering these two points, Van-der-Waals was the first, to propose a new equation of state for real gases in 1879. This equation, written below, is known as the Vander Waal's equation of state for real gases :

$$\left(P+\frac{a}{V^2}\right)(V-b)=RT$$

where a and b are two constants known as Van-der-Waals' constants.

Exercises

Essay type :

1. How do you know that the molecules of a substance are in constant random motion ?
2. What is Brownian motion ? What are the characteristics of the motion ? How do you account for such motion ? [*H. S. Exam 1979*]
3. What is kinetic theory of gases ? What are its basic assumptions ? [*H. S. Exam 1978, '82*]
4. Explain gas pressure according to the kinetic theory and write down an expression for it. [*H. S. Exam 1978*]
5. What is kinetic theory of gases ? Show that it leads directly to Boyle's law for a perfect gas and that it indicates the temperature of a gas is proportional to the mean *K.E.* of the gas molecules.

Short answer type :

6. In observing Brownian motion, do we observe the motion of the molecules of a liquid or of some other particles ?
7. What direct evidence is available to justify the belief that the molecules of a gas are in a continual state of random motion ? [*H. S. Exam 1979*]
8. What do you mean by R.M.S. velocity ?
9. Are the *r.m.s.* speeds of the molecules of different ideal gases maintained at the same temperature equal ? [*I.I.T. 1981*]
   [**Hints** : See art 8.7]
10. Explain what are meant by mean velocity and root-mean square velocity. Are they equal ? Which one is more important in kinetic theory of gases ?
11. How will the r.m.s. velocity of the molecules of a perfect gas vary (i) when temperature is increased (ii) density of the gas is decreased ?
12. Establish Boyle's law and Avogadro's hypothesis from the expression of pressure according to kinetic theory.
13. What is the interpretation of temperature according to the kinetic theory ? [*H. S. Exam 1978*]
14. Show that kinetic energy per gm-molecule is same for all gases at the same temperature.
15. Show, from the kinetic theory of gases that the average kinetic energy of a gas molecule is proportional to the absolute temperature of the gas. [*H. S. Exam 1979*]
16. Discuss the limitations of ideal gas equation.
17. Is ideal gas equation applicable to real gases ? Explain your answer.
18. Write down Van-der-Waal's equation of state. What led him to establish the equation ?
19. The average velocity of the molecules of a gas must be zero if the gas as a whole and the container are not in translational motion. How it can be that the average speed is not zero ?

Numerical problems :

20. A one litre flask contains $2 \cdot 5 \times 10^{25}$ molecules of a gas each of mass 5×10^{-25} gm. and having r.m.s. velocity 15×10^{3} cm/sec. Find the pressure of the gas.
[Ans. $9 \cdot 4 \times 10^{5}$ dynes/sq. cm. (nearly)]

21. At what temperature is the *r.m.s.* velocity of hydrogen molecule equal to that of carbon-dioxide molecules at 57°*C* ? A carbon-dioxide molecule is about 22 times heavier than a hydrogen molecule.
[Ans. 15°K]

22. (i) Calculate the *r.m.s.* velocity of hydrogen molecules at N.T.P. Density of hydrogen $= 9 \times 10^{-5}$ gm/c.c.
[Ans. $1 \cdot 85 \times 10^{5}$ cm./s]

(ii) Calculate the *r.m.s.* velocity of nitrogen molocules at 0°*C*. Density of nitrogen at N.T.P. $= 1 \cdot 25$ gm. per litre ; density of mercury $= 13 \cdot 6$ gm/c.c..
[*H. S. Exam 1982*[[Ans. $4 \cdot 86 \times 10^{4}$ cm/s]

23. Given that the gas constant $R = 8 \cdot 3 \times 10^{7}$ ergs per°*C* and the atomic weight of chlorine is 35·5, find the *r.m.s.* velocity of chlorine molecules at 0°*C*.
[Ans. $3 \cdot 1 \times 10^{4}$ cm/s]

[**Hints :** $C = \sqrt{3RT/M}$; $M = 2 \times 35 \cdot 5$ because chlorine is diatomic]

24. If the mean free path of nitrogen molecules at 0°*C* and one atmospheric pressure be $9 \cdot 44 \times 10^{-6}$ cm. what will be the time-interval between two consecutive collisions. $R = 8 \cdot 31 \times 10^{7}$ and the molecular weight of nitrogen $= 28$.
[Ans. $1 \cdot 9 \times 10^{-10}$ sec]

$$\left[\textbf{Hints :} \quad \text{time-interval} = \frac{\text{mean free path}}{r.m.s.\ \text{speed}} \right]$$

Harder Problems :

25. There are $5 \cdot 5 \times 10^{24}$ hydrogen molecules in a 100 c.c. flask. If the mass of each molecule be $3 \cdot 32 \times 10^{-24}$ gm. and *r.m.s.* speed 3×10^{4} cm/s, find the pressure of the flask in atmosphere.
[Ans. 54 atmos (nearly)]

26. Calculate the pressure, in mm of mercury, exerted by hydrogen gas if the number of molecules per c.c. is $6 \cdot 80 \times 10^{15}$ and the *r.m.s.* speed of the molecules is $1 \cdot 9 \times 10^{5}$ cm/s. Avogadro number $= 6 \cdot 02 \times 10^{23}$ and relative molecular mass of hydrogen $= 2 \cdot 02$. [Ans. 0·21 mm]

27. Calculate the number of molecule per c.c. of an ideal gas at 27°*C* and 1 cm. of mercury pressure. Mean kinetic energy per molecule at $27°C = 4 \times 10^{-4}$ ergs and density of mercury $= 13 \cdot 6$ gm/c.c.
[Ans. $4 \cdot 99 \times 10^{7}$]

28. The average kinetic energy of a molecule of hydrogen at 0°*C* is $5 \cdot 64 \times 10^{-14}$ ergs and molar gas constant $(R_0) = 8 \cdot 32 \times 10^{7}$ ergs/°*K*. Calculate Avogadro's number. [Ans. $6 \cdot 04 \times 10^{23}$]

$$\left[\textbf{Hints :} \quad \text{Average } K.E. = \frac{3}{2} \cdot \frac{R_0}{N_a} \cdot T \text{ where } N_a = \text{Avogadro number]}. \right]$$

29. Find the temperature at which the *r.m.s.* velocity of nitrogen molecules in the earth's atmosphere equals the velocity of escape from the earths gravitational field. Mass of nitrogen atom $= 23 \cdot 24 \times 10^{-24}$ gm ; mean radius of the earth $= 6370$ km, Boltzman costant $= 1 \cdot 37 \times 10^{-16}$ erg/0°*C* ; $g = 980$ cm/s^2.
[Ans. $14 \cdot 12 \times 10^{4}$ °*K*]

[**Hints :** Velocity of escape $= \sqrt{2gR}$, when R = radius of the earth].

30. Calculate (*a*) the *K.E.* of translation of an oxygen molecule at 27°*C* and (*b*) the total K.E. of a gm-molecule of oxygen at 27°*C*. Avogadro number $= 6 \cdot 023 \times 10^{23}$ and Boltzmann constant $= 1 \cdot 37 \times 10^{-16}$ erg/°*C* ; Oxygen mclecule is di-atomic
[Ans. (i) $6 \cdot 17 \times 10^{-14}$ erg (ii) $10 \cdot 28 \times 10^{-14}$ erg]

31. The temperature of a gas is increased from 27°*C* to 327°*C*. Show that the *r.m.s.* speed of air molecules at the higher temperature is $\sqrt{2}$ times that at the initial temperature.

32. The r.m.s. velocity of the molecules of an ideal gas at N.T.P. is 0·5 km/s ; find the density of the gas. Find also the density of the gas at 21C° when the pressure is kept unaltered.
[*Jt. Entrance 1983*] [Ans. 1·2 kg/m^3; 1.11 kg/m^3]

9

TRANSMISSION OF HEAT

9.1. Different ways of transmission of heat :

Heat may be transmitted from one place to another by three ways :—(i) **Conduction** (ii) **Convection** and (iii) **Radiation.**

Definition : *Conduction* : *The process in which heat is transmitted from the hotter part of a body to its colder part without any visible movement of the material of the body is called conduction.* Heat generally travels through a solid body by conduction.

Convection : *The process in which heat is transmitted from the hotter part of a body to its colder part by the actual movement of the material of the body, is called convection.* Heat is generally transmitted in liquids and gases by convection.

Radiation : *The process in which heat is transmitted from one place to another without the help of any material medium or without heating the intervening medium, if there be any, is called radiation.* Heat from the sun is transmitted to the earth by radiation.

Difference between the three processes : (i) Conduction and convection require some material medium either solid, liquid or gas but radiation takes place without the help of any material medium.

(ii) Conduction or convection is a slow process but radiation is a very rapid process. Radiant heat travels with the velocity of light.

(iii) Radiant heat, like light, travels in straight line but heat due to conduction or convection may travel in curved path. To protect us from sun's heat, we hold umbrella over our head. This shows that heat rays from the sun travel in straight line.

(iv) Radiant heat does not warm up the medium through which it passes, but heat due to conduction and convection warms up the medium through which it travels.

Conduction

9.2. Thermal conductivity and co-efficient of thermal conductivity :

Definition : *The ability of a given substance to conduct heat is called the conductivity of the substance.*

The conductivity of all substances are obviously not equal. A wooden rod can be held at one end for a pretty long time with the other end pushed into a fire but an iron rod similarly placed cannot be held for a long time. Soon the other end will be so hot that it will have to be dropped down. It shows that wood cannot conduct heat as fast as iron does. Iron is, therefore, a better conductor of heat than wood.

Definition : *Those substances which can conduct heat easily are called* **good conductors** *while those which cannot do so are called* **bad conductors** *or* **insulators.**

Most metals are good conductors of heat ; silver and copper are exceptionally good. On the other hand, wood, glass, cotton, rubber, cork etc. are bad conductors.

Co-efficient of thermal conductivity : The term 'co-efficient of thermal conductivity' or simply 'thermal conductivity' is usually used to denote quantitatively how far a substance is a conductor of heat. It will be clear from the following explanation.

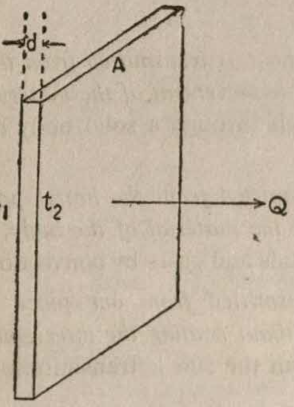


Fig. 9.1

Consider a rectangular plate of a certain material. The area of the plate$=A$, its thickness$=d$, and the temperatures of the two parallel faces$=t_1$, $t_2 (t_1 > t_2)$. Under this condition, heat will be conducted normally from the hotter surface to the colder through the plate (Fig. 9.1). It can be proved that the amount of heat Q conducted is (i) proportional to the area A *i.e.* $Q \propto A$ (ii) proportional to the temperature difference $(t_1 - t_2)$ between the faces *i.e.* $Q \propto (t_1 - t_2)$ (iii) inversely proportional to the thickness d of the plate *i.e.* $Q \propto \frac{1}{d}$ and (v) proportional to the time T for which the heat is allowed to flow *i.e.* $Q \propto T$.

Hence, $Q \propto \frac{A(t_1 - t_2)T}{d}$ or, $Q = \frac{K.A.(t_1 - t_2)T}{d}$, where K is a constant.

This constant 'K' is called the co-efficient of thermal conductivity or simply the thermal conductivity of a material.

If $A=1$, $(t_1 - t_2)=1$, $T=1$, $d=1$, then $Q=K$.

Definition : *The co-efficient of thermal conductivity of a material may be defined as the quantity of heat flowing per second through unit area of cross-section of an element of the material, of unit thickness when the difference of temperature between its faces is unity.*

For example, the co-efficient of thermal conductivity of copper is 0·92. It means that 0·92 calorie of heat will flow per second through 1 sq. cm. of an element of copper, of thickness 1 cm. when the difference of temperatures between its faces is 1°*C*.

It is obvious from the above definition that a high value of K makes a substance good conductor of heat while a low value of K a bad conductor. For a *perfect conductor*, $K = \infty$ and for a *perfect insulator*, $K=0$. No substance is however, a perfect conductor or a perfect insulator. So the extreme values of K are not found in reality.

In this connection, it is to be remembered that in the *C.G.S.* system,

Q should be expressed in calories
A ,, ,, ,, ,, sq. cm.
d ,, ,, ,, ,, cm.
T ,, ,, ,, ,, second
t_1, t_2 ,, ,, ,, ,, celsius

Hence, the unit of K is cal/cm/°C/sec (cal $cm^{-1}{}^{\circ}C^{-1}s^{-1}$)

And in the *F.P.S.* system,

Q should be expressed in Btu.
A ,, ,, ,, ,, sq. ft.
d ,, ,, ,, ,, ft.
T ,, ,, ,, ,, second
t_1, t_2 ,, ,, ,, ,, fahrenheit

Hence the unit of K is B.tu/ft/°F/sec (B.tu $ft^{-1}{}^{\circ}F^{-1}s^{-1}$)

In *M.K.S.* system, the unit of thermal conductivity$=Js^{-1}m^{-1}K^{-1}$ or since Joule second^{-1}=watt (W), the unit of K is $Wm^{-1}K^{-1}$.

9.3. Temperature variation with distance in a lagged and unlagged bar :

If the temperatures at the two ends of a bar of length l be θ_2 and θ_1 ($\theta_2>\theta_1$), then the quantity of heat conducted through the bar in time t is given by,

$$Q=\frac{K.A.(\theta_2-\theta_1)t}{l} \quad \text{or} \quad \frac{Q}{A.t}=\frac{K.(\theta_2-\theta_1)}{l}$$

Using the calculus symbols, $\frac{1}{A}\cdot\frac{dQ}{dt}=-K.\frac{d\theta}{dl}$ .. (i)

Negative sign appears because the temperature gradient, here, is a negative gradient (*i.e.* with the increase of l, the value of θ decreases).

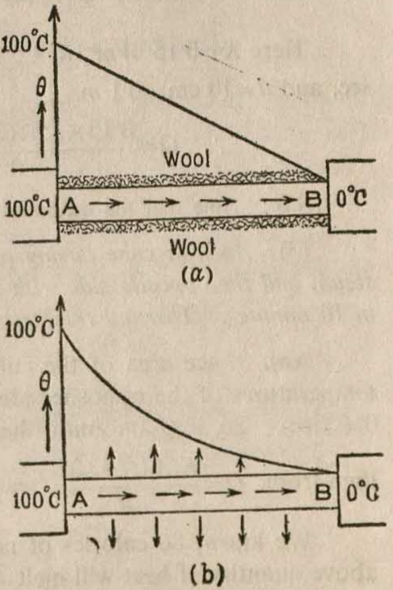


Fig. 9.2

(*a*) Suppose, the temperatures at the ends of the bar AB, which is lagged with non-conducting material wool, are 100°C and 0°C. Heat will not be radiated from the surface of the rod to the surrounding ; all the heat will be conducted through the bar from A to B. As a result, the rate of flow of heat *i.e.* $\frac{dQ}{dt}$ through any section of the bar will be same. From equation (i) it is seen that under this condition the temperature gradient $\left(\frac{d\theta}{dt}\right)$ along the bar remains constant. This is shown in Fig. 9.2(a) where the change of temperature along the length of the bar is a straight line.

(*b*) If the bar is unlagged, it will lose heat due to radiation from its surface to the surrounding. As a result, the rate of flow of heat $\left(\frac{dQ}{dt}\right)$ across any section of the bar will gradually decrease from the hot end to the cold end. Eqn. (i) shows that under this circumstances, the temperature gradient $\left(\frac{d\theta}{dl}\right)$ also decreases along the length of the bar. This has been shown in fig. 9.2(b) where the gradient at a point of the curve decreases with distance from the hot end to the cold end of the bar.

Examples : (1) *An iron plate is 4 mm. broad and its area is 150 sq. cm. The two opposite surfaces of the plate are at temperatures 100°C and 30°C and in 1 sec. 3940 calories of heat flow from one surface to the other. What is the thermal conductivity of iron ?* [*H.S. Exam. 1983*]

Ans. Here $d=4$ mm$=0{\cdot}4$ cm. ; $A=150$ sq. cm.

$(t_1-t_2)=100°-30°=70°C$; $Q=3940$ cal. $T=1$ sec. $K=?$

We know, $Q=\frac{K.A.(t_1-t_2).T}{d}$ or, $3940=\frac{K\times150\times70\times1}{0{\cdot}4}$

$$\therefore \quad K=\frac{3940\times0{\cdot}4}{150\times70}=0{\cdot}15 \text{ C.G.S. (nearly).}$$

(2) *Calculate the quantity of heat conducted through 1·5 m^2 of a brick wall 10 cm thick in half an hour, if the temperature on one side 10°C and on the other 30°C. Thermal conductivity of brick=0·15 $Wm^{-1}K^{-1}$.*

Ans. We know, $Q=\frac{K.A(t_1-t_2).T}{d}$

Here $K=0{\cdot}15\ Wm^{-1}K^{-1}$; $A=1{\cdot}5m^2$. $(t_1-t_2)=(30-10)=20°K$; $T=30\times60$ sec. and $d=10$ cm$=0{\cdot}1\ m$.

$$\therefore \quad Q=\frac{0{\cdot}15\times1{\cdot}5\times20\times30\times60}{0{\cdot}1} \text{ joule}=81\times10^3 \text{ joule}$$

[N.B : Note that the quantities are given in *m.k.s.* units]

(3) *An iron cube having an area of 4 sq. cm. has one side in contact with steam and the opposite side with ice. Calculate the amount of ice that would melt in 10 minutes. Thermal conductivity of iron=0·2.* [*Tripura H. S. 1981*]

Ans. Face area of the cube=4 sq. cm. ; hence its thickness=2 cm. ; the temperatures of the opposite sides of the cube are respectively 100°C (steam) and 0°C (ice). So, if Q amount of heat flows from one side to the other in 10 minutes, then from, $Q=\frac{K.A.(t_1-t_2)T}{d}$, we have, $Q=\frac{0{\cdot}2\times4\times100\times10\times60}{2}=24000$ calories.

We know, 80 calories of heat are necessary to melt 1 gm. of ice. So, the above quantity of heat will melt $\frac{24000}{80}=300$ gm. of ice.

(4) *The surface area of a glass window is 1 sq. metre. The thickness of glass in 3 mm. The temperatures outside and inside the room are respectively −5°C and 20°C. Calculate the rate of heat flowing out through the glass per second if the coefficient of thermal conductivity of glass be 0·002 c.g.s. unit.* [*Jt. Entrance 1982*]

Ans. Amount of heat conducted $Q=\dfrac{KA(t_1-t_2).T}{d}$

Here, $K=0{\cdot}002$ c.g.s. ; $A=1$ sq. metre$=10^4$ sq. cm. ; $(t_1-t_2)=[20-(-5)]=25°C$; $T=1$ sec and $d=3$ mm$=0{\cdot}3$ cm.

$$\therefore\quad Q=\frac{0{\cdot}002\times10^4\times25\times1}{0{\cdot}3}=1666{\cdot}6 \text{ cal.}$$

(5) *A cubical vessel of 10 cm. side is filled with ice at 0°C and is immersed in a water bath at 100°C. Find the time in which all ice will melt. Thickness of the vessel=0·2 cm. ; thermal conductivity of its material=0·2 C.G.S.; density of ice =0·92 gm/c.c.*

Ans. Volume of the ice$=10\times10\times10=1000$ c.c.

Mass ,, ,, ,, $=1000\times0{\cdot}92=920$ gm.

We know, 80 calories of heat are necessary for melting 1 gm. of ice. So, to melt 920 gm. of ice, the necessary amount of heat$=920\times80$ cal.

Suppose, the heat flows into the cubical vessel in T seconds. We know, $Q=\dfrac{K.A.(t_1-t_2).T}{d}$ Here, $K=0{\cdot}2$; $A=10\times10$ sq. cm.; $(t_1-t_2)=100°C$; $d=0{\cdot}2$ cm; since there are 6 sides of the vessel through which heat is entering, the total heat that enters into the vessel,

$$Q=6\times\frac{0{\cdot}2\times10\times10\times100\times T}{0{\cdot}2}=60000\times T \text{ cal.}$$

$$\therefore\quad 60000\times T=920\times80 \quad\text{or,}\quad T=\frac{920\times80}{60000}=1{\cdot}2 \text{ sec. (approx)}$$

(6) *A lagged copper bar, 10 cm long, whose ends are pressed against metal tanks at 0°C and 100°C, are separated from them by layers of dirt. The layer of dirt is 0·1 mm thick. Assuming that the conductivity of dirt is $\frac{1}{1000}$ that of copper find the temperature of each end of the bar.*

Ans. Let the conductivity of copper be K and the cross-section of the bar be α. Suppose the temperatures at the ends A and B are θ_2 and θ_1. Since the bar is lagged the rate of flow of heat $\dfrac{Q}{t}$ is constant from end A to end B.

Therefore, $\dfrac{Q}{t}=\dfrac{K}{1000}\cdot\dfrac{\alpha(100-\theta_2)}{0{\cdot}01}$

$$=\frac{K.\alpha(\theta_2-\theta_1)}{10}=\frac{K}{1000}\cdot\frac{\alpha(\theta_1-0)}{\cdot01}$$

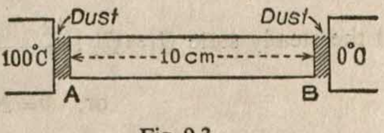


Fig. 9.3

Dividing by $K.\alpha$, we get,

$$\frac{100-\theta_2}{10}=\frac{\theta_2-\theta_1}{10}=\frac{\theta_1}{10} \text{ or } 100-\theta_2=\theta_2-\theta_1=\theta_1$$

$$\therefore\quad \theta_2=66{\cdot}7°C \text{ and } \theta_1=33{\cdot}3°C$$

(7) *One end A of a copper rod of uniform cross-section and of length 3 metres is in contact with a block of ice and the other end B is introduced in a water bath maintained at 100°C. The temperature of a point C on the rod is maintained at 300°C so that in the steady state, rate of melting of ice is equal to the rate of production of steam. Assuming the whole system to be insulated from the surroundings, find the distance of C from A. Take the latent heat of ice=80 cal/gm and latent heat of steam=560 cal/gm.*

Ans. Heat will flow from C to A as well as to B [fig. 9.4]. Let the mass of ice melted per sec. be m. Then the mass of steam produced per sec. is also m. For ice, the heat required$=m\times 80$ cal and for steam, the heat required$=m\times 560$ cal.

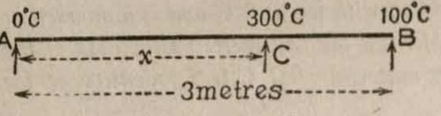


Fig. 9.4

Considering the flow of heat from C to A where the ice melts, $m\times 80=\frac{k.\alpha(300-0)}{x}$ [$k=$ conductivity and $\alpha=$cross-section of the rod]

Similarly considering the flow of heat from C to B where steam is produced, $m\times 560=\frac{k.\alpha(300-100)}{3-x}$. Dividing one by the other, $\frac{560}{80}=\frac{200}{3-x}\times\frac{x}{300}$

$\therefore\quad x=2{\cdot}74$ metres (nearly).

(8) *A sheet of rubber and a sheet of cardboard, each 2 mm. thick are pressed against each other. The free surface of rubber is at 0°C and that of cardboard is at 25°C. If the area of each sheet is 100 sq. cm., find the amount of heat conducted through the composite sheet per hour. Conductivity of rubber and cardboard are* 12×10^{-5} *and* 60×10^{-5} *respectively.*

Ans. Suppose the temperature of the common surface of the composite sheet is $\theta°C$. The temperature gradient along the rubber sheet$=\frac{\theta_2-\theta_1}{d}=\frac{\theta-0}{0{\cdot}2}=\frac{\theta}{0{\cdot}2}$ and that along the card-board sheet$=\frac{25-\theta}{0{\cdot}2}$. If Q_1 be the heat conducted through the rubber sheet in 1 sec, then $Q_1=\frac{12\times 10^{-5}\times 100\times\theta}{0{\cdot}2}$ cal.

Similarly for cardboard sheet $Q_2=\frac{60\times 10^{-5}\times 100\times(25-\theta)}{0{\cdot}2}$ cal.

At the steady state $Q_1=Q_2$; $\frac{12\times 10^{-5}\times 100\times\theta}{0{\cdot}2}=\frac{60\times 10^{-5}\times 100\times(25-\theta)}{0{\cdot}2}$

or, $\theta=20{\cdot}83°C$

If Q be the heat conducted per hour, then

$$Q=\frac{12\times 10^{-5}\times 100\times 20{\cdot}83\times 3600}{0{\cdot}2}=4499{\cdot}28 \text{ cal.}$$

(9) *Water is being boiled in a flat bottomed kettle placed on a stove. The area of the bottom is 300 sq. cm. and the thickness is 2 mm. If the amount of steam produced is 1 gm. per minute, calculate the difference of temperature between the inner and the outer surface of the bottom. The thermal conductivity of the material of the kettle=0·5 cal/°C/sec/cm: latent heat of steam=540 cal/gm.* [*I.I.T. 1970*]

Ans. Heat conducted to water through the bottom,

$$Q=\frac{KA(\theta_2-\theta_1).t}{d}=\frac{0\cdot 5\times 300\times(\theta_2-\theta_1)\times 60}{0\cdot 2}=\tfrac{9}{2}\times 10^4\times(\theta_2-\theta_1)\text{ cal.}$$

Heat absorbed by 1 gm of steam=1×540 cal.

$\therefore\ \ 540=\tfrac{9}{2}\times 10^4\times(\theta_2-\theta_1)$ or $\theta_2-\theta_1=0\cdot 012°C$

(10) *Three rods of material x and three rods of material y are connected as shown in the fig. 9.4(a). All the rods are of identical length and cross-sectional area. If the end A is maintained at 60°C and the junction E at 10°C, calculate the temperature of the junctions B, C and D. The thermal conductivity of x=0·92 and that of y=0·46* [*I.I.T. 1978*]

Ans. At the steady state, heat (Q_{AB}) flowing along *AB* per sec=sum of heats flowing along *BD* and *BC* per sec. ($Q_{BD}+Q_{BC}$).

i.e. $Q_{AB}=Q_{BD}+Q_{BC}$ [according to arrow heads]

or, $\dfrac{K_y.\alpha(60°-\theta_B)}{l}=\dfrac{K_y.\alpha(\theta_B-\theta_D)}{l}+\dfrac{K_x.\alpha(\theta_B-\theta_C)}{l}$ where l=length and α=cross-sectional area of each rod.

$\therefore\ \ K_y(60°-\theta_B)=K_y(\theta_B-\theta_D)+K_x(\theta_B-\theta_C)$

or $K_y(60°-2\theta_B+\theta_D)=K_x(\theta_B-\theta_C)$

or, $0\cdot 46(60°-2\theta_B+\theta_D)=0\cdot 92(\theta_B-\theta_C)$

or, $60°-2\theta_B+\theta_D=2\theta_B-2\theta_C$

or $4\theta_B-2\theta_C-\theta_D=60°$ .. (i)

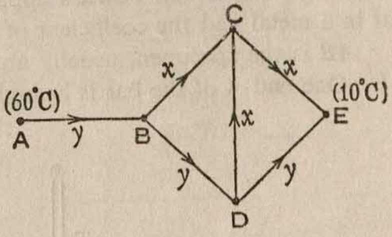


Fig. 9.4(*a*)

Considering the junction *D*, we have.

$Q_{BD}=Q_{DC}+Q_{DE}$

or. $\dfrac{K._y\alpha(\theta_B-\theta_D)}{l}=\dfrac{K_x.\alpha(\theta_D-\theta_C)}{l}+\dfrac{K_y(\theta_D-10°)}{l}$

or, $K_y(\theta_B-2\theta_D+10°)=K_x(\theta_D-\theta_C)$

or, $0\cdot 46(\theta_B-2\theta_D+10°)=0\cdot 92(\theta_D-\theta_C)$

or, $\theta_B-2\theta_D+10°=2\theta_D-2\theta_C$

or, $4\theta_D-2\theta_C-\theta_B=10°$ .. (ii)

Also considering the point *C*, we have $Q_{BC}+Q_{DC}=Q_{CE}$ [accordings to arrow heads]

$$\frac{K_x.\alpha(\theta_B-\theta_C)}{l}+\frac{K_x.\alpha(\theta_D-\theta_C)}{l}=\frac{K_x.\alpha(\theta_C-10°)}{l}$$

or, $(\theta_B-\theta_C)+(\theta_D-\theta_C)=(\theta_C-10°)$

or, $3\theta_C-\theta_B-\theta_D=10°$ .. (iii)

Solving equations (i), (ii) and (iii), we get $\theta_B=30°C$; $\theta_C=\theta_D=20°C$

9.4. Thermal resistance and thermal resistivity :

In the art 9·2, we have seen that $Q=\frac{k.A(t_1-t_2)T}{d}$

or, $\frac{Q}{T}=\frac{t_1-t_2}{\frac{1}{k}\cdot\frac{d}{A}}$ .. .. (i)

Now from electrical analogy, if we suppose that a potential difference of (V_1-V_2) exists between the opposite faces of the plate (fig. 9.1) and a current I flows normally from one face to the other, then from Ohm's law we have,

$I=\frac{V_1-V_2}{R}=\frac{V_1-V_2}{\rho.\frac{d}{A}}$ .. .. (ii)

where R=the electrical resistance of the plate and ρ=electrical resistivity. Comparing equations (i) and (ii), we can say $\frac{1}{k}\times\frac{d}{A}$=the thermal resistances (R) of the plate and $1/k$=the thermal resistivity of the material (ρ).

9.5. Determination of thermal conductivity of metals by Searle's apparatus :

Metals are good conductors of heat ; it is easy to get a rapid flow of heat along a metal bar. Searle's apparatus can be used to get such a good flow of heat in a metal and the coefficient of thermal conductivity therefrom.

AB is the specimen, usually about 4 cm. in diameter and 20 cm. long [Fig 9.5]. One end *A* of the bar is heated by passing steam through steam-chest *E* and

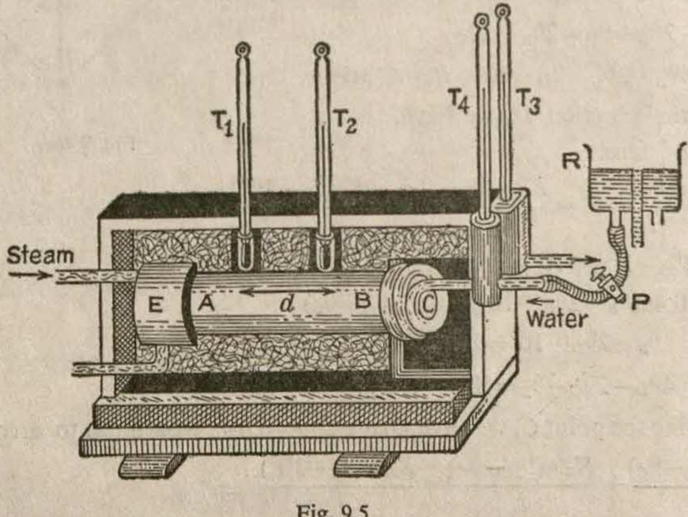


Fig. 9.5

the other end *B* cooled by circulating water from a reservoir *R*. The whole apparatus is heavily lagged with felt. To measure the temperature gradient, two thermometers T_1 and T_2 are placed in two mercury-filled cups. The cups are made of the same metal as the bar and are soldered to the specimen at a known distance d apart. The circulating water flows in through the inlet tube (shown by arrow head), goes round the copper coil which is soldered to the end B of the specimen and out through the outlet tube. The water leaving the outlet is warmer than the incoming water which shows that the temperature falls continuously along the bar.

The whole apparatus is left running, with a steady flow of water until the thermometers T_1, T_2, T_3 and T_4 read steady temperatures. The steady rate of flow of the cooling water is measured with a measuring cylinder and a stop-clock.

Let K=the coefficient of thermal conductivity of the specimen.

A=cross-sectional area of the bar.

θ_1 and θ_2=temperatures recorded by T_1 and T_2 respectively.

θ_3 and θ_4=temperatures recorded by T_3 and T_4 respectively.

t=time of flow of heat. Then, $Q=\frac{K.A(\theta_1-\theta_2)t}{d}$.

This heat is carried away by the cooling water. If m=mass of water collected in time t, then $Q=m(\theta_3-\theta_4)$.

$$\therefore \quad \frac{K.A(\theta_1-\theta_2)t}{d}=m(\theta_3-\theta_4) \quad \text{or} \quad K=\frac{m(\theta_3-\theta_4)d}{A(\theta_1-\theta_2).t}$$

All the quantities on the *R.H.S.* being known, K can be evaluated.

Table of thermal conductivities of some substances

Substance	Conductivity	Substance	Conductivity
Silver	0·97	Zinc	0·26
Gold	0·70	Lead	0·80
Copper	0·92	Iron	0·16
Aluminium	0·50	Glass	0·002

9.6. Comparison of conductivities of different substances :

The following experiments may be done to compare the conductivities of different materials.

Experiments : (1) Take three wires of copper, iron and aluminium each 50 cm. long and about 3 mm. in diameter. Put one end of each rod in the flame of a Bunsen burner as shown in the fig. 9.6(a) and bring a match-stick along the length of each rod from the cold end to the hot end. It will be seen that the match-stick catches fire at different lengths of the rods. In the copper rod, the match-stick will have to travel the least distance, then in the aluminium rod, while the stick will have to travel the furthest in the iron rod. This qualitatively shows that so far as conductivity is concerned, copper comes first, then aluminium and lastly iron.

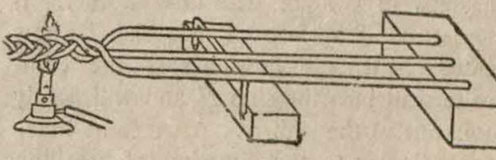


Fig. 9.6(a)

(2) **Ingenhausz's experiment** : Fig. 9.6(b) shows the arrangement for the experiment. A, B, C, D are rods of different materials but having the same length and diameter. The rods are equally coated with a layer of wax. They pass-through corks inserted in holes on the side of a metal trough. Boiling water is poured into the trough so that the ends of the rods inside the trough are all heated to the same temperature. Heat will be conducted through the rods and the coating of wax will melt. After some time has elapsed it is found that the wax has melted to different distances along the rods, indicating differences in their thermal conductivities. Evidently, the rod in which the wax melts to the longest distance, has the greatest thermal conductivity.

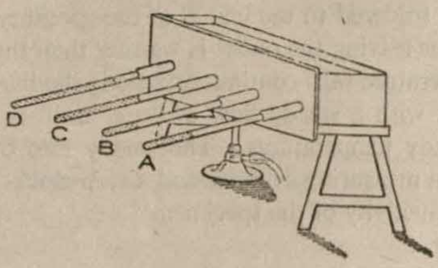


Fig. 9.6(b)

If the thermal conductivities of the rods be K_1, K_2, K_3 etc. and the lengths over which the wax melts in them be l_1, l_2, l_3 etc., then, it can be proved that

$$\frac{K_1}{l_1^2}=\frac{K_2}{l_2^2}=\frac{K_3}{l_3^2}=\ldots\text{etc.}$$

If the conductivity of one material is known, that of the other can be easily determined from the above relation. Two conditions are, however necessary for the establishment of the above relation. Firstly, the cross-sections of the rods should be all equal and secondly, the rods should have identical emissive power. Emissive power being dependent on the nature of the surface, to ensure equality of emissive power, each rod is given a similar coating of wax.

9.7. Conduction of heat through a composite wall :

Two slabs M and N with flat surfaces are kept in contact with each other as shown in fig. 9.7. The thickness and the thermal conductivity of the slab M are x_1 and k_1 respectively while those for the slab N are x_2 and k_2. The surfaces in contact are called composite walls and the slab as a whole as the composite slab. Clearly, a number of such composite walls may be formed by putting several blocks or slabs in contact with one another. If the left-hand surface of the slab be now heated, heat will flow across the composite wall into the slab N and will finally come out of the slab N. After some time, different layers of the composite slab perpendicular to the direction of heat-flow will attain a steady state. Then if θ_2 and θ_1 be the temperatures of the free surfaces of the composite slab and θ the temperature of the interface, the quantity of heat Q_1 flowing in time t through the slab M is given by. $Q_1=\dfrac{k_1A(\theta_2-\theta)t}{x_1}$ where A=area of each slab.

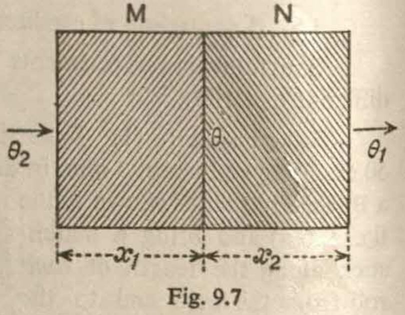


Fig. 9.7

Similarly, if Q_2 be the quantity of heat that flows through the slab N in the same time, $Q_2=\frac{k_2A(\theta-\theta_1)t}{x_2}$. In the steady state, $Q_1=Q_2=Q$(say). Hence,

$$\frac{Q}{t}=\frac{k_1A(\theta_2-\theta)}{x_1}=\frac{A(\theta_2-\theta)}{x_1/k_1}=\frac{A(\theta-\theta_1)}{x_2/k_2}=\frac{A(\theta_2-\theta_1)}{\frac{x_1}{k_1}+\frac{x_2}{k_2}} \quad \dots \text{(i)}$$

Now imagine that instead of the composite slabs, we have one single slab of thickness (x_1+x_2) and thermal conductivity k and its opposite faces are at temperatures θ_2 and θ_1 respectively so that the rate of flow of heat Q/t will be same as before. Then for the single slab, we have, $\frac{Q}{t}=\frac{A(\theta_2-\theta_1)}{(x_1+x_2)/k}$ . . . (ii)

Comparing eqn. (i) and (ii) we see that $\frac{x_1+x_2}{k}=\frac{x_1}{k_1}+\frac{x_2}{k_2}$

Hence, k is called the *equivalent thermal conductivity*. For n number of slabs, $\frac{x_1+x_2+\dots+x_n}{k}=\frac{x_1}{k_1}+\frac{x_2}{k_2}+\dots+\frac{x_n}{k_n}$.

Further from eqn. (i) we can find the temperature θ of the interface of the slabs in the following way,

$$\frac{\theta_2}{x_1/k_1}-\frac{\theta}{x_1/k_1}=\frac{\theta}{x_2/k_2}-\frac{\theta_1}{x_2/k_2} \text{ or } \theta\left(\frac{1}{x_2/k_2}+\frac{1}{x_1/k_1}\right)=\frac{\theta_2}{x_1/k_1}+\frac{\theta_1}{x_2/k_2}$$

$$\text{or } \theta\left(\frac{k_2}{x_2}+\frac{k_1}{x_1}\right)=\frac{k_1\theta_2}{x_1}+\frac{k_2\theta_1}{x_2} \quad \therefore \quad \theta=\frac{k_1\theta_2x_2+k_2\theta_1x_1}{k_2x_1+k_1x_2}$$

Examples : (1) *Four slabs of equal area have thickness 1 cm., 2 cm., 3 cm. and 4 cm. respectively. Being placed side by side, they form a composite slab. If their thermal conductivities are 0·2, 0·3, 0·02 and 0·04 respectively, find the equivalent thermal conductivity.*

Ans. If k be the equivalent conductivity, we know,

$$\frac{x_1+x_2+\dots+x_n}{k}=\frac{x_1}{k_1}+\frac{x_2}{k_2}+\dots+\frac{x_n}{k_n}; \text{ Hence } \frac{1+2+3+4}{k}=\frac{1}{0{\cdot}2}+\frac{2}{0{\cdot}3}+\frac{3}{0{\cdot}02}+\frac{4}{0{\cdot}04}$$

$$\text{or } \frac{10}{k}=261{\cdot}6 \quad \therefore \quad k=0{\cdot}038 \text{ (nearly)}$$

(2) *Two equal plates with thermal conductivities k_1 and k_2 are joined together to form a single plate of double thickness. Show that the equivalent thermal conductivity of the composite plate is $2k_1k_2/(k_1+k_2)$.*

Ans. If k be the equivalent conductivity, then for two plates,

we have, $\frac{x_1+x_2}{k}=\frac{x_1}{k_1}+\frac{x_2}{k_2}$; Since $x_1=x_2$,

$$\frac{2}{k}=\frac{1}{k_1}+\frac{1}{k_2} \quad \therefore \quad k=\frac{2k_1k_2}{k_1+k_2}$$

(3) *A bar of copper of length 75 cm. and a bar of steel of length 125 cm. are joined together end to end. Both are of circular cross-section with a diameter 2 cm.*

The free ends of copper and steel are maintained at 100°C and 0°C respectively. The surface of the bars are thermally insulated. What is the temperature of the copper-steel junction ? What is the heat transmitted per unit time across the junction ? Thermal conductivity of copper$=9{\cdot}2\times10^{-2}$k.cal/m/°C/s and that of steel$=1{\cdot}1\times10^{-2}$ k.cal/m/°C/s. [*I.I.T.* 1977]

Ans. Conductivity of copper, $k_1=\dfrac{9{\cdot}2\times10^{-2}\times10^3}{10^2}=0{\cdot}92$ cal./cm./°C/s

„ „ steel $k_2=\dfrac{1{\cdot}1\times10^{-2}\times10^3}{10^2}=0{\cdot}11$ cal./cm./°C/s

Area of cross-section $A=\pi r^2=\pi(1)^2=\pi$ sq. cm.

Heat conducted per sec is given by, [Fig. 9.8]

$$Q=\frac{k_1A(\theta_1-\theta)}{l_1}=\frac{k_2A(\theta-\theta_2)}{l_2}$$

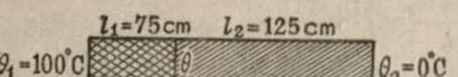


Fig. 9.8

or, $k_1l_2(100-\theta)=k_2l_1(\theta-0)$

or, $0{\cdot}92\times125\times(100-\theta)=0{\cdot}11\times75\times\theta$

or, $11500-115\ \theta=8{\cdot}25\ \theta$

$\therefore\ \theta=93{\cdot}3$°C (nearly)

Again, $Q=\dfrac{k_2A(\theta-\theta_2)}{l_2}=\dfrac{0{\cdot}11\times\pi\times93{\cdot}3}{125}=0{\cdot}258$ cal./sec.

9.8. Conduction of heat through a slab of varying thickness ; Freezing of lake :

The temperature of air in cold countries very often goes below 0°C and the water in the lakes, ponds etc. freezes into ice. At first, the surface layer of water in a lake, being in contact with ambient cold air, gives up latent heat to the atmosphere and freezes into thin layer of ice. Then the lower layer of water, in its attempt to give up latent heat, has to send heat through the layer of ice on the surface. In this way, as the thickness of ice increases, the water below has to transmit heat through such increasing thickness of ice.

Let the temperature of the ambient air over a lake be$-\theta$°C and the initial thickness of the layer of ice is x cm. Further, let the thickness of the layer of ice on the lake increases by a very small amount dx in time dt. If A be the area of the surface of the lake, the volume of ice formed$=A\times dx$ and its mass$=A\times dx\times\rho$ where ρ=density of ice at 0°C.

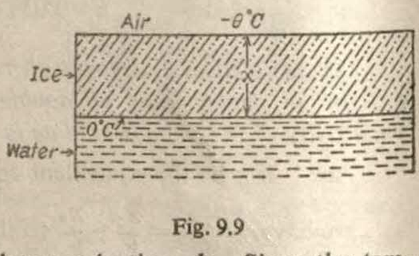


Fig. 9.9

If L be the latent heat of ice, the quantity of heat given up by ice$=A\times dx\times\rho\times L$ and this heat is conducted to air outside through a slab of ice of thickness x in time dt. Since the temperature at the bottom of the ice slab is 0°C and that on the upper surface$-\theta$°C, we have, $A\times dx\times\rho\times L=\dfrac{K.A.[0-(-\theta)].dt}{x}=\dfrac{K.A.\theta}{x}\cdot dt$ or $x.dx=\dfrac{K\theta}{\rho L}\cdot dt.$

If the thickness of the slab increases from x_1 to x_2 in time t then by integration, we get, $\int_{x_1}^{x_2} x.dx=\frac{k.\theta}{\rho.L}\int_0^t dt.$ or, $\frac{1}{2}(x_2^2-x_1^2)=\frac{K\theta}{\rho L}.t$ $\therefore$ $t=\frac{\rho L(x_2^2-x_1^2)}{2K\theta}$ ..(i)

If $x_1=0$ and $x_2=x$ (say) *i.e.* if in time t, a slab of thickness x is formed then

$$t=\frac{\rho.Lx^2}{2K.\theta} \quad \text{..} \quad \text{(ii)}$$

Example : *Find the time in which a layer of ice 3 cm. thick on the surface of a pond will increase its thickness by 1 mm. when the temperature of the surrounding air is—20°C, from the following data : thermal conductivity of ice=0·005 c.g.s. ; latent heat of fusion of ice=80 cal/gm. ; density of ice at 0°C=0·91 gm/c.c.*

Ans. From eqn. (i) of the preceding article, we have, $t=\frac{\rho.L(x_2^2-x_1^2)}{2k.\theta}$

Here, $\rho=0{\cdot}91$ gm/c.c. ; $L=80$ cal/gm. ; $k=0{\cdot}005$; $\theta=20°$C ; $x_1=3$ cm. ; $x_2=3{\cdot}1$ cm. $\therefore$ $t=\frac{0{\cdot}91\times80}{2\times{\cdot}005\times20}[(3{\cdot}1)^2-(3)^2]=\frac{0{\cdot}91\times80\times6{\cdot}1\times0{\cdot}1}{2\times{\cdot}005\times20}$ sec.

=3 min. 42 sec.

9.9. Variable state, steady state and thermal diffusion :

When one end of a metal rod is left in a fire, the other end inevitably becomes hot after some time, and the whole rod acquires what might be called a *steady state* which is reached when the various parts of the rod acquire maximum temperatures. The rod then loses heat at the same rate that the rod gains it. The end in and near the fire is visibly hot, the remote end hot but visibly not so, and intermediately there is a temperature gradient along the length of the rod.

Prior to reaching steady state a number of events occur. At the beginning, the heat received by any transverse layer of the rod is spent in three ways : (*a*) a portion of the heat received is absorbed and thereby the temperature of the layer is raised. This rise of temperature of the layer is proportional to the specific heat of the material of the bar, (*b*) a portion is radiated from the exposed surface of the layer and (*c*) the remaining portion is transferred to the next layer by conduction.

Hence, at the beginning, the temperature of each layer gradually rises. This change of thermal state of the rod with time is called the *variable state* when both conduction and absorption of heat take place. Their combined effect is what is called *diffusivity*.

The rise of temperature of different layers cannot continue indefinitely. After some time, a *steady state* is reached when each layer acquires a fixed temperature which gradually falls from hot end to cold end of the rod. During this time, no heat is absorbed by the rod.

In the variable state, the rate of diffusion of heat through a rod, does not only depend on the thermal conductivity of the material of the rod but also on the specific heat of the material. It can be proved that

$$\text{Diffusivity}=\frac{\text{Coeff}^n\text{ of thermal conductivity}}{\text{Sp. heat}\times\text{density}}=\frac{K}{S.\rho}$$

[K=coefficient of the thermal conductivity ; S=sp. heat and ρ=density]

Definition : *Diffusivity or thermometric conductivity is defined as the rate of rise of temperature of a body when the body is passing through the variable state.*

In the variable state, both conductivity and specific heat have importance but in the steady state, since there is no absorption of heat, only conductivity is significant in the transference of heat.

A substance may have a high diffusivity and a low conductivity. On the other hand, substances are available having a low diffusivity and a high conductivity. Thus, bismuth has greater diffusivity than iron but the conductivity of bismuth is less than that of iron.

9.10. Different substances have different diffusivity :

Four flat bars of different materials such as copper, iron, zinc and slate are fixed to the rim of a circular wooden frame in such a manner that they almost touch one another at the centre of the circular frame. When the bars are heated by a flame kept at the centre, each end of the different bars will be heated to the same temperature and heat will travel along the bars. A piece of phosphorus is placed in each bar in a cup at the extreme end of the bars at equal distances from the centre. It will be found that the phosphorus at the end of the copper bar very soon ignites, followed by that on the zinc ; later that at the end of the iron bar catches fire but the phosphorus on the slate never burns. This experiment shows that the rise of temperature is the fastest in copper and slowest in slate. In other words, *the diffusivity of different materials is different.*

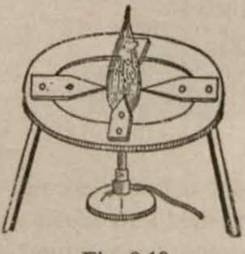


Fig. 9.10

9.11. Good and bad conductors and their uses :

Almost all metals are good conductors of heat. Of different metals, silver is the best and copper the next best conductor of heat. In general, liquids are better conductors than gas ; solids, on the other hand, better than liquids. For example, the heat conductivity of copper is about 700 times more than that of water, which is again nearly, 25 times more conductive than gas. That air is a bad conductor of heat can be realised from the following simple experiment.

Take some coarse powder of chalk in one hand and some very fine powder in the other. If you now transfer a hot metallic ball from one hand to the other, you will feel it hotter in the hand which contains fine powder. The reason is as follows. The coarse grains of chalk confine some air in it, but the fine powder does not do so. Air being a bad conductor of heat, the hot metallic ball cannot conduct heat through air to the hand. So it does not appear very hot. On the other hand, there being no air in the fine powder, heat is quickly conducted to the hand and the ball appears very hot.

Glass is a bad conductor of heat. When a piece of glass is heated in one place, the neighbouring parts of the glass do not warm up with it, because heat is not quickly conducted on all sides. Consequently, the colder parts resist expansion of the heated part and the force thus set up cracks the glass. Similarly glass which has been heated should be made to cool slowly and uniformly to avoid cracking. For this reason, special care is taken to prepare chimneys made of glass.

Wool, cotton, asbestos, rubber etc. are all non-conductors of heat. As heat insulators, they are used extensively. Asbestos fibres are used as lining to the walls of refrigerators and cookers and asbestos cement prepared with plastics, are used as heat insulating covers of boiler and steam pipes. Wool, cotton etc. are used in making warm clothes, quilts, rugs etc. which we use in winter.

Cooking utensils, small boilers, calorimeters are made of copper because copper is a good conductor of heat. Aluminium is used in making cylinder and piston head of engines. Davy's safety lamp which the miners use, is made of copper gauge.

The following simple experiments amply illustrate the properties of good conductors and bad conductors of heat.

(1) **Boiling of water in a paper bag :**

The temperature at which paper catches fire is much higher than that of boiling water. This makes it possible to boil a small quantity of water in a *thin* paper bag, without charring the paper [Fig. 9.11]. Much of the heat supplied to the paper passes through it. While this heat raises the water to its boiling point the temperature on the lower side of the paper remains below the value at which the paper ignites.

If the bag is made of *thick* paper, the paper will burn out before the water boils because thick paper can not conduct heat to water as quickly as thin paper.

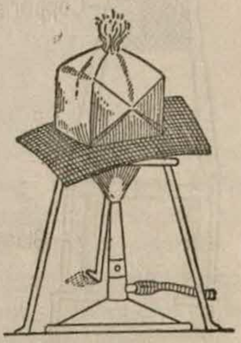


Fig. 9.11

(2) **Flame and wire-gauge experiment :**

When a dirt-free copper gauge is lowered on the flame of a bunsen burner, the flame can not get through the gauge [Fig. 9.12 (i)]. The flame burns below the gauge. Copper being a very good conductor of heat, the gauge conducts the heat

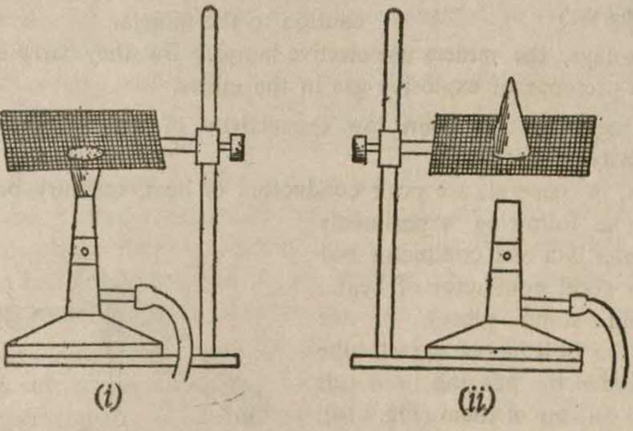


Fig. 9.12

away from the flame so quickly that the temperature on the other side of the gauge remains below the ignition temperature of the gas. Combustion is therefore, confined to the part of the flame below the gauge. After some time, of course, the gauge itself is heated to the ignition temperature and the flame spreads to the gas above it.

When the gauge is held a few inches above the burner and the flame lighted from the top, it burns above the gauge [Fig. 9.12(ii)] and can not spread below the gauge. Here also, the good conductivity of the gauge prevents the flame from extending below the gauge.

(3) **Davy's safety lamp :**

The principle illustrated in flame and gauge experiment above was utilised by Sir Humphry Davy for the construction of safety lamp for coal miners. It can be used in mines without any risk of explosion even when explosive gases are present in the mine.

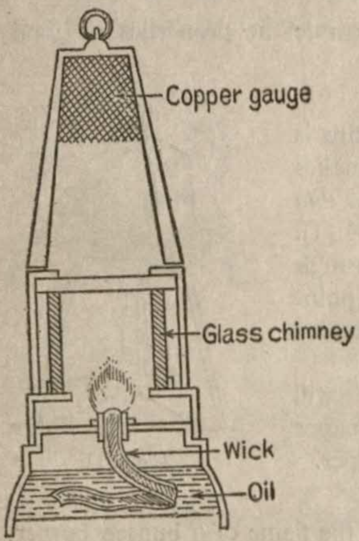


Fig. 9.13

Fig. 9.13 shows the appearance of a safety lamp. It is an oil lamp with a copper wire gauge surrounding the flame. In an explosive atmosphere, the gases which penetrate into the flame are ignited and burn inside the lamp. But the flame cannot extend outwards through the gauge because the gauge conducts the heat away from the burnt gases very rapidly. The explosive gas outside the lamp takes a long time to become hot enough to explode but by that time the oil becomes exhausted and the lamp is put out. If the gas outside is explosive, it burns with a distinctive colour inside the lamp, the colour being a caution to the miners.

Now -a-days, the miners use elective lamps. But they carry a safety lamp to detect the presence of explosive gas in the mines.

9.12. Experiment to show low conductivity of water and comparatively high conductivity of mercury :

Liquids, in general, are poor conductors of heat, mercury being the only exception. The following experiments show that water is a bad conductor but mercury is a good conductor of heat.

(1) Hold some pieces of ice in place at the bottom of a test tube containing water by pushing in a tuft of steel wool on top of them (Fig.9.14). Heat the water near the top of the tube by means of a gas flame. Soon the water will start to boil, yet the ice will not

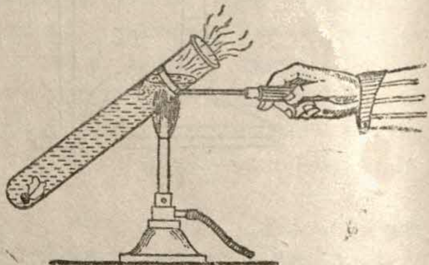


Fig. 9.14

melt. Water being a bad conductor of heat, very little heat is transferred to the ice which remains at the bottom unmelted.

(2) Take equal quantities of water and mercury in two thin walled glass test tubes and attach a small piece of cork to the bottom of each with melted wax (Fig. 9.15). A piece of thick copper wire bent twice at right angles is then inserted in the test tubes with an arm in each of the two liquids. Apply heat to the centre of the wire with a burner. Heat is conducted through the copper wire equally into the two liquids. In a very short time wax on the mercury-filled tube melts and the cork is dislodged. Very prolonged heating is necessary before the same occurs in the water-filled tube. This simple experiment shows that mercury is a much better conductor of heat than water.

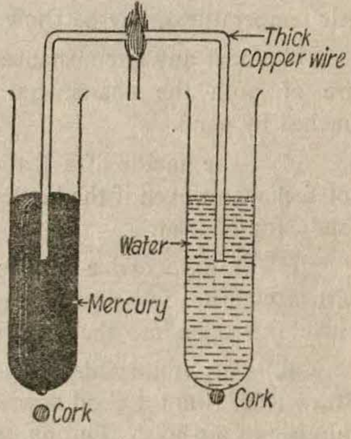


Fig. 9.15

9.13. Some practical illustrations of conduction of heat :

(1) The warm clothes we wear in winter are not really warm. They are as cold as ordinary clothes. But as these clothes keep our body warm during winter, we call them warm clothes. These are generally made of wool and contain many air-pockets. Air is a bad conductor of heat. These air-pockets in woollen garments prevent body-heat from being conducted away and hence keep our body warm. The fibres of a cotton garment are not so loose and do not possess air-holes. So, they cannot effectively keep our body warm.

For the same reason, it is comfortable to wear two shirts in winter than one shirt of equal thickness as the two. Two shirts will confine an air layer between them which prevents body-heat from being conducted away. So our body is kept warm. If one shirt of equal thickness is used instead, it will not prevent passage of heat as effectively as before.

You may have noticed that an old quilt is not as comfortable as a new one. In a new quilt, a lot of air is enclosed in the cotton which prevents conduction of heat, whereas in an old quilt, the air is squeezed out due to constant use.

(2) Glass stoppers in bottles often get tightly fixed and attempt to remove them by force generally results in breakage. Such stoppers can be easily removed if the mouth of the bottle is slightly heated. The neck expands on heating but as glass is a bad conductor of heat, the stopper does not get the heat quickly and therefore, does not expand like the neck.

(3) In a cold room, metallic substances appear very cold to touch but not the wooden substances, although a thermometer will show that both of them have the same temperature. Metals being good conductors of heat, metallic substances draw heat from the hand as soon as they are touched and we get a feeling of coldness. But wood is not a good conductor and hence wooden substances do not appear cold to touch.

In the same way, if an iron chair and a wooden chair be left in the sunshine for some time, iron chair will appear much hotter than wooden chair although their temperatures may be shown to be equal.

If under any circumstances, the body temperature be equal to the temperature of both the chairs, both will appear equally hot or equally cold when touched by hand.

(4) The handle of a kettle is wrapped up with cane and the handle does not feel warm even if the kettle is filled with boiling water, because cane is a bad conductor of heat.

(5) Lumps of ice can be kept without melting for some time if covered with saw-dust. Saw-dust being bad conductor of heat does not allow heat from outside to enter into the lump which, therefore, remains unmelted for a while.

(6) In countryside, many dwelling houses have thatched or straw roof. Straw itself is not a good conductor of heat. Further straw roof encloses a large number of air-holes. During summer, heat from outside cannot enter the house, the inside of which consequently, remains cool. In winter again, the heat from within the house cannot go out due to the bad conductivity of straw and air and the house remains warm. Such things do not happen in a house with tin roof. Tin being a good conductor of heat, the house becomes too hot in summer and too cold in winter.

CONVECTION

9.14. Some experiments of convection of heat :

(1) Take a rectangular glass vessel as shown in fig. 9.16 and fill with water. Drop a piece of potassium permanganate crystal at the mouth of the vessel and apply heat below a vertical arm *AB*. You will see that a stream of colourless water ascends along the arm *AB* while a coloured stream descends along the arm *CD*. In this way, a current of water will be found to circulate. After some time, of course the whole mass of water will attain the same temperature. Heat is here actually carried from one place to another in the liquid by the movement of the liquid itself. This phenomenon is called **convection.**

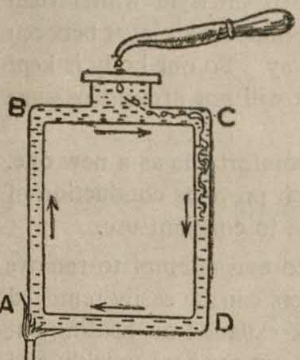


Fig. 9.16

It is to be noted that by the process of convection, heat can be transferred through a fluid *only in the upward direction* and not in the downward or lateral direction. The process of conduction, however, transfers heat in all directions.

(2) Like water, convection current may be set up in air. The following simple experiment can be performed.

Pour some water in a vessel and put a burning candle in it. A chimney

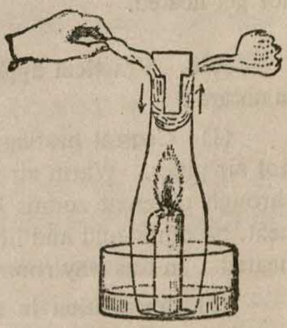


Fig. 9.17

is placed over the candle in such a way that air cannot enter the chimney from the bottom (Fig. 9.17). The flame is soon extinguished owing to lack of oxygen. If now a *T*-shaped card-board is placed over the chimney as shown in the figure, the candle continues to burn. The card-board divides the chimney into two compartments which facilitates setting up of convection current. If a piece of smoking paper (a piece of blotting paper soaked in turpentine and dried) be held over the mouth of the chimney, profuse smoke will be found to come from one chamber and pass out from the other showing that a convection current has been set up in the air. Receiving fresh supply of oxygen from convection current the flame continues to burn.

(3) Convection current in air is also responsible for the burning of a table lamp or a hurricane lamp. The frame on which the chimney of the lamp is fitted contains several holes. When the flame burns, the air above the flame becomes hot and light and goes out of the chimney through the opening at the top. Fresh air enters into the chimney through the holes in the frame and maintains the supply of oxygen. The lamp, therefore, continues to burn.

If the holes be closed by wax, the supply of oxygen is cut off and the flame extinguishes after some time.

(4) It is hotter the same distance over the top of a fire than it is in front. The reason is the convection current. Air over the top of the fire, being heated goes higher up while the colder air from the sides rushes towards the fire. In this way, a convection current in air is set up. For this, it feels very hot at the top of the fire. Very little heat is conducted sideways because air itself is a bad conductor of heat. The little heat that passes sideways is due to radiation. The flow of cold air from the sides again reduces this heat.

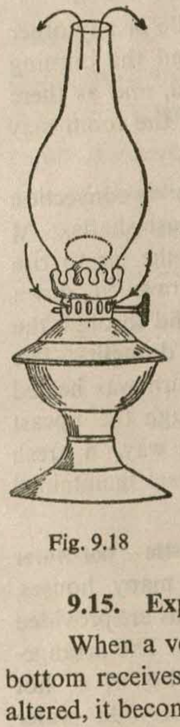


Fig. 9.18

9.15. Explanation of convection current :

When a vessel containing a liquid is heated, a portion of the liquid near the bottom receives the heat first and it expands. As the mass of the liquid remains unaltered, it becomes lighter and floats up just as a piece of wood floats up or a hydrogen-filled balloon left in the air, rises up. In effect, convection may be considered as an application of Archimedes' principle.

On the other hand, if the upper portion of the liquid in the vessel is heated, the liquid there expands and becomes lighter while liquid below is denser. The upper portion of the liquid, therefore, remains floating at the top and no convection current is set. The only way for the heat to travel downwards is conduction but liquid being a bad conductor of heat the lower portion of the liquid does not get heated.

9.16. Practical applications of convection of heat for different purposes and in nature :

(1) **Central heating system** : Room heating in winter is usually done by hot air pipes. Warm air being less dense than cold air, goes up and is circulated through different rooms by means of supply pipes. The hot air, on giving up heat, becomes cold and heavy and comes down by means of exit pipe and is again heated. In this way rooms are kept hot by setting up convection current in air.

(2) **Ventilation in a room** : Convection plays an important part in the ventilation of rooms. The air in the room becomes warm due to constant breathing of the inhabitants and for other reasons. This impure air should be ventilated out and fresh air from outside be allowed to enter the rooms. For this, ventilators are fitted almost near the roof of the rooms. The impure hot air goes up and passes out through the ventilators and fresh pure air from outside enters the rooms through the windows and doors.

It is unwise to sleep in a completely closed room with a candle or any other light burning, because it may cause death. Due to breathing and the burning of candle, the oxygen of the enclosed air soon becomes exhausted, and as there is no fresh supply of oxygen from outside, the persons sleeping in the room may die of suffocation.

Formerly ventilation in coal-mines was arranged by setting up convection current with the help of two shafts—known as upcast and downcast shafts. At the bottom of the upcast shaft a fire was ignited. This caused the air to rise up the shaft. Air travelled down the downcast shaft and through the mine to maintain up draught. The incoming air in its turn was heated and passed out through the upcast shaft again. In this way, a fresh air-flow was always maintained through the mine.

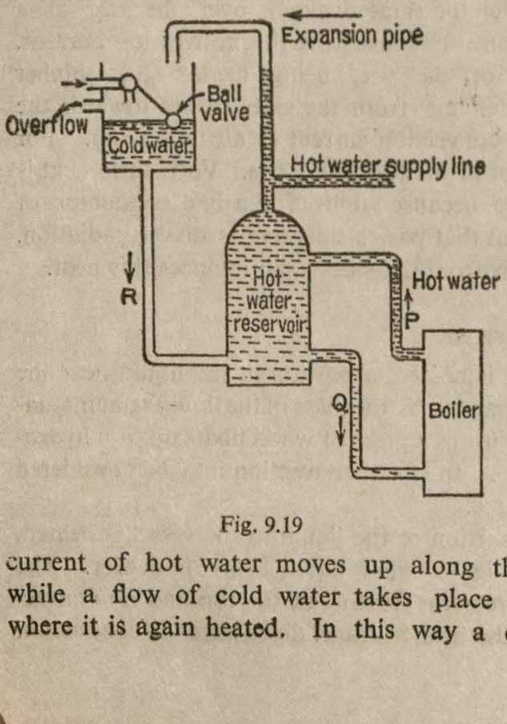


Fig. 9.19

(3) **The domestic hot-water supply system** : In many houses, kitchens and bath-rooms are provided with hot-water supply. The arrangement consists of a boiler, a hot water reservoir and a cold water reservoir, inter-connected by pipes [Fig. 9.19]. When the water in the boiler is heated a convection current of hot water moves up along the pipe *P* into the hot water reservoir while a flow of cold water takes place through the pipe *Q* into the boiler where it is again heated. In this way a circulation of water is set up with

the result that the hot water reservoir is gradually filled up with hot water from the top downwards. When the level of hot water in the hot-water reservoir reaches the supply line pipe, hot water is supplied to the kitchen etc. Taps are provided with the supply line so that hot-water may be drawn whenever necessary. It is to be noted that the pipe *P* leaves the boiler at the top and enters the hot-water reservoir at the top while cold water pipe leaves the hot-water reservoir at the bottom and enters the boiler at the bottom.

When water (hot) is drawn through the supply line, equal volume of cold water enters the hot reservoir through the pipe *R* from the cold water reservoir. The whole system is thus kept full of water and no air can enter into the pipe. A supply of cold water is maintained through the ball-valve to keep the level there constant.

If, the water in the boiler boils vigorously at any time, the hot water may rush up. As a precaution, expansion pipe is provided which discharges the excess water into the cold water reservoir.

(4) **Wind** : Winds are convection currents set up in the atmosphere as a result of unequal heating of the earth by the sun. The air over the more heated area of the earth becomes less dense than that of the other parts and goes up while colder air from the neighbouring regions rushes in to fill up the space. This causes what we call a wind.

(5) **Land and Sea breezes** : In sea-coasts a breeze blows from sea to land during the day and from land to sea during night. Land, having a lower specific heat than water, gets warmed up more quickly than water in the sea during day-time. Consequently, the air above the land gets heated and rises and the cooler air from the sea flows in to take its place, thus causing a **sea-breeze** [Fig. 9.20(*a*)]. The sea-breeze starts blowing in the day and becomes intense towards the evening.

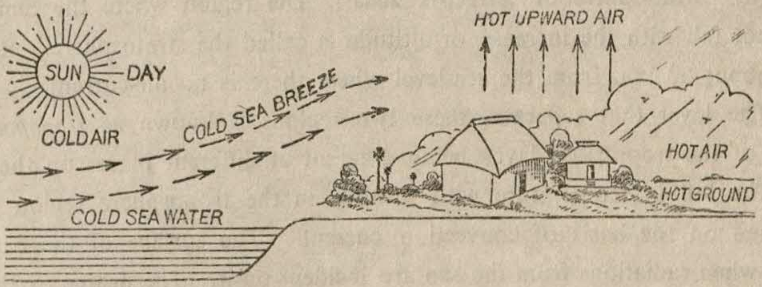


Fig. 9.20(a)

After sunset, however, the land which has acquired less heat than the sea on account of its lower specific heat, cools more quickly than the sea and the air above it becomes cooler than that above the sea ; the direction of the current of air is then reversed and during the night a **land-breeze** sets in [Fig. 9.20(*b*)]. This breeze becomes intense towards the dawn.

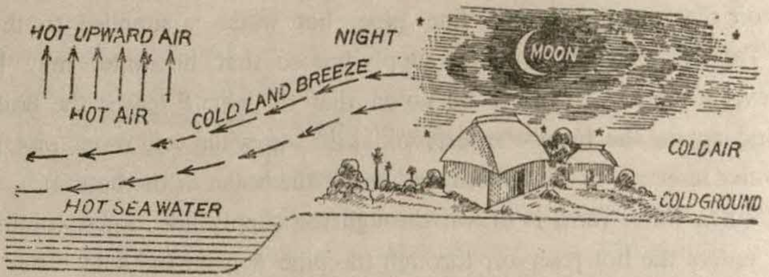


Fig. 9.20(b)

9.17. Convective equilibrium of the atmosphere :

The temperature of air over the earth's surface decreases with altitude upto a height of about 20 km. The rate of fall of temperature has been found to be about 5°C per km. height from the sea-level. In this respect, the atmosphere is divided into two regions.

(i) **Troposphere or the convective zone** : The region where the temperature falls with the increase in height from the surface of the earth is called the troposphere. It extends upto about 20 km. from the surface of the earth. The density of the atmosphere in this region falls off as the height increases and the temperature falls to a minimum of about −55°C.

(ii) **Stratosphere or advective zone** : The region where the temperature does not fall with the increase of altitude is called the stratosphere. It extends upto about 50 km. from the sea-level where there is no air-current.

The layer that separates these two regions is known as *tropopause*, the height of the tropopause layer being different at different places on the earth.

The fall in temperature with altitude in the troposphere region may be explained on the basis of convection current. The surface of the earth gets heated when radiations from the sun are incident on it. The heated earth warms up the air surrounding it by conduction and radiation. The warm air then moves up and a convection current is set up. The heated air moves from the regions of higher to lower pressure and is adiabatically cooled. In the same way, when colder air from above moves down from lower to higher pressure regions, it is adiabatically heated. Thus a convective equilibrium is set up and there is a gradual fall of temperature with increase in height. This is why we find the top of a mountain much cooler than near sea-level.

RADIATION

9.18. Properties of radiant heat :

As already mentioned, heat is said to be propagated by radiation, when it is transmitted through a material medium without the medium playing an active part by itself getting heated. The heat from the sun reaches the earth by means of radiation.

As a matter of fact, all heated bodies radiate heat and the radiant heat bears a close similarity with light. The following properties of radiant heat will establish its identity with light :

(1) Heat radiations, like light, travel in all directions from a hot body.

(2) Thermal radiation travels through vacuum with the velocity of light. This is why they are both cut off at the same instant in a solar eclipse.

(3) Thermal radiation, like light, travels in straight line. For instance, the heat coming from the sun can be cut off by interposing an umbrella because it travels in straight line.

(4) Thermal radiation is reflected or refracted like light. You may have seen how a piece of paper may be charred by concentrating solar rays by a lens like light rays.

(5) Thermal radiations have other optical properties, like interference, diffraction and polarisation.

(6) Radiant heat, like light, obeys the law of inverse square. The law states that the intensity of the radiation at a point due to a given source, is inversely proportional to the square of the distance of the point from the source.

9.19. Nature of radiant heat :

In discussing the properties of thermal radiation, it has been mentioned in the preceding article that there are close similarities between radiant heat rays and visible light rays. As a matter of fact, they belong to the same family known as the family of **electromagnetic waves.** Gamma rays, X-rays, visible light, radiant heat, wireless waves—all of them belong to the family of electromagnetic waves, having the same properties and velocity (about 1,86,000 miles/sec) except that their wave-lengths are different. When the wave-length lies between 5000 metres to 25 metres, we have wireless waves which are used for broadcasting radio programmes. These waves produce no sensation of sight or heat to us. When the wave-length becomes smaller and lies between 4×10^{-2} cm. to 8×10^{-5} cm., the waves produce sensation of heat and are known as radiant heat waves. As wave-length becomes gradually smaller, we get one after another, visible light, X-rays and gamma rays.

A body, before being heated to redness, emits heat waves of comparatively longer wave-length, known as **infra red rays** which have many useful applications. When the body is red hot, it emits heat rays as well as visible red light rays which have the largest wave-length among the seven visible light waves. For this reason, a very hot body looks red. When the body is heated still further so that it is white hot, it emits, along with the heat rays, all the seven colours of visible spectrum. Seven colours being mixed together make the body look white.

9.20. Instruments for detection and measurement of radiant heat :

(*a*) **Differential air thermometer** : This instrument, devised by Leslie, is now only of historical importance and may be used for demonstration purpose.

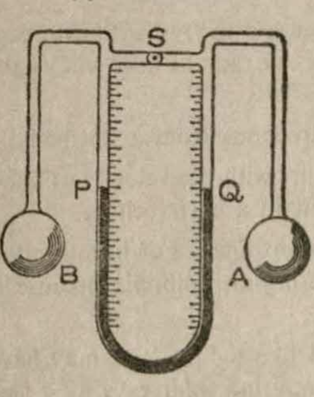


Fig. 9.21

It consists of two equal bulbs *A* and *B*, containing air, connected by a *U*-tube serving as a manometer [Fig. 9.21]. A non-volatile liquid, like sulphuric acid, is put in the manometer. Before measuring the radiant heat, the levels of the acid in the manometer are made equal as *P* and *Q* by opening the stop-cock *S*. Now, the radiation to be measured is allowed to fall on any one of the bulbs, say *B* while the other bulb *A* is shielded from the radiation. The air in *B* is heated and expands pushing the manometer liquid to a lower level. The difference in the levels *P* and *Q* of the manometer thus created is a measure of the intensity of the incident radiation.

(*b*) **Thermopile** : *See heating effect of current*, second volume of this book.

(*c*) **Bolometer** : Langley was the first to utilise the uniform change of resistance of a platinum strip with temperature to construct a very useful and sensitive instrument for measuring thermal radiation. Bolometer is the name given to this instrument.

It consists mainly of very thin platinum strips joined together in series, resembling a grid of parallel elements [Fig. 9.22]. The strips are coated with platinum black so that it may absorb all the radiation incident on it.

Two such grids, identical in all respects are arranged to form the two ratio arms of a wheat-stone bridge. Before measuring the thermal radiation, the two other arms of the bridge are adjusted so that the galvanometer shows no deflection. Thermal radiation, then, is allowed to fall on one of the grids, the other being shielded from the radiation. Its resistance increases instantaneously, the balance is upset and the galvanometer shows a deflection. The bridge is again balanced and the increase in resistance is found out. It is possible to calculate the intensity of the radiation incident on the bolometer for the increase of its resistance. Langley used his bolometer to measure the feeble intensity of the radiation reaching the earth from the moon.

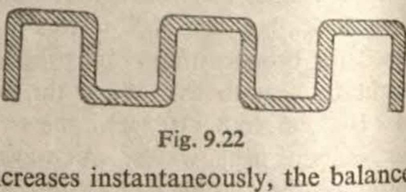


Fig. 9.22

9.21. Some important illustrations in connection with radiation and absorption :

Every body radiates and absorbs heat, more or less. It depends upon several factors *viz.*, the temperature of the body and its surroundings, the nature of its surface, its material etc. It may, however, be shown very easily that *good radiators are good absorbers and bad radiators are bad absorbers.* For example, a black body is a good radiator ; it is a good absorber too. On the other hand a polished metallic surface is a bad radiator as well as a bad absorber. If we hold a piece of white card, with a patch of black drawing ink on it, in front of a fire, the black patch soon comes to feel warmer than its white surrounding, showing that a black body is a good absorber of heat radiation.

In this connection, a few common examples are cited below :—

(1) If the outer surface of a kettle is polished, it will take more time to boil water than one whose outer surface is rough and blackened. A rough and blackened surface will absorb more heat from the fire and will make the water boil quickly, while most of the heat will be reflected back by the polished surface. For this reason, the bottom of cooking vessels should be made black and rough instead of white and polished.

For the same reason, a pair of polished and shining shoes gives greater comfort.

(2) Black clothes should not be worn on a hot day since black is a good absorber of sun's radiant heat. While black is also a good emitter, the external temperature is higher than the body temperature and the exchange rate is, therefore, such as to heat the body. White clothes are worn in hot climates because white is a good reflector and therefore a poor absorber.

(3) Umbrella cloth is usually made black for it can radiate sun's radiant heat quickly in all directions, keeping the space inside the umbrella comparatively cooler.

(4) Humid air is a good absorber of heat as compared with dry air. For this reason, in a winter day when the sky is cloudy, the air becomes humid and absorbs more heat from the sun. On such days, we do not feel very cold. When the atmosphere becomes clear and the air dry, absorption of heat lessens and the temperature falls.

We generally feel warm at night when the sky is cloudy. The surface of the earth receives heat during the day-time, but cools down at night by radiation of heat in the atmosphere. If the sky be cloudy, the radiant heat emitted by the hot surface at night, is reflected back by the cloud which makes the air warm.

(5) Take two thermometers of identical nature, having one of the bulbs coated with lampblack. Expose them to sun rays. After some time, the thermometer with black bulb will be found to read a higher temperature than the other. This shows that under similar conditions, a black body absorbs more heat than a white one.

(6) Transparency of different substances with respect to radiant heat rays is different. Those substances which are transparent to heat-rays of given wave-

lengths, are called *diathermanous* with respect to those wave lengths and those substances which are opaque to those given wave lengths, are called *athermanous* with respect to the wave lengths. In this regard, glass is a very good example. Ordinary glass is transparent to visible light, but not to the longer, invisible waves given off, say, by soil that has been warmed by the sun. This fact is made use of in a *greenhouse* built for the purpose of preserving rare and valuable plants and flowers. The sun's rays of smaller wave lengths pass readily through the glass roof and are absorbed in the soil within. This, being warmed, then emits rays of its own. But these are mainly long waves which cannot pass through the glass. So the greenhouse acts like a heat trap. In cold countries, the inside of a greenhouse is sufficiently warm.

(7) In desert regions, it is extremely hot during day-time but extremely cold at night.

Air over a desert land being very dry act as a diathermanous substance to sun-rays which easily penetrate the atmosphere and fall on the ground. The sandy ground, as a result, becomes very hot in the day. At night, the hot ground radiates heat which easily passes through the atmosphere into the space and the ground becomes cold very quickly.

(8) If the outside surface of a tea-cup is polished, tea remains hot in the cup for a long time. The radiating power of a polished surface is insignificantly small. So hot tea retains its heat. No radiation takes place from the surface of the cup.

9.22. Emission and absorption of radiation :

Experiments show that the thermal radiation emitted by a hot surface depends on the following :

(i) *the temperature* (θ_1) *of the surface*

(ii) *the temperature* (θ_2) *of the surroundings*

(iii) *the nature and area* (A) *of the surface*

(iv) *the interval* (t) *during which the radiation takes place.*

If R be the total thermal radiation emitted by the surface, then $R = eA(\theta_1 - \theta_2).t$, where e is called the *emissive* or *radiating power* of the surface.

Emissive power is different for different substances, although their temperatures may be equal. For a given temperature of a body if e_λ be the emissive power of a surface, then the radiant energy emitted per sq. cm. per sec. between the wavelengths λ and $\lambda + d\lambda$ is given by $E = e_\lambda \, d\lambda$.

From our daily experience, we know that a polished surface reflects most of the radiation incident on it while a rough and black surface absorbs most of it. Different substances, however, have different absorptive powers, just as they have different emissive powers. The absorptive power a_λ of a surface for a given temperature and wavelength λ may be defined as the ratio of the radiation absorbed to the radiation incident on it between λ and $\lambda + d\lambda$.

For a given temperature of a body if a_λ be the absorptive power of a surface, then the radiant energy absorbed per sq. cm. per sec between the wavelengths λ and $\lambda+d\lambda$ is given by $E=a_\lambda\, d\lambda$.

9.23. Good radiators are good absorbers ; Ritchie's experiment :

This experiment demonstrates that a good emitter (or a radiator) is a good absorber of heat. The apparatus used by Ritchie is shown in the fig. 9.23.

A and *B* are metal vessels with flat faces connected to each other by means of a bent glass tube which contains some coloured liquid. The vessels *A* and *B* contain air only. *C* is another hollow metal vessel placed between *A* and *B*. The face of *C* turned towards *A* is lampblacked and the other face is polished. The faces of *A* and *B* turned towards the cylinder *C* are respectively polished and lampblacked. Thus, of two opposite faces, one is lampblacked and the other is polished.

If the central vessel *C* be filled with boiling water, *no change in the level of the coloured liquid in the bent tube is observed.* The reason is that the lampblack surface of *C* emits considerable amount of heat radiation and the polished surface of *A* opposite to it absorbs a very little of it. Again the polished surface of *C* opposite to *B* emits only a little radiation which is completely absorbed by the blackened surface of *B* facing the cylinder *C*. Thus the greater emissive power of the black surface of *C* is compensated by a small absorptive power of the polished face of *A* while the feeble radiating power of the polished surface of *C* is balanced by the greater absorbing power of the black face of *B*. Consequently the level of the coloured liquid in the bent tube is not changed.

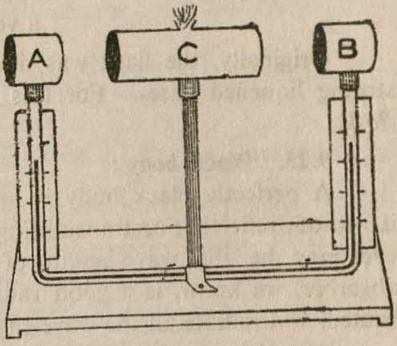


Fig. 9.23

If the central cylinder *C* is now rotated through 180° so that the two polished faces of *A* and *C* face each other while the two blackened surfaces of *B* and *C* also face each other, there will at once be a marked difference in levels of the coloured liquid in the bent tube. This is evidently due to the fact that a good radiator now faces a good absorber and a bad radiator a bad absorber. This experiment, therefore, conclusively proves that a good emitter is a good absorber of heat and a bad emitter is a bad absorber of heat.

9.24. Thermosflask :

The thermosflask or the vacuum flask keeps a hot liquid (tea, milk etc.) hot or a cold drink cold for a long time. It can do so because its manufacturing technique seals all three ways of exchange of heat between the inside and outside of the vessel. The hot liquid inside can, therefore, retain its heat while a cold drink can retain its coldness.

Fig. 9.24 shows a section of the flask. It consists of a double-walled glass vessel having a vacuum between the walls. It has a rather narrow neck and its mouth is closed by a cork. The glass vessel is kept inside a metallic cover and the two are separated by a spring. The metallic cover is evidently a protection to the glass vessel. No heat can enter or leave the inner flask by conduction or convection across the vacuum. A certain amount of heat may enter the flask through radiation but that is also reduced to a minimum by making the inside surface of the outer glass wall and outside surface of inner glass wall silver-polished. As glass itself is a bad conductor of heat, the glass wall conducts very little heat. Only a little heat may be transmitted by conduction through the mouth which is again closed by a piece of cork—a non-conducting material. The sum-total of the heat transfer being very small, cold liquid remains cold and a hot liquid hot for a very long period.

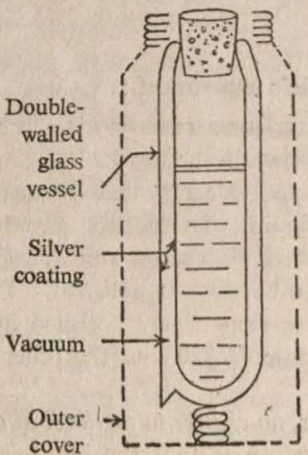


Fig. 9.24

Originally, the flask was devised by Sir James Dewar for the purpose of storing liquefied gases. For this reason, the flask is sometimes called ***Dewar flask***.

9.25. Black body :

A perfectly black body is one which absorbs all the radiations falling on it without reflecting or transmitting any of it. It will always look black, whatever may be the wave-length of thermal radiation incident on it. A good absorber, we know, is a good radiator. Hence, a perfectly black body, when heated, will radiate all the wavelengths from ultra-violet to infra-red.

It is, however, impossible to get a perfectly black body. Lampblack absorbs about 95% and platinum black about 98% of the incident radiation. Kirchoff proved that if an enclosure be made with walls of a material impervious to all radiations and if the enclosure be kept at a constant temperature, it will behave like a perfectly black body. The radiation emitted by such an enclosure at a particular temperature depends on the temperature alone and not on the material of the wall. If a substance be kept in the enclosure, it will attain the temperature of the enclosure at the steady state and it will, then, emit black body radiation at that temperature.

On the basis of the above fact, F′ery designed a black body, known as F′ery's black body (Fig. 9.25). It consists of a hollow, double-walled, metallic sphere with a small opening *O* at one side. The inside of the sphere is blackened with

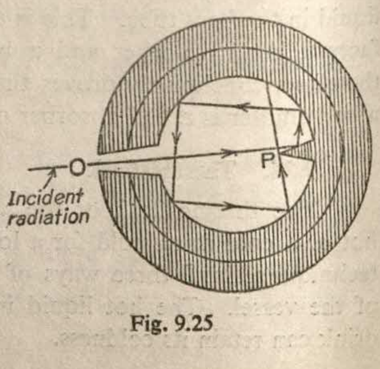


Fig. 9.25

lamp-black and the outside surface is nickel-polished. The space between the two walls of the sphere is evacuated to prevent loss of heat by conduction and convection. If any thermal radiation enters the sphere, it will suffer multiple reflections in the sphere as shown in the figure and will ultimately be completely absorbed by the sphere. A conical projection P is arranged at the back wall exactly opposite to the opening O. It prevents any radiation from coming out of the orifice O if it happens to suffer direct reflection at the back wall opposite to O. When this sphere is maintained at a desired high temperature, the orifice O (not the walls) of the sphere acts as a perfect black body and emits radiation.

9.26. Kirchoff's law :

Kirchoff first established a law connecting the emissive power and absorptive power of a body. The law states that *the ratio of the emissive power and the absorptive power for a given wave length is constant for all bodies at the same temperature and is equal to the emissive power of a perfectly black body at that temperature.*

If e_λ and a_λ be the emissive power and the absorptive power of a body at a given temperature T and for a given wave length λ, the law gives, $\frac{e_\lambda}{a_\lambda}=E_\lambda$ where E_λ is the emissive power of a perfectly black body at temperature T and wave length λ.

From the above relation we may conclude that a_λ *i.e.*, the absorptive power of a substance increases with the increase of e_λ *i.e.*, the emissive power. In other words, a good radiator is also a good emitter. Ritchie's experiment described in art. 9.23 is an experimental verification of Kirchoff's law.

Kirchoff's law finds one of the most important applications in the explanation of the Fraunhoffer lines in the solar spectrum. (See **Optics, 2nd volume**).

9.27. Prevost's theory of exchange :

Since fire produces a sensation of warmth and a piece of ice that of coldness, the idea that hot bodies radiate heat and cold bodies emit cold radiation was prevalent for some time until in 1972, Prevost pointed out the incorrectness of the conception and proved that thermal radiation was essentially an exchange process.

If a cold block of metal is brought into a warm room, *it radiates heat to the walls* of the room and the walls also radiate heat to the block. Because, the walls are at a higher temperature, they emit more heat per second to the block than the block gives up in return. Due to this unequal exchange of heat, the temperature of the cold block rises while that of the hot wall falls until the temperature of the block becomes equal to the temperature of the wall. Even then the block and wall, each radiates and absorbs at exactly the same rate.

Prevost's theory of exchange states that *a body at the temperature of the surroundings radiates and absorbs heat at equal rates.*

According to this theory, a body cools when it emits more radiation than it receives from its surroundings and warms up when it absorbs more radiation than it emits. When the radiation emitted is equal to the radiation absorbed, the temperature of the body remains constant. Even when the temperature is equalised, an exchange of radiation continues.

9.28. Stefan's law :

The total energy emitted by a body depends only on its temperature. The relation between the radiant heat emitted by a body and its temperature was first discovered through extensive laboratory experiments carried out by Stefan. The same law was later derived from theoretical consideration by Boltzmann and is now known as *Stefan-Boltzmann law*. Symbolically, the law may be stated as : $E=\sigma T^4$

where E represents the energy radiated per second by a body at an absolute temperature T and σ is a constant, known as *Stefan's constant* and has the value $1{\cdot}37\times10^{-12}$ cal/cm²/sec.

The law, however, applies only to so-called "black bodies".

The Stefan-Boltzmann law may, otherwise, be stated as : The total amount of heat (for all wave-lengths) radiated by a perfectly black body per second per unit area is directly proportional to the fourth power of its absolute temperature,

i.e., $E\propto T^4$ or $E=\sigma T^4$.

This law is sometimes called Stefan's *fourth power law*.

In order to calculate the net energy lost by a hot body, the above law can be generalised as follows : "If a black body at absolute temperature T is surrounded by another black body at absolute temperature T_0, the amount of net energy lost per second per sq. cm. by the body at the higher temperature is given by,

$$E=\sigma(T^4-T_0^4) \quad \cdot\cdot \quad \text{(i)}$$

9.29. Newton's law of cooling from Stefan-Boltzmann law :

Newton's law of cooling states that the rate of cooling of a body is proportional to its excess temperature over its surroundings provided the excess temperature is small. The law may be regarded as a special case of Stefan-Boltzmann law ; for it can be proved in the following way that for a hot body at temperature T kept in an enclosure of temperature T_0, Stefan-Boltzmann law would reduce to Newton's law of cooling, provided $(T-T_0)$ is small.

We know that the net rate of heat lost by a hot body at temperature T, placed in an enclosure at temperature T_0 is given by

$$E=\sigma(T^4-T_0^4)=\sigma(T-T_0)(T^3+T^2.T_0+T.T_0^2+T_0^3)$$

If $(T-T_0)$ is small *i.e.* if $T=T_0$, the above equation may be written in the following way :

$$E=\sigma(T-T_0)\times4T_0^3=K(T-T_0) \text{ where } K=4.\sigma.T_0^3$$

If the temperature T_0 of the enclosure be kept constant, K becomes a constant quantity. Hence, $E\propto(T-T_0)$.

In other words, the rate of cooling of a body is directly proportional to its excess temperature over its surroundings. This is Newton's law of cooling.

9.30. Application of the law :

From the mathematical expression of the law, it is apparent that by measuring the total heat radiated by a body, its temperature can be determined, provided the value of the Stefan's constant is known. As a matter of fact, solar temperature is measured in this way. The instrument used for this purpose is called radiation pyrometer.

(i) Solar temperature :

The central part of sun is intensely hot. It is called *photosphere.* Surrounding the photosphere there is a comparatively cooler atmosphere, known as *chromosphere.* Solar temperature means the temperature of the photosphere.

Suppose the radius of the photosphere is R and its temperature is T°K. If the photosphere be regarded as a perfect black body, then according to Stefan's law, the energy radiated by the sun per minute : $E=4\pi R^2\sigma T^4\times 60$ [σ=Stefan's constant].

If a sphere be imagined to be drawn with the radius equal to the distance between the sun and the earth, then the above energy will fall normally on the sphere. If the mean distance be r, the surface area of the sphere$=4\pi r^2$.

So, the energy incident per unit area per minute

$$=\frac{4\pi R^2\sigma T^4\times 60}{4\pi r^2}=\frac{R^2\sigma T^4}{r^2}\times 60$$

Now, the *solar constant* is defined as the average amount of energy which will fall on one sq. cm. of a black surface per minute, held normal to the sun's rays and placed at mean distance of the earth from the sun. If S be the solar constant, then according to its definition, $S=\frac{R^2.\sigma.T^4}{r^2}\times 60$

$$\therefore\quad T^4=\frac{S}{60\sigma}.\left(\frac{r}{R}\right)^2 \quad \text{or} \quad T=\left\{\frac{S}{60\sigma}.\frac{r^2}{R^2}\right\}^{\frac{1}{4}}$$

Experimental data show that $S=1{\cdot}937$ cal/ sq. cm./min.

$\sigma=1{\cdot}37\times 10^{-12}$ cal/sq. cm./sec.

$r=9{\cdot}28\times 10^7$ miles

$R=4{\cdot}3\times 10^5$ miles

These values give the solar temperature as $T=5755$°K (nearly)

(ii) Radiation pyrometer :

Radiation pyrometer may be used to estimate temperature of hot bodies. The basic principle of this instrument is Stefan's law. This instrument has the advantage that unlike thermometers, thermo-couples and other common temperature measuring devices, it need not be brought in contact with the hot body and hence can measure very high temperature.

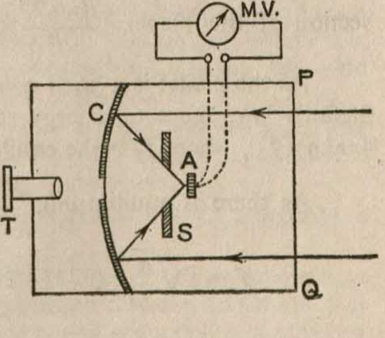


Fig. 9.26

Fig. 9.26 shows a section of the radiation pyrometer. Radiant heat rays from the hot body are allowed to enter the instrument through the aperture PQ. The rays are sharply focussed through a diaphragm S on to a blackened plate A by a concave mirror C. The exact focussing is adjusted by viewing the image of the source through a telescope T fixed behind a hole in the concave mirror. One end of a thermo-couple is in contact with the plate A and a sensitive millivoltmeter (M.V.) is connected at the other end. When radiation falls on the plate A, the receiving end of the couple is heated and a current flows through the thermo-couple. The e.m.f. of the thermo-couple is recorded by the millivoltmeter.

The thermo e.m.f. developed is proportional to the temperature difference between the hot and cold ends of the couple. If T_1 and T_2 be the temperatures of the hot body and the receiving junction of couple respectively, then according to Stefan's law, $E=a(T_1^4-T_2^4)$ where a is a constant.

Since T_1 is very large compared to T_2, T_2^4 can be neglected in comparison to T_1^4.

It is to be noted that the e.m.f. is not exactly proportional to the fourth power of the absolute temperature of the hot body for the following reasons : (i) Stefan's law is strictly applicable to black bodies and the actual hot bodies are seldom perfectly black (ii) T_2^4 is not zero and (iii) Conduction of heat through the connecting wire and stray radiation raises the temperature of the cold junction. For this reason, the pyrometer is calibrated in comparison with a standardised thermo-couple and the value of the power is found to vary from 3·8 to 4·2.

Example : *Assume the sun to be a sphere of radius* $7{\cdot}5\times10^5$ *km. having a surface temperature 5727°C, radiating out energy following Stefan's law. A planet at a distance of* 15×10^7 *km from the centre of the sun, is assumed to be a conducting sphere of radius 6400 km., completely absorbing the incident solar radiation and radiating energy outwards from its surface following Stefan's law. Calculate the equilibrium temperature of the planet.* [*Jt. Entrance 1978*]

Ans. The total energy radiated by the sun per second, $E=4\pi R^2\sigma T^4$.

Now, the energy incident per unit area at a distance d from the sun

$$=\frac{E}{4\pi d^2}=\frac{4\pi R^2.\sigma.T^4}{4\pi d^2}=\frac{R^2.\sigma.T^4}{d^2}$$

Hence, the energy received by the planet $=\dfrac{R^2.\sigma.T^4}{d^2}\times$ area of the cross-section of the planet $=\dfrac{R^2.\sigma.T^4}{d^2}\times\pi r^2$ [r=radius of the planet].

As the planet is a good conductor and as it radiates energy according to the Stefan's law, the total energy radiated from the entire surface of the planet$=4\pi r^2.\sigma\times T_1^4$, where T_1 is the equilibrium temperature of the planet.

As there is equilibrium, $\dfrac{R^2.\sigma.T^4}{d^2}\times\pi r^2=4\pi r^2.\sigma\times T_1^4$ $\quad\therefore\quad T_1^4=\dfrac{R^2.T^4}{4.d^2}$

$$\text{or,}\quad T_1=T\sqrt{\frac{R}{2d}}=(5727+273)\times\sqrt{\frac{7{\cdot}5\times10^5}{2\times15\times10^7}}=6000\times\sqrt{\frac{1}{20\times20}}$$

$$=6000\times\tfrac{1}{20}=300°\text{K}.$$

Exercises

Essay type :

1. What are the different modes of transference of heat ? Explain them with suitable illustrations. What are their differences ?

2. What are good and bad conductors of heat ? Give a few examples. Cite some applications of good and bad conductors of heat.

3. One end of a metal rod of uniform cross-section is inserted into a steam chamber and the other end is in contact with a lump of ice. Explain the variation of temperature along the rod with a graph when (i) the rod is well lagged and (ii) when the rod is exposed to air.

4. Show, with the help of an experiment, that different substances have different thermal conductivities.

5. Write notes on the properties and nature of thermal radiation. What is the difference between radiant heat and light ?

6. Describe one experiment in each case to show that (i) copper is a better conductor of heat than iron (ii) water is a bad conductor of heat (iii) a black body is a better absorber of heat than a polished body (iv) mercury is a better conductor of heat than water.

7. "Good emitters are good absorbers and bad emitters are bad absorbers". Discuss the statement with suitable experiment.

8. Describe, with a neat diagram, the construction of a thermosflask and explain its action.

9. Explain how the temperature of the sun can be estimated by the application of Stefan's law. Define solar constant.

10. Explain (*a*) black body (*b*) black body radiation.

Describe an instrument which can be used to measure the temperature of a furnace at about 1200°K, using the radiation from the furnace. Explain how the instrument is used to obtain a value of the temperature.

11. State and explain Stefan-Boltzmann Law. Using the law deduce Newton's law of cooling which states that if the temperature difference between a hot body and its surroundings be small, the rate of cooling of the body is proportional to the excess temperature of the body over the surroundings.

Short answer type :

12. What are 'conductivity' and 'coefficient of thermal conductivity' ? Is conductivity different for different materials ? What is its unit in the c.g.s. system ?

13. Thermal conductivity of glass is ·002—explain the statement fully.

14. Two metal bars *A* and *B* of the same size but of different materials are coated with equal thickness of wax and placed each with one end in a hot bath. It is noted that at first the wax on *A* melts at a greater rate than that on *B* but that when a steady state has been reached, a greater length of wax has been melted on *B* than on *A*. Explain.

15. Answer the following questions :—(i) If you touch a piece of iron and a piece of wood lying exposed to the heat of sun, which feels hotter and why ? (ii) Why are roofs of houses in countryside made of straw ? (iii) Why are woollen garments called 'warm' ? (iv) Why is the handle of a kettle wrapped with cane ?

16. A test tube filled with water is held in inclined position and the upper part of the tube is then heated. On touching the lower part of the test tube, it does not feel hot. But when heat is applied to the lower part, the upper part feels hot. What does it indicate ?

17. When a piece of metal cooled by liquid air (temperature = −180°C) is touched, a sensation of scorching is produced. Why ?

[**Hints :** The piece is so much cooled that when touched by finger, heat very quickly flows from the finger to the metal. As a result, the portion of the finger in contact with the piece is frozen and sticks to it. To keep the finger away, the skin is bruised and a feeling of scorching is produced.]

18. What connections are there between an object's feeling hot or cold and its neat capacity as well as its thermal conductivity ?

19. A block of wood and a block of metal are at the same temperature. When the blocks feel cold, the metal feels colder than wood ; when the blocks feel hot, the metal feels hotter than the wood. Explain. At what temperature will the blocks feel equally cold or hot ?

20. On a winter day, the temperature of the inside surface of a wall is much lower than the room temperature and that of the outside surface is much higher than the outdoor temperature. Explain.

[**Hints** : Brick is a bad conductor of heat.]

21. Answer the following questions :—(i) Why does the 'greenhouse' keep hot in winter ? (ii) It is hotter the same distance over the top of a fire than it is in front. Why ? (iii) Which will give you better protection against cold—one thick shirt or two of half the thickness ; the material being the same ? (iv) White clothes are not preferred in winter. Why ? (v) Why is the umbrella cloth usually black ? (vi) You are given two identical mercury thermometers and the bulb of one of them is blackened. What difference in their readings will be noticed if they are exposed to a clear cloudless night ? (vi) A room with large glass windows requires more heating in winter than one where walls are more modestly pierced. Why ?

22. If hot air rises, why is it cooler at the top of a mountain than near sea level ?

23. Two identical metallic cans contain some hot water. The walls of one can are polished while those of the other are blackened. In which can, the water will cool rapidly ? Give reasons for your answer.

24. Two rods *A* and *B* are of equal length. The temperatures at the two ends of each bar are θ_1 and θ_2. Under what condition will the rate of flow of heat in the rods be equal ?

[**Hints** : the condition is $K_1\alpha_1=K_2\alpha_2$]. [*Jt. Entrance 1985*]

25. What do you mean by diathermanous and athermanous bodies ?

26. You have taken a very fine powder of chalk in one hand and a coarse powder in the other. If you exchange a hot metal ball from one hand to the other, which hand will feel warmer ?

27. Answer the following questions :—(i) Why does a squirrel wrap its bushy tail round its body during its winter sleep ? (ii) Why are saucepans usually made of metal ? (iii) Why do birds puff their feathers out on a cold day ? (iv) Why are motor car radiators provided with metallic fins ? (v) A house with a straw roof keeps cool in summer and warm in winter. How ? (vi) In summer the upper surface of water in a tank is warm but lower surface is cold. In winter, the lower surface of water is hot but the water at the upper surface is cold. Why ? (vii) Bodies of human beings wrapped up in rugs keep warm in winter but a piece of ice similarly wrapped up keeps cold in summer. Why ? (viii) A stone floor feels very cold to bare feet in winter but a carpet in the same room feels comfortably warm. Why ? (ix) What should be the nature of the bottom of a cooking vessel—smooth, black or white ? [*H. S. Exam. 1982*]

28. Explain why felt rather than air is used for thermal insulation even though the thermal conductivity of air is less than that of felt ? [*Jt. Entrance 1985*]

[**Hints** : Felt is itself an insulator ; it also prevents loss of heat due to convection.]

29. A sphere, a cube and a thin circular plate are made of same material and are of same mass. They are heated to a temperature of 200°C and are left in a room. Which one will cool most rapidly and which one most slowly ?

30. A thermosflask contains milk. The flask is vigorously shaken. Consider the milk as the system. Does its temperature rise ? Has heat been added to it ? Has work been done on it ? Has its internal energy changed ?

31. What do you mean by a perfectly black body ? Can a very hot body like a furnace be regarded as a perfectly black body ?

32. An enclosure contains a black body *A* which is in thermal equilibrium with it. A second black body *B*, at a higher temperature than *A*, is then introduced in the enclosure. If the enclosure is maintained at constant temperature and all heat-exchange is by radiation, state and explain how the temperatures of *A* and *B* will change with time.

33. State Stefan's law. What is Stefan's constant ? What is its application ?

Objective type :

34. Select the correct answer in the following questions :—

(i) A wooden rod can be held at one end for a pretty long time with the other end pushed into a fire but an iron rod similarly placed cannot be held for long time because iron is denser/a better conductor of heat/has less diffusivity than wood.

(ii) A cloudy night is found to be warmer than a clear night because cloud is opaque to radiant heat/air is bad conductor of heat/cloud is a bad conductor of heat.

(iii) It is hotter the same distance over the top of a fire than it is in front because air is a good absorber/convection current takes hot air up/conductivity of air is poor.

(iv) In vacuum electric lamps, the heat emitted by the filament travels the space between the filament and the wall of the bulb by conduction/convection/radiation.

(v) In day time, land is heated to a higher temperature than the sea water because water has a higher specific heat/water is a poor conductor of heat/water is less dense than earth.

(vi) A perfectly black body is one which absorbs all incident radiation/reflects all incident radiation/absorb most of the incident radiation/is coated with lamp black.

(vii) A bolometer measures heat radiation/thermal conductivity/thermo e.m.f./specific heat.

(viii) A wall has two layers A and B, each made of a different material. Both the layers have the same thickness. The value of K for A is twice that of B. Under thermal equilibrium, the temperature difference across the wall is 30°C. The temperature difference across the layer A is 6°C/12°C/18°C/24°C. [*I.I.T. 1980*]

Numerical type :

35. A copper plate 1 metre long, 1 metre broad and, 1 cm. thick has two opposite faces at a difference of temperature of 10°C. How much heat will flow through the plate per minute ? Thermal conductivity of copper$=0{\cdot}96$ c.g.s. [Ans. 576×10^4 cal]

36. Two vessels made of different material are similar in shape and dimension. They are fully filled with ice at 0°C. By the heat from outside all the ice melts in 25 minutes in one vessel and in 20 minutes in the other vessel. Compare their thermal conductivities. [Ans. 4:5]

37. The inside and outside temperatures of a glass window of a room are 20°C and 5°C respectively. The glass is 0·3 cm. thick and has an area of 2 sq. metres. If thermal conductivity of glass is ·0012, calculate the rate at which heat flows into the room from outside through the glass window. [*H. S. Exam. 1982*] [Ans. 1200 cal/sec]

38. When water is boiled in a metallic vessel whose bottom is 5 mm. thick, the level of water in the vessel is lowered by 1 cm. in every 5 minute. What is the temperature below the vessel ? Thermal conductivity of the metal$=0{\cdot}12$ and the latent heat of steam$=540$ cal/gm. [Ans. 107·5°C]

39. A hollow metallic cube has each side 10 cm. long and 1 cm. thick. It is completely filled with ice and then placed in water at 100°C. How much ice will melt in a minute ? Thermal conductivity of the metal$=0{\cdot}5$. Latent heat of ice$=80$ cal/gm, [Ans. 22·5 Kg.]

40. The thickness of a metal vessel is 1 mm. and the walls have a total area of 1000 sq. cm. The vessel is surrounded by ice and is then filled up with boiling water at 100°C. If 80 gm. of ice melts per minute, find the coeff^n of thermal conductivity of the metal. [Ans. 0·106]

41. An iron cube of 4 sq. cm. total area has one face in contact with steam and the other with ice. How much ice will melt in 10 minutes ? Thermal conductivity of iron is 0·2.

[Ans. 300 gm.]

42. A round vessel having external diameter 10 cm. is made by sheet copper of 2 mm. thickness. The vessel is filled up by ice and is then placed in boiling water at 100°C. Find the rate at which ice will melt. Conductivity of copper$=0{\cdot}72$ c.g.s. and $L=80$ cal/gm.

[Ans. 1·41 kg/sec.]

43. A square sheet of cork, 3 mm. thick is pasted again a sheet of glass 5 mm. thick. The area of each sheet is 20 sq. cm. The temperatures of the two surfaces of the combined square sheet are respectively 100°C and 20°C, the glass surface being at higher temperature. Find the temperature of interface between the cork and glass. The conductivities of glass and cork are respectively 0·002 and 0·0006. [Ans. 73·3°C]

44. An ice-box is built of wood 1·75 cm. thick, lined with cork 3 cm. thick. If the temperature of the inner surface of the cork is 0°C and that of the outer surface of wood 12°C, what is the temperature of the interface ? Thermal conductivity of wood = 0·0006 and of cork = 0·000012. [Ans. 10·75°C]

[Hints : $\theta = \dfrac{K_1 x_2 \theta_2 + K_2 x_1 \theta_1}{K_2 x_1 + K_1 x_2}$]

45. One half *AB* of a rod, 30 cm. long and 5 sq. cm. in cross-section, is made of copper, while the other half *BC* is made of iron. The temperature of the end *A* is 100°C and that at the *C* is 0°C. If the bar is lagged, what will be the rate of flow of heat along the bar at the steady state ? Thermal conductivity of copper = 0·9 and that of iron = 0·12. [Ans. 7·06 cal/*s*]

46. The two ends of a lagged copper bar length 50 cm. are exposed to 100°C and 0°C bath. There is a layer of water 0·1 mm. thick at each end of the bar. What are the temperatures at the ends of the bar ? Thermal conductivity of copper = 1·04 and of water = 0·0014. [Ans. 88·54°C ; 11·46°C]

47. Two rods of circular cross-sections, having radii r_1 and r_2, are of equal length. Each rod has its ends at temperatures θ_1 and θ_2. What is the condition that ensures equal rates of flow of heat through the rods ? Thermal conducitivity of the rods are K_1 and K_2 respectively. [Ans. $k_1 r_1^2 = k_2 r_2^2$]

48. Suppose, a pond is covered with ice 10 cm. thick and the temperature of overlying air is −5°C. Find the time in which the thickness of ice will increase by 1 mm. Thermal conductivity of ice = 0·008 ; Latent heat = 80 cal/gm. and density = 0·92 gm/c.c. [Ans. 30 mnt. 40 sec.]

49. Find the equilibrium temperature of a perfectly black disc insulated on back side, exposed normally to sun's rays on the surface of the earth. Effective temperature of sun = 6200°K ; radius of sun = $4{\cdot}2 \times 10^5$ miles and the distance of the earth to sun = $9{\cdot}8 \times 10^7$ miles. [Ans. 421·6°K]

Harder problems :

50. A bar, 2 cm^2 in cross-section has one end in contact with a piece of ice and the other end projected into a steamchest. How much ice will melt in 2 minutes ? Length of the bar = 10 cm. and thermal conductivity = 0·25. [Ans. 15 gm.]

51. A person has covered his body with a flannel shirt 4 mm. thick. If the outside temperature and body temperature be 27°C and 98·6°F, how much heat will he lose per hour per sq. metre of his body ? Thermal conductivity of flannel = 0·00012 c.g.s. [*Jt. Entrance 1974*] [Ans. 101×10^3 cal]

52. Assume that the thermal conductivity of copper is four times that of brass. Two rods of copper and brass having the same length and cross-section are joined end to end. The free end of the copper rod is kept at 0°C and the free end of the brass rod is kept at 100°C. Calculate the temperature of the junction of the two rods at equilibrium. Neglect radiation loss. [*I.I.T. 1970*] [Ans. 20°C]

53. A room is 7 by 5 by 3 metres and the walls are of brick 20 cm. thick and have windows of glass 0·05 cm. thick and of total area 5 sq. metres. The temperature of the room is 20°C higher than that outside. Calculate approximately the quantity of heat that is passing out per minute through the walls and the windows of the room. The thermal conductivity of brick and glass may be taken as 12×10^{-4} and 17×10^{-4} c.g.s. units respectively. [*Jt. Entrance 1978*] [Ans. 25×10^4 cal.]

54. A copper calorimeter with a copper heating element fitted inside weighs 300 gm. It is filled to a certain mark with 200 gm. of a liquid. With 41 watt electrical input to the heater, the temperature rises from 20°C to 45°C in 10 minutes. When 140 gm. of the liquid is replaced by 1250 gm. of copper rivets such that the liquid level remains the same and the calorimeter heater is supplied with the same electrical input as before, the temperature rises from 20°C to 45°C in 9 minutes 5 sec. It is found that a steady temperature of 45°C can be maintained in either case with an electrical input of 2 watt, the room temperature remaining constant at 20°C throughout the experiment. Calculate sp. heats of (i) copper and (ii) the liquid.

[*Jt. Entrance 1980*] [Ans. (i) 0·095 (ii) 1·00]

55. A closed cubical box made of perfectly insulating material has walls of thickness 8 cm., and the only way for heat to enter or leave the box is through two solid, cylindrical metallic plugs, each of cross-sectional area 12 cm^2 and length 8 cm. fixed in the opposite walls of the box (Fig.9.27). The outer surface A of one plug, is kept at 100°C while that of the outer surface B of the other plug is maintained at 4°C. The thermal conductivity of the material of the plug is 0·5 c.g.s. A source of energy generating 36 cal/sec. is enclosed inside the box. Find the equilibrium temperature of the inner surface of the box, assuming that it is the same at all points on the inner surface.

[*Jt. Entrance, 1972*] [Ans. 76°C]

[**Hints :** Heat flowing in per sec through the plug A+heat generated by the source per sec=Heat flowing out per sec. through the plug B.]

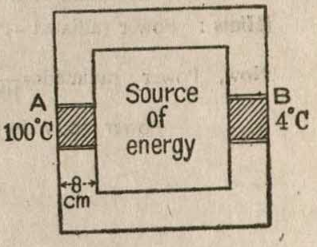


Fig. 9.27

56. A room is maintained at 20°C by a heater of resistance of 20 ohms connected to 200 volts mains. The temperature is uniform throughout the room and the heat is transmitted through a glass window of area 1m^2 and thickness 0·2 cm. Calculate the temperature outside. Thermal conductivity of glass is 0·2 cal/m/°C/sec and mechanical equivalent of heat is 4·2 joules/cal.

[*I.I.T. 1978*] [Ans. 15·23°C]

[**Hints :** Heat generated per sec.$=\frac{i^2R}{4\cdot2}$ cal$=\left(\frac{V}{R}\right)^2\cdot\frac{R}{4\cdot2}=\frac{V^2}{4\cdot2\times R}=\frac{(200)^2}{4\cdot2\times20}$ cal.]

57. Two slabs with parallel sides are in contact with each other. The thickness, conductivity and temperatures on the outside surfaces of the blocks are respectively x_1, x_2, K_1 and K_2 and θ_1, θ_2. Find the temperature θ of their interface.

$\left[\text{Ans. } \theta=\frac{k_1\theta_1x_2+K_2\theta_2x_1}{K_2x_1+K_1x_2}\right]$

58. Assume that the thermal conductivity of copper is twice that of aluminium and four times that of brass. Three metal rods, made of copper, aluminium and brass are each 6 inches long and 1 inch in diameter. Three rods are placed end to end, with aluminium between the other two. The ends of copper and brass rods are maintained at 100°C and 0°C respectively. Find the equilibrium temperatures of the copper-aluminium junction and the aluminium-brass junction.

[Ans. 85·7°C, 57·2°C]

59. The thickness of ice on a lake is 5 cm and the temperature of air above is −20°C. Find the time taken for the thickness of ice to be doubled. Thermal conductivity of ice=0·005 density of ice=0·91 gm/c.c. and latent heat=80 cal/gm.

[Ans. 7 hr. 40 mnt.]

60. The temperature gradient in the earth's crust is 32°C per km., and the mean conductivity of rocks is 0·008 c.g.s. units. Taking the radius of the earth as 6000 km, calculate the daily loss of heat by the earth by conduction.

[Ans. 10^{18} cal (nearly)]

61. The temperature of a blackened copper sphere is 127°C. It loses heat at the rate of 2·8°C per minute when placed in an enclosure of temperature 27°C. At what rate will a copper sphere of double the radius lose heat at temperature 227°C placed in an identical surrounding?

[Ans. 4·34°C/min.]

62. Assume that a planet radiates heat at a rate proportional to the fourth power of its surface temperature *T* and that the planet assumes such a steady temperature that this loss of heat is exactly compensated by the heat gained from the sun, show that other things remaining the same, a planet's surface temperature will vary inversely as the square root of its distance from the sun. [*Jt. Entrance 1982*]

63. Estimate the temperature of the earth, assuming that it is in radiative equilibrium with the sun. Radius of the sun $=7\times10^{8}$ metre, temperature of solar surface $=6000$°K, distance of the earth from the sun $=1{\cdot}5\times10^{11}$ metre. [Ans. 290°K]

64. The tungsten filament of an electric lamp has a length of 0·5 metre and a diameter 6×10^{-5} metre. The power rating of the lamp is 60 watt. Assuming the radiation from the filament is equivalent to 80% that of a perfect black body radiator, at the same temperature, find the steady temperature of the filament. $\sigma=5{\cdot}7\times10^{-8}\ \text{Wm}^{-2}\text{K}^{-4}$. [Ans. 1933°K]

[**Hints** : Power radiated = Power received

Now, Power radiated $=\frac{80}{100}\times\sigma\times2\pi rhT^{4}=8\times5{\cdot}7\times10^{-8}\times2\pi\times3\times10^{-5}\times0{\cdot}5\times T^{4}$

Power received $=60$.]

SOUND

1

SIMPLE HARMONIC MOTION

1.1. Periodic motion :

Definition : *Any motion, simple or complex, which repeats itself in equal interval of time, is called periodic motion.*

We know that the earth moves round the sun in a year and the earth repeats this motion over and over again. Hence motion of the earth round the sun may be called periodic motion. Similarly the hour hand or the minute hand of a clock executes periodic motion.

Again, if a simple pendulum be prepared by suspending a small metallic sphere with the help of a thread and the pendulum be released after being pulled aside a little, it will oscillate to and fro. It will be found that the pendulum describes the same path repeatedly in a definite time. So, the motion of the pendulum is also periodic. Since, this motion takes place in a straight line, it is called *linear periodic motion* or *oscillations.* The motion of the earth or of the hour hand of a clock is called *rotational periodic motion,* because it takes place along a circle.

1.2. Oscillation or vibration :

When a body moves back and forth repeatedly about a mean position, its motion is called an *oscillation* or *vibration.* An essential requirement for oscillation is a position of or a state of equilibrium. If the system is disturbed from its position of equilibrium, and left to itself, it oscillates about the position of equilibrium as the mean position.

Take the case of a long and thin steel spring, one end of which is fastened to a rigid support and a load is suspended from the other. The spring will elongate a little due to the load. Suppose it is in equilibrium at the position A. Here, the elastic force of the spring and the weight of the load balance each other. Now pull the load down a little, say upto *B* [Fig. 1.1] and then release it. It will be seen that the spring contracts and overshoots the point *A* to *C* and then oscillates up and down with the point *A* as the mean position. Such motion of the spring will be called oscillation or vibration. With the passage of time, the oscillation will gradually die down and finally the load will come to rest at *A* again.

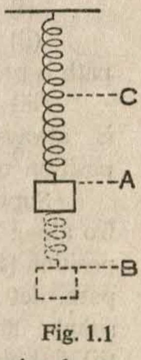


Fig. 1.1

Oscillation may take place in another way. Pull the bob of a simple pendulum aside a little and let it go. The bob will move back and forth with the position of rest *O* as the mean position [Fig 1.2]. Such motion of the bob is also called oscillation.

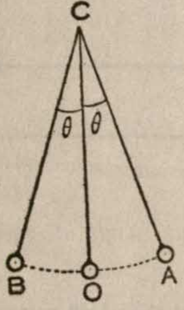


Fig. 1.2

It is to be noted that *an oscillation is, by and large, a periodic motion but all periodic motions are not oscillations.* Further, it will be seen later that an oscillation can be expressed in terms of harmonic functions *viz*, sine and cosine functions. An oscillation which can be expressed in terms of one single harmonic function, is known as *harmonic oscillation.*

In the case of oscillation of a spring or of a pendulum bob, one *complete oscillation* will be executed when the load makes one upward and one downward journey (from *B* to *C* and back to *B*) or the bob makes a complete to-and-fro motion (from *A* to *B* and back to *A*). Half of one complete oscillation (*i.e.* from *B* to *C* or from *A* to *B*) is called *half oscillation* or *half vibration.*

The time taken by the load or the bob to make one complete oscillation is called its *periodic time.* The number of complete oscillations executed by the load or the bob in 1 second is called its *frequency* of oscillation. The maximum displacement of the oscillating body measured from the mean position is known as the *amplitude* of oscillation. The amplitude of the load in fig. 1.1 is *AB* or *AC* while that of the pendulum bob in fig. 1.2 is *OA* or *OB*. The amplitude is *linear* if the displacement takes place along a straight line as in the case of a loaded spring ; it is *angular* if the displacement takes place along the arc of a circle. The angular amplitude of the pendulum bob in fig. 1.2 is θ.

1.3. Simple harmonic motion :

Definition : *If a force acts on an oscillating body in such a way that the force is always directed towards the mean position of its path and is always proportional to the displacement of the body measured from the mean position, the motion of the body under such force is called simple harmonic.*

Characteristics of simple harmonic motion :

(i) The motion is periodic as well as oscillatory.

(ii) The acceleration of the particle executing S.H.M. at any point of its path is proportional to its displacement at that point.

(iii) The acceleration of the particle is always directed towards the mean position of the path.

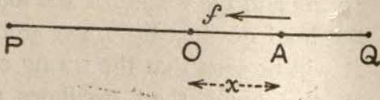


Fig. 1.3

Suppose a particle is moving to and fro along the line *PQ* with *O* as the mean position [Fig. 1.3]. At any point *A* on its path, let the acceleration of the particle be *f*. For the motion to be simple harmonic (i) *f* should be directed towards the mean position *O* and (ii) *f* should be proportional to *x*, the displacement of the particle at *A*.

From the second condition, we can write, $f \propto -x$ or $f = -k \times x$ where *k* is the constant of proportionality. Note that a minus sign has been put before *x*.

This is because of the fact that displacement (x) and acceleration (f) are oppositely directed.

Now, if a force P acting on the particle produces the above acceleration, then from Newton's law of motion, we get

$$P = m.f = -k.m.x. = \text{constant} \times (-x) \ldots\ldots\ldots (\text{i})$$

This shows that the force acting on a particle executing S.H.M. is also proportional to its displacement and is directed always towards the mean position. As the force tends to bring the particle back to its mean position, *the force is a restoring force.*

The constant mentioned in the eqn. (i) above is generally called the *force constant* and has unit dyne/gm.

1.4. Relation between S. H. M. and circular motion :

Simple harmonic motion is closely related with circular motion. Fig. 1.4 shows an illustration how a circular motion can generate simple harmonic motion.

The arrangement consists of a disc D capable of rotating in a vertical plane about a horizontal axis passing through its centre O. A piston P is attached to the rim of the disc at Q. If the disc D rotates in the clockwise direction as shown by the arrow heads, the piston P moves up and down through the cylinder. If the disc rotates at a uniform speed, the piston P executes linear simple harmonic motion. In this case, the simple harmonic motion of the piston P may be regarded as the projection of the circular motion of Q on the vertical axis YY'. As a matter of fact, the projection of uniform circular motion on any of the diameters of the circle may be regarded as a simple harmonic motion along that diameter.

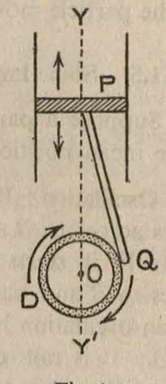


Fig. 1.4

An everyday example of conversion of linear simple harmonic motion into circular motion occurs in locomotives. Here, the 'back and forth' or reciprocating motion of the crankshaft is changed to a regular circular motion of the wheels of the train.

The relation between S. H. M. and circular motion is very helpful in studying the different aspects of the simple harmonic motion.

Suppose, a particle is moving round the circumference $ABCD$ of a circle having centre at O, with uniform angular velocity [Fig. 1.5]. If at every instant a perpendicular is drawn from the position of the moving point P to the diameter AOB or COD, then the foot of the perpendicular N will be found to move along the diameter with simple harmonic motion. Suppose the particle starts from A and moving anti-clockwise, arrives at P at an instant. The foot of the perpendicular will be at N travelling towards the centre O. When the particle is at P_1, the foot crossing over the centre and travelling towards B, will arrive at N_1, when the particle goes to the lower half of the circle at P_2, the foot will return from B towards O and will reach the point N_2. In this way, when the foot N arrives at A again, the particle also returns to A. Thus with the continual rotation of the particle P along the circumference of the circle, the foot N executes oscillatory motion along the diameter AOB. As the particle P possesses uniform angular velocity, it takes a definite time to make one complete revolution. So, the foot N also takes the same time to make one complete oscillation from A to B and back to A. In other words, the motion of the foot N is periodic.

In the same way, if perpendicular PN' be drawn from various positions of the rotating particle on the diameter COD, the foot N' will also move in a periodic manner along the diameter COD. Further, it may be proved that the motion of the foot N or N' has other characteristics of S.H.M. too.

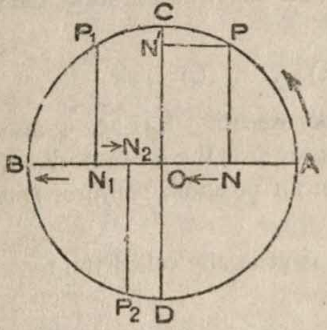


Fig. 1.5

The circle $ABCD$ mentioned above is referred to as the *circle of reference* and the particle moving along the circumference of the circle as *reference particle.*

1.5. Some important terms in connection with S.H.M :

Suppose a particle is executing S.H.M. along the straight line AB with O as the mean position (Fig. 1.6).

Oscillation : If an oscillation starts at A, it is not completed until the particle moves across to B and back again to A. If it starts from O and moves to B and back to O, only half an oscillation has been completed. It is not completed until the particle moves to A and back to O again.

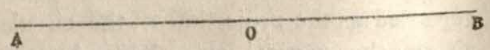


Fig. 1.6

Time period : Time taken by the particle to make one complete oscillation is called its time-period.

Frequency : The number of complete oscillations executed by the particle in one second is called its frequency. The unit '1 cycle per sec' is now-a-days called 1 hertz (Hz). Thus 200 c.p.s.$=200Hz$.

If T be the time-period and n the frequency of the particle, then, $n=\frac{1}{T}$.

Again, the reference particle (Fig. 1.5) describes an angle of 2π radians at the centre of the circle of reference in time T. Hence $T=\frac{2\pi}{\omega}$, where ω is the angular velocity of the reference particle. So $n=\frac{1}{T}=\frac{\omega}{2\pi}$ or $\omega=2\pi n$.

Amplitude : The maximum displacement of the particle either on the right or on the left of the mean position O, is called the amplitude of the particle. In fig. 1.6 OA or OB represents the amplitude.

Phase : Phase of a vibrating particle at any instant is its state of motion at that instant. State of motion of a vibrating particle at any instant is determined by its displacement, velocity and acceleration at that instant.

Phase at any instant may be measured by the angle traced out at the centre of the circle of reference by the revolving particle, starting from C say, and travelling in the given time. For this reason it is called phase angle. Thus 'phase angle at any time is $\frac{\pi}{2}$,' this means that the particle starting from C (Fig. 1.7) has moved to A while the particle executing S. H. M. on COD has arrived at O, the mean position. Hence its displacement is zero.

Similarly 'phase angle at any time is π'—this means that the particle is at D, having equal but opposite displacement than at C. For this reason a phase difference of π indicates exactly opposite phase.

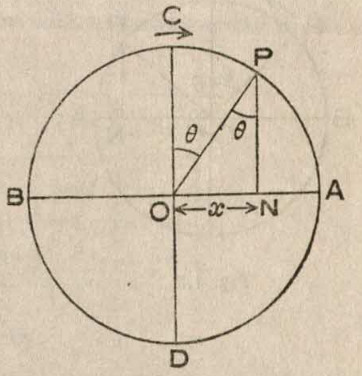


Fig. 1.7

Phase-difference : The term phase-difference very often comes when we consider two simultaneous simple harmonic motions. If two simple harmonic motions start from the same point so that the states of their vibration at every instant are identical, they are said to be in same phase or to have zero phase-difference. On the other hand, if the states of their motion at every instant be opposite they are said to have opposite phase or a phase-difference of π. This is true irrespective of their amplitudes and velocities being equal or not. We shall see presently that simple harmonic motion of a particle in the x-direction may be represented by $x_1 = a \cos \omega t$. Similarly, S.H.M. of another particle in the same direction may be written as $x_2 = b \cos (\omega t + \phi)$. Here, ωt is the phase angle of the first motion while $(\omega t + \phi)$ is that of the second. The phase-difference between the two motions is, therefore, ϕ.

When $\phi = 0$, the motions are in same phase *i.e.* their maximum and minimum displacements take place at the same instant and they cross the mean position simultaneously in the same direction.

When $\phi = \pi$, the motions are in opposite phase *i.e.* their maximum displacements take place at the same instant but in the opposite directions and they cross the mean position simultaneously in the opposite directions.

1.6. Displacement, velocity, acceleration etc. of a particle executing S.H.M. :

(i) **Displacement** : Consider a particle revolving along the circumference of the circle of reference $ACBD$, with an angular velocity ω in the direction of the arrow [Fig. 1.7]. Suppose, the radius of the circle$=OP=a$; also suppose that the particle, starting from C, arrives at P after a time t. The angle traced out by the revolving particle at the centre of the circle$=\angle POC=\theta=\omega.t.$ and the displacement of the foot N of the perpendicular$=ON=x$ (say).

Here, $x=OP.\sin\theta=a\sin\omega t$.

If the period of oscillation of the particle N be T, then $\omega=\frac{2\pi}{T}$; hence,

$$x=a\sin\frac{2\pi}{T}.t=a\sin 2\pi nt$$

[n=the frequency of the particle N]

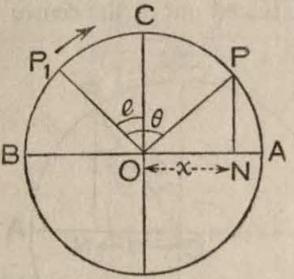


Fig. 1.8

Instead of counting time from the position C, suppose, time is counted from the moment the particle was at P_1 and after an interval t the particle arrives at P [Fig. 1.8]. In this condition, $\angle P_1OP=\theta=\omega t$ which is the phase angle of the motion.

$$\text{Now, } x=ON=OP\sin OPN=a\sin\angle COP$$
$$=a\sin(\angle P_1OP-\angle P_1OC)$$
$$=a\sin(\omega t-e) \quad \ldots \quad \ldots \quad \text{(i)}$$

Here, $\angle P_1OC=e$ and it is called 'epoch' (*i.e.* initial phase or phase at $t=0$).

The initial phase depends on where we choose to fix the zero for time measurement. With a suitable choice of the instant from which time is counted, (*i.e.* when the point P_1 is taken on the right of C) the expression of displacement can be written as follows : $x=a\sin(\omega t+e)$..(ii).

(ii) **Velocity** : When the reference particle, travelling in the direction of the arrow, arrives at P, its velocity V at that point is directed along a tangent PQ drawn at that point to the circle of reference [Fig. 1.9]. Since the particle N executes S.H.M. due to the motion of the reference particle, the component of V along the direction OA will give the velocity of N at that instant. Now, drawing a line PR through P parallel to OA, we get $\angle RPQ=\theta$.

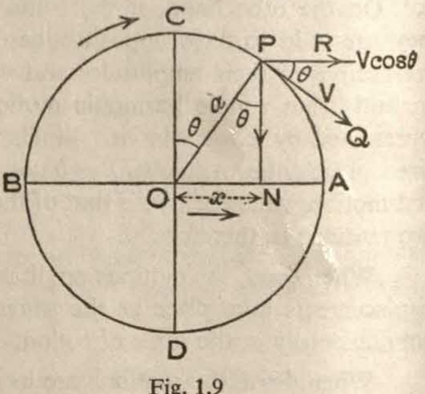


Fig. 1.9

$\therefore$ the component of V along $PR=V.\cos\theta=\omega.a.\cos\theta$.

$$\left[\therefore\ V=\frac{2\pi a}{T}=\omega.a\right]$$

Hence, the velocity v of N at the instant is given by,

$$v=\omega.a\cos\theta=\omega.a\times\frac{\sqrt{a^2-x^2}}{a}=\omega.\sqrt{a^2-x^2}$$

From the above expression, we know that the velocity of N is dependent on its displacement (x).

Now, when $x=0$ *i.e.* when the particle N crosses the mean position, its velocity $v=\omega\sqrt{a^2}=\omega.a$ and it is maximum.

Again, when $x=a$, the velocity $v=\omega.\sqrt{a^2-a^2}=0$ and it is minimum.

So, we can say, *when a particle, executing S.H.M. passes through its mean position, it acquires maximum velocity which is the product of its angular velocity* (ω) *and its amplitude* (a) *and when it arrives at the extreme ends of the amplitude it is momentarily at rest.*

The above variation of velocity and displacement with time as recorded in the table below has been shown graphically in fig. 1.10.

Time (t)	0	$T/4$	$T/2$	$3T/4$	T
Displacement (x)	0	a	0	$-a$	0
Velocity (v)	$+a\omega$	0	$-a\omega$	0	$+a\omega$

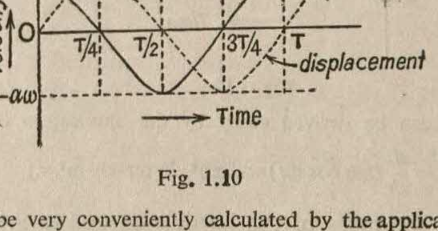


Fig. 1.10

[N.B. Velocity can be very conveniently calculated by the application of calculus in the following way. $x=a\sin\omega t$; $\therefore\ v=\frac{dx}{dt}=\frac{d}{dt}(a\sin\omega t)=a\omega\cos\omega t=a\omega\cos\theta=\omega\sqrt{a^2-x^2}$.]

(iii) **Acceleration :** If the generating particle, revolving along the circumference of the circle of reference, arrives at P at any instant, its acceleration (f_r) will be directed along the radius PO towards the centre of the circle. The component of this acceleration along NO will give the acceleration of the particle N at that instant. It will be directed along NO [Fig. 1.11].

So, at the given instant the acceleration of N is $f=f_r.\cos(90°-\theta)=f_r.\sin\theta$

$$=-\omega^2 a\sin\theta=-\omega^2 x\ [\because\ f_r=\omega^2 a]$$

Since the acceleration is directed opposite to the displacement, minus sign has been put before it.

From the above expression of the acceleration, we come to know that the acceleration of N is proportional to its displacement (x).

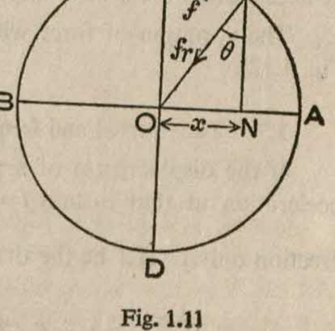


Fig. 1.11

Now, when $x=0, f=0$ *i.e. the acceleration is zero when the particle N passes through the mean position.*

Again, when $x=a$, $f=-\omega^2 a$ and *it is maximum.* The negative sign shows that the acceleration and the displacement are oppositely directed.

The above variation of acceleration and displacement with time as recorded in the table below has been shown graphically in fig. 1.12.

Time (t)	0	$T/4$	$T/2$	$3T/4$	T
Displacement (x)	0	a	0	$-a$	0
Acceleration (f)	0	$-\omega^2 a$	0	$+\omega^2 a$	0

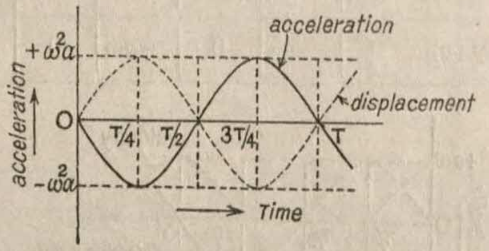


Fig. 1.12

[N.B. Acceleration can be derived easily by the application of calculus ; we know $v=a\omega\cos\omega t$ ∴ $f=\frac{dv}{dt}=\frac{d}{dt}(a\omega\cos\omega t)=-a\omega^2\sin\omega t=-\omega^2.x.$]

(iv) **Force :** If the mass of the particle executing S.H.M. be m and f be its acceleration at any instant, then the force P acting on the particle at that instant is given by, $P=m.f=-\omega^2.x.m.=-m.\frac{4\pi^2}{T^2}.x.$

This shows that the force and the displacement are also oppositely directed. Since the displacement is always measured *from* the mean position, the force and the acceleration are always directed *towards* the mean position.

The variation of force with time is same as that of acceleration with time [Fig. 1.12].

1.7. Time-period and frequency of S.H.M. :

If the displacement of a particle, executing S.H.M. be x at any instant, its acceleration at that instant $f=\omega^2 x$ [Negative sign is ignored since it gives the direction only.] If T be the time period, $T=\frac{2\pi}{\omega}$ or $\omega=\frac{2\pi}{T}$

$$\therefore \quad f=\frac{4\pi^2 x}{T^2} \quad \text{or} \quad T=2\pi\sqrt{\frac{x}{f}}=2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}.$$

This shows that the time-period is not dependent on the amplitude. In other words, simple harmonic motion is *isochronous*.

If, again, n be the frequency, $n=\frac{1}{T}=\frac{1}{2\pi}\sqrt{\frac{\text{Acceleration}}{\text{Displacement}}}$

Examples : (1) *The time period of a particle, executing S.H.M. is 12 seconds and the amplitude is 10 cm. Find the acceleration of the particle when it is 2 cm. away from its mean position. If the mass of particle is 2 gm, what is the force acting on the particle then ? Find also, the maximum velocity of the particle.*

Ans. We have, $f=\omega^2x=\left(\frac{2\pi}{T}\right)^2x=\left(\frac{2\times3\cdot14}{12}\right)^2\times2=0\cdot548$ cm/sec².

The force acting at that instant, $P=mf=0\cdot548\times2=1\cdot096$ dynes.

Again, the maximum velocity, $v_{max}=\omega.a=\frac{2\pi}{T}.\ a=\frac{2\times3\cdot14}{12}\times10$

$=5\cdot23$ cm/sec.

(2) *A particle executes simple harmonic motion whose amplitude is 4 cm and time-period is 1·2 sec. What time will the particle take to get a displacement of 2√3 cm. from the mean position ?*

Ans. If ω be the angular velocity, then $\omega=\frac{2\pi}{T}=\frac{2\pi}{1\cdot2}$ rad/sec. Suppose, initially, the particle is at the mean position *i.e.* $x=0$ at $t=0$. Taking the equation of displacement as $x=a\sin\omega t$ we get at $t=t$, $x=2\sqrt{3}$ cm., and $a=4$ cm.

$\therefore\ 2\sqrt{3}=4\sin\omega t$ or $\sin\omega t=\frac{\sqrt{3}}{2}$ or $\omega t=\frac{\pi}{3}$ rad.

Hence, $\frac{2\pi t}{1\cdot2}=\frac{\pi}{3}$ or $t=0\cdot2$ sec.

(3) *A particle is executing S.H.M. along a straight line. When it is 2 ft. and 3 ft. away from the mean position, its velocities are respectively 5 ft/sec and 4 ft/sec. What is the amplitude of the motion? What is its time-period ?*

Ans. We know, $v=\omega\sqrt{a^2-x^2}$ In the first case, $5=\omega(\sqrt{a^2-4})$ and in the second case, $4=\omega(\sqrt{a^2-9})$.

Dividing, $\frac{5}{4}=\frac{\sqrt{a^2-4}}{\sqrt{a^2-9}}$ or $\frac{25}{16}=\frac{a^2-4}{a^2-9}$ or $a^2=\frac{161}{9}$ $\therefore\ a=\sqrt{\frac{161}{9}}=4\cdot23$ ft.

Again, from the first equation, we get,

$5=\frac{2\pi}{T}\sqrt{\frac{161}{9}-4}=\frac{2\pi\times5\sqrt{5}}{3T}$ $\therefore\ T=\frac{2\times3\cdot14\times\sqrt{5}}{3}=4\cdot68$ sec (approx).

(4) *A small coin rests on a horizontal table which is moving vertically in simple harmonic motion with an amplitude of 10 cm. but with gradually increasing frequency. Above a certain frequency, the thrust between the coin and the table would become zero at some point in the motion. What is this frequency and at what point in the motion does the thrust become zero at this frequency ?*

Ans. As the frequency of oscillation is increased, let the thrust between the coin and the table vanish at a frequency n_0 when there will be no contact between the two. This will occur at every cycle so that the coin loses contact periodically with the table. We know the maximum acceleration in S.H.M. occurs at the end of the oscillation and it is also clear that the contact will be broken at this instant. If a be the amplitude and ω the angular velocity of S.H.M., then maximum acceleration$=\omega^2.a$. The coin loses contact with the table when it is moving *down* with acceleration 'g'. So, $\omega^2 a=g$ or $(2\pi n_0)^2.a=g$

$$\therefore \ n_0=\sqrt{\frac{g}{4\pi^2 a}}=\sqrt{\frac{980}{4\pi^2\times 10}}=\frac{7\sqrt{2}}{2\pi}=1{\cdot}6.$$

(5) *The equation of S.H.M. of a particle is* $x=a.\sin\omega t$; *its velocities are* u *and* v *when the displacements are* x_1 *and* x_2 *respectively. Show that the amplitude of the motion is given by* $a=\left(\frac{v^2x_1^2-u^2x_2^2}{v^2-u^2}\right)^{\frac{1}{2}}$.

Ans. If $x=a\sin\omega t$ is the equation of S.H.M. of a particle, its velocity is given by $v=\omega\sqrt{a^2-x^2}$, where x is the displacement [ref. art. 1.6(ii)].

Since the velocity is u when the displacement is x_1 we have $u=\omega\sqrt{a^2-x_1^2}$. Similarly when the velocity is v, we have, $v=\omega\sqrt{a^2-x_2^2}$.

$$\text{Dividing, } \frac{u}{v}=\sqrt{\frac{a^2-x_1^2}{a^2-x_2^2}} \quad \text{or} \quad u^2(a^2-x_2^2)=v^2(a^2-x_1^2)$$

$$\therefore \ a^2(v^2-u^2)=v^2x_1^2-u^2.x_2^2 \quad \text{or} \quad a=\left(\frac{v^2x_1^2-u^2x_2^2}{v^2-u^2}\right)^{\frac{1}{2}}.$$

(6) *If the displacement of a particle at any instant is given by* $x=a\cos\omega t+b.\sin\omega t$, *show that the motion of the particle is simple harmonic. If* $a=3$ *cm.,* $b=4$ *cm., and* $\omega=2$, *find the time-period, amplitude, maximum velocity and maximum acceleration.*

Ans. $x=a\cos\omega t+b.\sin\omega t$; So, velocity $v=\frac{dx}{dt}=\frac{d}{dt}(a\cos\omega t+b\sin\omega t)$ $=-a.\omega.\sin\omega t+b.\omega.\cos\omega t$.

$$\text{Again, acceleration } f=\frac{dv}{dt}=\frac{d}{dt}(b.\omega.\cos\omega t-a.\omega.\sin\omega t)$$
$$=-b.\omega^2\sin\omega t-a.\omega^2\cos\omega t$$
$$=-\omega^2(a\cos\omega t+b.\sin\omega t)=-\omega^2.x$$

$\therefore$ $f\propto x$; this shows that the motion is simple harmonic. Amplitude of motion$=\sqrt{a^2+b^2}=\sqrt{(3)^2+(4)^2}=5$ cm. Time-period $T=\frac{2\pi}{\omega}=\frac{2\pi}{2}=3{\cdot}14$ sec.

Maximum velocity, $v_{max}=\omega\times\text{amplitude}=2\times 5=10$ cm/s

" acceleration $f_{max}=\omega^2\times\text{amplitude}=(2)^2\times 5=20$ cm/s^2.

1.8. Illustrations of simple harmonic motion :

(i) **Motion of a simple pendulum** : If a small metallic sphere be suspended by a light, inextensible string, we get a simple pendulum. The sphere is known as

If the *bob* of the pendulum. Suppose, the bob is at rest at A (Fig. 1.13). In this rest position, the weight of the bob is neutralised by the tension in the string. If the bob be displaced through a small angle θ to B, the component $mg\cos\theta$ of the weight balances the tension T while the other component $mg\sin\theta$ tries to bring the bob back to the position of rest.

So, the effective force F on the bob acting towards the mean position$=-mg\sin\theta=-mg.\frac{x}{l}$.

[If θ be small, according to radian measurement, $\sin\theta=\theta=\frac{AB}{l}=\frac{x}{l}$.]

The negative sign shows that the force is opposite to the displacement or that the force is directed towards O. Also, $AB=x$. The acceleration of the bob $=\frac{F}{m}=-\frac{g}{l}.x$.

Hence, the acceleration $\propto x$ [$\because$ 'g' and 'l' are constants.]

So, it is seen that the acceleration of the bob at any instant is proportional to its displacement at that instant and is directed towards the mean position. These are the two main conditions of S.H.M. So, we can say that the motion of a simple pendulum is simple harmonic. The time-period of the pendulum

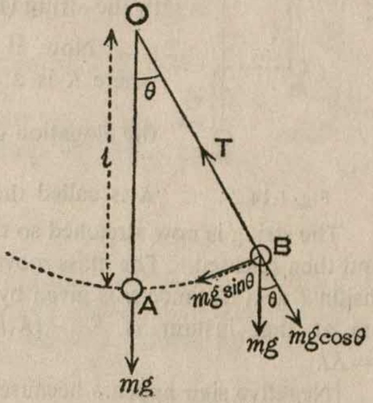


Fig. 1.13

$$T=2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}=2\pi\sqrt{\frac{x}{gx/l}}=2\pi\sqrt{\frac{l}{g}}$$

Example : *A small sphere of mass 10 gm is oscillating like a simple pendulum with amplitude 7 cm and time-period 2 sec. What is the tension in the thread when its velocity is maximum? What is the value of the maximum velocity?* $g=980$ *cm/s*^{2}.

Ans. When the pendulum bob crosses the mean position (the point A in fig. 1.13) its velocity becomes maximum. In S.H.M. we know $v_{max}=\omega.a$.

Again, $\omega=\frac{2\pi}{T}=\frac{2\pi}{2}=\pi$ $\therefore$ $v_{max}=\pi\times a=\frac{22}{7}\times 7=22$ cm/s.

Let the tension in the thread be P when the bob crosses the point A. Here, the resultant force acting along the thread towards $O=P-mg$ [Fig. 1.13]. Again, the centripetal force on the bob$=\frac{mv^2_{max}}{l}$. So, $P-mg=\frac{m.v^2_{max}}{l}$

$$\text{or } P=m\left(g+\frac{v^2_{max}}{l}\right) \quad .. \quad .. \quad \text{(i)}$$

Now, $T=2\pi\sqrt{\frac{l}{g}}$ or $l=\frac{gT^2}{4\pi^2}=\frac{g\times 4}{4\times\pi^2}=\frac{980}{(3{\cdot}14)^2}$

Putting this value in eqn (i) $P=10\left[980+\frac{(22)^2\times(3{\cdot}14)^2}{980}\right]$

$$=10\,(980+4{\cdot}88)=9848{\cdot}8 \text{ dynes.}$$

(ii) **Vertical oscillations of an elastic string supporting a weight :** Suppose an elastic string of length L is suspended from a rigid support and a mass 'm' hangs from the free end. The string will elongate a little due to the weight of the mass and will remain in a steady state, the weight of the mass having been counter-balanced by the tension in the string (Fig. 1·14).

Now, if T be the tension in the string, $mg=T=K.l$, where K is a constant, because tension is proportional to the elogation of the string. $\therefore\ K=\frac{mg}{l}$

K is called the force constant of the string.

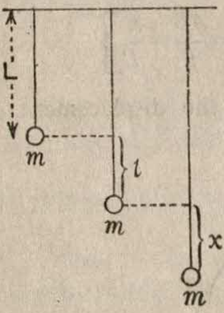


Fig. 1.14

The string is now stretched so that the mass m is displaced through a distance x and then released. The mass moves up and down with simple harmonic motion. Tension T at a distance x is given by $T=K(l+x)$; the resultant force acting on the mass at that instant is $F=-[K(l+x)-mg]=-[Kl+Kx-mg]=-Kx$, because $mg=Kl$.

[Negative sign appears because the tension is opposite to displacement.]

Hence, the acceleration of the mass$=\frac{F}{m}=-\frac{Kx}{m}$ or, acceleration $\propto x$.

So, the vertical oscillations of the loaded elastic string satisfy the conditions of S.H.M. The time-period of the oscillations,

$$T=2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}=2\pi\sqrt{\frac{x}{Kx/m}}=2\pi\sqrt{\frac{m}{K}}$$

Now since $K=\frac{mg}{l}$, we can write, $T=2\pi\sqrt{\frac{m.l}{mg}}=2\pi\sqrt{\frac{l}{g}}$.

[**N.B.** It is to be noted that l is not the length of the thread ; it is the elongation of the thread due to the mass m.]

Examples : (1) *A mass is suspended from one end of an elastic thread. The elongation of the thread is 1·5 cm. If the mass is pulled a little and then released so that the mass oscillates up and down, find the time-period of oscillation.*

Ans. If l be the elongation of an elastic thread, then, the time-period of its vertical oscillation is $T=2\pi\sqrt{\frac{l}{g}}$.

In the present case, therefore, $T=2\pi\sqrt{\frac{1{\cdot}5}{980}}=0{\cdot}25$ sec.

(2) *Two masses m_1 and m_2 are attached to the ends of a massless spring of force constant K. The mass m_2 is resting on a horizontal table and the mass m_1 is held at a certain height such that the spring remains unstretched as shown in the fig. 1.14(a). Find the time-period of oscillations of the spring (i) when m_1 is released and (ii) when the table is removed.*

Ans. (i) When m_1 is released and m_2 rests on the table, the spring will be acted on by the weight of the mass m_1 only. Hence its time-period of oscillation

$$T=2\pi\sqrt{\frac{m_1}{K}}.$$

(ii) When the table is removed, the weight of both the masses will act on the spring. It may be shown that a system consisting of two masses m_1 and m_2 oscillates along the axis of the spring with the same time-period as one single mass of value μ, where $\mu=\frac{m_1m_2}{m_1+m_2}$. Here μ is called the *reduced mass* of the system which is smaller than either m_1 or m_2. Hence, the time-period of oscillation is given by

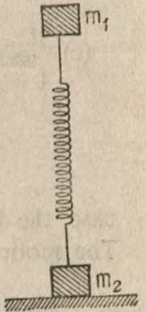


Fig. 1.14(*a*)

$$T=2\pi\sqrt{\frac{\mu}{K}}=2\pi\sqrt{\frac{m_1m_2}{(m_1+m_2)K}}.$$

(3) *A mass M is suspended from one end of an elastic thread. If it is pulled down a little and then released, it oscillates up and down simple harmonically. If an additional mass m is added to M, the time period of simple harmonic motion changes in the ratio $\frac{5}{4}$. Find the ratio m/M.*

Ans. If the elongation of the thread due to a mass be l, its time-period of vertical oscillations $T=2\pi\sqrt{\frac{l}{g}}$. If, now, l_1 be the elongation due to the mass M and l_2 that due to the mass $(M+m)$, then, $T_1=2\pi\sqrt{\frac{l_1}{g}}$ and $T_2=2\pi\sqrt{\frac{l_2}{g}}$.

$$\therefore \quad \frac{l_2}{l_1}=\frac{T_2^2}{T_1^2}=\left(\frac{5}{4}\right)^2=\frac{25}{16}.$$

If Y be the Young's modulus of the thread, then

$$Y=\frac{Mg.L}{\alpha.l_1}=\frac{(M+m)g.L}{\alpha.l_2} \quad [L= \text{initial length of the thread}]$$

$$\therefore \quad \frac{M+m}{M}=\frac{l_2}{l_1}=\frac{25}{16} \quad \text{or} \quad \frac{m}{M}=\frac{25-16}{16}=\frac{9}{16}$$

(iii) **Horizontal oscillations of an elastic spring :** Suppose a body of mass m is attached to one end of a spring and the mass rests on a smooth horizontal surface. The other end of the spring is fastened to a rigid support. In fig. 1.15(a), the mass remains at rest and there is no force acting on the mass due to the spring. If the mass is displaced to the right [Fig. 1.15(b)], the spring becomes *stretched* and it exerts a force on the mass directed to the left. If K be the force-constant of the spring and x be the displacement of the mass to the right, then the force on the mass, $F=-Kx$. [Art. 2.3, Elasticity, Vol. I] If, on the other hand, the body is displaced to the left [Fig. 1.15(c)], the spring becomes *compressed* and it again exerts a force on the mass directed to the right. Here, also, $F=-K.x$. In each case, the force is a restoring force and is proportional to the displacement (x). The motion of the oscillating mass is, therefore, simple harmonic.

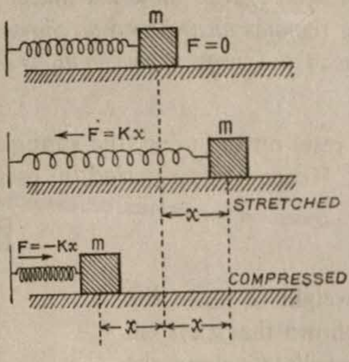


Fig. 1.15

Now, when the displacement of the mass is x, its acceleration $f=\frac{\text{force}}{\text{mass}}$ $=\frac{Kx}{m}$ [Negative sign is omitted because it simply denotes that acceleration is directed opposite to the displacement].

So, the time-period of oscillation, $T=2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}=2\pi\sqrt{\frac{x}{K.x/m}}$ $=2\pi\sqrt{\frac{m}{K}}$. So, the time-period of horizontal oscillations of the spring is determined by the mass attached to the spring and the force constant of the spring.

Example : *Two springs are connected to a mass m and to two rigid supports as shown in fig. 1.16. Show that the frequency of horizontal oscillation of the mass is* $n=\frac{1}{2\pi}\sqrt{\frac{K_1+K_2}{m}}$ *where* K_1 *and* K_2 *are the force constants of the springs.*

Ans. When the mass m moves to the right, the first spring is stretched while the second one is compressed. On the other hand, when the mass moves to the left, the first spring is compressed and the second one is stretched.

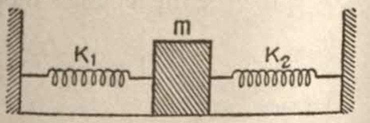


Fig. 1.16

Now, the time-period of oscillations of the first spring, $T_1=2\pi\sqrt{\frac{m}{K_1}}$

or the frequency $n_1=\frac{1}{2\pi}\sqrt{\frac{K_1}{m}}$. Similarly the frequency of oscillation of the second spring $n_2=\frac{1}{2\pi}\sqrt{\frac{K_2}{m}}$.

Hence the frequency of oscillation of the mass is given by

$$n^2=n_1{}^2+n_2{}^2=\frac{K_1}{4\pi^2m}+\frac{K_2}{4\pi^2m}\quad\therefore\quad n=\frac{1}{2\pi}\sqrt{\frac{K_1+K_2}{m}}$$

(iv) Oscillation of a liquid in a U-tube : Taking some liquid in a U-tube, if pressure be applied on the liquid column in one arm of the tube and then released, the liquid column will begin to oscillate in a simple harmonic manner.

Suppose a liquid of density ρ is poured in a U-tube whose cross-section is α. Let A and B represent the liquid levels in the two arms of the U-tube, when the liquid is at rest [Fig. 1.17]. Let the column of the liquid in left arm be depressed to C so that $AC=x$. Since the liquid is incompressible, the level in the other limb will rise equally to D so that $DB=x$. So, the difference of the heights of the liquid levels in the two limbs$=2x$. The weight of this column will try to bring the two levels again at the same height. So, the force responsible for the motion of the liquid column$=2x.\alpha.\rho.g$. The mass of the entire liquid in the U-tube$=2L.\alpha.\rho$, where L is the height of the level AB from the bottom.

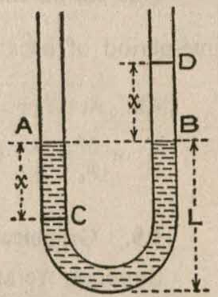


Fig. 1.17

Now, the acceleration f towards A or B is given by

$$f=\frac{\text{force}}{\text{mass}}=-\frac{2x.\alpha.\rho.g.}{2L.\alpha.\rho.}=-\frac{g}{L}.x.\quad\therefore\ f\propto x.$$

The minus indicates that the force towards A is opposite to the displacement measured from A at that instant.

This shows that the motion of the liquid about A or B is simple harmonic. Time-period of oscillation is given by

$$T=2\pi\sqrt{\frac{\text{displacement }(x)}{\text{acceleration }(f)}}=2\pi\sqrt{\frac{x.L}{g.x.}}=2\pi\sqrt{\frac{L}{g}}$$

(v) Oscillation of a gas enclosed in a cylinder : Suppose some gas of volume V is confined in a cylinder of cross-section α by a smooth piston P kept at the position A [Fig. 1.18]. Suppose the pressure exerted on the gas by the piston is P_0. If the piston be now pressed inward to the position B and then released, the compressed gas will try to expand and will execute simple harmonic motion.

Let the additional pressure exerted by the piston be p so that the present pressure$=P_0+p$. The volume of the gas will be decreased by, say, v so that the present volume$=V-v$. If the change takes place under isothermal condition, then

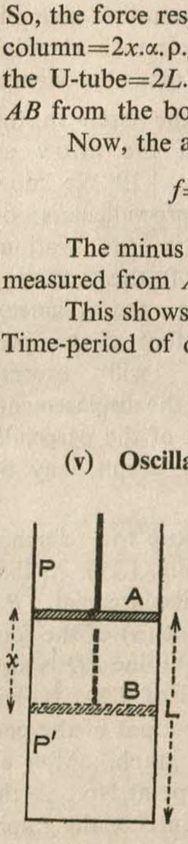


Fig. 1.18

$$P_0V=(P_0+p)(V-v)=P_0V-P_0v+p.V-p.v.$$

or $p=\frac{P_0.v}{V}=\frac{P_0\alpha.x.}{V}$ [*p.v.* being very small is neglected]

So, the additional force exerted by the gas on the piston $=-p.\alpha=-\frac{P_0.\alpha^2.x.}{V}$

[minus because force and displacement are opposite]

The acceleration of the mass of the gas, $f=\frac{\text{force}}{\text{mass}}=-\frac{P_0\alpha^2 x}{V.M}$.

where M is the mass of the gas. $\therefore\ f\propto x$

This shows that the motion of the enclosed gas is simple harmonic. The time-period of oscillation is $T=2\pi\sqrt{\frac{\text{displacement }(x)}{\text{acceleration }(f)}}=2\pi\sqrt{\frac{xV.M}{P_0\alpha^2.x}}=2\pi\sqrt{\frac{VM}{P_0\alpha^2}}$

[**N.B.** At the position *A*, the force on the piston = wt. of the gas *i.e.* $P_0.\alpha=M.g$

or $\frac{M}{P_0}=\frac{\alpha}{g}$. Further. $L.\alpha=V$. $\therefore\ T=2\pi\sqrt{\frac{\alpha}{g}\cdot\frac{L\alpha}{\alpha^2}}=2\pi\sqrt{\frac{L}{g}}$.]

1.9. Graphical representation of S.H.M. :

Suppose, we are to represent graphically a simple harmonic motion of amplitude '*a*' and time-period *T*. For this purpose, draw a circle of reference of radius '*a*' and imagine a reference particle moving with a time-period *T* along the circumference of the circle in the direction of the arrow as shown in fig. 1.19. We know that if perpendiculars be drawn from the various positions of the rotating particle on the diameter *COD*, the foot of the perpendicular will execute *S.H.M.* If the displacements of the foot of the perpendicular at different times be known, then the time-displacement graph may be drawn.

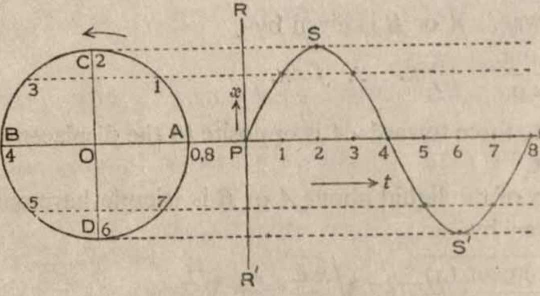


Fig. 1.19

Let us divide the circle into eight equal parts and suppose the reference particle starts revolving from *A* in the direction of the arrow (Fig. 1.19). At that instant the foot will be at *O*, the mean position, having zero displacement. *PR* and *PQ* are two mutually perpendicular lines. The displacement (x) of the foot will be plotted along *PR* and the time (t) along *PQ*. The straight line *PQ* is also divided into eight equal parts, each part signifying an interval of $T/8$. It has already been mentioned that at the start, the displacement of the foot of the perpendicular is zero. This is represented by the point *P* on the graph. After an interval of time $T/4$, when the reference particle moves to the point No. 2, the displacement of the foot is maximum and it is denoted by the point *S* on the graph. In this way, plotting points on the graph according to different time and corresponding displacement, we get a graph as shown in fig. 1.19. It is to be noted that when the displacements of the foot take place along OD, they are to be regarded negative and plotting of points corresponding to negative displacements should be made along PR'. For example, after an interval of $\frac{T}{8} \times 6$ *i.e.* $3T/4$, the displacement of the foot is OD—maximum in the negative direction. The corresponding point on the graph is S'.

Note that the time-displacement curve representing a *S.H.M* is a *sine curve.*

If the starting point of the reference particle be C instead of A, the time dislacement curve will be like one as shown in fig. 1.20. Note that the curve, in this case, is a *cosine curve.*

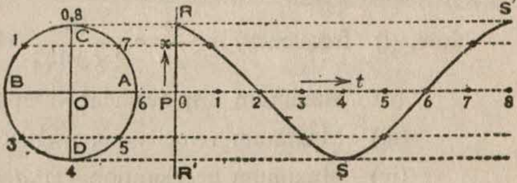
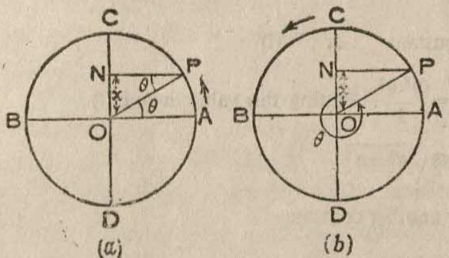


Fig. 1.20

So, a sine curve or a cosine curve can represent a $S.\ H.\ M.$ graphically.

1.10. Mathematical equation of S.H.M. :

Let the reference particle revolve with uniform angular velocity ω along the circle of reference of radius $OP(=a)$ in an anticlockwise direction. If the particle starts from A, the foot of the perpendicular N starts from O, the mean position. Suppose, after an interval of time t, the reference particle arrives at P, so that $\angle POA = \theta$ [Fig. 1.21(a)].



(a) (b)

Fig. 1.21

Here, the displacement of N is given by,

$$x = ON = OP.\sin\theta = a.\sin\theta = a.\sin\omega t \quad [\therefore\ \theta = \omega t]$$

This equation represents a simple harmonic motion mathematically. It is to be noted that the relation between the displacement and the time is a sine relation.

Again, if the reference particle starts from C and arrives at P after time t, revolving in the direction of the arrow [Fig. 1.21(b)] the displacement of N is given by, $x = ON = OP.\cos(360 - \theta) = OP.\cos\theta = a\cos\theta = a\cos\omega t$.

Here, the relation between the displacement and the time is a cosine relation.

Any of the above two equations will represent a simple harmonic motion mathematically. The equations representing the displacement are different because the starting point of the generating particle is different in two cases.

If 'e' be the epoch, the equation of S.H.M will be

$$x = a\sin(\omega t \pm e) \text{ and } x = a\cos(\omega t \pm e)$$

Example : *A simple harmonic motion is represented by the equation $x=10 \sin (10.t-30°)$. Here x is measured in centimetre, t in second and phase angle in degree. Find (a) the frequency (b) the maximum displacement (c) the maximum velocity and (d) the maximum acceleration of the motion.*

Ans. General equation of S.H.M. is $x=a \sin (\omega t+e)$. The equation given is $x=10 \sin (10.t-30°)$.

Comparing the equations, we get, the amplitude$=a=10$ cm. Angular velocity $\omega=10$ rad/s and epoch or initial phase $=-30°$.

Now, (i) frequency, $n=\frac{\omega}{2\pi}=\frac{10}{2\times 3{\cdot}142}=1{\cdot}63$ per. sec.

(ii) Maximum displacement=amplitude=10 cm.

(iii) Maximum velocity$=a.\omega=10\times 10=100$ cm/s.

(iv) Maximum acceleration$=\omega^2.a=(10)^2\times 10=1000$ cm/s^2.

[Equation of S.H.M—alternative derivation :

Equation of S.H.M. can be derived directly from its definition by calculus without using the circle of reference. With usual notation, acceleration $f=\frac{dv}{dt}=\frac{dx}{dt}.\frac{dv}{dx}=v.\frac{dv}{dx}$.

According to the definition of S.H.M. $f \propto -x$ or $f=-\omega^2 x$ [art. 1.6(iii)]

$$\therefore\quad v.\frac{dv}{dx}=-\omega^2.x \text{ or } v.dv=-\omega^2.x.dx.$$

Integrating, $\frac{v^2}{2}=-\omega^2.\frac{x^2}{2}+c$, where c is a constant .. (i)

Now, $v=0$ when $x=a$, the amplitude. So, $c=\frac{\omega^2.a^2}{2}$. Putting this value in eqn. (i)

$$v^2=\omega^2(a^2-x^2) \text{ or } v=\omega\sqrt{a^2-x^2} \text{ or } \frac{dx}{dt}=\omega\sqrt{a^2-x^2}$$

$$\text{or, } \frac{1}{\omega}\int\frac{dx}{\sqrt{a^2-x^2}}=\int dt+k \text{ where } k \text{ is another constant}$$

$$\text{or, } \frac{1}{\omega}\sin^{-1}\left(\frac{x}{a}\right)=t+k. \quad .. \text{ (ii)}$$

When $t=0$, $x=0$; so $k=0$. Substituting this value in eqn. (ii)

$$\sin^{-1}\left(\frac{x}{a}\right)=\omega t \quad \text{or} \quad x=a \sin \omega t$$

This is the equation of S.H.M. When t increases to $t+2\pi/\omega$, $x=a \sin (\omega t+2\pi)=a \sin \omega t$, which is the same displacement value as at t. Hence this motion is periodic and the time-period$=2\pi/\omega$.]

1.11. Energy in a simple harmonic motion :

Let m=mass of the particle vibrating simple harmonically and v=its velocity at any instant. $\therefore$ Its *K.E.* at that instant$=\frac{1}{2}mv^2$.

If x=displacement of the particle at the instant, its velocity $v=\omega\sqrt{a^2-x^2}$

$\therefore$ Kinetic energy, $E_k=\frac{1}{2}m\,\omega^2(a^2-x^2)$.

If $x=0$, $E_k=\frac{1}{2}m.\,\omega^2.\,a^2$ and this is the maximum value of E_k.

Again, if $x=a$, $E_k=0$ and this is the minimum value of E_k. So, *K. E.* is maximum at the mean position and zero at the extreme ends of the amplitude.

The particle also possesses potential energy. When the particle is at the mean position, no force acts on it. When it has a displacement x, the force acting on it$=m\omega^2x$. If x be supposed very small, we can assume that a mean force of value$=\dfrac{0+m\omega^2x}{2}=\frac{1}{2}\,m\omega^2x$ is always acting on the particle bewteen $x=0$ and $x=x$. So, the work done in displacing the particle through a distance x against this mean force=mean force×displacement$=\frac{1}{2}m\omega^2x\times x=\frac{1}{2}m\omega^2x^2$. This is the *P.E.* of the particle at that position.

[**Alternative calculus method** : With the help of calculus, P.E. of a particle executing S.H.M. can be very easily calculated. We have seen that when the particle has a displacement x, the force acting on it$=m.\omega^2x$. Now suppose the particle undergoes a very small displacement dx during which the force may be supposed to be constant. The work done$=m.\omega^2x\times dx$. So, the total work done for a displacement $x=\int_0^x m.\omega^2x.dx=\frac{1}{2}m\omega^2x^2$. In other words, P.E. of the particle $E_p=\frac{1}{2}m.\omega^2x^2$.]

Clearly, potential energy, $E_p=\frac{1}{2}m\omega^2a^2$, when $x=a$ and $E_p=0$ when $x=0$ *i.e. P.E.* is maximum at the extreme ends of the amplitude and zero at the mean position.

Also, Total energy $E=E_p+E_k$ $=\frac{1}{2}m\omega^2x^2+\frac{1}{2}m\omega^2(a^2-x^2)=\frac{1}{2}m\omega^2a^2$ =constant.

This proves that *the principle of conservation of energy holds good in the case of simple harmonic motion.* Fig. 1.22 shows the variation of kinetic and potential energies with displacement x.

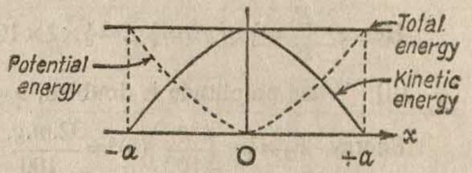


Fig. 1.22

Let a simple pendulum *AO* execute simple harmonic motion along a path *POQ* (Fig. 1.23). *O* is its position of rest. If it is pulled aside to *P*, some work is to be done against gravity which remains stored in the bob as its potential energy. When released, the bob will move towards *O* with increasing velocity. This means that the bob is acquiring greater and greater kinetic energy at the expense of its potential energy. While crossing the mean position, the velocity of the bob becomes maximum and the whole of its potential energy is then converted into kinetic energy. Due to inertia of motion, the pendulum with this maximum velocity now swings to the other side. As it approaches the other extreme end *Q*, its velocity slowly decreases. This means that the kinetic energy of the pendulum gradually diminishes, yielding place to potential energy again, until at the extreme end *Q*, the whole of the kinetic energy is transformed into potential energy and the bob momentarily comes to rest. It has been proved that at any point during its motion, the total energy

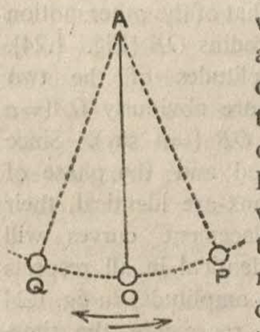


Fig. 1.23

of the pendulum remains always constant. So, if there be no resistive forces like friction, air resistance etc, a pendulum once set into motion, will go on oscillating for ever, transforming *K.E.* into *P.E.* and vice versa.

Example : *The energy of a simple pendulum of length 100 cm is 2×10^6 ergs when its displacement is 4 cm. Find its energy when (i) the length of the pendulum is doubled and (ii) the amplitude is doubled.*

Ans. If the amplitude be a, the total energy of the bob$=\frac{1}{2}m.\omega^2.a^2$

Now, $\omega=\frac{2\pi}{T}=\frac{2\pi}{2\pi}\sqrt{\frac{g}{l}}=\sqrt{\frac{g}{l}}$. In the first case when the length is 100 cm. and amplitude $a=4$ cm., the energy of the bob, $E=\frac{1}{2}m\left(\frac{g}{100}\right)\times(4)^2=\frac{8.m.g.}{100}$.

According to the problem, $E=2\times 10^6 \text{ ergs}=\frac{8.m.g.}{100}$

(i) When the length is doubled, $l=200$ cm. and $\omega=\sqrt{\frac{g}{200}}$

$\therefore$ Energy $E_1=\frac{1}{2}m\left(\frac{g}{200}\right)(4)^2=\frac{8\ mg}{200}$

Hence, $\frac{E_1}{E}=\frac{1}{2}$ or $E_1=\frac{1}{2}\ E=\frac{1}{2}\times 2\times 10^6=10^6$ ergs.

(ii) When amplitude is doubled, $a=8$ cm ; length is unaltered.

Energy $E_2=\frac{1}{2}m\left(\frac{g}{100}\right)(8)^2=\frac{32.m.g.}{100}$

$\therefore\ \frac{E_2}{E}=4$ or $E_2=4.E=4\times 2\times 10^6=8\times 10^6$ ergs.

1.12. Composition of two collinear S.H.M.'s by graphical method :

(a) When they are in phase : Suppose two simple harmonic motions of same period and phase but of different amplitudes are taking place simultaneously along the vertical axis *ROR′*. The circle of reference of one motion has its radius *OA* and that of the other motion has its radius *OB* [Fig. 1.24]. The amplitudes of the two motions are obviously *OA*($=a$ say) and *OB* ($=b$ say). Since the period and the phase of the motions are identical, their time-displacement curves will also be identical in all respects except in amplitude. In fig. 1.24 $XP_1P_2P_3$ represents the time-displacement curve of the simple harmonic motion of amplitude 'a' and $XQ_1Q_2Q_3$ represents the time-displacement curve of the other motion of

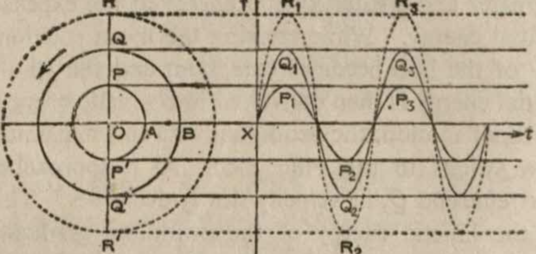


Fig. 1.24

amplitude 'b'. From the curves, it is clear that the maximum displacements in both cases, either in the positive or in the negative direction, take place in the same time. Similarly, the minimum displacements also take place in the same time.

To find the resultant time-displacement curve, in this case, the displacements of the two motions at every instant are to be added algebraically. If $OR=OP+OQ$, the radius of the circle of reference of the resultant S.H.M. is OR, and its time-displacement curve is $XR_1R_2R_3$. The amplitude of the resultant simple harmonic motion is obviously $(a+b)$ and it is in the same phase with the component motions.

(b) When they are in opposite phase : Suppose two S.H.M's of same time-period but of different amplitudes having opposite phases take place simultaneously along the vertical axis ROR'. The radius of the circle of reference of one motion is OA while that of the other is OB (Fig. 1.25). The amplitudes of the motions are evidently OA ($=a$,say) and OB ($=b$, say). Since the phases of the motions are opposite, the maximum displacement of one motion in the positive direction will coincide with the maximum displacement of the other in the negative direction. If the reference particle of one motion starts from Q on its circle of reference, the reference particle of other motion starts from P' on its circle of reference. Moreover, the amplitudes of the motions are also different. In fig. 1.25, $XP_1P_2P_3$ is the time-displacement curve of the S.H.M. whose amplitude is 'a' and $XQ_1Q_2Q_3$ is the time-displacement curve of the S.H.M. whose amplitude is 'b'. From the curves, it is apparent that when maximum displacement due to one motion takes place in the positive direction, the maximum displacement due to the other takes place in the negative direction, because the phases are opposite.

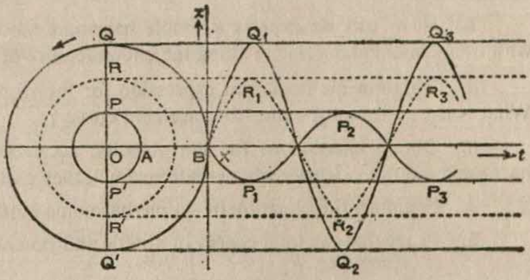


Fig. 1.25

To find the resultant time-displacement curve, in this case, the displacements of the two motions at every instant are to be added algebraically. Since the displacements are, at every moment opposite, the algebraic addition, in this case, means subtraction of the displacements. If $OR=OQ-OP$, the radius of the circle of reference of the resultant S.H.M. is OR and $XR_1R_2R_3$ is its time-displacement curve. The amplitude of the resultant S.H.M. is $(b-a)$ and its phase is same as that of the component of bigger amplitude.

[Mathematically the composition can be done very conveniently. Although it is not within the syllabus, yet it will be an interesting exercise for the students. Two S.H.M's of same phase but of different amplitude can be represented by the equations (i) $y_1=a \sin \omega t$ and (ii) $y_2=b. \sin \omega t$.

The composition will be given by their algebraic sum. If y be the resultant displacement, then $y=y_1+y_2=a \sin \omega t+b. \sin \omega t=(a+b) \sin \omega t$.

This shows that the resultant motion is also simple harmonic having amplitude $(a+b)$ and phase same as the phase of the constituent motions.

Similarly two S.H.M's of opposite phase but of different amplitude may be represented by the equations (i) $y_1=a \sin \omega t$ and (ii) $y_2=b. \sin(\omega t+\pi)=-b. \sin \omega t$. The resultant displacement y is given by $y=y_1+y_2=a \sin \omega t-b. \sin \omega t=(a-b) \sin \omega t$.

This shows that resultant motion is also simple harmonic having amplitude $(a-b)$ and phase same as the phase of the first constituent motion.]

Exercises

Essay type :

1. How can we express a simple harmonic motion by the projection of a point moving with uniform angular velocity along the circumference of a circle of reference on any diameter ?

2. Obtain a mathematical expression for the displacement of a particle executing S.H.M. What is the nature of the time-displacement curve ?

3. Obtain expressions for the velocity, acceleration and force in the case of a simple harmonic motion. Under what conditions will they assume minimum and maximum values ?

4. Establish the equation of simple harmonic motion. [*H. S. Exam. 1981*]

5. Prove that the total energy of kinetic and potential at any instant in a S.H.M. is constant. [*H. S. Exam. 1982*]

6. Explain graphically how you can compose two collinear S.H.M.'s of same period and phase but of different amplitude ? What will happen, if the phases are opposite ? [*H. S. Exam. 1981*]

Short answer type :

7. Define 'simple harmonic motion' and explain the terms period, amplitude and phase. [*H. S. Exam. 1982*]

8. Explain clearly what you understand by simple harmonic motion and state the conditions for which a particle will execute such motion. [*H. S. Exam. 1980*]

9. State with reasons, whether the following are simple harmonic motions :—(i) the motion of planets round the sun (ii) the vertical oscillations of a loaded spring (iii) the motion of the clock hands (iv) the motion of a simple pendulum.

10. Prove that the motion of a simple pendulum is simple harmonic. Is there any condition for it ? What will happen if the condition is violated ? What property of matter takes the bob of pendulum across the mean position to the other side ?

11. What is the difference between the motion of a simple pendulum and that of a loaded spring ?

12. Prove that kinetic energy of a particle executing S.H.M. is maximum when the particle crosses the mean position. Obtain an expression for the maximum K. E. of the particle. When is the K.E. minimum ?

13. Show that the potential energy of a particle executing S.H.M. in minimum when the particle crosses the mean position. When will the potential energy be maximum ? What is the value of it ?

Objective type:

14. Fill in the blanks in the following sentences by selecting suitable terms from the right-hand column :

(i) Any motion, simple or complex, which repeats itself in equal intervals of time, is called a — motion.

(ii) The quantity which gives the state of motion at any instant of a particle executing S.H.M. is called its —.

(iii) A particle, executing S.H.M. has maximum acceleration when it is at the end of its —.

(iv) The phase at the beginning of the simple harmonic motion of a particle is called its —.

(v) A particle, executing S.H.M. has maximum velocity when it passes through the —.

(i) mean position
(ii) amplitude
(iii) periodic
(iv) phase
(v) epoch.

Numerical type :

15. A particle is executing S.H.M. along a line 4 cm long. Its velocity when it crosses the middle point of its path is 12 cm/sec. Find its time-period. [Ans. 1·05 sec.]

16. The amplitude of a particle executing S.H.M. is 5 ft. When the particle is 2 ft. away from the mean position of its path, its acceleration is 8 ft./sec^2. What will be its velocity when it is 3 ft. away from the mean position ? What is the force acting on the particle at that position, if its mass is 2 lb. [Ans. 8 ft./sec : 24 poundal]

17. A particle is executing linear simple harmonic motion. When it is 2 ft. and 3 ft. away from the mean position, its velocities are respectively 4 ft/s and 3 ft/s. What is the amplitude of the motion ? What is its time-period ? [*H. S. Exam. 1981*] [Ans. 3·98 ft. ; 5·3 sec.]

18. A particle executes linear harmonic motion about the point $x=0$; at $t=0$, it has displacement $x=0·37$ cm. and zero velocity. If the frequency of motion is 0·25/sec., determine (*a*) the amplitude (*b*) the maximum speed and (*c*) the maximum acceleration.
[Ans. (*a*) 0·37 cm. ; (*b*) 0·58 cm./s (*c*) 0·91 cm/s^2]

19. Prove that when a particle, executing S.H.M., is $\frac{\sqrt{3}}{2}$ times the amplitude away from its mean position, it has a velocity half its maximum value.

[Hints : $\omega\sqrt{a^2-x^2}=\frac{1}{2}a\omega$; max^m velocity$=a\omega$].

20. (a) At what displacement of a particle executing S.H.M. will have its kinetic energy equal to the potential energy ? [Ans. $\frac{1}{\sqrt{2}}\times$amplitude]

(b) A particle O is oscillating along a straight line between two fixed points X and Y. If the *K.E.* of the particle is always proportional to the product of the distances OX and OY, show that the motion of the particle is simple harmonic.

21. A particle of mass 0·8 kg is executing S.H.M. with an amplitude of 1 metre and a periodic time of 11/7 sec. Calculate the velocity and the kinetic energy of the particle at the moment when the displacement is 0·6 metre. [Ans. 3·2 metre/s ; 5·12 joule]

22. An S.H.M. is represented by the equation : $y=2 \sin 40\,\pi\, t$ cm. Find the amplitude, time-period, and acceleration at the extreme end of the amplitude.
[*H. S. Exam. 1982*] [Ans. 2 cm. ; $\frac{1}{20}$ sec. ; 31584 cm/s^2]

23. A simple harmonic motion is represented by the equation : $x=15 \sin (5t-45°)$. Find the amplitude, angular velocity and the epoch of the motion.
[Ans. 15 cm. ; 5 rad/sec., 45°]

24. A particle of mass 5 gm is executing S.H.M. The effective force on it is 1280 dynes when its displacement is 25 cm. What is its time-period ? [Ans. 1·96 sec.]

25. A spring, hanging vertically, has a mass 10 gm. attached to its lower end. A force of 8 kg. extends the spring through 50 cm. Find the frequency of the oscillations of the spring.
[Ans. 20 (nearly)]

26. A helical spring elongates 10 cm. when subjected to a tension of 5×10^5 dynes. Find the mass which should be attached to the bottom of the spring so that when pulled down and released the mass will vibrate exactly twice per second : Find also its maximum velocity when the amplitude of vibration is 1 cm. [*Jt. Entrance 1979*] [Ans. 5 kg. ; 12·5 cm/s]

27. (i) A mass M attached to a spring oscillates with a period of 2 sec. If the mass is increased by 2 kg, the period increases by 1 sec. Find the initial mass M, assuming that Hooke's law is obeyed. [*I.I.T. 1979*] [Ans. 1·6 kg]

[Hints : $T=2\pi\sqrt{\dfrac{M}{K}}$ where M is the load and K the force constant of the spring.]

(ii) Two mases m_1 and m_2 are suspended together by a massless spring of spring constant K [Fig. 1.26]. When the masses are in equilibrium, m_1 is removed without disturbing the system. Find the angular frequency of oscillation of m_2. [*I.I.T. 1981*] [Ans. $\dfrac{1}{2\pi}\sqrt{\dfrac{K}{m_2}}$]

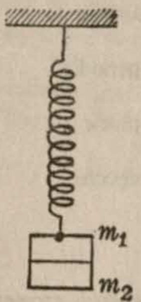


Fig. 1.26

28. A very small particle is kept on a horizontal platform, which is vibrating simple harmonically in a vertical plane. If the frequency of vibration is gradually increased, the thrust of the particle on the platform becomes zero at a particular frequency and at a particular instant of its motion. If the amplitude of motion be 0·08 metre, find the particular frequency. $g=9{\cdot}8$ metre/s². [Ans. 1·8]

29. A small coin is placed on a horizontal platform which vibrates vertically in simple harmonic motion with a period of 0·5 sec. Find the maximum amplitude of the motion which will allow the coin to remain in contact with the platform throughout the motion. $g=980$ cm/s². [Ans. 6·3 cm.]

30. A uniform rod of wood is floating vertically with 30 cm. of its length immersed in water. If the rod is slightly pressed down and then released, what will be the time-period of its harmonic oscillation ? [Ans. 1·09 sec.]

31. A vertical U-tube of uniform cross-section contains water to a height of 30 cm. The water column on one side is depressed and then released. Find the time-period of its oscillation. [Ans. 1·09 sec.]

32. The total energy of a particle executing S.H.M. of period 2π sec. is 10240 erg. At an instant π/4 sec. after the particle passes the mid-point of the swing, its displacement is $8\sqrt{2}$ cm. Calculate the amplitude of motion and the mass of the particle. [Ans. 16 cm. ; 80 gm]

[Hints : Total energy$=\dfrac{2\pi^2.m.a^2}{T^2}=\dfrac{2\pi^2.m.a^2}{(2\pi)^2}=10240$ $\therefore$ $m.a^2=20480$.

Now, $x=a\sin\omega t=a\sin\dfrac{2\pi}{T}.t=a\sin\dfrac{2\pi}{2\pi}t=a\sin t$. At $t=\pi/4$, $x=8\sqrt{2}$ cm.

$\therefore$ $8\sqrt{2}=a.\sin\pi/4$ or $a=16$ cm. Again, $ma^2=20480$ or $m\times(16)^2=20480$

$\therefore$ $m=80$ gm.]

33. A bob of mass 20 gm of a simple pendulum is oscillating simple harmonically with an amplitude of 5 cm. and periodic time of 2 sec. Find the tension in the thread when the bob has maximum velocity. What is the value of the maximum velocity ? [Ans. 16 cm/s ; 19·6×10⁵ dynes]

34. A simple pendulum has a period of 4·2 second. When the pendulum is shortened by 100 cm. the period is 3·7 second. Calculate the value of g and the original length of the pendulum. [Ans. 1000 cm/s² ; 450 cm.]

Harder problems :

35. A particle of mass 10 gm. is describing a simple harmonic motion along a straight line with a period of 2 sec. and amplitude of 10 cm. What is the kinetic energy when it is (i) 2 cm. from its equilibrium position (ii) 5 cm. from the equilibrium position ?

[*I.I.T. 1972*] [Ans. (i) 4800 ergs (ii) 3750 ergs]

36. When a man weighing 60 kg. on the earth gets into a car, the centre of gravity of the car is depressed by 0·3 cm. Assuming the mass of the car to be 1000 kg., find the frequency of vibration of the car when it is empty. [*Jt. Entrance 1978*] [Ans. 2·22]

37. The positions of a particle executing S.H.M. along the x-axis are $x=A$ and $x=B$ at time t and $2t$ respectively. Show that its period of oscillation is given by $T=\dfrac{2\pi t}{\cos^{-1}(B/2A)}$.

38. A particle moving in a straight line with S.H.M. of period $2\pi/\omega$ about a fixed point O has a velocity $\sqrt{3}\,b.\omega$ when at a distance b from O. Show that its amplitude is $2b$ and that it will cover the rest of its distance in time $\pi/3\omega$.

39. A simple harmonic motion is represented by the equation $y=6 \sin 10\,\pi t+8 \cos 10\,\pi t$ where y is in centimetre, and t is in second. Find the (i) amplitude (ii) the period and (iii) the initial phase. [Ans. (i) 10 cm. (ii) 0·2 sec. (iii) 53°8′]

40. A particle executes S.H.M. about a point $x=0$; at $t=0$, it has a displacement $x=0{\cdot}37$ cm. and zero velocity. If the frequency of motion is 0·25/sec., determine (*a*) time-period (*b*) angular frequency (*c*) the amplitude. [Ans. (*a*) 4 sec. (*b*) $\pi/2$ rad/s (*c*) 0·37 cm.]

41. A particle of mass 0·06 kg. executes S.H.M. along a path of length 0·2 metre at the rate of 300 oscillations/minute. Find (i) its total energy (ii) its potential and kinetic energy when the displacement is $x=a/2$ where a is the amplitude. [Ans. (i) 2·96 joule (ii) 0·74j and 2·22j]

42. A small sphere is kept on a concave glass of radius of curvature 109 cm. placed on a horizontal table. Show that if the sphere is displaced from the mean position and left to itself, it executes S.H.M. Find also, the time period of oscillation. $g=981$ cm/s^2 [Ans. 2·09 sec.]

43. An ideal gas is enclosed in a vertical cylinder and supports a freely moving piston of mass M. The piston and cylinder have equal cross-sectional area A. Atmospheric pressure is P_0 and when the piston is in equilibrium, the volume of the gas is V_0. The piston is now displaced slightly from its equilibrium position. Assuming that the system is completely isolated from its surroundings, show that the piston executes simple harmonic motion and find the frequency of oscillation. γ=ratio of two sp. heats. [*I.I.T. 1981*] [Ans. $n=\dfrac{A}{2\pi}\sqrt{\dfrac{\gamma P_0}{V_0 M}}$]

44. A cylindrical wooden block of cross-section 15 cm^2 and mass 230 gm. is floated over water with an extra weight 50 gm. attached to its bottom. The cylinder floats vertically. From the state of equilibrium, it is slightly depressed and released. If the specific gravity of wood=0·3 and $g=980$ cm/s^2, find the frequency of oscillation of the block. [Ans. 0·79]

45. 6·8 kg of mercury (density=13·6 gm/c.c.) is poured into a U-tube with uniform bore of 2 cm. diameter. When slightly disturbed, the mercury oscillates up and down freely about the position of equilibrium. Find the period of oscillation. [Ans. 1·78 sec (nearly)]

46. A test tube of weight 6 gm and of external diameter 2 cm is floated vertically in water by placing 10 gm of mercury at the bottom of the tube. The tube is depressed by a small amount and then released. Find the time of oscillation. [Ans. 0·453 sec.]

47. A 4 kg block extends a spring 16 cm from its unstretched position. The block is removed and a 0·5 kg. body is hung from the same spring. If the spring is then stretched and released, what is its period of motion ? [Ans. 0·28 sec.]

48. A simple pendulum of length l is suspended from the roof of a room. Its time-period is T_1. When its length is shortened by a, the time-period becomes T_2. Show that $g=\dfrac{4\pi^2 a}{T_1^2-T_2^2}$.

49. An elastic cord of length l is suspended from a rigid support. A mass m, attached to its lower end, stretches it by a length a. An additional mass M attached to it, stretches it further by a length b. When the system has come to rest, the mass M gets detached. Show that the distance of mass m from the upper end of the cord, at an instant t is given by $l+a+b\cos\sqrt{\frac{g}{a}}.t$.

50. Two springs are joined together. The free end of one is connected to a mass m and the free end of the other to a fixed support. If the springs separately have force constants K_1 and K_2 show that the frequency of oscillation of m is $n=\frac{1}{2}\sqrt{\frac{K_1K_2}{m(K_1+K_2)}}$.

51. Two light springs of force constants K_1 and K_2 and a block of mass m are in one line AB on a smooth horizontal table such that one end of each spring is fixed on rigid supports and the other end is free as shown in fig. 1.27. The distance CD between the free ends of the springs is 60 cm. If the block moves along AB with a velocity 120 cm/sec, in between the springs, calculate the period of oscillation of the block. ($K_1=1{\cdot}8$ N/m ; $K_2=3{\cdot}2$ N/m ; $m=200$ gm) [*I.I.T. 1985*] [Ans. 4·66 sec]

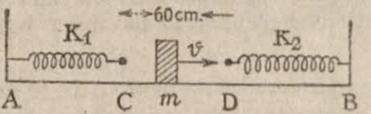


Fig. 1.27

[**Hints** : For the spring AC, $T_1=2\pi\sqrt{m/K_1}=2\pi\sqrt{\frac{0{\cdot}2}{1{\cdot}8}}=2{\cdot}09$ sec.

" " DB, $T_2=2\pi\sqrt{\frac{0{\cdot}2}{3{\cdot}2}}=1{\cdot}57$

Between C and D. $T_3=\frac{2\times60}{120}=1$ sec.]

52. A horizontal spring is found to be stretched 3 inches from its equilibrium position when a force of 0·75 lb acts on it. Then a 1·5 lb body is attached to the end of the spring and is pulled 4 inches along a horizontal smooth table from equilibrium position. Find the period of oscillation of the body when it is released. [Ans. 0·79 sec]

2

VIBRATIONS

2.1. Nature of vibrations :

Vibrations are mainly of two types:—(i) *transverse* and (ii) *longitudinal.*

Transverse vibrations : *If different particles of a body vibrate in a direction perpendicular to the length of the body, the vibrations are called transverse vibrations.*

If the middle point of a wire stretched at two ends be plucked, the wire will vibrate up and down (Fig. 2.1). It will be seen that each particle of the wire vibrates up and down in a direction perpendicular to the length of the wire. Such vibrations of a stretched string or wire are transverse vibrations.

Pull the bob of a simple pendulum a little at right angles to the length of the string. On releasing the bob the pendulum goes on swinging to-and-fro. The vibrations of the bob take place perpendicular to the length of the string and hence they are transverse vibrations.

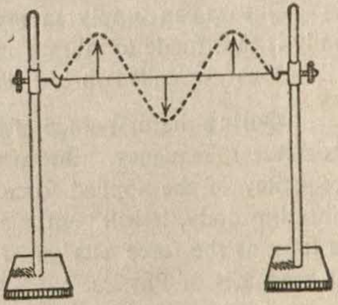


Fig. 2.1

Longitudinal vibrations : *If different particles of a body vibrate in a direction parallel to the length of the body, the vibrations are called longitudinal vibrations.*

One end of a spring S is fixed to a rigid support and from the other end hangs a weight *W* [Fig. 2.2]. If the spring be now stretched a little and then released the spring will alternately elongate and contract in length and the weight *W* will move up and down [Fig. 2.2(*c*)]. Here different particles of the spring vibrate up and down parallel to the length of the spring. Hence the vibrations of the spring are longitudinal.

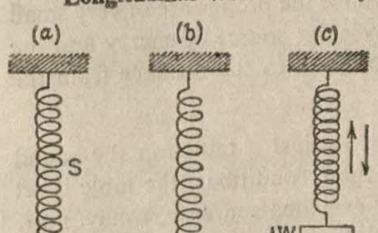


Fig. 2.2

When a tuning fork vibrates, the prongs of the fork move towards and away from each other but the bend along with the stem is alternately depressed and elevated. Hence, the vibrations of the prongs are transverse but those of the stem or the bend are longitudinal.

2.2. Free and Forced vibrations :

We can set up vibrations or oscillations in a body by imparting to it some initial displacement or some initial velocity. In the first case, the energy supplied to the body is potential while in the second case, it is kinetic. For example, when the bob of a pendulum is pulled aside, it is given some initial potential energy. When released it goes on oscillating. When a spring, suitably clamped at one end, is pulled at the other end, it is also given some initial potential energy and when released, it goes on vibrating. These vibrations are called *free* or *natural vibration* in the sense that the system is left free after giving some energy initially. A mechanical system which is free to move, like a pendulum or a spring has a frequency of vibration, called the *natural frequency*, which depends on the dimensions of the body. Thus the natural frequency of a pendulum depends on its length and that of a spring depends on its extension etc. In practice however, the amplitude of natural vibration diminishes as time passes because the system will always have some resistance *i.e.* a cause for the loss of energy.

But we can supply energy to an oscillator to keep it oscillating. When a body is thus made to vibrate under the influence of an external periodic force, the vibrations are called *forced vibration.*

During the first stage of forced vibration, the body is found to vibrate with its natural frequency. But after some time, it will be found to vibrate with the frequency of the applied force. Whatever may be the natural frequency of the vibrating body, it will continue vibrating with the frequency of the applied force as long as the force acts on it. Forced vibrations play a very important part in all branches of Physics.

Illustrations of forced vibrations are not far to seek. When a vibrating tuning fork is held close to the ear, the sound heard is quite loud, but the loudness falls off very rapidly as the fork is moved away. Since the prongs are of very small area, we may regard the fork as an approximately point source of energy and the loudness of sound in that case is inversely proportional to the distance from the source.

If, however, the stem of the fork is pressed against a table-top the sound can be heard clearly all over the room. Under these conditions, the table is set into forced vibration which then acts as a large or extended source. It may, now, be considered as equivalent to a large number of point sources, all of which contribute to the loudness of sound at any point in the room.

It may be pointed out that although the sound is louder, it does not last long. The amplitude of vibration of table-top is very much less than that of the fork but its large area transmits energy to the air at a much greater rate than the small area of the prongs.

Motion of pendulums offers very good illustrations of forced vibration. Two simple pendulums of equal length are suspended from a flexible support

(Fig. 2.3). One of them is pulled to one side and released to swing freely. The horizontal support responds to the motion and in swaying back and forth in step with the vibrating pendulum, acts to set the other pendulum in forced vibration. The energy of the first pendulum being thus transmitted to the second, the first pendulum will come to rest after some time and the second one will continue oscillating. Now, the second pendulum will induce forced vibration in the first through horizontal support and its energy will be gradually transmitted to the first. As a result, after some time, the second pendulum will come to rest and the first one will continue oscillation. In this way, several exchanges of oscillations will take place between the pendulums due to forced vibrations.

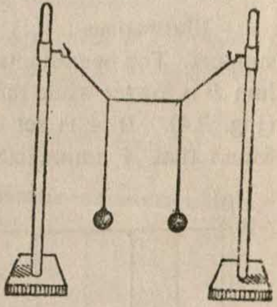


Fig. 2.3

[**Mathematical notes :**

Suppose the periodic force to which an oscillator is subjected is F_0 sin pt where F_0 is the amplitude and $p/2\pi$ the frequency. In the case of a damped oscillator, the damping force is, to a first approximation, proportional to its velocity and may be represented by $\gamma . \frac{dx}{dt}$ where γ is called the damping coefficient of the medium. Further a restoring force proportional to the displacement x also acts on the body. The equation of motion of the body may, therefore, be represented by :

$$m \frac{d^2x}{dt^2} = -\gamma . \frac{dx}{dt} - cx + F_0 \sin pt$$

$$\text{or} \quad \frac{d^2x}{dt^2} + \frac{\gamma}{m}\frac{dx}{dt} + \frac{c}{m}x = \frac{F_0}{m} \sin pt$$

$$\text{or} \quad \frac{d^2x}{dt^2} + 2k . \frac{dx}{dt} + \omega_0^2 . x = f_0 . \sin pt \quad .. \quad \text{(i)}$$

Here, $\frac{\gamma}{m} = 2k$, where k is called the damping coefficient of the medium and $\sqrt{\frac{c}{m}} = \omega_0$, the natural angular frequency of the oscillator and $F_0/m = f_0$.

The complete solution of eqn (i) may be written in the form :

$$x = a_0 e^{-kt} \sin(\omega t + \theta) + A \sin(pt - \theta) \quad .. \quad \text{(ii)}$$

The first term on the R.H.S. of eqn. (ii) represents the natural damped oscillation of frequency $\omega/2\pi$ or $\frac{1}{2\pi}\sqrt{k^2 - \omega_0^2}$ with amplitude $(a_0 e^{-kt})$ decaying exponentially to zero and the second represents a forced oscillation of frequency $p/2\pi$ (same as the frequency of the forcing applied) and a constant amplitude A, where $A = f_0/\sqrt{(\omega_0^2 - p^2)^2 + 4k^2p^2}$.

The former oscillation dies out quickly and the oscillator finally oscillates with constant amplitude A and frequency $p/2\pi$ of the forcing. The equation, then, becomes $x = A \sin(pt - \theta)$.

2.3. Resonance :

If the frequency of the applied force is identical with the natural frequency of the body, a large vibration may be set up by the application of even a small force. This kind of forced vibration in which the frequency of the forcing coincides with the natural frequency of the vibrating body, is known as *resonance* or *sympathetic vibration*.

Definition : *Resonance is said to occur whenever a particular body or system is set in oscillation at its own natural frequency as a result of impulses received from some other system which is vibrating with the same frequency.*

Illustrations : (1) Suspend four pendulums from a horizontal flexible support. The pendulums A and A' have equal lengths but the length of the pendulum B is longer while that of the pendulum C is shorter than the length of A or A' (Fig. 2.4). If A is set oscillating at right angles to the support XY, it will be found that A' immediately takes up the oscillation and begins to oscillate with the same amplitude as A. The other two pendulums will also oscillate but their amplitudes will be different and their swinging will be out of step with those of A or A'. After such irregular vibration for some time, the pendulums B and C, will come to rest and afterwards they will again start vibrating with the same frequency as A and A' but with a smaller amplitude. Such behaviour of the pendulums is due to forced vibration and resonance. The pendulums A and A' have their natural frequencies equal because their lengths are equal. The vibration of A exerts a periodic force on the support XY. This is transmitted to other pendulums which are set into periodic vibration. The frequency of the applied force agrees only with the natural frequency of A'. So, A' is thrown into resonance or sympathetic vibration, while the pendulums B and C undergo forced vibration of smaller amplitude.

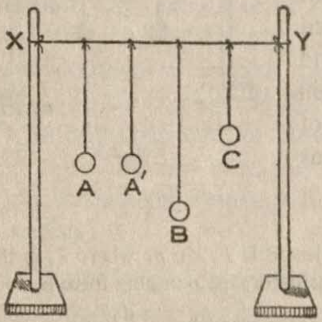


Fig. 2.4

(2) Cases have been known in the past where suspension bridges have been damaged by the resonant vibrations caused by marching military columns. This can occur if the rate of marching happens to bear a simple numerical relationship to the natural frequency of oscillation of the bridge determined by its shape, size and material. Now-a-days, in order to guard against accident, soldiers are always given the order to break step while crossing a suspension bridge.

When a car runs along an undulated road, the oscillations of the body of the car on its springs may match, in frequency, the jerks from the wavy road. Such resonance is disastrous for the car and the driver, sensing the resonance, at once changes the speed of the car.

Illustrations of resonance are readily available in acoustics, too.

(3) **Hollow box in musical instruments :** In stringed instruments like *sitar*, *esraj*, *violin* etc, strings are stretched on a hollow wooden box which is full of air. The vibrations of the strings excite forced vibration in the box and thence in air which vibrates in resonance with the strings. This intensifies the sound of the instruments.

Tuning forks are also mounted on hollow box whose size is such that the air inside it has natural frequency same as that of the tuning fork. As a result, when the tuning fork vibrates, the air in the box is thrown into resonance producing a loud sound.

(4) **Helmholtz's resonator** : German scientist Helmholtz devised a resonator in order to analyse a composite sound consisting of several frequencies. The function of the resonator is based on forced vibration and resonance.

The resonators are made of brass and look like spheres with two protuberances on the two ends of a diameter (Fig. 2.5). The mouth '*a*' is slightly bigger than the mouth '*b*'. The source of sound is held near the mouth '*a*'. Depending on the size of the resonator, the air inside it has a characteristic frequency. The incident sound sets the air into forced vibration. Resonance will occur if the incident composite sound has, among its many frequencies, a frequency equal to the natural frequency of the air inside the resonator. Keeping ear in contact with the smaller mouth '*b*', this frequency will be heard clearly and loudly. Taking a number of such resonators of different frequencies, it is easy to analyse a composite sound.

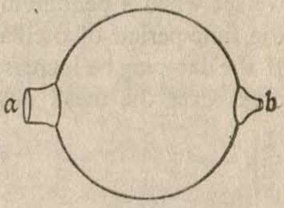


Fig. 2.5

If l= length of the neck a of the resonator, α=its cross-section, v=volume of the sphere and V=the velocity of sound in air, it may be proved that the natural frequency of the resonator is given by $n=\frac{V}{2\pi}\sqrt{\frac{\alpha}{l.v}}$.

(5) **Resonant air column** : The air column in a tube, open or closed, has a natural frequency depending upon its length. If a vibrating tuning fork be held near the mouth of the tube, forced vibration will be produced in the air column. If the frequency of the tuning fork agrees with the natural frequency of the column, resonance will occur and a loud sound will be produced. Such an arrangement is called a resonant air column.

2.4. Damped oscillation :

In discussing natural vibration of a body it has been mentioned that the natural vibrations die out steadily and the body finally comes to rest on account of viscous, frictional and other resistances. To maintain oscillations against such resistive forces, the body has to do some work which entails a loss of energy. Due to gradual loss of energy in this way, the amplitude of vibration of the body diminishes and the body finally comes to rest. Such oscillations are called *damped oscillations.*

A simple pendulum, set into oscillation and left to itself, will be found to have its amplitude gradually diminishing and after some time it will be found to have stopped oscillating. During oscillation, the pendulum faces viscous resistance of air as well as frictional resistance at the point of suspension which damp the vibration of the pendulum.

When a tuning fork is struck with a hammer, it emits a sound. But the sound, with the passage of time, becomes fainter and fainter and finally stops. This shows that the amplitude of vibration of the fork gradually diminishes. This is due to the frictional forces called into play among the layers of the material of the fork as well as to viscous resistance of the air. The vibrations of a tuning fork may, therefore, be called damped vibrations.

As a matter of fact, all practical cases of free oscillations are damped because they always face some resistance. If the resistive force *i.e.*, damping be small, the oscillator will oscillate but its amplitude will slowly diminish. This is what we see when a pendulum is set oscillating in air. With the increase of damping, the time-period of oscillations also increases and the oscillations die down quickly. If the damping be increased to such an extent that the body after deflection can not cross over the mean position during its return journey, the limit of damping is called *critical damping*. It is needless to mention that at the critical damping stage, the body stops oscillating and comes to the rest position in quickest possible time. If the damping be large, the body will not oscillate at all ; it will slowly come back to its position of rest from the deflected position. If a pendulum, for example, be pulled aside and allowed to oscillate in a liquid of high density, it will not oscillate at all because of high damping. If, on the other hand, the damping is small, the body will oscillate but with decreasing amplitude. A graph showing how the amplitude of a damped oscillation dies out is given in fig. 2.6. The dotted line shows how the amplitude steadily dies out though the motion is mainly simple harmonic.

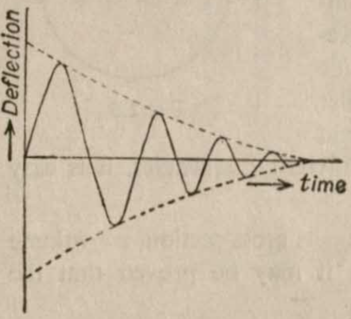


Fig. 2.6

Such damped oscillations are also found in electrical circuits like inductive circuits, ballistic galvanometers etc.

[Mathematical notes :

In damped oscillation, the body is set into vibration by giving an initial displacement or velocity and then allowed to oscillate freely. No external force is applied on the body. Only a resistive force of amount $\gamma.\frac{dx}{dt}$ and a restoring force of amount $c.x$ act on it. The equation of motion of a damped harmonic oscillator may, therefore, represented by,

$$\frac{d^2x}{dt^2}+\frac{\gamma}{m}.\frac{dx}{dt}+\frac{c}{m}.x=0 \quad \text{[See mathematical notes of art. 2.2]}$$

$$\text{or,}\quad \frac{d^2x}{dt^2}+2k.\frac{dx}{dt}+\omega_0^2 x=0 \quad .. \quad \text{(i)}$$

The general solution of the equation (i) is,

$$x=Ae^{(-k+\sqrt{k^2-\omega_0^2})t}+B.e^{(-k-\sqrt{k^2-\omega_0^2})t} \quad .. \quad \text{(ii)}$$

So, the displacement contains two terms which diminish exponentially and hence the oscillator comes to the rest position very quickly. Now three cases may arise as follows :

(i) when $k>\omega_0$; $\sqrt{k^2-\omega_0^2}$ is a real quantity with a positive value. So, each of the two terms on the R.H.S. of eqn. (ii) has an exponential term with negative power. The displacement.

after attaining its maximum value, therefore, dies out exponentially with time without oscillation. Such motion is called *overdamped*, *aperiodic* or *dead-beat*.

(ii) when $k=\omega_0$; $\sqrt{k^2-\omega_0^2}=0$ and $x=(A+B)e^{-kt}$

In this case also, the motion is non-oscillatory and the displacement diminishes exponentially at a very quick rate. Damping, in this case, is called *critical damping* because the oscillator *just ceases to oscillate.*

(iii) when $k<\omega_0$; $\sqrt{k^2-\omega_0^2}$ will be an imaginary quantity. Solution of eqn. (ii) can be obtained in the following form : $x=a_0e^{-kt}.\ \sin(\omega t+\phi)$ when $\omega=\sqrt{k^2-\omega_0^2}$.

This is the equation of a damped harmonic motion with amplitude a_0e^{-kt} and frequency $\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{K^2-\omega_0^2}$. It is so called because the sine term in the solution indicates that the motion is harmonic in nature and the exponential term indicates that the amplitude is gradually damping out.]

2.5. Sharpness of resonance :

If has been mentioned earlier that the amplitude of forced vibration depends upon the relation between the natural frequency of the vibrating body and the frequency of the periodic forcing. The amplitude also depends upon the damping. Fig. 2.7 shows the response curves of a vibrating body under heavy, medium and small damping.

From the figure it is seen that the amplitude of forced vibration, in all cases, becomes maximum at a particular frequency of the forcing. This frequency is called the *resonant frequency*. At a frequency slightly less than or greater than the resonant frequency, the amplitude of the forced vibration diminishes appreciably. In the case where the damping is small, the resonant frequency coincides with the natural frequency of the vibrating body, but as the damping increases, the resonant frequency becomes increasingly less than the natural frequency. Furthermore, the resonance is very sharp when the damping is small but with the increase of damping, the curve becomes flatter near the resonance region. This is known as the *sharpness of resonance*. In the case of resonant air column, for example, the damping is high. For this reason the resonance is obtained for a certain length of the air column instead of a sharp definite length. It can be proved that the amplitude of the vibrating body becomes infinite when the damping is completely absent.

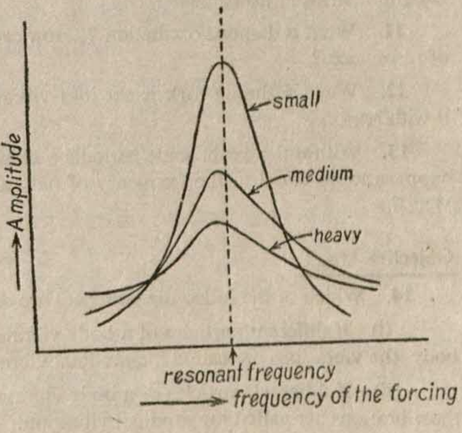


Fig. 2.7

Exercises

Essay type :

1. What are different types of vibration ? Explain their differences with examples.

2. What is the difference between natural vibration and forced vibration ? Explain their nature with the help of mechanical and acoustical illustrations.

3. What do you understand by free and forced vibration ? Under what condition will forced vibration produce resonance ? [*H. S. Exam. 1980*]

4. What are forced vibration and resonance? Give one example of each.

Short answer type :

5. A vibrating tuning fork is kept in air. Discuss the nature of vibration of the fork and those of the air medium. When the stem of the fork is pressed on a wooden board, loud sound is heard. Why does it happen so ? Does it make any difference in the duration of vibration of the fork ? If so, why ?

6. Why is the stem of a tuning fork usually mounted on a wooden box ?

7. What is resonance ? Explain with illustrations.

8. What is the utility of the hollow box of a violin ? What is Helmholtz's resonator ? [*H. S. Exam. 1960, '76*]

9. Describe a suitable arrangement for producing resonance between a tuning fork and an air column.

10. Answer the following questions :—(*a*) The intensity of sound increases when the handle of a vibrating tuning fork is pressed on a table top. Why ? (*b*) Why orders are given to soldiers to break steps while crossing a suspension bridge ? (*c*) Why are a number of strings fixed in a stringed instrument ?

11. What is damped oscillation ? How can you express it graphically ? What is sharpness of resonance ?

12. When a tuning fork is set into vibration in air, the vibrations are damped. Explain it with reasons.

13. Within the earth, some periodic oscillations are often present. If the frequency of these happens to be equal to the frequency of natural waves on a lake, disastrous results may follow. Why ?

Objective type :

14. Which of the following sentences are correct ?

(i) If different particles of a body vibrate in a direction perpendicular to the length of the body, the vibrations are called longitudinal vibration.

(ii) If different particles of a body vibrate in a direction parallel to the length of the body, the vibrations are called longitudinal vibration.

(iii) When a body is made to vibrate under the influence of an external periodic force, the vibrations are called transverse vibration.

(iv) Resonance occurs when the natural frequency of an oscillator happens to match the frequency of the forced oscillations.

(v) If the damping of a pendulum increases, its time period decreases and it oscillates for a long time,

3

WAVE MOTION

3.1. Wave :

When a stone is dropped into a quiet pond, it creates a disturbance at the point of contact and this disturbance spreads outward in the form of crests and troughs over the entire surface of the pond.

The water particles at the point of impact are first disturbed and they hand over the disturbance to the adjoining particles. In this way, disturbance, being transferred from particle to particle reaches the edge of the pond. It is to be noted in this connection that although the disturbance travels over the surface of the pond from one place to another, the water particles are not bodily moved; they merely bob up and down, scarcely moving from its place. Whenever a portion of a medium —solid, liquid or gas—is disturbed, it tries to come back to its former position of rest due to elastic forces called into play by the displacement. When the particles of the disturbed portion of the medium regain their normal level, they possess considerable momentum due to inertia of motion, for which the particles overshoot their mark and are deflected in the opposite direction. In other words, the particles of the medium oscillate like a pendulum. Because of cohesive forces existing in material medium, a displaced particle drags down or pulls up its neighbouring particles. Consequently, disturbance created at any part of the medium, spreads throughout the medium. Such disturbances are called *waves.*

Mechanical wave motion is possible only in material media (solid, liquid or gas) which possess inertia as well as elasticity. Water waves and sound waves are both examples of this type of wave motion and are, therefore, called mechanical waves or elastic waves.

Non-mechanical or *electromagnetic wave motion* requires no material medium for its propagation. Light waves, wireless waves, X-ray waves etc can travel through empty space. They, therefore, belong to the category of electromagnetic waves.

However, when electromagnetic waves travel through *material medium* the velocity of the waves depends upon the nature of medium. The velocity of the electromagnetic waves is the highest in vacuum.

3.2. Types of mechanical waves :

Waves generated by the simple harmonic motion of the particles of the medium are called simple harmonic waves. They are of two types :— (i) *transverse* and (*ii*) *longitudinal.*

Transverse wave : Transverse waves are those in which each particle of the medium vibrates up and down along a line perpendicular to the direction of propagation of the wave.

Fig. 3.1 shows the nature of a transverse wave. Here the particles of the medium vibrate up and down whereas the wave travels in a horizontal direction. The directions of motion of different particles situated on the wave at any instant have been shown in the diagram by small arrow heads. The points where the particles have maximum displacement in the positive directions are called *crests* and the points where the particles have maximum displacement in the negative direction are called *troughs*. In the diagram, the point *A* is situated on a crest while the point *C* on a trough. Note that the phase of *A* is exactly opposite to the phase of *C* while *A* is in same phase with *D*. This type of wave motion is possible in media which possess elasticity of shape or rigidity *i.e.* solids. Liquids and gases possess neither rigidity nor do they resist any vertical displacement of their particles and hence a transverse wave motion is not possible in liquid or gaseous medium. Question may be raised that light travels through liquids and gases and light is a transverse wave motion. How is it possible? It is to be remembered that light is not a mechanical wave motion ; it is electromagnetic wave. Electromagnetic waves can be propagated through solid, liquid and gases or even through empty spaces.

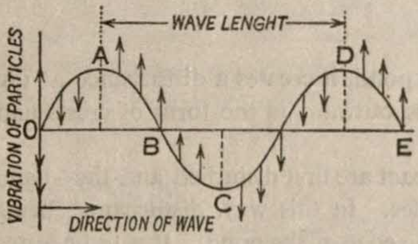


Fig. 3·1

Examples : (1) Probably everyone has, at some time or another, thrown a stone into a pond or other smooth sheet of water and noticed the circular ripples which spread out from the spot where the stone strikes the water. A floating chip of wood does not move forward with the waves that strike it, but merely moves up and down. Here the waves travel along the surface of water in a horizontal direction while water particles move simple harmonically in a line perpendicular to the direction of motion of the wave. So, the ripples on the surface of water constitute transverse waves. In this way, light, heat, wireless waves are all examples of transverse waves.

[**N.B.** : Although water waves have been cited as examples of transverse waves, yet they are not truly transverse waves . In the production of water-waves, gravitation and surface tension play an important part. Still, for the sake of understanding the wave-motion at the first stage, it is better to regard them as simple transverse waves.]

(2) When one end of a piece of rope or string is moved up and down in a direction perpendicular to its length, the particles of the rope near the end exert a drag on their neighbours so that these begin to oscillate as well. This process continues throughout the rope, until finally any particular particle is oscillating up and down slightly later than one immediately before it. The net result is that a transverse wave, consisting of crests and troughs, travels forward with a certain velocity.

Longitudinal wave : Longitudinal waves are those in which the particles of the medium oscillate to and fro about their mean position along the direction of propagation of the wave motion itself.

Fig. 3.2 shows the nature of a longitudinal wave in air. A column of air is divided into several equidistant layers and when longitudinal waves pass by, each layer vibrates back and forth about some equilibrium position. Since layers of air are moving simple harmonically along straight lines parallel to the direction of propagation, some layers come very close to each other at any instant, producing what is called a *compression* followed by some layers having separated from each other producing what is called a *rarefaction*. Longitudinal wave motion is possible in media possessing elasticity of volume *i.e.* in solids, liquids as well as in gases.

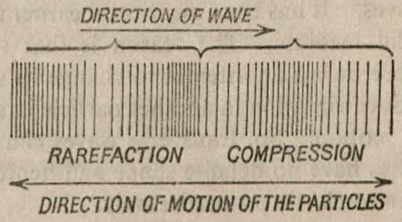


Fig. 3.2

Examples : (1) The apparatus shown in fig. 3.3. consists of a helical coil of thin soft copper wire containing several turns. Each turn of the coil is supported by a fine silk thread in the form of a V, the ends of the thread being fixed to a wooden framework. Holding one end of the coil by hand if a to and fro endwise motion be given to it, a wave will pass along the coil. This wave is called longitudinal because the small excursions of the separate coils are executed longitudinally or along the line of advance of the disturbance. This longitudinal wave is characterised by alternate compressions and rarefactions of the coils.

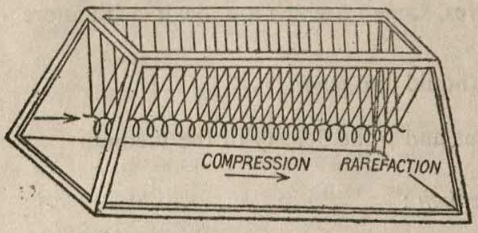


Fig. 3.3

(2) Take a thin but long spring and fasten one end of it to a rigid support and the other end to one of the prongs of a tuning fork. When the fork does not vibrate, the coils of the spring remain equally spaced. When the tuning fork vibrates, it sends a longitudinal wave along the spring. Some of the coils will be found to be compressed and some rarefied [Fig. 3.4].

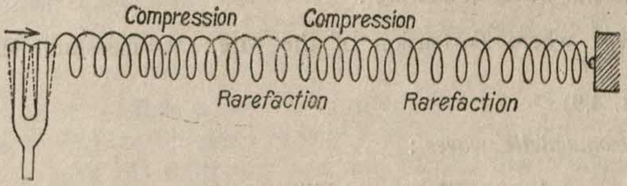


Fig. 3.4

Transverse and longitudinal waves are together known as *progressive waves* ; these waves progress from one point to another through a medium. There is another type of waves, having opposite properties known as *stationary waves*, which will be discussed later on.

3.3. Nature of the medium :

An elastic medium is necessary for the production and propagation of elastic waves. It has been mentioned earlier that only transverse waves can travel through solid medium. The reason is that solids have a definite shape and can resist more or less an attempt to change its shape. Any vibrating particle in a solid can induce vibration in another particle in a direction at right angles to its own direction of motion. So, transverse waves can be set up in a solid medium. Liquids and gases have no definite shape and hence transverse waves are not produced in them due to elastic properties only.

Longitudinal waves, however, can be set up in all the three types of media—solid, liquid and gas. The reason is that each one of them can resist volume strain. A vibrating particle in a material medium can induce vibration in another particle in a direction parallel to its own direction of motion. So, longitudinal waves can travel in all material media.

It is to be noted that the velocity of elastic waves (transverse or longitudinal) depend on the modulus of elasticity and the density of the medium. We list below the velocity of various types of waves, some of which are dealt with more elaborately in other sections of the book.

1. *Longitudinal waves in a medium* (liquid and gas) :

   $V=\sqrt{\frac{E}{\rho}}$, where E=bulk modulus and ρ the density of the medium

2. *Longitudinal waves in a solid medium* :

   $V=\sqrt{\frac{Y}{\rho}}$, where Y=Young's modulus of the solid.

3. *Sound waves in gases* :

   $V=\sqrt{\frac{\gamma P}{\rho}}$ where P=pressure of the gas, ρ=density of the gas and γ= ratio of sp. heats of the gas.

4. *Transverse waves in a string* :

   $V=\sqrt{\frac{T}{m}}$ where T=tension of the wire ; m=mass/unit length of the wire. (See art. 4.9)

5. *Electromagnetic waves* :

   $V=\frac{1}{\sqrt{\mu k}}$ where μ=the permeability and k=permittivity of the medium.

3.4. Characteristics of progressive waves :

The characteristics of progressive waves may be summarised in the following way :

(i) Progressive waves are generated by continuous disturbance of a portion of the medium and they advance through the medium with a velocity determined by the density and the elasticity of the medium.

(ii) Each particle of the medium undergoes same type of vibration about its position of rest with identical frequency and amplitude. The vibrations may be transverse or longitudinal relative to the line of propagation of the waves.

(iii) The phase of one particle is transmitted to the next along the line of propagation. The phase difference between two particles along this line is proportional to the distance between the particles.

(iv) Progressive wave carries energy from one point to another along a line perpendicular to the wave-front without any bodily motion of the medium.

(v) As the wave proceeds, every point of the medium undergoes same changes in pressure and density.

3.5. Some definitions in connection with a wave :

Wavelength : *The wavelength is defined as the distance between two successive particles which are exactly at the same phase of motion.*

In fig. 3.1, the points A and D or the points B and E on the transverse wave have the same phase. Hence the distance AD or BE is the wavelength of the wave. Note that the distance AD or BE is the distance between the centres of two nearest crests or two nearest troughs. So, the wavelength in a transverse wave may alternatively be defined as the distance between the centres of two nearest crests or two nearest troughs.

Similarly, in fig. 3.2, two points at the middle of two successive rarefactions or two successive compressions are in the same phase and their distance gives the wave length of a longitudinal wave. Note also that this distance includes one complete rarefaction and one complete compression in succession. Hence, for a longitudinal wave, the total distance between consecutive rarefaction and compression may be taken as its wave length.

Amplitude of wave : *The maximum displacement of a wave particle from the rest position is called the amplitude of the wave.* In fig. 3.1 the displacement of the point A or D from the line OBE is the amplitude of the wave.

Period of wave : *The time taken for a wave particle to make one complete oscillation is called the period of wave.*

Frequency of wave : *The number of complete oscillations made by a wave particle in one second is called the frequency of wave.*

Wave front : When a stone is dropped into a quiet pond, a set of waves spreads outwards from the point of impact in ever widening circles, consisting of crests and troughs. In other words, it may be said that water particles situated either on the crests or on the toughs lie on concentric circles, and have the same phase. *The continuous locus of points in the same phase of vibration in a medium through which a wave is passing, is called a wavefront.* When transverse waves are produced on the surface of water, the wave fronts are evidently circular in shape.

In any homogeneous medium—not to speak of water alone—waves will spread in all directions with the same velocity from the point of disturbance and all particles in space equidistant from the point of disburbance will be in same phase. Hence, when a wave is propagating through space, the wave-front is spherical.

Wave velocity : *The distance travelled by a wave in 1 second is called the wave velocity.*

We have seen that when a wave passes through a medium, the particles of the medium do not move forward ; they simply move up and down. Then, whose velocity do we mean by wave velocity ?

When a wave is produced on the surface of water, the water particles do not bodily move forward but the wavefront does not remain stationary. It advances through the medium with a definite velocity. This velocity of the wave front is called the wave velocity.

3.6. A few important relations :

(*a*) *Relation between wave velocity, frequency, time-period and wave length* :

There is a simple relation between the frequency n, the wave length λ and the wave velocity V. Suppose the source vibrates for exactly 1 sec. In this time, just n complete waves will be sent out. Each of these waves has a length λ ; so the first wave will be at a distance $n\lambda$ from the source. But in one second's time, the first wave will have travelled a distance V, the wave velocity. So we have, $V=n\lambda$. *i.e. wave velocity=frequency × wave length*

Again $n=\frac{1}{T}$ $\therefore$ $V=\lambda/T$ or $\lambda=VT$.

(*b*) *Relation between the velocity of a wave in two different media* :

Let A and B be two different media. The velocity of a wave in the medium A is, suppose V_A and that in the medium B is V_B.

For the medium A, we have, $V_A=n.\lambda_A$ where λ_A is the wave length in the medium A. Similarly for the medium B, we have $V_B=n.\lambda_B$, where λ_B is the wave length in the medium B. Dividing, $\frac{V_A}{V_B}=\frac{\lambda_A}{\lambda_B}$.

This is the relation between the velocities of a wave in two different media.

(*c*) *Relation between wave length and frequency* :

Suppose, two different waves are propagating in a given medium. The frequency and wave length of one of them are n_1 and λ_1 respectively while those of other are respectively n_2 and λ_2.

For, the first wave, we have $V=n_1\lambda_1$ where V=velocity of the waves in the medium. For, the second wave, $V=n_2\lambda_2$ (medium being the same, velocity of the waves will be the same) $\therefore$ $n_1\lambda_1=n_2\lambda_2$ or, $\frac{\lambda_1}{\lambda_2}=\frac{n_2}{n_1}$

This is the relation between the wavelength and frequency.

Examples : (1) *A wave of length 580 cm. is produced in water. If the velocity of the wave in water is 145,000 cm/sec, what is the frequency of the wave ?*

Ans. Here, $\lambda=580$ cm ; $V=145{,}000$ cm/sec ; $n=$?

We know, $V=n\lambda$ or $n=\frac{V}{\lambda}=\frac{145{,}000}{580}=250$

(2) *The frequency of a tuning fork is 256 and sound travels to a distance of 20 metres while the fork executes 16 vibrations. Find the wave length of the note emitted and also the velocity of sound.*

Ans. Suppose, the wave length of the sound wave$=\lambda$ cm. So, during one complete vibration of the tuning fork, the wave will travel a distance λ cm.

$$\therefore \quad 16\,\lambda=20\times100 \text{ or, } \lambda=\frac{20\times100}{16}=125 \text{ cm.}$$

Again $n=256$; $\lambda=125$ cm. ; $V=$?

We have, $V=n\lambda=256\times125=32{,}000$ cm/sec.$=320$ metres/sec.

(3) *If the frequency of a tuning fork is 560, find how far the sound will have travelled at the instant when the fork just completes 50 vibrations. Velocity of sound=1120 ft/sec.*

Ans. We have, $V=n\lambda$; here $V=1120$ ft/sec. ; $n=560$.

$\therefore$ $1120=560\,\lambda$ or, $\lambda=2$ ft.

So, in 50 complete oscillations, the sound travels$=2\times50=100$ ft.

(4) *A piece of cork is floating on water in a small tank. When ripples pass over the water surface, the piece of cork moves up and down. What will be the maximum velocity of the piece if the velocity of the wave is 0·2 m/s, wavelength of the wave is 15 mm and amplitude of the wave is 5 mm ?*

Ans. When ripples flow across the piece of cork, it oscillates up and down with the same amplitude and frequency as the ripples. Now, for the ripples, $V=n\lambda$. Here, $V=0{\cdot}2$ m/s ; $\lambda=\frac{15}{1000}$ metre. So, $n=\frac{V}{\lambda}=\frac{0{\cdot}2\times1000}{15}$

The maximum velocity of the piece of cork with amplitude $a=\frac{5}{1000}$ metre is

$$V_{max}=a.\omega=2\pi n.a=2\times\frac{22}{7}\times\frac{0{\cdot}2\times1000}{15}\times\frac{5}{1000}=0{\cdot}42 \text{ m/s. (nearly)}$$

(5) *A rod hanging from a helical spring dips partly in a sheet of water and executes 120 vibrations in a minute. The ripple generated in the water is such that the distance between 11 consecutive crests is 20 cm. Calculate the velocity of propogation of the ripple.*

Ans. We know, $V=n\lambda$. In this case, $n=\frac{120}{60}=2$. Now, the distance between 2 consecutive crests is equal to the wave-length of the waves. So, 11 consecutive crests will include 10 waves which cover a distance of 20 cm. So, $\lambda=\frac{20}{10}=2$ cm.

$$\therefore \quad V=2\times2=4 \text{ cm/sec.}$$

3.7. Difference between transverse and longitudinal waves :

Transverse Wave	*Longitudinal Wave*
1. If the particles of an elastic medium vibrate up and down producing a disturbance in the medium which travels in a direction *perpendicular* to the motion of the particles, the wave so generated is known as transverse wave.	1. If the particles of an elastic medium vibrate back and forth producing a disturbance in the medium which travels in a direction *parallel* to the motion of the particles, the wave so generated is known as longitudinal wave.
2. When transverse waves pass over a medium, crests and troughs are formed.	2. When longitudinal waves pass over a medium, compressions and rarefactions are formed.
3. The distance between the centres of two successive crests or troughs is called the wave length of a transverse wave.	3. The distance between the centres of two successive compressions or rarefactions is called the wave length of a longitudinal wave.
4. Transverse waves can be polarised.*	4. Longitudinal waves can not be polarised.

3.8. Equation of a progressive harmonic wave :

Progressive harmonic waves, we know, are formed when the particles of the medium through which the wave progresses, vibrate simple harmonically. We shall now try to find out an equation of such waves.

When a progressive wave travels through a medium, the particles of the medium undergo simple harmonic motion. Since some time is needed for the disturbance to reach one particle from another, there is a gradual fall of phase of the particles along the line of propagation of the wave. If the wave is supposed to be travelling from left to right, then the particles on the left will be disturbed first and those on the right later. Consequently the particles will have different phases.

Consider a progressive wave originating at *A*, travels towards *C* [Fig. 3.5]. If we start counting time when the particle at *A* just passes through its mean position in the positive direction (*i.e.* upward in the case of a transverse wave and forward in the case of a longitudinal wave), the equation of motion of the particle *A* may be written as $y=a \sin \omega t$ where a=amplitude of vibration, y=displacement of the particle from the line *ABC* at any time t and ω=angular velocity of the particle.

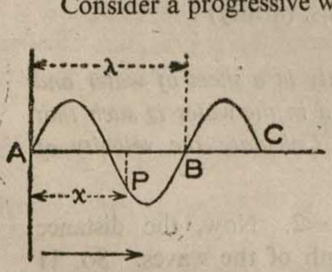


Fig. 3.5

If the frequency of the particle be n, then $\omega=2\pi n$. So, $y=a \sin 2\pi nt$.

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*See chapter six.

Since the successive particles to the right of A receive and repeat its movements after definite intervals of time, the phase lag goes on increasing as we proceed further from A towards the right. Suppose, the phase of a particle at P, distant x from A, be ϕ. Then, the equation of motion of the particle at P is $y=a\sin(\omega t-\phi)$.

We know, the wave length of the wave is the distance between two consecutive particles having same phase. In this case, the wave length λ=the distance AB. Further, we know, that the change of phase between two particles situated at a distance of λ is 2π (*i.e.* of the same phase). Hence, a particle at any point P situated at a distance x from A will have a change of phase ϕ given by $\phi=\frac{2\pi}{\lambda}.x$.

So, the displacement y of the particle at P is given by

$$y=a\sin(\omega t-\phi)=a\sin\left(2\pi nt-\frac{2\pi}{\lambda}.x.\right)$$

$$=a\sin\left(2\pi.\frac{Vt}{\lambda}-\frac{2\pi}{\lambda}.x.\right) \quad \text{[velocity of the wave } V=n\lambda]$$

$$=a\sin\frac{2\pi}{\lambda}(Vt-x). \quad .. \quad \text{(i)}$$

This equation represents a progressive wave* for, it gives the displacement of any particle on the wave situated at a known distance from the arbitrary point A.

Similarly, the equation of a progressive wave travelling in the opposite direction (*i.e.* $-x$) is, $y=a\sin\frac{2\pi}{\lambda}(Vt+x)$. .. (ii)

[**Note** : In deducing the above equation of progressive wave, we assumed that the particle at A just passes through its mean position in the positive direction at $t=0$. If this be not the case, the particle is said to have an *initial phase* θ, say. The equation of wave then becomes

$$y=a\sin\frac{2\pi}{\lambda}(Vt-x+\theta)]$$

Particle velocity : It is to be remembered that the velocity of a wave through a medium and the velocity of a particle of the medium are entirely different

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*Differentiating eqn. (i) of art. 3.8 twice, we get the particle acceleration, which is given by,

$$\frac{d^2y}{dt^2}=-\frac{4\pi^2.V^2}{\lambda^2}.a\sin\frac{2\pi}{\lambda}(Vt-x)=-\frac{4\pi^2V^2}{\lambda^2}.y$$

Differentiating the same equation twice with respect to x, we get, for strain or compression

$$\frac{d^2y}{dx^2}=-\frac{4\pi^2}{\lambda^2}a\sin\frac{2\pi}{\lambda}(Vt-x)=-\frac{4\pi^2}{\lambda^2}.y. \quad \text{From these we have, } \frac{d^2y}{dt^2}=V^2.\frac{d^2y}{dx^2}$$

This is referred to as the differential equation of a progressive harmonic wave.

things. Suppose a transverse wave *OABC* is travelling from left to right with a velocity *V* [Fig. 3.5(*a*)]. A particle of the medium, say *a* will vibrate transversely up and down about a certain point as the mean position. If *v* be its velocity at any instant, it is clear that the velocity changes with time. For a transverse wave, the direction of *v* is always at right angles to the direction of *V*.

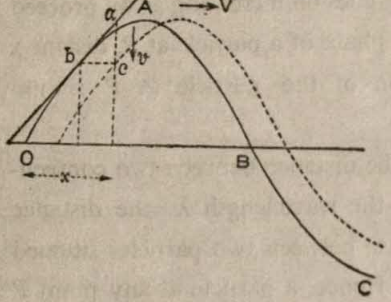

Fig. 3.5(a)

If the wave travels a small distance *bc*, the particle *a* in the mean time will come down through a distance *ac*. Here the ratio *ac*/*bc* is called the slope of the curve *OAB*.

$$\therefore \quad \frac{\text{Particle velocity }(v)}{\text{Wave} \quad ,, \quad (V)} = \text{slope of the wave curve.}$$

For example, the instantaneous velocity of the particle at *A* is zero but those at *O* and *B* are maximum.

From the equation (i) above we can find the velocity *v* of the particle (as distinct from the wave velocity *V*) at a general position *x* at any time *t*.

$$y = a \sin \frac{2\pi}{\lambda}(Vt - x) = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad \left[\because \ \frac{V}{\lambda} = \frac{1}{T}\right]$$

$$\therefore \quad v = \frac{dy}{dt} = +\frac{2\pi}{T}.a \cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad .. \quad \text{(iii)}$$

$$= +v_0 \cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad \left[v_0 = \frac{2\pi}{T}.a\right] \quad .. \quad \text{(iv)}$$

Note that the particle velocity *v* is a varying quantity, while the wave velocity *V* is a constant quantity for a given medium. The time and space variation of the particle velocity is of the same nature as of the displacement and the maximum particle velocity, known as *velocity amplitude* (v_0) is $\frac{2\pi}{T}$ times the displacement amplitude *a*.

Examples : (1) *A progressive wave is represented by the equation :* $y = 10 \sin(120\pi t - 0{\cdot}04\pi x)$, *where x and y are measured in cm. and mm. respectively and t in sec. Find the amplitude, frequency, wave length, speed of the wave and velocity amplitude of the particle.*

Ans. The general equation of a progressive wave is given by

$$y = a \sin\left(2\pi nt - \frac{2.\pi}{\lambda}.x.\right)$$

The equation given in the question is $y = 10 \sin(120\pi t - 0{\cdot}04\pi x)$. Comparing these two equations, we can write

(i) amplitude $a = 10$ mm. [sin function is a pure number; so *a* has same unit as *y*]

(ii) frequency (n) $2\pi n = 120\pi \quad \therefore \quad n = 60$.

(iii) wave length (λ) $\frac{2\pi}{\lambda} = {\cdot}04\pi \quad \therefore \quad \lambda = \frac{2}{{\cdot}04} = 50$ cm.

(iv) speed $v = n\lambda = 60 \times 50 = 3000$ cm/sec $= 30$ metres/sec.

(v) velocity amplitude : $\frac{2\pi}{T}.a = 2\pi n.a = 2 \times 3{\cdot}14 \times 60 \times 10$mm/s $= 3{\cdot}77$ m/s.

(2) *A progressive wave is moving with a velocity 2m/s through a medium in x-direction. At $x=0$, the displacement of a particle at any time t is 7 sin $10\pi t$ mm. What will be the displacement of a particle at $x=60$ cm. at a time 0·75 sec. later.*

Ans. Angular frequency of the harmonic motion of the particle $\omega = 10\pi$. Wave velocity $V = 2$m/s $= 200$ cm/s ;

$$\text{Wave length } \lambda = \frac{V}{n} = \frac{V}{\omega/2\pi} = \frac{2\pi \times 200}{10\pi} = 40 \text{ cm.}$$

If ϕ be the phase of the particle at $x = 60$ cm. with respect to the particle at $x = 0$, then $\phi = \frac{2\pi}{\lambda}.x = \frac{2\pi}{40} \times 60 = 3\pi$ behind. So, the displacement of the second particle $= 7 \sin(10\pi t - 3\pi) = 7 \sin(10 \times \pi \times 0{\cdot}75 - 3\pi) = 7 \sin(4{\cdot}5\pi)$

$= 1{\cdot}71$ mm. (nearly).

(3) *A transverse wave represented by $y = 0{\cdot}25 \sin(4\pi t - 0{\cdot}2\pi x)$, where x and y are in feet and t in sec, passes along a string. Each particle of the string moves up and down at right angles to the direction of the wave motion. Find the velocity and acceleration of a particle at $x = 10$ ft. from the end and $t = 0{\cdot}5$ sec.*

Ans. General equation of a transverse wave, having amplitude a, frequency n and wave length λ, may be written as $y = a \sin(2\pi nt - \frac{2\pi}{\lambda}x)$. Comparing the given equation of the wave with this standard equation, we find the following:

$$a = 0{\cdot}25 \text{ ft} \; ; \; 2\pi n = 4\pi \text{ or } n = 2 \text{ and } \frac{2\pi}{\lambda} = 0{\cdot}2\pi \text{ or } \lambda = 10 \text{ ft.}$$

Now, velocity of the particle at $x = x$ and $t = t$, is given by,

$$v = 2\pi.n.a \cos 2\pi\left(nt - \frac{x}{\lambda}\right) \quad \text{[See eqn. (iii) of art. 3.8]}$$

$$= 2\pi \times 2 \times 0{\cdot}25 \cos 2\pi\left(2t - \frac{x}{10}\right)$$

$$= 2\pi \times 2 \times 0{\cdot}25 \cos 2\pi\left(2 \times 0{\cdot}5 - \frac{10}{10}\right) = \pi \cos 2\pi \times 0 = 3{\cdot}14 \text{ ft/sec.}$$

$$\text{And acceleration, } f = \frac{dv}{dt} = -2\pi na \times 2\pi n \sin 2\pi\left(nt - \frac{x}{\lambda}\right)$$

$$= -4\pi^2.n^2.a \sin 2\pi\left(2 \times 0{\cdot}5 - \frac{10}{10}\right) = 0.$$

This shows that the particle is passing through the mean position at that instant.

3.9. Phase of a wave :

As in the case of a simple harmonic motion, the term phase of a wave is that quantity which gives complete information regarding the wave motion. It is obvious from the equation (i) of art. 3.8 that if we know the quantity $\frac{2\pi}{\lambda}(Vt-x)$ we know all about the wave motion. We may, therefore, say that the phase of the wave at any point x at time t is given by $\delta=\frac{2\pi}{\lambda}(Vt-x)$

$$=\frac{2\pi t}{T}-\frac{2\pi x}{\lambda} \qquad \left[\because V=\frac{\lambda}{T}\right]$$

It should be noted that the phase of a wave is a function of both x and t. In other words, phase of waves at a fixed point (x=constant) changes with time t. Also at a particular instant (t=constant) the phase is different at different points.

Let us take two points at distances x_1 and x_2 from A, the point of origin of the waves [Fig. 3.5]. Then the phase δ_1 of the first point at any instant t is given by $\delta_1=\frac{2\pi t}{T}-\frac{2\pi x_1}{\lambda}$. Similarly, the phase of the second point at the same instant is $\delta_2=\frac{2\pi t}{T}-\frac{2\pi x_2}{\lambda}$.

$\therefore$ Phase difference between the points at any time $t=\delta_1-\delta_2=\frac{2\pi}{\lambda}(x_2-x_1)$

$=\frac{2\pi}{\lambda}\times$path-difference between the points.

If $x_2-x_1=\lambda/2$ *i.e.* path-difference be half the wave length then the phase-difference$=\frac{2\pi}{\lambda}\times\frac{\lambda}{2}=\pi$. The points are said to be out of phase with one another.

Example : *The equation of a progressive wave is* $y=2{\cdot}5\sin(400t-0{\cdot}68x+\pi/2)$. *Find (i) the phase at* $x=0$, $t=0$, *(ii) the phase-difference between two points separated by 20 cm. along X-axis (iii) the change in phase at a given position in 0·4 millisecond.*

Ans. Here the phase $\phi=400t-0{\cdot}68x+\pi/2$

(i) at $t=0$, $x=0$, $\phi_0=\pi/2$

(ii) phase-difference$=-0{\cdot}68\times(x_2-x_1)=-0{\cdot}68\times20=13{\cdot}6$ rad.

(iii) change of phase$=400\times0{\cdot}4\times10^{-3}=0{\cdot}16$ rad.

3.10. Reflection of waves :

When a progressive wave travelling in a homogeneous medium, falls on a surface separating two media, a part of the wave is thrown back into the original medium in the same form and travels with the same velocity. This phenomenon is known as reflection of wave. It has been seen that if the reflector is a rigid one, the incident wave gets reflected with its nature unchanged *i.e.* for a transverse wave, a crest is reflected as a crest and a trough as a trough and for a longitudinal wave, a compression is reflected as a compression and a rarefaction as a rarefaction. But if the reflector is a yielding one, the reflection of wave takes place with a change of phase. In the case of transverse waves, a crest is reflected as a trough and vice versa. In the case of longitudinal waves, a compression is reflected as a rarefaction and a rarefaction as a compression.

Take a large, flat pan and pour some water in it. Hold a hard, flat surface (wooden or metallic) or a card partly immersed in water at one end of the pan (Fig. 3.6). Ripples may be produced by falling drops of water and when they strike the surface, they get reflected. They can be clearly seen if lighted by an unshaded lamp placed some distance above the pan.

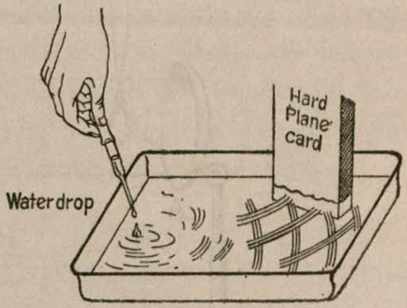

Fig. 3.6

In the same way, any wave like light waves, sound waves etc. may be reflected by suitable reflectors. Wave length of sound is much longer than that of light. For this reason, larger reflectors are necessary for reflection of sound than for reflection of light. Further, for reflection of light, the reflector must have very smooth surface. For sound reflection, this is not necessary. For all these reasons, brick walls, a row of trees, mountains, clouds etc. act as good sound reflectors. It may be proved by Huyghens' wave theory that waves obey laws of reflection when they get reflected by suitable reflectors.

Laws of reflection of sound waves :

The reflection of sound waves obeys the same laws as in the case of reflection of light waves. The laws are :

(i) The incident ray, the reflected ray and the normal to the reflecting surface at the point of incidence lie in the same plane.

(ii) The angle of incidence is equal to the angle of reflection.

Experiments to demonstrate reflection of sound :

(i) **By a plane reflector :** Two metal tubes *A* and *B* are set up in a horizontal plane inclined to one another and pointing towards a vertical flat surface of a wooden board *C* [Fig. 3.6(*a*)]. A wooden partition *P* is kept between the tubes to cut off the direct sound waves.

A ticking watch is placed near the end of the tube *A* and ear is presented to the end of the tube *B*. Sound waves travel down the tube *A* and are reflected by the board *C*. The loudness of the sound heard through the tube *B* is found to be maximum when the board *C* is adjusted so that the normal to it lies in the plane of the tubes and makes equal angles with their axes *i.e.* when $\angle PCA = \angle PCB$.

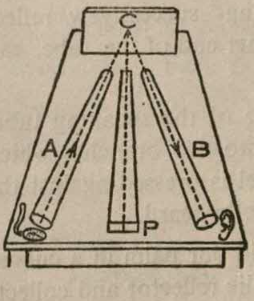

Fig. 3.6(a)

Under these conditions, the incident and the reflected waves and the normal are in the same plane and the angle of incidence ($\angle PCA$) is equal to the angle of reflection ($\angle PCB$). The experiment thus verifies the laws of reflection of sound waves.

(ii) **By a concave reflector :** *A* and *B* are two metallic concave reflectors, mounted co-axially and facing each other (Fig. 3.7). If a ticking watch is placed at the focus of the reflector *A*, distinct sound will be heard if ear is presented at the focus of the other reflector *B*. If the ear be slightly displaced, the sound will be inaudible. The sound waves, being reflected by the reflector *A*, are rendered parallel to the axis of the reflectors and after second reflection at the reflector *B*, converge at its focus, just like light waves.

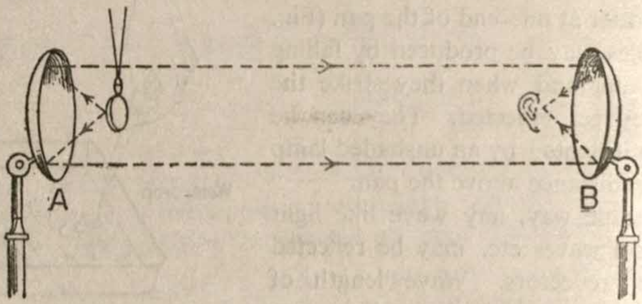

Fig. 3.7

3.11. Practical applications of reflection of sound :

(i) Before the advent of microphones and loudspeakers, big halls were provided with large parabolic sound reflectors. The speaker used to stand at the focus of the reflector and the sound waves were rendered parallel by the reflector and hence could travel over large distances before fading out.

(ii) In large automobiles, steamships etc., speaking tubes are used for communication between the driver and the passenger. Sound can be transmitted over short distances without much loss of intensity by this tube. Sound waves, passing into the tube, cannot spread out but travel down the tube, being successively reflected at the walls of the tube, so that ear placed at the other end of the tube, can hear the sound distinctly [Fig. 3.8].

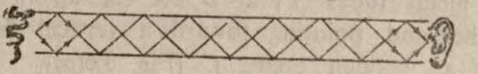

Fig. 3.8

(iii) Physician's stethoscope works on the principle of the speaking tube. A little above the collecting funnel, the tube bifurcates into two branches which separately go to the two ears. When the collecting funnel is pressed against the chest of a patient, the sound of his heartbeat can be distinctly heard.

(iv) To hear a distant, feeble sound, we usually hold our palm in a curved fashion near our ear. The curved palm acts like a parabolic reflector and collects sound wave into our ear.

The horn of a gramophone, the ear-trumpet used by deaf persons etc, use reflection of sound in their action.

(v) Auditoriums meant for public meeting, musical conference, lectures etc. are generally acoustically designed. The ceilings of such halls are suitably curved so as to concentrate sound by reflections at various points of the hall. To deaden reverberation, cushions are placed on the seats and the walls are covered with soft pad. The soft clothing of the audience also absorbs sound to some extent instead of reflecting it. For this reason, speeches are heard more distinctly when the hall is full of audience than when it is almost empty. On the other hand it is convenient for a speaker to deliver his lecture in a room than in an open space where the speaker will not get any assistance from the reflection of sound.

3.12. Echo :

Echoes are repetition of sound and are produced when sound waves are reflected from hard substances like a cliff, an extended wall, a row of trees etc. You may have the experience of hearing an echo by making a sound in large empty hall. After a while, the sound is repeated, being reflected by the distant wall of the hall.

Minimum distance of the reflector : When we hear a sound, its sensation persists in our ear for about $\frac{1}{10}$*th* of a second after its production. This is called *persistence of hearing*. Hence, to hear an echo distinguished from the original sound, at least $\frac{1}{10}$*th* of a second must pass between the production of sound and the arrival of the reflected wave at the ear. In a room of ordinary size, the reflected wave reaches the ear within the aforesaid interval, for which we cannot ordinarily distinguish between the original sound and the echo. Since the velocity of sound under normal conditions is about 1120 ft/sec, the distance traversed by sound in $\frac{1}{10}$*th* of a second is 112 ft/sec. Hence, the minimum distance between the source and the reflector for distinct echo of abrupt sound (such as that produced by a gunshot, hand clapping, explosion etc.) should be 56 ft.

Echo of articulate sound : Every word in human speech consists of a number of syllables. Each syllable takes about $\frac{1}{5}$*th* of a second to be uttered distinctly. Hence to hear the echo of a monosyllabic word, the distance of the reflector from the speaker should be at least 112 ft, because sound travels a total distance of $1120\times\frac{1}{5}=224$ ft in $\frac{1}{5}$*th* of a second. For disyllabic word, the distance should be 224 ft *i.e.* double than the first, for trisyllabic word three times and so on.

Multiple echoes : Sometimes, more than one echo may be heard for a sound due to multiple reflections. If a sound be made between two parallel cliffs, multiple reflections occur at the cliffs and a person standing between the cliffs will hear a number of echoes. Near Verdun in France, there are two parallel towers at a distance of about 164 ft. A person stationed between them hears about a dozen echoes by making a single sound. The rumbling of thunder is due to multiple reflections of sound by different layers of cloud.

Reverberation : In big halls it is often seen that a continuous rolling of sound persists for sometime after a loud sound has stopped. This is known as *reverberation.* When the reflecting surface is at a distance less than the minimum distance required for a distinct echo, reflected sound waves follow so closely upon the direct sound wave that they cannot be distinguished as separate sounds. One merely receives the impression that the original sound has been prolonged and a reverberation is caused. Curtains, cushions and furniture of a room absorb sound waves and minimise reverberation.

3.13. Echo depth-sounding :

One of the most important practical uses of echoes is to find the depth of sea as well as the height of an aeroplane. The methods used are as follows.

(*a*) Depth of sea :

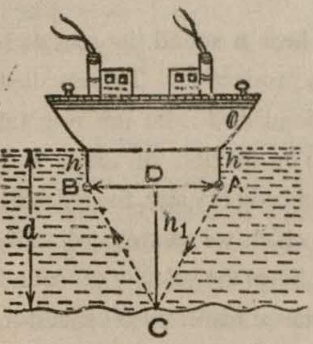

Fig. 3.9(i)

From one end of a ship floating on the sea, an explosive substance *A* is immersed at a depth *h* and from the other end an electrical receiver of sound, known as hydrophone *B* is kept immersed at the same depth *h* [Fig. 3.9(i)]. When a big sound is produced by setting fire to the explosive substance, a part of the sound wave travels straight to the hydrophone along the path *AB*. The hydrophone records the time of arrival of sound wave. Another part of sound wave reaches the bottom of the sea through water along the path *AC* and then gets reflected there, finally reaching the hydrophone *B* along the path *CB*. The hydrophone also records the time of arrival of the reflected sound wave.

Suppose t_1 is the time taken by sound to reach *B* from *A* along the straight path *AB*. If *V* be the velocity of sound in sea-water, then $AB=Vt_1$.

Now, $DB=\frac{1}{2}AB=\frac{1}{2}Vt_1$ .. (i)

Again, suppose t_2 is the time taken by sound to reach *B* from *A* along the reflected path. Then $AC+CB=2BC=Vt_2$ $\therefore$ $BC=\frac{1}{2}Vt_2$

$$\text{So, } h_1=\sqrt{(BC)^2-(BD)^2}=\sqrt{(\tfrac{1}{2}Vt_2)^2-(\tfrac{1}{2}Vt_1)^2}=\frac{V}{2}\sqrt{t_2^2-t_1^2}$$

$$\therefore \text{ Depth of sea } d=h+h_1=h+\frac{V}{2}\sqrt{t_2^2-t_1^2}$$

So, determining t_1 and t_2, the depth of the sea can be found out.

(*b*) **Height of an aeroplane :**

The height of a moving aeroplane is also measured in this way. Consider an aeroplane flying horizontally along the line XY [Fig. 3.9(ii]. When it is at X, it produces a loud sound. Suppose, the aeroplane receives the echo of the sound reflected by the ground, when it is at Y. From the diagram, it is clear that the sound waves have been reflected along the paths XO and OY. If t be the time-interval between the original sound and its echo and v cm/sec be the velocity of the aircraft, then $XY = v.t$.

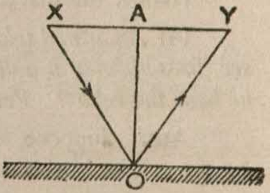

Fig. 3.9(ii)

Again, $XA = \frac{1}{2}XY = \frac{1}{2}.vt$. If the velocity of sound in air be V sm/sec, then $XO = OY = \frac{1}{2}V.t$.

Let h cm. be the height of the air-craft from the ground. Then $AO = h$. Now, from the right-angled triangle AOX, we have,

$$h = \sqrt{(XO)^2 - (XA)^2} = \sqrt{(\tfrac{1}{2}Vt)^2 - (\tfrac{1}{2}vt)^2} = \tfrac{1}{2}t\sqrt{V^2 - v^2}$$

So, knowing the velocity of the air-craft (v), the velocity of sound (V) and the time-interval between the original sound and its echo (t) we can find the height of the air-craft.

Examples : (1) *The sound of dropping an anchor from a boat was reflected by a cliff in front and came 2 sec. after. If the velocity of sound be 1120 ft/sec, what was the distance of the cliff ?*

Ans. Total distance travelled by sound in 2 sec $= 2 \times 1120 = 2240$ ft.

To produce echo, the sound should travel to the cliff and back to the boat

So, the distance of the cliff $= \frac{2240}{2} = 1120$ ft.

(2) *An echo repeated five syllables. If the velocity of sound be 1120 ft/sec., find the distance of the reflector.*

Ans. We know that to hear echo of a monosyllabic word, $\frac{1}{5}$*th* of a second should be allowed to elapse between the sound and its echo. So, for a five syllable word, the minimum time interval necessary $= 5 \times \frac{1}{5} = 1$ sec. In this time, the sound travels 1120 ft. So, the distance of the reflector $= \frac{1120}{2} = 560$ ft.

(3) *A man hammering at regular intervals of 0·4 second hears echoes of the hammer blow exactly half way between successive blows. When he stops hammering, he hears two further echoes. Calculate the distance between the man and the reflector, assuming the velocity of sound in air be 332 metres/sec.*

Ans. As the echoes are produced half-way between successive blows, the time interval between two successive echoes is equal to the time-interval between two successive blows *i.e.* 0·4 sec. Since 2 echoes are heard after the last blow, the time-interval between the last blow and the last echo $= 0{\cdot}2 + 0{\cdot}4 = 0{\cdot}6$ sec. During this time, the sound will travel twice the distance between the man and the reflector. Hence time taken by sound to travel from the man to the reflector $= \frac{0{\cdot}6}{2} = 0{\cdot}3$ sec.

The distance travelled by sound during this time = velocity × time
$= 332 \times 0{\cdot}3 = 99{\cdot}6$ metres.

Hence, the distance between the man and the reflector = 99·6 metres.

(4) *An armed soldier running towards a cliff with a uniform velocity of 4 metres/sec, fires his gun at a distance of 2·4 kilometres from the cliff. When and where will he hear the echo ? Velocity of sound=330 metres/sec.*

Ans. Suppose, after t sec, the soldier hears the echo. The distance moved by the soldier in this time $= 4.t$ metres.

So, the distance travelled by the sound from the moment it is produced and the moment its echo is heard $= 2400 + (2400 - 4t)$ metres. [2·4 km = 2400 metres]

Now, sound covers this distance in t sec. Hence,

$$\text{velocity of sound} = \frac{2400 + (2400 - 4t)}{t} \text{ metres/sec}$$

$$\therefore \quad \frac{2400 + (2400 - 4t)}{t} = 330 \text{ (given) or, } \quad 4800 = 334t \quad \therefore \quad t = 14{\cdot}3 \text{ sec.}$$

So, the soldier will hear echo 14·3 sec after he fires his gun. The distance of the soldier from the cliff $= (2400 - 14{\cdot}3 \times 4) = 2342{\cdot}8$ metres $= 2{\cdot}34$ km. (nearly).

(5) *A man stationed between two parallel cliffs fires a gun. He hears the first echo after 2 seconds and the next after 5 seconds. What is his position between the cliffs and when will he hear the third echo ?* [*H. S. Exam. 1983*]

Ans. The first echo will be produced when sound is reflected by the nearer cliff and the second echo when the sound is reflected by the furthest cliff. Now,

suppose d_1 = the distance of the man from the nearer cliff
and d_2 = ,, ,, ,, ,, ,, ,, ,, furthest cliff.

If V be the velocity of sound, then for the first echo $2d_1 = V \times 2$ and for the second echo, $2d_2 = V \times 5$. Dividing, $\frac{d_1}{d_2} = \frac{2}{5}$.

So, the ratio of the distances of the cliffs from the man = 2 : 5.

Now, the sound reflected by the nearer cliff will proceed towards the furthest cliff and that reflected by the furthest cliff towards the nearer cliff. When these two waves after second reflection arrive simultaneously to the person, the third echo will be heard.

It is clear that time required for this to happen = 2 + 5 = 7 seconds.

(6) *A short sound is made in front of a wall and the echo is heard after 1·6 sec. On moving closer to the wall by 33 metres, the sound is repeated and the echo is heard after 1·4 sec. Calculate the velocity of sound and the distance of the wall.*

Ans. Suppose, velocity of sound $= V$ metres/sec and the distance of the wall $= D$ metres. So, in the first case, $2D = V \times 1{\cdot}6$........(i)

and in the second case $2(D - 33) = V \times 1{\cdot}4$........(ii)

Dividing eqn. (ii) by eqn. (i) we have, $\frac{D-33}{D} = \frac{1{\cdot}4}{1{\cdot}6} = \frac{7}{8}$

or $8D - 264 = 7D \quad \therefore \quad D = 264$ metres.

Substituting this value of D in eqn. (i), we have, $2\times 264 = V\times 1{\cdot}6$

$$\therefore \quad V = \frac{2\times 264}{1{\cdot}6} = 330 \text{ metres/sec.}$$

(7) *A road runs between two parallel rows of buildings. A motorist moving with a velocity of 36 km/hr, sounds the horn. He hears the echo 1 sec after he sounded the horn. Find the distance between the two rows of buildings. When will he hear the second echo ? Velocity of sound=330 metres/sec.*

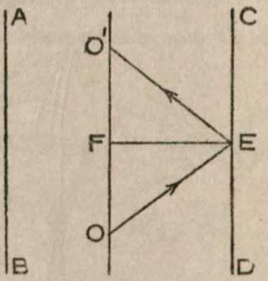

Fig. 3.10

Ans. Suppose AB and CD are the two rows of buildings and O the position of the car when the horn is sounded (Fig. 3.10). O' is the position of the car after 1 sec when the motorist hears the first echo.

OE is the incident wave and EO' its reflected wave. Hence, $OE = O'E$;

the speed of the car$= \dfrac{36\times 1000}{60\times 60} = 10$ metres/sec.

i.e. $OO' = 10$ metres.

Now, OE=the distance travelled by sound in $\frac{1}{2}$ sec=165 metres.

Again, $(EF)^2 = (EO)^2 - (OF)^2 = (165)^2 - (5)^2 = 27200$

$[OF = \frac{1}{2}OO' = \frac{1}{2}\times 10 = 5 \text{ metres}]$

$\therefore \quad EF = \sqrt{27200} = 164{\cdot}9$ metres.

So, the distance between the two rows of buildings$=2\times 164{\cdot}9=330$ metres. The motorist will hear the second echo 2 seconds after the horn is sounded.

3.14. Refraction of waves :

When a progressive wave, travelling through a homogeneous medium, falls on a second medium, a part of it enters the second medium and travels with a different velocity. Passing into the second medium, the wave also changes the course if the incidence is not normal. The phenomenon is called refraction, which can be explained by Huyghens' wave theory. We shall discuss here, the refraction of sound waves.

In general, if incident waves move in any other direction than normal, the directions of incident and refracted waves will be different. However, it is found that the angle of incidence and angle of refraction are related to the velocity of waves in the two media. If i and r denote the angles of incidence and refraction (Fig 3.11) and V_1 and V_2 are the velocities of the waves in the medium No. 1 and

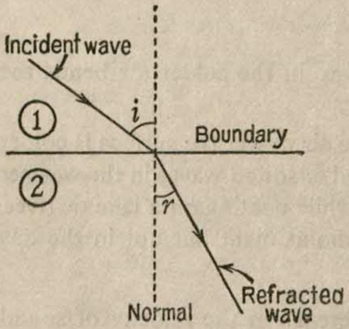

Fig. 3.11

No. 2 respectively, then $\frac{\sin i}{\sin r}=\frac{V_1}{V_2}$=constant. The above equation is known as Snell's law of refraction. The constant is known as the refractive index of medium No. 2 with respect to medium No. 1.

The laws of refraction of sound waves are same as those of light waves.

Laboratory arrangement : The arrangement consists of a round brass ring covered on both sides by thin rubber sheets. It is filled with carbon dioxide gas to form a bi-convex lens. Carbon dioxide is denser than air. If a ticking watch is placed at some point on the axis of the lens, its sound will be heard distinctly at some other point on the axis on the other side of the lens. If ear be placed some where else, the sound of ticking will not be heard. Here sound waves are refracted by the denser medium and like light rays, converge to a particular point (Fig. 3.12).

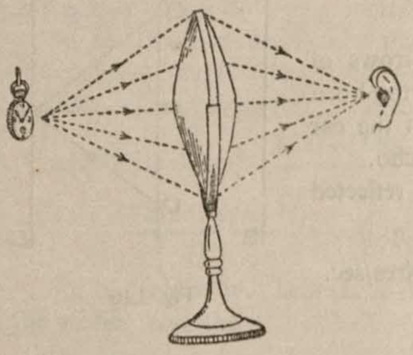

Fig. 3.12

3.15. Refraction of sound waves in atmosphere :

(*a*) **Effect of temperature :** In day time, the surface of the earth becomes heated by the sun and the air near the surface is hotter than that which is higher up so that the lower speed of sound waves in the colder air bends the waves up and away from the ground (Fig. 3.13).

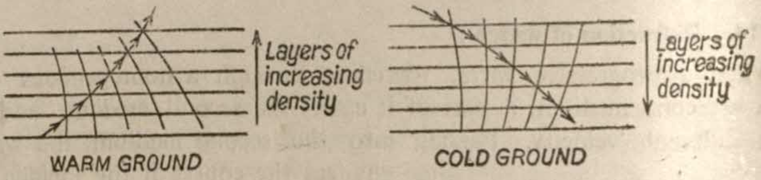

Fig. 3.13

At night, the earth surface cools down and the air nearer the surface is colder than that which is higher up, so that the higher speed of sound-waves in the warmer air bends the waves back down. For this reason, while boating on a lake or river, it is possible to hear the music from a distant radio at night but not in the day time.

(*b*) **Effect of wind :** Wind also makes difference in the velocity of sound and can thereby cause refraction. When sound travels in the direction of the wind, the upper part of the waves travels faster than the lower part and the waves bend down. Consequently conversation of distant people can be heard if the wind blows in that direction. On the other hand, if the wind blows in the opposite direction, the upper part of the waves travels slower than the lower part and the waves bend up and away from the ground (Fig. 3.14) and hence conversation of distant people cannot be heard distinctly.

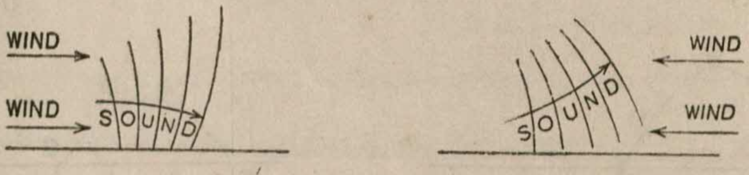

Fig. 3.14

Exercises

Essay type :

1. What do you understand by a wave ? What are transverse and longitudinal waves ? Explain with illustrations.

2. Show by an experiment that sound can be reflected like light.

3. Show that a progressive wave of amplitude 'a' and velocity 'v' moving in the positive x direction is represented by the equation $y=a\sin\frac{2\pi}{\lambda}(vt-x)$. How will the wave be represented if it moves in the opposite direction ?

4. Compare transverse wave with longitudinal wave.

5. At a point inside a fluid medium a to and fro periodic motion (a disturbance) along the x-direction is impressed. Explain briefly using a neat sketch how this disturbance will be propagated along the x, y and z directions and the type of elasticity modulus involved in each. [*Jt. Entrance 1979*]

6. Mention some practical applications of multiple reflections of sound.

7. Explain, with neat diagrams, how sound waves are refracted due to variation of temperature and wind.

Short answer type :

8. Define the following terms in connection with a progressive wave :—(i) Frequency (ii) Wave length (iii) Period (iv) Wave front.

9. What is the relation between the wave length and wave velocity ? [*H. S. Exam. 1982*]

10. The following functions in which A is a constant are of the form $f(x+vt)$:

$y=A(x-vt)$; $y=A(x+vt)^2$; $y=A\sqrt{x-vt}$; $y=A\log_e(x+vt)$

Explain why these functions are not useful in wave motion.

[**Hints** : None of the functions represents a periodic motion]

11. A transverse wave is travelling along a string from left to right. Fig. 3.15 represents the shape of the string at a given instant. A, B, C, D, E, F, G and H are points on the string. At this instant state (*a*) which of the above points have an upward velocity (*b*) which points have a downward velocity (*c*) which points have zero velocity (*d*) which points have maximum magnitude of velocity ? [*I.I.T. 1970*]

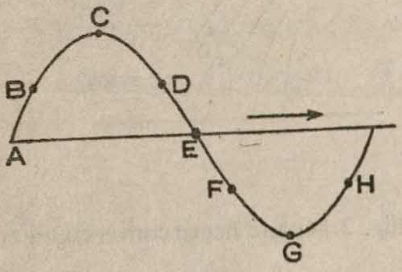

Fig. 3.15

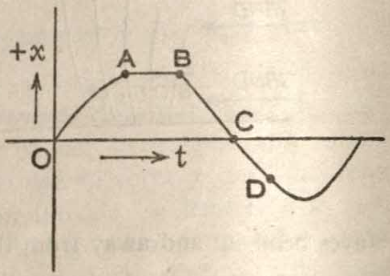

Fig. 3.16

12. Fig. 3.16 represents the displacement of a particle along the x-axis as a function of time. Find the direction of velocity and acceleration of the particle between the following points : (i) between O and A (ii) between A and B (iii) between C and D. [*I.I.T. 1973*]

13. What is an echo ? How is depth sounding done with the help of echo ? Why cannot echoes be heard at short distances ?

14. What is reverberation ? How is reverberation minimised in a modern cinema hall ?

15. Answer the following questions :—(i) The wall of a room can reflect sound waves but not light waves. On the other hand, a simple plane mirror can reflect light waves but not sound waves. Why ? (ii) Why are roofs of auditoriums meant for public meetings bent like arches ? (iii) Why do we hold our palm in a curved fashion near our ear to catch distant sound ? (iv) Distant sound can be heard at night but not in day time. Why ?

Objective type :

16. (*a*) Mark the correct answer in the following questions :—

(i) When particles of a medium vibrate along a line perpendicular to the direction of propagation of wave, the wave is called longitudinal/transverse/stationary.

(ii) When a wave is generated in a medium, having wave length 10 cm. and frequency 10, the velocity of the wave is 10 cm. per sec/5 cm. per sec/100 cm. per sec.

(iii) While boating on a lake or river, it is possible to hear the music from a distant radio at night but not in day time because of refraction of sound waves/temperature/velocity of sound/echo.

(iv) A wall of a building can reflect sound waves but not light waves because the wall is large/sound has greater wave length/sound has less velocity.

(*b*) In the statement given below, several alternatives are provided. Place a tick mark (√) in the bracket [ ] against each correct alternative :

A wave equation which gives the displacement along the y-direction is given by $y=10^{-4}\sin(60t+2x)$ where x and y are in metres and t is time in second. This represents a wave—

(i) [ ] travelling with a velocity 30 m/s in the negative x direction. [√]
(ii) [ ] of wave length π metres. [√]
(iii) [ ] of frequency $30/\pi$ Hertz. [√]
(iv) [ ] of amplitude 10^{-4} metre travelling along the negative x direction. [×]
[*I.I.T. 1982*]

(*c*) A transverse wave is described by the equation $y=y_0 \sin 2\pi(ft-x/\lambda)$. The maximum particle velocity is equal to four times the wave velocity if (i) $\lambda=\pi y_0/4$ (ii) $\lambda=\pi y_0/2$ (iii) $\lambda=\pi y_0$ (iv) $\lambda=2\pi y_0$. Which is correct ? [*I.I.T. 1984*]

[Hints : $\lambda=2\pi y_0$]

Numerical problems :

17. Write the equation for a wave travelling in the negative direction along the x-axis and having an amplitude 0·01 metre, a frequency 550 vib/sec and a speed 330 m/s.

[Ans. $y=0{\cdot}01 \sin 10\pi\left(110t+\frac{x}{3}\right)$]

18. Light is a wave, having velocity 3×10^{10} cm/sec. If the wave length of green light is 6×10^{-6} cm., what will be its frequency ? [Ans. 5×10^{14}]

19. The frequencies of two tuning forks are 128 and 384. Compare their wave lengths in air. [Ans. 3 : 1]

20. A tuning fork can produce 2·5 ft. long waves. If the wave velocity in air be 1100 ft/s, what is the frequency of the fork ? [Ans. 440]

21. If the frequency of a tuning fork is 400 and the velocity of sound in air is 320 metres/sec., find how far sound travels when the fork executes 30 vibrations. [Ans. 24 metres]

22. The frequency of a tuning fork is 560. If the velocity of sound in air be 1100 ft/sec., how far will the sound travel while the fork completes 100 vibrations ? [Ans. 196·4 ft]

23. Taking 1120 ft/sec as the speed of sound in air, find the number of vibrations which a fork of frequency 264 must make before the sound is heard at a distance of 154 ft ? [Ans. 36·3]

24. A body vibrating with a constant frequency sends waves 10 cm. long in a medium A and 15 cm. long in another medium B. Compare the wave velocities in A and B.

[*H. S. Exam. 1962*] [Ans. 2 : 3]

25. A wave is represented by $y=a\sin\left(2000\pi t-\frac{\pi x}{17}\right)$ where t is in seconds, y and x in cm. Find the (i) wave length (ii) the velocity and (iii) the frequency of the wave. Also find the phase-difference between two points on the wave separated by a distance of 180 cm.

[Ans. (i) 34 cm (ii) 340 metre/s (iii) 1000 (iv) $\frac{10\pi}{17}$ rad.]

26. A man standing away from a cliff hears the echo of a sound 2 sec. after it is produced by him. What is the distance of the cliff from the man ? Velocity of sound in air=320 metres/sec. [Ans. 320 metres]

27. Two persons A and B are standing in a line perpendicular to the length of a cliff. A is nearer to the cliff and B is 825 ft. away from A. When A fires a gun, he hears the echo 1 sec. after but when B fires the gun, A hears the echo 1·75 sec. after. Find the velocity of sound and the distance of A from the cliff. [Ans. (i) 1100 ft/sec. (ii) 550 ft.]

28. A man fired a gun standing between two parallel cliffs. He heard two successive echoes after $1\frac{1}{2}$ sec and $2\frac{1}{2}$ sec. respectively. What was the distance between the cliffs ? When did he hear the third echo ? Velocity of sound=1120 ft/sec. [Ans. 2240 ft. ; 4 sec.]

29. An echo repeated six syllables. What was the distance of the reflector ? The velocity of sound is 1120 ft/sec. [Ans. 672 ft]

30. A pilot flying horizontal at the rate of 120 miles/hr. fired a gun. The sound reflected from the ground reached him 3 sec later. What was the height of the aeroplane ? Velocity of sound=1120 ft/sec. [*H. S. Exam. 1981*] [Ans. 1659 ft. (nearly)]

31. An engine is approaching a tunnel surrounded by a cliff and emits a short whistle when half a mile away. The echo reaches the engine 4·5 sec. after the sound is made. Calculate the velocity of sound, the speed of the engine being 50 miles/hr. [Ans. 1100 ft/sec.]

32. An aeroplane is flying parallel to the ground with a spend v. A sound produced from the plane is reflected back from the ground in t sec. If the velocity of sound be V, show that the height of the aeroplane from the ground is $\frac{1}{2}t\sqrt{V^2-v^2}$.

33. A bullet is fired from an aeroplane travelling horizontally at a speed of 300 miles/hr. The pilot hears the echo of the sound reflected from the ground after 4 seconds. Find the height of the plane from the ground. Velocity of sound in air=1120 ft/s.

[*Jt. Entrance 1985*] [Ans. 2060 ft (nearly)]

[**Hints** : Apply the formula, $h=\frac{1}{2}t\sqrt{V^2-v^2}$.]

34. Some explosives are kept at a depth x from the sea level. At the same level but a certain distance away, a hydrophone is kept. When the explosives burst, the hydrophone receives two sounds after intervals of t_1 and t_2 from the time of explosion. Show that the depth of the ocean is $x+\frac{v}{2}(\sqrt{t_2^2-t_1^2})$ where v is the velocity of sound in water.

Harder problems :

35. A rifle shot is fired in a valley formed between two parallel mountains. The echo from one mountain is heard after 2 seconds and the echo from the other mountain is heard 2 seconds later. What is the width of the valley ? Is it possible to hear the subsequent echoes from the two mountains simultaneously at the same point ? If so, after what time ?

[*I.I.T. 1973*] [Ans. 1080 metres ; yes ; 6 sec.]

36. A man standing at one end of a closed corridor 57 metre long blew a short blast on a whistle. He found that the time from the blast to the sixth echo was 2 seconds. If the temperature of air was 17°C, what was the velocity of sound at 0°C ? [Ans. 332 metres/sec]

37. A man standing in front of a mountain at a certain distance beats a drum at regular intervals. The drumming rate is gradually increased and he finds that the echo is not heard distinctly when the rate becomes 40 per minute. He then moves nearer to the mountain by 90 metres and finds that the echo is again not heard when the drumming rate becomes 60 per minute. Calculate (i) the distance between the mountain and the initial position of the man and (ii) the velocity of sound. [*I.I.T. 1974*] [Ans. 270 metres ; 360 m/s]

38. A person, standing 112 ft from a wall uttered 6 syllables in succession. Will he hear the echoes of all the syllables ? If not, which syllable will produce echo ? Velocity of sound in air=1120 ft/s. [*Jt. Entrance 1973*] [Ans. last syllable]

39. It is noticed that a sharp tap made in front of a flight of stone steps gives rise to a ringing sound. If each step is 0·25 metre deep, calculate the frequency of the sound. Velocity of sound in air is 340 m/s. [Ans. 680]

40. Approaching a hill, a steamer sounded its siren and heard the echo 6 seconds later. On making a sound again in its siren 3 minutes after, the echo was heard 4 seconds later. What was the speed of the steamer ? The speed of sound=1120 ft/s. [Ans. 6·26 ft/s]

41. Two observers A and B were standing on a line in between a bell and a wall. A heard an echo produced by the wall 1 sec. after the bell sounded while B heard the echo 0·2 sec later. Find the distance between A and B, given the velocity of sound in air=1100 ft/s. [Ans. 440 ft]

42. A man standing on the shore of a lake saw the smoke of the siren of a steamer anchored in the lake but heard its sound 3 seconds later and 4 seconds after he heard the sound, he received the echo reflected by a hill on the other side of the lake. What was the width of the lake ? Velocity of sound=1100 ft/s. [Ans. 5500 ft.]

43. A gun is fired on a sea-shore in front of a line of cliffis. A man standing 300 ft. away from the gun and equidistant from the cliffs notices that the echo takes twice as long to reach him as does the direct report. Find the distance of the gun from the cliff. [Ans. 225 ft]

44. A displacement wave is represented by $y=0{\cdot}25\times10^{-3}\sin(500t-0{\cdot}025x)$ where y, t and x are in cm., sec and metre respectively. Determine (i) the amplitude (ii) the period (iii) the angular frequency (iv) the wave length. Find also the amplitudes of particle velocity and particle acceleration. [Ans. (i) $0{\cdot}25\times10^{-3}$ cm (ii) $\pi/250$ sec (iii) 500/s (iv) 80π cm. 0·125 cm/s ; 62·5 cm/s^2]

45. A displacement wave is represented by $y=20\sqrt{3}\sin\left(\frac{2\pi t}{T}-\frac{2\pi x}{\lambda}\right)$. Find the displacement of a particle situated at $x=\frac{\lambda}{6}$ at $t=\frac{T}{3}$. [Ans. 30 cm.]

46. A plane progressive wave is represented by the equation $y=0{\cdot}1\sin(200\pi t-20\pi x/17)$ where y is the displacement in millimetres, t is in seconds and x is the distance from a fixed origin O in metres. Find the phase difference in radians between a point 25 cm from O and a point 110 cm from O. Also find the equation of a wave with double the amplitude and double the frequency but travelling exactly in the opposite direction.

$$\left[\text{Ans. } \pi\ ;\ y=0{\cdot}2\sin\left(400\pi t+\frac{20\pi x}{17}\right)\right]$$

4

SUPERPOSITION OF WAVES AND ALLIED PHENOMENA

4.1. Principle of superposition :

The principle of superposition of waves was first proposed by Thomas Young in 1801. It states that when two or more waves are superposed, the resultant displacement is equal to the algebraic sum of the individual displacements provided they are small. Suppose, the displacement at any point in the medium due to one wave is y_1 and that due to the other wave at the same instant is y_2. The resultant displacement at the point, according to the principle of superposition, will be $y=y_1 \pm y_2$. If the individual displacements are in the same direction, the positive sign is to be taken. If the displacements are in the opposite direction, the negative sign is to be taken.

The above principle is generally applicable to waves like sound, light, X-rays etc, regardless of direction or frequency or amplitude variations of the wave trains. But cases of practical importance are those where the waves have equal or nearly equal frequencies and amplitudes. The following three cases may arise:

(a) **Stationary or standing waves :** A stationary or standing wave is formed when two *equal* progressive waves are superimposed on one another when travelling in opposite directions.

(b) **Interference :** Interference occurs when two waves of *same* frequency and same or nearly equal amplitudes are superimposed.

(c) **Beats :** Beats occur when two waves of *nearly equal* frequencies and amplitudes moving in the same direction are superimposed.

We shall discuss, in this chapter, these phenomena of waves, especially of sound waves.

4.2. Stationary waves :

It has been mentioned before that transverse and longitudinal waves are known as *progressive waves* because these waves move from one place to another through a medium. But by superposition of progressive waves, another kind of wave may be produced which has no forward motion but remains steady in space.

Definition : *Waves produced by the superposition of two progressive wave trains of same frequency and amplitude travelling in the same medium in opposite directions, are called stationary waves.*

Under these circumstances, the displacement of any particle of the medium depends upon the individual displacements produced by the two wave trains and it is seen that under the influence of the two waves, the particles of the medium are arranged in particular wave forms, the chief characteristic of which is that these waves do not move forward but remain confined to the portion of the medium where the superposition takes place. For these reasons, the waves are called *stationary waves.* Further, it is found that certain particles of the medium are permanently at rest while certain others always vibrate with maximum amplitude.

Demonstration of stationary waves with transverse and longitudinal vibrations :

(a) To demonstrate the production of stationary waves, the following experiment may be done. One end of a rope is fastened to a post and the other end, held taut, is moved up and down with simple harmonic motion. Transverse waves will travel along the rope. As the waves reach the fixed end of the rope, they are reflected back to meet succeeding waves just coming up. If the waves have just the right frequency, they combine to form a stationary or standing wave and the rope is seen to vibrate in a series of equal segments (Fig. 4.1). The points N_1, N_2, N_3 etc are called *nodes* and there the rope remains at rest; in fact it makes no difference to the vibration of the rope if it is touched with the edge of a thin card at any of the nodes. Between each pair of nodes, the rope vibrates with increasing amplitude towards the centre, where it reaches a maximum. The central points marked A_1, A_2, A_3 etc are called *antinodes*. The entire wave section between two consecutive nodes is called a *loop*.

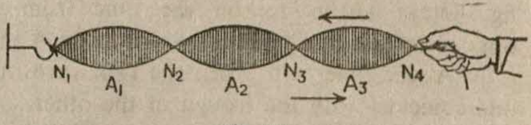

Fig. 4.1

In a stationary wave, the distance between two consecutive nodes or antinodes is taken as half the wave length of the wave. If λ be the wave length of a stationary wave, then $A_1A_2=N_1N_2=\lambda/2$ (Fig. 4.1). Hence, $N_1N_3=A_1A_3=\lambda$.

(b) The previous experiment with a rope demonstrates the production of stationary waves with transverse vibrations. Stationary waves with longitudinal vibrations may be demonstrated with a long flexible spring as shown in fig. 4.2(a). The right hand end of the spring is fixed and the left hand end is moved back and forth with simple harmonic motion. If the impressed vibration has the proper frequency, the waves travelling to the right, in meeting the reflected waves travelling to the left, set up nodes and antinodes. The nodes N_1, N_2. etc. correspond to points where there is no motion and the antionodes A_1, A_2 etc. to points where the motion is the maximum. The dots and arrows [Fig. 4.2(b)] show the relative positions and motions of each individual coil of the spring. The dots at the nodes remain fixed at all times while those at the antionodes move back and forth as shown by the arrows.

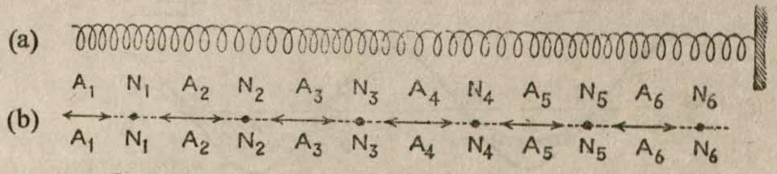
(a)
A₁ N₁ A₂ N₂ A₃ N₃ A₄ N₄ A₅ N₅ A₆ N₆
(b)
A₁ N₁ A₂ N₂ A₃ N₃ A₄ N₄ A₅ N₅ A₆ N₆

Fig. 4.2

Here, also, the distance between two consecutive nodes or antinodes is taken as half the wave length of the wave. If λ be the wave length, then $N_1N_2=A_1A_2=\lambda/2$. Hence $N_1N_3=A_1A_3=\lambda$.

It may be mentioned here that nearly all sounds emanating from musical instruments are the result of stationary waves. In stringed instruments like *sitar*, *esraj* etc, stationary waves are produced by the superimposition of transverse waves while in wind instruments like flute, organ etc, they are produced by the superimposition of longitudinal waves.

4.3. Explanation of the formation of stationary waves by graphical method : Two progressive waves of same amplitude, time period and velocity travelling in opposite directions are depicted by continuous and dotted curves in the fig. 4.3(a). Let us reckon the time from the moment the two waves meet in opposite phase at a point as shown in fig. 4.3 (a).

At the time $t=0$, when the two waves meet in opposite phase, the crest of one coincides with the trough of the other so that the resultant displacement at all points will be zero because the displacements due to the individual waves are equal and opposite. The resultant curve is, therefore, a straight line which is shown by a thick line [Fig. 4.3.(a)].

As the wave trains advance in their respective directions, after a quarter period ($t=T/4$), each wave has moved through a quarter of wave length. This would mean that the crest of one comes at the place where the crest of the other has come and similarly the troughs also arrive coinciding with each other. At points marked *A*, the displacement is maximum. All the particles pass through their extreme positions. Again at the points marked *N*, the displacement is zero. Particles at those points are at rest. The resultant wave, under this condition, has been shown by thick curve [Fig. 4.3. (b)].

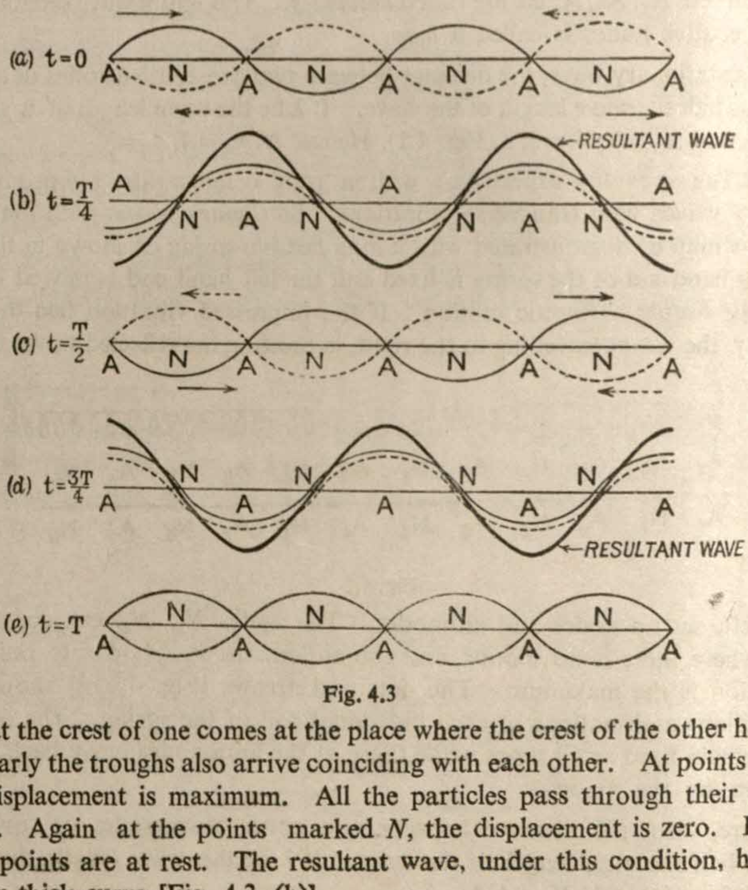

Fig. 4.3

After half a time-period ($t=T/2$) from the time of reckoning, the two waves move through a distance equal to half the wave length in the opposite directions so that crest of one falls over the trough of the other. All the particles pass through the mean position and the resultant displacement is zero and the resultant curve is again a straight line [Fig. 4.3(c)].

After another quarter period has elapsed (*i.e.* when $t=3T/4$), the waves move through a distance of three quarter of wave length and the conditions are reversed as in fig. 4.3(d). The particles pass through the extreme positions giving maximum displacement. Here also, the points marked *A* have maximum displacement and the points marked *N* have no displacement [Fig. 4.3(d)].

When $t=T$ *i.e.*, after one complete time period, the initial state of affairs as in fig. 4.3 (a) is repeated [Fig. 4.3(e)].

Thus, we see that during one complete cycle of events there are certain points marked *A* which have always maximum displacement and some other points marked *N*, which are always at rest. The former points are called *antinodes* while the later points *nodes*.

4.4. Mathematical analysis of stationary waves :

We have seen that the displacement of a particle in a medium traversed by a progressive wave at an instant t and at a distance x from the source of disturbance is given by $y_1=a \sin \frac{2\pi}{\lambda}(vt-x)$ and that due to a similar wave moving in the opposite direction with the same velocity is given by, $y_2=a \sin\frac{2\pi}{\lambda}(vt+x)$.

By the principle of superposition, the resultant displacement of a particle at x at time t is $y=y_1+y_2=a \sin \frac{2\pi}{\lambda}(vt-x)+a \sin \frac{2\pi}{\lambda}(vt+x)$

$$=2a \sin\frac{2\pi}{\lambda} vt \cos\frac{2\pi}{\lambda} x=A \sin \frac{2\pi}{\lambda}. vt \left[A=2a \cos\frac{2\pi}{\lambda}. x\right]$$

The equation represents a simple harmonic vibration of same wave length λ as the superposed waves but its amplitude $A=2a \cos \frac{2\pi}{\lambda} x$ which depends upon the position (x) of the particle. *The simple harmonic vibration does not, however, represent a progressive wave* because its phase does not contain term like $(vt-x)$. So the above equation represents a stationary wave.

Antinodes : Antinodes will be formed at points where the resultant amplitude $A=\pm 2a$ *i.e.* at points where $\frac{2\pi x}{\lambda}=0, \pi, 2\pi \ldots n\pi$ or $x=0, \frac{\lambda}{2}, \frac{2\lambda}{2} \ldots \frac{n\lambda}{2}$ ($n=0, 1, 2, 3$ etc.). Clearly, the distance between two consecutive antinodes $=\frac{\lambda}{2}$=half the wave length.

Nodes : Nodes will be formed at points where the resultant amplitude $A=0$ *i.e.*, at points where $\frac{2\pi x}{\lambda}=\frac{\pi}{2}, \frac{3\pi}{2}..(2n+1)\frac{\pi}{2}$

or $x=\frac{\lambda}{4}, \frac{3\lambda}{4}. \frac{5\lambda}{4} ....(2n+1)\frac{\lambda}{4}$ ($n=0, 1, 2, ..$etc). Here also, the distance between two consecutive nodes$=\frac{\lambda}{2}=$ half the wave length.

Example : *A sound wave of frequency 100 is incident normally on a wall and retraces its path on reflection. At what distances from the wall, the amplitudes of vibration of air particles will be (i) maximum and (ii) minimum. Velocity of sound in air=332 m/s.*

Ans. The incident wave and the reflected wave will be superposed on each other, producing standing waves. The wall will be the position of one node because there the air particles will have the minimum freedom of movement. Other points of node will be at such distances from the wall that the distance between each two conescutive nodes$=\lambda/2$. Suppose, the distance$=x$.

Now, $\lambda=\frac{V}{n}=\frac{332}{100}=3{\cdot}32$ metres ; hence $x=\lambda/2=\frac{3{\cdot}32}{2}=1{\cdot}66$ metre.

Hence, the distances of nodes (*i.e.* the positions of minimum displacements) from the wall are 1·66, 3·32, 4·98 metres. .. etc.

One antinode will be formed between each two consecutive nodes. So, the points of maximum displacements from the wall are 0·83, 2·49, 4·19 metres ....etc.

4.5. Characteristics of stationary waves :

From our discussion on stationary waves in the preceding articles, we may assign the following characteristics to the waves :

(i) Crests and troughs or condensations and rarefactions do not progress forward but simply appear and disappear at the same place alternately and so there is no transference of energy.

(ii) All the particles, except those at the nodes, execute simple harmonic motion. The amplitude of vibration of these particles varies from maximum positive value to maximum negative value passing through the intermediate zero values. The points where the amplitude is always zero are called nodes. The points midway between the nodes where the amplitudes have maximum values are called antinodes.

(iii) The intervening distance between two consecutive nodes is called a loop where the displacements of the particles are all in the same direction. The displacements of particles in two consecutive loops are in the opposite directions.

(iv) The maximum displacement is achieved by all the points at one time, zero displacement is also achieved at one time and so on. This means that the phase does not vary with the distance x.

(v) Twice during each cycle the whole medium has zero displacement, so that the potential energy is zero and all the wave energy is kinetic. Again twice during each cycle the whole medium attains the maximum displacement (which is, of course, different at different points) and at these instants, the kinetic energy is zero and all the wave energy is potential.

4.6. Distinction between progressive and stationary waves :

Progressive wave	Stationary wave
1. This wave is produced due to continuous periodic vibrations of a portion of a medium.	1. This wave is produced due to superposition of two identical progressive wavetrains moving along the same line in opposite directions in a medium.
2. The vibration of one particle is handed over to the next along the line of propagation. Hence the waveform progresses forward.	2. The waveform does not progress forward through the medium. They appear and disappear in the portion of the medium where the superposition of oppositely moving progressive waves takes place.
3. Each particle of the medium undergoes the *same cycle of changes* in respect of amplitude, velocity, acceleration, pressure-variation etc.	3. In stationary waves, amplitude, velocity, pressure-variation etc vary from point to point but are constant at a given point.
4. Along the line of propagation, there is a phase difference between the vibration of particles, but their amplitude is the same. Phase difference between two particles is proportional to the distance between them.	4. All particles between two nodes vibrate in the same phase but with different amplitudes. Particles on two sides of a node vibrate in opposite phases.
5. In a complete time period all the particles of the medium never come to rest together.	5. In a complete time period, all the particles of the medium come to rest twice together.
6. In progressive waves, the energy averaged over one wave length of the medium is always half kinetic and half potential.	6. The energy alternately becomes wholly kinetic and wholly potential.

4.7. Melde's experiment for demonstrating stationary waves :

In 1860 Melde devised a simple experiment to demonstrate the formation of stationary waves in a stretched string.

(i) **Longitudinal mode of vibration** : The arrangement consists of a light cord or a thin string AB, about 1 metre long, one end of which is attached to one of the prongs of a tuning fork, the other end passing over a pulley and carrying a hanger on which weights may be placed as shown in fig. 4.4. Keeping different weights on the hanger, the tension of the string may be suitably altered. The arrangement is so made that the movement of the prong is parallel to the length of the wire *i.e. the vibration is longitudinal.* When the fork vibrates, waves are set up in the wire, which travelling along the wire are reflected at the fixed end. The reflected waves, when superposed on the direct waves, give rise to stationary waves in the string. By suitably changing the tension, the same length of the string may be set in vibration consisting of two or more loops as shown in the figure. When the string vibrates in two loops. the points A,C,B of the string will have no displacement *i.e.* they are nodal points. The points D and E of the wire, on the other hand, will have maximum displacement and therefore, they are anti-nodal points..

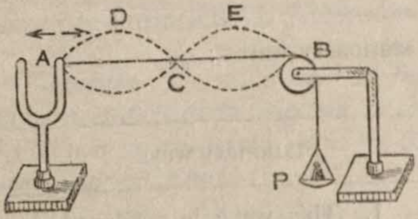

Fig. 4.4

(ii) **Transverse mode of vibration** : To produce stationary waves in the string by the transverse vibration of the tuning fork, the fork is rotated through 90° so that the prongs vibrate up and down in a vertical plane and at right angles to the length of the string [Fig. 4.5]. When the fork vibrates, transverse waves are set up in the string. These waves, being reflected at the fixed end are superposed on the direct waves and stationary waves are produced in the string as before. Here also, the same length of the string may be set in vibration consisting of two or more loops by suitably altering the tension.

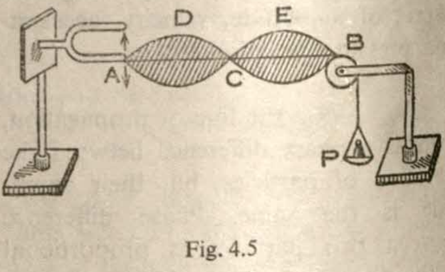

Fig. 4.5

It is to be noted that the frequency of the tuning fork may be determined by Melde's experiment.

4.8. Interference of waves :

Let us consider the case of two waves having the same amplitude and frequency travelling simultaneously in the same direction in a medium. Each particle of the medium is acted upon by two forces due to each wave. The resultant displacement of each particle will be the algebraic sum of the two displacements caused by the two waves. As a result, at certain points where a crest of one arrives at the same instant on a crest of the other (or, a compression of one on the compression of the other), larger crests (or larger compressions) will be formed. Similarly at other points, where a crest of one arrives at the same instant on a trough of the other (or, a compression of one on the rarefaction of the other), there shall be no displacement. In other words, when two waves meet in the *same phase* at a particular point, they combine to give maximum displacement and so maximum intensity. This is known as *constructive interference.* On the other hand when they meet in *opposite phase*, they combine to give minimum displacement and so minimum intensity. This is known as *destructive interference.* The whole phenomenon is known as interference of waves.

A mathematical analysis (art. 6.8) of interference shows that the following conditions are necessary for a sustained interference pattern :

(i) The amplitude and wave-length of the interfering waves must be identical.

(ii) The waves must cause displacements of the particles of the medium in the same straight line.

(iii) They must maintain a constant phase relation between them.

(iv) The waves must meet each other at the points of destructive interference with a path difference of odd multiple of $\lambda/2$ and at the points of constructive interference with a path difference of even multiple of $\lambda/2$.

Satisfying the above conditions, sound waves, light waves, X-ray waves etc. can exhibit the phenomenon of interference. At present we shall confine our attention only to the interference of sound waves. Interference of light waves will be taken up in chapter six.

4.9. Interference of sound waves and its demonstration :

If two sound waves of equal frequency and amplitude simultaneously strike a receiver of sound in same phase, they will multiply each other's effect and a loud sound will be heard. On the other hand, if they meet in opposite phase, they will destroy each other's effect and no sound will be heard. This phenomenon of producing alternately a loud sound and silence by the superposition of two sound waves is called *interference of sound.* It can be demonstrated by an apparatus devised by Quincke.

The apparatus consists of a U-shaped tube *B* made of glass or any metal like brass (Fig. 4.6). There is one side tube (*A* or *D*) attached to each of the limbs of the U-shaped tube. There is another U-shaped tube *C* which can be made to slide inside the previous one. A tuning fork of high frequency is made to vibrate over the end of the side tube *A*. Since the side tubes *A* and *D* divide the whole apparatus into two branches, sound waves starting from *A* travel along the paths *ABD* and *ACD* to reach the other side tube *D* where the listener puts his ear.

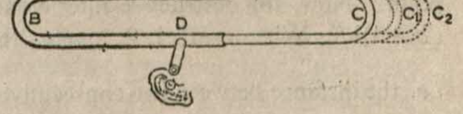

Fig. 4.6

In this way two identical sound waves may be artificially produced. If the two paths are equal, the two sound waves, arriving at D, will have zero path-difference and therefore, will have same phase. Consequently, the listener will hear a loud sound. The tube C is now made to slide in or out of the other tube so that the two paths differ by a length equal to $\lambda/2$, where λ is the wave-length of the sound wave. The waves will now meet each other at opposite phase and the listener will hear no sound although the tuning fork continues to vibrate. If the tube C is made to slide further so that the path-difference is now λ, the two waves will again meet in phase and produce a loud sound.

In this way, by adjusting the position of C, we shall come across alternate positions of maximum and minimum sound.

Silence zone : Production of silence zone is a good example of interference of sound. Fog-siren is usually sounded from a high cliff in the shore in a foggy morning to warn an approaching ship of the presence of the cliff. It is found that as the ship approaches the cliff, it passes alternately through regions of maximum and minimum sound.

Suppose, sound is being made by a fog-siren from the point A of a cliff (Fig. 4.7). Two wave trains will arrive at the ear of a person at B in the ship by two different paths—one coming directly along the path AB and the other along ACB after reflection on the surface of water. These two paths are obviously different in length. When the waves arrive at B, they will have a phase-difference. Drawing a perpendicular CD on AB, it is clear that the path-difference in this case$=CB-DB$.

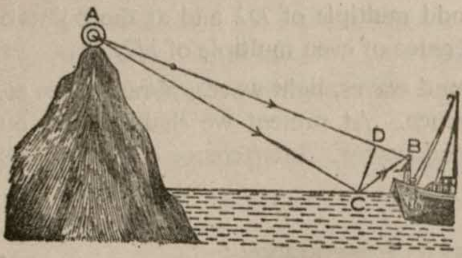

Fig. 4.7

If $(CB-DB)=\lambda/2$ or odd multiple $\lambda/2$, the person in the ship will hear no sound. The ship is said to be in the silence zone then. As the ship moves forward, the path-difference between the waves changes and the person again hears the sound of the siren.

Example : *Two sound waves, starting from a sounding body, met each other at a point, travelling along different paths. Silence is produced at the point when the path-difference is 12 cm. or 36 cm. If the velocity of sound in air be 330 metres/s, find the frequency of the sounding body.*

Ans. Silence means that destructive interference takes place at the meeting point. Now, for destructive interference, the condition is that path-difference$=(2n+1)\lambda/2$. When $n=0, 1, 2, 3$..etc., the path-differences are $\lambda/2$, $3\lambda/2$, $5\lambda/2$..etc.

i.e., the distance between two consecutive silent points$=\frac{3\lambda}{2}-\frac{\lambda}{2}=\lambda$.

Here, $\lambda=36-12=24$ cm. Again $V=n\lambda$ or $n=\frac{V}{\lambda}=\frac{330\times100}{24}=1375$.

4.10. Beats :

Take two tuning forks of equal frequency, each mounted on a hollow wooden box. Now, vibrate the tuning forks one after another. In each case, a continuous sound will be heard. Now, vibrate them simultaneously. In this case also a continuous sound will be heard but the intensity of the sound will be much greater than before. Now, attach a little wax in one of the prongs of one tuning fork. Its frequency will be lowered a little because it becomes a little heavier. If the two tuning forks are now sounded simultaneously, the sound heard will not be continuous as before. It will rise and fall periodically. Such periodical waxing and waning of sound is called *beats*.

So, when two notes of nearly equal pitch are sounded at the same time, a regular waxing and waning in the loudness of the tone is heard. These alterations in loudness are called beats.

Explanation : Consider two sources of sound having frequencies six and five respectively. The first one will produce six waves and the second one five in one second. The forms of the two waves are shown in the fig. 4.8(a), where the thick line curve denotes the wave of frequency six and the dotted line curve that of frequency five. The resultant wave produced by their superposition has been shown in fig. 4.8(b).

Consider an instant when both the sources are exactly in step so that they are simultaneously producing compressions. Under these conditions they reinforce each other and produce a resultant sound of maximum intensity. It is shown by the point A' in the fig. 4.8(b). There, the amplitude of the resultant wave is maximum. After one second, the first source will have made 6 vibrations while the second one makes 5 so that the waves differ by a length equal to one complete wavelength. As a result, the sources are once more in step with each other, producing a resultant wave of maximum amplitude and a sound of maximum intensity. This is represented by the point C' in the fig. 4.8(b). But in between the two *i.e.* after $\frac{1}{2}$ sec. the waves differ by a length equal to half the wave length and at this stage their phases will be opposite—one producing a compression at exactly the same moment that the other is producing a rarefaction. The resultant disturbance of the air will now be a minimum and the intensity of sound will also be minimum. This is represented by the point B' in the diagram.

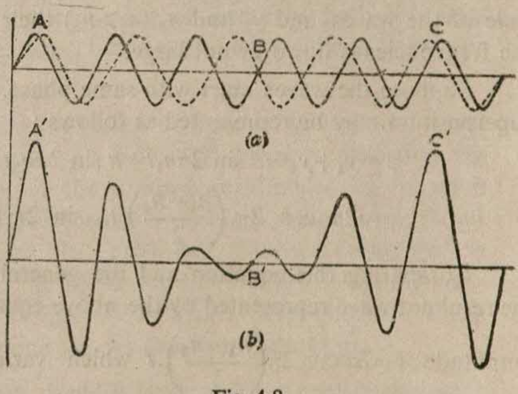

Fig. 4.8

As the sources continue their vibrations, the intensity of the resultant sound will rise and fall periodically, producing a throbbing sound.

Beat frequency : *The number of waxings or wanings of resultant sound in* 1 *second is called beat frequency.*

We have just now seen that when the frequencies of the sources are 6 and 5 (difference of frequencies=1), the number of rise or fall of intensity of sound in 1 sec is 1. Hence, the beat frequency, in this case, is 1 which is also equal to the difference of frequencies of the sources. As a matter of fact, in all cases, it can be proved (art. 4.11) that the *beat frequency=difference of frequencies of the sources.*

If n_1 and n_2 be the frequencies of the sources, then beat frequency, $N = n_1 \sim n_2$.

Perception of beats : To hear beats clearly, the difference in frequency of the two notes should be small. If the difference be high, the number of beats per second increases. It then becomes difficult for the ear to distinguish between the rise and fall of intensity of sound. The ear apparently hears a continuous sound. It has been found experimentally that perception of beats becomes impossible if the beat frequency exceeds 15 or 16 per second. Further, perception of beats also becomes difficult if the difference in amplitude of the two notes is high because the weaker sound cannot appreciably affect the louder one.

4.11. Analytical treatment of beats :

Consider two simple harmonic waves of same amplitude but of slightly different frequencies travelling in the same direction. They can be represented by the following equations : $y_1 = a \sin 2\pi n_1 t$ and $y_2 = a \sin 2\pi n_2 t$, where 'a' is the amplitude of the waves and n_1 and n_2 ($n_1 > n_2$) their frequencies. The difference of the frequencies is however not large.

Suppose the waves start with same phase. The resultant wave due to their superposition may be represented as follows :

$$y = y_1 + y_2 = a \sin 2\pi n_1 t + a \sin 2\pi n_2 t$$

$$= 2a \cos 2\pi \left(\frac{n_1 - n_2}{2}\right) t. \ \sin 2\pi \left(\frac{n_1 + n_2}{2}\right) t$$

Comparing this equation and the general equation of a wave, we see that the resultant wave represented by the above equation is a simple harmonic wave of amplitude $A = 2a \cos 2\pi \left(\frac{n_1 - n_2}{2}\right) t$ which varies with time t. Consequently, the amplitude of the resultant wave will change with time, becoming maximum at one time and minimum at the other. This makes the intensity of the resultant sound varying and a throbbing sound is heard. This throbbing sound is known as beats.

Beat frequency : When $t = 0, \dfrac{1}{n_1 - n_2}, \dfrac{2}{n_1 - n_2}$ ....etc., the amplitude of the resultant wave $A = 2a$ and maximum, which produces loud sound. So, the time-interval between two consecutive loud sounds$= \dfrac{1}{n_1 - n_2}$ or in 1 second $(n_1 - n_2)$

number of loud sounds will be heard. So, beat frequency$=n_1-n_2=$difference of frequencies.

Again, when $t=\frac{1}{2(n_1-n_2)}, \frac{3}{2(n_1-n_2)}, \frac{5}{2(n_1-n_2)} \ldots\ldots$ etc. the amplitude of the resultant wave $A=0$ and minimum, which produces silence. So, the time-interval between two consecutive minimum sounds$=\frac{1}{n_1-n_2}$ or in 1 second (n_1-n_2) number of minimum sounds will be heard. So beat frequency$=n_1-n_2=$difference of the frequencies.

4.12. Application of beats :

(a) *Determination of unknown frequency* : When two tuning forks of nearly equal frequencies are sounded together they produce beats equal to the difference in their frequencies. The number of beats is first counted and let it be x. If the frequency of one of the forks is known (say n_1), then the frequency of the other is $n_2=n_1+x$ or n_1-x. In order to ascertain which is the correct value of n_2, the tuning fork of unknown frequency is slightly loaded with wax. This would cause a slight lowering of frequency of the unknown fork and hence the number of beats now will either increase or decrease. If the unknown frequency n_2 is less than n_1 (*i.e.* $n_2=n_1-x$) the number of beats will increase. On the other hand, if the unknown frequency n_2 is higher than n_1 (*i.e.* $n_2=n_1+x$) the number of beats will decrease. This is how by counting the number of beats in both the cases before and after loading the fork of unknown frequency, it is possible to determine the frequency of the fork.

(b) *Detection of poisonous gases in mines* : This is a very important practical application of the phenomenon of beats. It helps in detecting the presence of harmful or poisonous gases in the atmosphere of mines. Two organ pipes, one filled with pure dry air and the other with the air of the mine are blown together. If the mine air is also pure, no beats are heard. But if it is impure, the velocity of sound in it changes and so the frequency also changes while the wave length remains unchanged. As a result beats will be heard. This gives a warning before-hand to the miners before the mine air can cause an explosion.

Examples : (1) *Two tuning forks A and B produce 5 beats per second when sounded together. The frequency of B is 252 ; it is found that number of beats increases when the fork A is slightly filed. What was the frequency of A before filing ?*

Ans. Since 5 beats are produced, the frequency of A is either $252+5=257$ or, $252-5=247$.

Now, after filing, the frequency of A will increase and the number of beats heard also increased. In other words, increase of frequency of A due to filing increased the difference of the frequencies of A and B. Hence, before filing, the frequency of A was higher than that of B *i.e.* the frequency of A was 257.

(2) *Three tuning forks of frequency $n+x$, n and $n-x$ respectively are sounded together. Assuming their amplitudes to be equal, show that the number of beats produced by them is x per sec.*

Ans. Since, the amplitudes are equal, the waves produced by the forks may be represented by the following equations :

$$y_1=a \sin 2\pi(n+x)t$$

$$y_2=a \sin 2\pi nt$$

and $$y_3=a \sin 2\pi(n-x)t.$$

The resultant wave due to their superposition is given by

$$y=y_1+y_2+y_3=a[\sin 2\pi(n+x)t+\sin 2\pi nt+\sin 2\pi(n-x)t]$$

$$=a\ [2.\sin\frac{2\pi(n+x)t+2\pi(n-x)t}{2}.\cos\frac{2\pi(n+x)t-2\pi(n-x)t}{2}+\sin 2\pi nt]$$

$$=a\ [2.\sin 2\pi nt.\cos 2\pi xt+\sin 2\pi nt]$$

$$=a(1+2\cos 2\pi xt)\sin 2\pi nt.$$

$$=A.\sin 2\pi nt,\text{ where } A=a(1+2\cos 2\pi xt)$$

This represents a simple harmonic wave with amplitude A which, at a particular place (x=constant) depends on time t. This means that the amplitude and hence intensity of the resultant sound at a place varies with time. As a result beats will be produced.

Now, A is maximum when $\cos 2\pi xt$ is maximum *i.e.* when $\cos 2\pi xt=1$ *i.e.* $2\pi xt=2s\pi$ where $s=0, 1, 2..$etc.

This means that maximum sound is obtained at times $t=0, \frac{1}{x}, \frac{2}{x}, \frac{3}{x}$ etc. Hence the no. of beats per sec.$=x$.

Again A is minimum when $2\cos 2\pi xt+1=0$ *i.e.*, $\cos 2\pi xt=-\frac{1}{2}$ *i.e.*, $2\pi xt=2s\pi\pm2\pi/3$ $[\cos 2\pi/3=-\frac{1}{2}]$

This means minimum sound is obtained at times $t=\frac{1}{3x}, \frac{4}{3x}, \frac{7}{3x}$ .. .. etc.

Hence, the no. of beats per sec$=1\Big/\left(\frac{4}{3x}-\frac{1}{3x}\right)=x.$

(3) *64 tuning forks are arranged in order of increasing frequency, the last one being an octave higher than the first and each two consecutive forks produce 4 beats per second. What is the frequency of the first tuning fork ?*

Ans. Let the frequency of the first fork be n. Since each two consecutive forks produce 4 beats per sec., and the forks are arranged in order of increasing frequency, the frequency of the second fork$=n+1\times4$, that of the third fork$=n+2\times4$ etc. Accordingly, the frequency of the last fork$=n+63\times4$.

But according to the problem, $2n=n+63\times4$

$$\therefore\quad n=63\times4=252.$$

4.13. Transverse vibration of a stretched string :

When a long, thin flexible string or wire stretched tightly between two fixed points A and B as shown in fig. 4.9 is plucked down* and let go, the string as a whole is found to vibrate up and down. Here, each portion of the string vibrates at right angles to the length of the string *i.e.*, each portion vibrates *transversely*. This transverse vibration of the string takes place mainly due to the stretching force or tension applied at the ends of the string and not so much to the elasticity of the material of the string. If T be the tension and m the mass per unit length (*i.e.* linear density) of the string, then the velocity V of the transverse wave along the string is given by $V=\sqrt{\frac{T}{m}}$. The waves originating at the point of plucking, travel along the wire with the above velocity towards the respective fixed ends where they are reflected back. The reflected waves meet each other and get superposed. Stationary waves are, as a result, produced with nodes and antinodes. Now, if the length of the string is so adjusted that only one antinode is formed at the middle of the string and two nodes at the two fixed ends *i.e.* if the string vibrates in a single loop (Fig. 4.9), the vibration is called *fundamental* vibration and the sound emitted by the string is called fundamental tone.

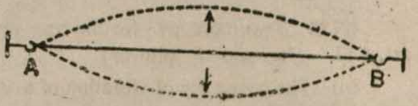

Fig. 4.9

We know that in a stationary wave, the distance between two consecutive nodes $=\lambda/2$ where λ is the wave length of the wave. So, if l be the length of the wire AB, then for the fundamental tone, $l=\lambda/2$ or $\lambda=2l$.

If n be the fundamental frequency, $n=\frac{V}{\lambda}=\frac{1}{2l}\sqrt{\frac{T}{m}}$ .. .. (i)

It is to be remembered that in the C. G. S. system, l is to be expressed in centimetre, T in dynes and m in gm/cm. In the F. P. S. system, l is to be expressed in ft, T in poundals and m in lb/ft.

Further, if r be the radius of cross-section of the string and ρ the density of its material, $m=\pi r^2\times 1\times\rho$ $\therefore$ $n=\frac{1}{2l}\sqrt{\frac{T}{\pi r^2\rho}}=\frac{1}{2lr}\sqrt{\frac{T}{\pi\rho}}$ .. .. (ii)

4.14. Laws of transverse vibration of string :

Marsenne, a noted French mathematician, first established certain laws relating to the fundamental frequency of transverse vibration of strings in 1636. They are as follows :

(i) **Law of length** : The frequency of vibration of a string is inversely

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*Transverse waves can be set up in a stretched string by (i) plucking, (ii) bowing and (iii) striking. In musical instruments like *Guitar*, *Setar* etc., strings are plucked, in *Esraj* and *Violin*, bowing of the string is resorted to while in *Piano*, the wire is struck by a soft padded hammer.

proportional to its length provided the tension T and mass per unit length m are kept constant *i.e.* $n \propto \frac{1}{l}$ if T and m are fixed.

(ii) **Law of tension** : The frequency of vibration of a string is directly proportional to the square root of the tension provided the vibrating length and mass per unit length are kept fixed. *i.e.* $n \propto \sqrt{T}$, if l and m are fixed.

(iii) **Law of mass** : The frequency of vibration of a string is inversely proportional to the square root of the mass per unit length of the string provided the length and tension are kept fixed *i.e.* $n \propto \frac{1}{\sqrt{m}}$ if l and T are fixed.

The above three laws are known as the laws of transverse vibration of strings. It is to be noted that the above laws follow from the equation of the fundamental frequency *viz*, $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$.

[**N.B.** Two more laws for the transverse vibration of string follow from the equation (ii) of art. 4.13. They are as follows :

(*a*) The frequency of vibration of a string is inversely proportional to the radius of the wire provided the vibrating length, tension and density of the string are kept fixed *i.e.* $n \propto 1/r$ if l, T and ρ are fixed. This is known as the *law of radius*.

(*b*) The frequency of vibration of a string is inversely proportional to the square root of its density provided, the vibrating length, tension and radius of the string are kept fixed, *i.e.* $n \propto \frac{1}{\sqrt{\rho}}$ if l, T and r are fixed . This is known as the *law of density*.

The laws of transverse vibration of string, however, refer to the first three laws established by Marsenne.]

4.15. Sonometer :

The laws of transverse vibration of a string can be verified experimentally with the help of a sonometer. It consists of a thin metallic wire stretched across two fixed bridges A and B placed over a hollow wooden box. One end of the wire is fixed rigidly to a peg at one end of the box and the other passes over a smooth, frictionless pulley P fixed at the other end of the box carrying a hook. Weights W can be placed on the hook. There is a movable bridge M between the two fixed bridges. If a portion of the string between the bridges A and M be plucked, the string will vibrate transversely and emit a sound whose frequency depends upon the length of the string between the bridges A and M, its tension and its mass per unit length. There is provision for stretching another wire side by side with the experimental wire, whose tension can also be varied as desired. This wire is known as reference wire.

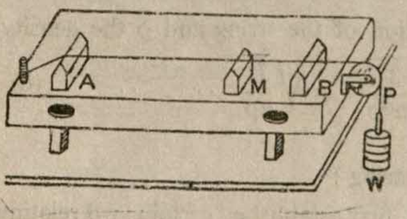

Fig. 4.10

4.16. Experimental verification of the laws of transverse vibration of string :

(i) **Verification of the law of length :** A suitable tension is applied to the experimental wire and by moving the movable bridge M, the length of the portion AM of the string is adjusted in unison with a tuning fork of known frequency. For exact adjustment, the following procedure may be adopted. Press the stem of the vibrating fork against the wooden board of the box. The vibration of the fork will be transmitted to the wire. Place a light paper rider folded in the form of V on the middle point of the wire AM. Now move the bridge M till the paper rider is thrown out. Note down this length . Next a number of tuning forks of different frequencies are tuned with the experimental wire by adjusting the proper length of the vibrating wire keeping the tension unchanged. In each case, the length of the vibrating wire is noted. Let $n_1, n_2, n_3..$ be the frequencies of the tuning forks and $l_1, l_2, l_3..$ the corresponding lengths. It will be found that $n_1l_1=n_2l_2=n_3l_3=....$ This shows that when T and m are kept fixed $n\propto\frac{1}{l}$.

(ii) **Verification of the law of tension :** This law can be verified by tuning the experimental and the reference wires simultaneously. A known tension T is applied to the reference wire. Now, tension T_1 is applied to a fixed length l of the experimental wire. The reference wire is now tuned with the experimental wire by changing the length of the reference wire. Let it be l_1. Next, tension of the experimental wire of the fixed length l is changed to $T_2, T_3, T_4..$and in each case the corresponding unison lengths of the reference wire l_2, l_3, l_4 .. are found without changing its tension. It is found that for the reference wire if $n_1, n_2,..$are its frequencies in different cases, then, $n_1 : n_2 : n_3..=\frac{1}{l_1} : \frac{1}{l_2} : \frac{1}{l_3}..$

For the experimental wire, it will be found that

$T_1 : T_2 : T_3 ..=\frac{1}{l_1^2} : \frac{1}{l_2^2} : \frac{1}{l_3^2}..$

Combining these two, we get, $n_1^2 : n_2^2 : n_3^2 .. =T_1 : T_2 : T_3 ..$

In other words, when l and m are kept fixed, $n\propto\sqrt{T}$.

(iii) **Verification of the law of mass :** Apply a known tension T to the reference wire having mass per unit length m. Now the same tension T is also applied to the known fixed length l of the experimental wire having mass per unit length m_1 . Tune this wire with the reference wire by changing its length. Let the length of the reference wire be l_1. Next replace the experimental wire with other wires successively having $m_2, m_3..$as their mass per unit length and find out the tuned length of the reference wire in each case without changing the tension. It is found that for the reference wire, $n_1 : n_2 : n_3..=\frac{1}{l_1} : \frac{1}{l_2} : \frac{1}{l_3}..$

and for the experimental wires, $m_1 : m_2 : m_3..=l_1^2 : l_2^2 : l_3^2..$

Combining the two, we get $n_1^2 : n_2^2 : n_3^2..=\frac{1}{m_1} : \frac{1}{m_2} : \frac{1}{m_3}..$

This shows that when l and T are kept fixed, $n\propto\frac{1}{\sqrt{m}}$.

4.17. Determination of frequency of a tuning fork by a sonometer :

Stretch the sonometer wire by a suitable load (Fig. 4.10). Strike the tuning fork with a padded hammer and press its stem against the board. The vibration of the fork will be transmitted to the wire. Now the length of the wire *AM* is to be so adjusted by moving the bridge *M* that it is in unison with the tuning fork. For exact adjustment, the same procedure as was adopted in the verification of the law of length, may be followed—*viz*, placing a light paper rider of the shape V at the middle of the portion *AM* and adjusting the position of *M* till the rider is thrown out. Measure the length of the wire with the help of a scale and let it be *l* cm.

Next, take a certain length of the sonometer wire (say, 50 cm.) as sample and find its weight in a balance. From the length and mass of the sample wire its mass per unit length (*m*) can be found out. Suppose, a load of *M* kg. was used for stretching the wire. Then, $T=M\times1000\times980$ dynes.

$$\text{So,}\quad n=\frac{1}{2l}\sqrt{\frac{T}{m}}=\frac{l}{2l}\sqrt{\frac{M\times1000\times980}{m}}$$

All the quantities on the righthand side of the above equation being known, *n* can be found out.

Examples : (1) *A string 100 cm. long and 32 gm. in weight is stretched by a weight of 16 kg. Calculate the frequency of the fundamental vibration of the string.*

Ans. We know, the fundamental frequency $n=\frac{1}{2l}\sqrt{\frac{T}{m}}$.

Here, $l=100$ cm. ; $M=\frac{32}{100}=0{\cdot}32$ gm/cm. ; $T=16\times1000\times980$ dynes.

$$\text{So,}\quad n=\frac{1}{2\times100}\sqrt{\frac{16\times1000\times980}{0{\cdot}32}}=\frac{1}{2\times100}\sqrt{\frac{16\times1000\times980\times100}{32}}$$

$$=\frac{4\times1000}{2\times100}\sqrt{\frac{98}{32}}=20\times\frac{7}{4}=35$$

(2) *A wire 50 cm long vibrates 100 times a second. If the length is shortened to 30 cm. and the stretching force quadrupled, what will be the frequency ?*

Ans. Suppose, *T* dyne is the tension in the first case : then

$$100=\frac{1}{2\times50}\sqrt{\frac{T}{m}}\qquad [m=\text{mass/unit length of the wire}].$$

When the tension is quadrupled, it becomes $4T$ dynes. In this case,

$$n=\frac{1}{2\times30}\sqrt{\frac{4T}{m}}=\frac{1}{30}\sqrt{\frac{T}{m}}.\quad \text{Dividing,}\ \frac{n}{100}=\frac{2\times50}{30}\quad \therefore\ n=\frac{2\times50\times100}{30}=333{\cdot}3$$

(3) *A sonometer wire emits a note of frequency 150. What will be the frequency of the note emitted by it if its tension is increased in the ratio 9 : 16 and the length is doubled?* [*H. S. Exam. 1981*]

Ans. Suppose, the length and tension in the first case are l_1 and T_1 respectively. Their values in the second case are l_2 and T_2 respectively. So, in the first case, $150=\frac{1}{2l_1}\sqrt{\frac{T_1}{m}}$ and in the second case, $n=\frac{1}{2l_2}\sqrt{\frac{T_2}{m}}$. Dividing we get, $\frac{n}{150}=\frac{l_1}{l_2}\sqrt{\frac{T_2}{T_1}}$ But according to the question, $l_1 : l_2=1 : 2$ and $T_2 : T_1=16 : 9$.

So, $\frac{n}{150}=\frac{1}{2}\sqrt{\frac{16}{9}}=\frac{4}{2\times3}$ $\quad\therefore\quad n=\frac{4\times150}{2\times3}=100.$

(4) *Two steel wires of equal length emit fundamental tones when stretched by 225 gm and 256 gm respectively. The ratio of the frequencies is 1 : 4. Find the ratio of their diameters.*

Ans. We know, $n=\frac{1}{2lr}\sqrt{\frac{T}{\pi\rho}}=\frac{1}{ld}\sqrt{\frac{T}{\pi\rho}}$ [d=diameter]

The lengths of the wires are equal and the densities of the material are also equal because both are made of steel. If the diameters of the wires are d_1 and d_2, then in the first case, $n_1=\frac{1}{ld_1}\sqrt{\frac{225\times g}{\pi\rho}}$

and in the second case, $n_2=\frac{1}{ld_2}\sqrt{\frac{256\times g}{\pi\rho}}$. Dividing, $\frac{n_1}{n_2}=\frac{d_2}{d_1}\sqrt{\frac{225}{256}}=\frac{d_2}{d_1}\times\frac{15}{16}$

But according to the problem, $n_1 : n_2=1 : 4$;

$$\text{So, } \frac{1}{4}=\frac{d_2}{d_1}\times\frac{15}{16} \text{ or } \frac{d_1}{d_2}=\frac{15}{4}.$$

(5) *A sonometer wire of length 76 cm is maintained under a tension of 4×10^6 dynes and an alternating current is pased through the wire. A horse shoe magnet is placed with its poles above and below the wire at its mid-point and the resulting force set the wire in resonant vibration. If the density of the material of the wire is 8·8 gm/c.c. and the radius of the wire is 0·5 mm, what is the frequency of the alternatning current ?*

Ans. As the wire carries a current and is placed in a magnetic field, a force will be applied on the wire and it will vibrate transversely. Resonance will occur when the frequency of the alternating current is equal to the frequency n of the transverse vibration of the string. Now,

$$n=\frac{1}{2lr}\sqrt{\frac{T}{\pi\rho}}=\frac{1}{2\times76\times\cdot05}\sqrt{\frac{4\times10^6}{\pi(8\cdot8)}}=\frac{2\times10^3}{2\times76\times\cdot05}\sqrt{\frac{1}{\pi(8\cdot8)}}=50.$$

∴ The frequency of the alternating current=50.

(6) *A vertical sonometer wire is loaded at one end. A 60 cm of the wire, when vibrating transversely, is in unison with a tuning fork. When an additional load of 10 gm is put to the wire, the length has got to be increased by 2 cm to restore unison. What was the initial load on the wire?*

Ans. Let the initial load be M gm. If n be the frequency of the wire, then

$$n=\frac{1}{2l}\sqrt{\frac{T}{m}}=\frac{1}{2\times60}\sqrt{\frac{M.g}{m}} \quad .. \quad .. \quad \text{(i)}$$

When the load and length are increased, the frequency remains the same.

Hence $n=\dfrac{1}{2\times62}\sqrt{\dfrac{(M+10)g}{m}}$ .. .. (ii)

From eqns. (i) and (ii) we get,

$$\frac{1}{2\times60}\sqrt{\frac{M.g}{m}}=\frac{1}{2\times62}\sqrt{\frac{(M+10)g}{m}} \text{ or, } \frac{62}{60}=\sqrt{\frac{M+10}{M}}$$

Squaring, $\dfrac{3844}{3600}=\dfrac{M+10}{M}$ $\therefore$ $M=147{\cdot}5$ gm (nearly)

(7) *A steel wire of length 1 metre, mass 0·1 kg. and uniform cross-sectional area $10^{-6}m^2$ is rigidly fixed at both ends. The temperature of the wire is lowered by 20°C. If transverse waves are set up by plucking the string in the middle, calculate the frequency of the fundamental mode of vibration. Young's modulus (Y) of steel=2×10^{11}N/m^2 ; coefficient linear expansion of steel=$1{\cdot}21\times10^{-5}$ per°C.* [*I.I.T. 1984*]

Ans. Thermal stress due to fall of temperature$=Y.\alpha.\theta$. where α is the coefficient of linear expansion and θ the fall of temperature.

$\therefore$ Stress$=2\times10^{11}\times1{\cdot}21\times10^{-5}\times20=4{\cdot}84\times10^7 N/m^2$.

Hence, tension=stress×cross-sectional area$=4{\cdot}84\times10^7\times10^{-6}=48{\cdot}4N$.

Now, frequency of fundamental $n=\dfrac{1}{2l}\sqrt{\dfrac{T}{m}}=\dfrac{1}{2\times1}\sqrt{\dfrac{48{\cdot}4}{0{\cdot}1}}=\dfrac{22}{2}=11.$

(8) *A sonometer wire fixed at one end has a solid mass M hanging from its other end to produce tension in it. It is found that a 70 cm. length of the wire produces a certain fundamental frequency when plucked. When the same mass M is hanging in water, completely submerged in it, it is found that the length of the wire has to be changed by 5 cm in order that it will produce the same fundamental frequency. Calculate the density of the material of the mass M hanging from the wire.* [*I.I.T. 1972*]

Ans. The fundamental frequency of transverse vibration is given by

$$n=\frac{1}{2l}\sqrt{\frac{T}{m}}=\frac{1}{2\times70}\sqrt{\frac{T_1}{m}}=\frac{1}{140}\sqrt{\frac{T_1}{m}} \quad .. \quad .. \quad \text{(i)}$$

When the mass M is submerged in water, $n=\dfrac{1}{2\times65}\sqrt{\dfrac{T_2}{m}}$ .. .. (ii)

Dividing (i) by (ii) we get, $1=\dfrac{2\times65}{140}\sqrt{\dfrac{T_1}{T_2}}$ or $\dfrac{T_1}{T_2}=\dfrac{196}{169}$

But $\frac{T_1}{T_2}=\frac{Mg}{\left(M-\frac{M}{\rho}\right)g}=\frac{\rho}{\rho-1}$ [ρ=density of the material of M]

$$\therefore \quad \frac{\rho}{\rho-1}=\frac{196}{169} \quad \therefore \quad \rho=\frac{196}{27}=7{\cdot}26 \text{ gm/c.c}$$

4.18. Overtones in a stretched string :

We have seen in art. 4.13 that when a stretched string AB vibrates in one segment [Fig. 4.11(a)], two nodes are formed at the fixed ends and an antinode is formed in the middle. This is the simplest mode of vibration of the string. In this condition the length of the wire $l=\frac{\lambda_1}{2}$ and $n_1=\frac{1}{2l}\sqrt{\frac{T}{m}}$. The mode of vibration of the string may, however, be changed. The string may be made to vibrate in two, three or four etc. segments as shown in fig. 4.11. When the string is touched gently by the finger at its middle point and bowed or plucked half way between one fixed end and the midpoint,the string begins to vibrate in two segments [Fig. 4.11 (b)]. Under this circumstances, the length of the wire $l=\lambda_2$ and $n_2=\frac{2}{2l}\sqrt{\frac{T}{m}}=2n_1$. Likewise when the string is touched lightly at one third of its length from one end and plucked half way between the fixed end and the touched point, the string begins to vibrate in three segments [Fig. 4.11(c)]. Here, $l=\frac{3\lambda_3}{2}$ and $n_3=\frac{3}{2l}\sqrt{\frac{T}{m}}=3n_1$. In this way, by producing nodes at different points of the string and plucking it at suitable points, the string may be made to vibrate in four, five etc segments. When an instrumentalist plays with a stringed instrument he touches the strings lightly at various points and sets each one vibrating in two or more segments. Generally if the wire vibrates in s segments $n_s=\frac{s}{2l}\sqrt{\frac{T}{m}}=s.n_1$.

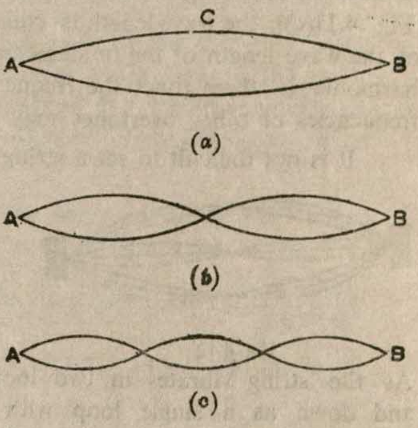

Fig. 4.11

It has been mentioned earlier that when the string vibrates in one segment, the vibration is the simplest and the frequency of the note emitted is the lowest. This note is called the *fundamental* note. The notes emitted by a string when it vibrates in more than one segment are called *overtones*. Those overtones whose frequencies are simple multiples of the fundamental frequency are called *harmonics*. Harmonics play an important part in the production of musical sound.

Fig. 4.11(a) shows the string vibrating in one segment. It is emitting the fundamental or first harmonic. In fig. 4.11(b), the string vibrates in two segments, emitting the second harmonic. In this condition, the length of the wire equals the wave length of the second harmonic, or half the wave length of the fundamental tone. Since the product of the frequency and the wave length is constant, it is clear that the frequency of the second harmonic is double of the fundamental frequency or of the first harmonic.* In other words, *the pitch of the second harmonic is an octave higher than the pitch of the fundamental*. For the third harmonic [Fig. 4.11(c)], the wave length is equal to $\frac{2}{3}$rd of the length of the wire or $\frac{1}{3}$rd of the wave length of the fundamental tone. Hence, the frequency of the third harmonic is three times the frequency of the fundamental. In this way, the frequencies of other overtones may be found out.

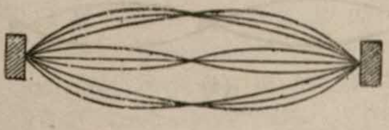

Fig. 4.12

It is not difficult to set a string vibrating with its fundamental and several of its higher modes at the same time. This is accomplished by plucking or bowing the string vigorously. As an illustration, a diagram of an ideal string vibrating with two normal modes at the same time is shown in fig. 4.12. As the string vibrates in two loops with a frequency $2n$, it also moves up and down as a single loop with the fundamental frequency n. The sound wave sent out by such a vibrating string as shown in fig. 4.13 is composed of two frequencies, the fundamental or the first harmonic of frequency n, [Fig. 4.13(a)] and the second harmonic with the frequency $2n$ [Fig. 4.13 (b)]. The sound waves combine as they travel through the air and upon their arrival at the ear, set the eardrum vibrating as shown in Fig. 4.13(c).

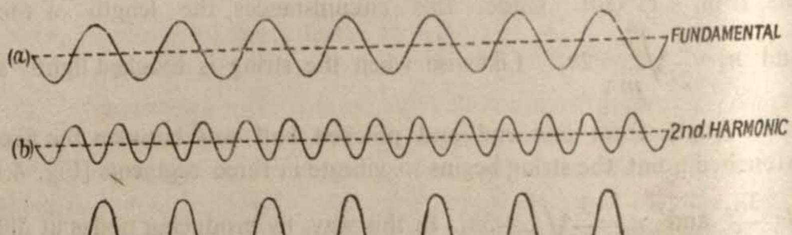

Fig. 4.13

It may be pointed out here that the overtones by their presence in a note, enhance the musical value of the note (see art. 5.15). In the case of the sound produced by a vibrating string, the number of overtones present is determined by locating the point of plucking. For example, if a string is plucked at the middle, the point will be an antinode. Hence, all harmonics which have antinodes at the middle will be present and other harmonics will be absent. Fig. 4.11 shows that 1st, 3rd, 5th harmonics have antinode at the middle. So these harmonics will be present in the note while 2nd, 4th, 6th *i.e.* all even harmonics will be absent. Musicians control the quality of sound emitted by stringed instruments like *sitar* etc., by plucking the string at suitable places.

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* Some are of opinion that fundamental should not be called the first harmonic. The tone emitted by the string vibrating in two segments should be called the Ist harmonic and the subsequent tones as 2nd, 3rd, etc harmonics. In this book, however, this opinion has not been accepted.

On the other hand if a vibrating string is touched lightly at any point, a node will be produced at that point. Consequently all overtones except those which have nodes at that particular point will be suppressed. Thus, if a vibrating string is touched lightly at $\frac{1}{3}$rd of its length, 3rd, 6th, 9th etc. harmonics, having nodes at the point, will be emitted and others will be suppressed. Musicians also use their fingers to suppress unwanted harmonics while playing on a stringed instrument.

Examples : (1) *A sonometer wire of length 1 metre is stretched by a force of 4 kg. wt. Another wire of same material and diameter has been stretched by the side of the first with a force of 16 kg. wt. The frequency of the second harmonic of the second wire is to be made equal to that of fifth harmonic of the first wire. What should be length of the second wire ?*

Ans. Since the material and diameter of the wires are equal, their linear density *i.e.* mass per unit length (m) will also be equal. If the length of the first wire be l_1, tension T_1 and the frequency of the fifth harmonic n_5 then $n_5=\frac{5}{2l_1}\sqrt{\frac{T_1}{m}}$.

Similarly if, for the second wire, the length is l_3, tension T_2 and the frequency of the second harmonic n_2', then, $n_2'=\frac{2}{2l_2}\sqrt{\frac{T_2}{m}}$

Since $n_5=n_2'$, $\frac{1}{l_2}\sqrt{\frac{T_2}{m}}=\frac{5}{2l_1}\sqrt{\frac{T_1}{m}}$ or $\frac{\sqrt{T_2}}{l_2}=\frac{5}{2}\cdot\frac{\sqrt{T_1}}{l_1}$

or $5\times l_2\times\sqrt{T_1}=2\times l_1\times\sqrt{T_2}$ or, $5\times l_2\times\sqrt{4}=2\times100\times\sqrt{16}$ $\therefore$ $l_2=80$ cm.

(2) *A stretched string 60 cm. long vibrates emitting its fundamental. A tuning fork of frequency 295 when placed by the side of the string produces 5 beats per second. The beat-frequency increases if the tension of the string is increased. What is the velocity of sound ?*

Ans. Since 5 beats are produced in a second, the fundamental frequency of the string is either $295+5=300$ or, $295-5=290$.

When the tension is increased, the frequency of the string also increases. In this condition increased beat-frequency indicates that the actual fundamental frequency of the string=300. As the string emits its fundamental, the length of the string, $l=\frac{\lambda}{2}$ $\therefore$ $60=\frac{\lambda}{2}$ or $\lambda=120$ cm.

Now, $V=n\lambda=300\times120=36000$ cm/sec=360 metres/sec.

(3) *A tuning fork of frequency 200 is in unison with a sonometer wire. If the tension of the wire is increased 1%, how many beats will be heard ?*

Ans. Other factors remaining same, the frequency of the note emitted by a sonometer wire is proportional to the square root of its tension. If the tensions be T and $(T+T/100)$ and the corresponding frequencies are n_1 and n_2 then,

$$\frac{n_2}{n_1}=\sqrt{\frac{T+T/100}{T}}=\left(1+\frac{1}{100}\right)^{\frac{1}{2}}=1+\frac{1}{200}=\frac{201}{200}$$

Since the initial frequency of the wire (n_1) was in unison with the tuning fork, $n_1=200$ and therefore $n_2=201$.

$\therefore$ No. of beats heard$=201-200=1$ per sec.

(4) *A tuning fork produces 10 beats per second with sonometer wires of lengths 95 cm. and 100 cm. What is the frequency of the fork ?*

Ans. Let n_1 and n_2 be the frequencies of sonometer wires of length 95 cm. and 100 cm. respectively. Let n be the frequency of the fork.

Since the frequency of a stretched string varies inversely as the length of the string, it is clear that $n_1>n_2$.

Now, we can write $n_1=\frac{k}{95}$ and $n_2=\frac{k}{100}$ [k=a constant of proportionality].

Further, we have $n_1-n=10$ and $n-n_2=10$.

So, from the 1st relation, we get $\frac{k}{95}=10+n$

and „ „ 2nd „ „ „ $\frac{k}{100}=n-10$

Dividing, $\frac{95}{100}=\frac{n-10}{n+10}$ or $1-\frac{5}{100}=\frac{n-10}{n+10}$ or $\frac{5}{100}=1-\frac{n-10}{n+10}=\frac{20}{n+10}$

$\therefore$ $n=390$.

4.19. Stringed instruments :

The transverse vibrations of strings find their application in the production of musical sound in stringed instruments. In these instruments, a number of metallic or catgut strings having different lengths and thickness are stretched over a wooden board. The frequencies emitted by these strings depend upon the tension, the mass per unit length of the strings as well as the point of plucking or bowing. In instruments like *sitar*, *esraj*, *veena*, *guitar* etc., the strings are set into vibration by plucking while in violin, it is done by bowing. In the piano, however, the strings are of thin steel wire and they are set into vibration by striking with soft, felt-covered hammer.

In all these instruments, the strings are stretched over a hollow wooden box known as sound chest, which acts as a resonant body. The vibrations of the strings excite forced vibration in the air of the sound chest, which intensifies the sound.

4.20. Longitudinal waves in a stretched string :

If a stretched string is stroked along its length with the help of a piece of resin-soaked leather, longitudinal vibrations are set up in the wire which produce a high-pitched sound. This note should be clearly distinguished from the note produced by the *transverse* vibration when the string is plucked. In art. 3.3 we saw that the velocity V of a longitudinal wave in a string is, $V=\sqrt{\frac{Y}{\rho}}$ where $Y=$ Young's modulus of the wire and $\rho=$its density. The two ends of the stretched string being the points of nodes, the wave length λ of the longitudinal wave is equal to $2l$ where l is the length of the string. If n be the frequency of the longitudinal vibrations of the string, then $n=\frac{V}{\lambda}=\frac{1}{2l}\sqrt{\frac{Y}{\rho}}$. Thus the frequency of longitudinal vibrations of a stretched string depends on (i) the Young's modulus and (ii) the density of the string.

Comparison between transverse and longitudinal wave velocities :

We have just now seen that the velocity of *longitudinal waves* in a stretched string is $V_L=\sqrt{\frac{Y}{\rho}}$. In art. 4.8 we saw that the velocity of *tranverse waves* in a stretched string is $V_T=\sqrt{\frac{T}{m}}$.

If α be the cross-section of the wire, $m=\alpha.\rho$. Hence, $V_T=\sqrt{\frac{T}{\alpha\rho}}$. If, now, $V_L=V_T$, then $\sqrt{\frac{T}{\alpha\rho}}=\sqrt{\frac{Y}{\rho}}$ or $Y=\frac{T}{\alpha}$.*i.e.* Young's modulus$=\frac{\text{longitudinal tension}}{\text{cross-section}}$ $=\frac{\text{longitudinal stress}}{1}$.

So it comes out that when $V_L=V_T$, the longitudinal strain is 1. This means that under the given tension, the increase in length is equal to the original length of the string. But no material is known to us which possesses so high an elastic limit. So V_L *cannot be equal to* V_T. Since the strain is always less than 1, $Y>\frac{T}{\alpha}$ or $Y.\alpha>T$ or $V_L>V_T$.

So, we may conclude that for any real tension, V_L is always greater than V_T.

4.21. Waves in rods ; Tuning fork :

Consider a rod AB fixed rigidly at its mid-point O [Fig. 4.14]. If the rod is stroked along its length, a stationary longitudinal wave is set up in the rod due to the reflection at its ends and a high-pitched sound is produced. Since boththe ends are free and the mid-point is fixed, antinodes are formed at the ends and a node at the middle. Thus the length l of the rod is equal to half the wave length ($\lambda/2$) of the wave in the rod. Hence $l=\lambda/2$. Therefore, the frequency of the note $n=\frac{V}{\lambda}=\frac{V}{2l}$ where V is the velocity of longitudinal wave in the rod, given by $V=\sqrt{\frac{Y}{\rho}}$.

[$Y=$Young's modulus and $\rho=$density of the rod].

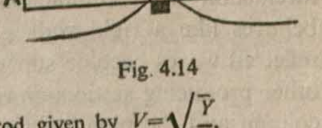

Fig. 4.14

The same rod can, however, be set into transverse vibrations with $\lambda=2l$ and $n=\frac{V}{2l}$, but in this case, V depends not only on the elasticity of the rod but also on the cross-section of the rod.

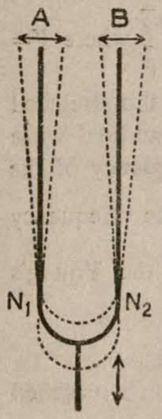

Fig. 4.15

A very important case of transverse vibration in a rod is obtained when the rod is bent in the form of U and a stem is provided to it at the middle. This is the well-known tuning fork [Fig. 4.15] widely used in acoustical experiments. Since the free ends are antinodes, there are two nodes N_1 and N_2 in between.

When the tuning fork is vibrated, the prongs A and B move to and fro sideways but the points N_1 and N_2 have no motion because they are nodes. When the prongs A and B vibrate to and fro, the mid-point together with the stem move up and down. So *the vibrations of the prongs are transverse but those of the stem are longitudinal.* Although the frequency of a tuning fork is given by a complicated formula yet the important fact is that it gives a *pure note* ; harmonics of higher frequency which may occur quite appreciably in the vibrations of a straight rod, are totally absent in the vibrations of a tuning fork. For this reason, it is widely used in acoustical experiments.

Vibrations of air column

4.22. Introduction :

You know that sweet sounds are emitted when you blow at the open end of a small glass phial or a key. During pujas, you must have seen that sweet sounds are produced by blowing a conchshell. This shows that when an enclosed air-column is set into vibration, stationary waves are produced in it and it emits a sweet sound. Such vibrations of air column find application in many wind instruments. Two types of pipes or tubes are used in wind instruments, *viz*, (i) *Open pipe*, where both the ends of the pipe are open and (ii) *Closed pipe* where one of the ends of the pipe is closed and the other open. In the following sections, we shall discuss the nature and mode of vibrations of air column in the open and closed pipes and the frequencies of sound emitted by them.

4.23. Vibrations of air column in a closed pipe :

When the open end A of a closed pipe is blown, longitudinal waves consisting of compressions and rarefactions travel down the pipe towards the closed end N. These waves are reflected from the closed end as such—a compression as a compression and a rarefaction as a rarefaction because the end behaves like a rigid wall. The incoming and reflected waves become superimposed over each other producing stationary vibrations in the air column and the pipe emits a sound. With the variation of the wave length of the stationary waves, the frequency or the pitch of the emitted sound changes.

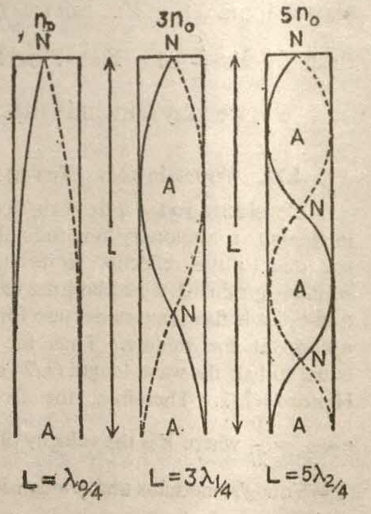

Fig. 4.16

[It is useful to compare longitudinal stationary waves in a tube with transverse stationary waves in a string. In both cases the nodes and antinodes are separated by a distance equal to one-quarter of a wave length. Owing to the difficulty of representing longitudinal stationary waves diagrammatically, it is customary to represent them symbolically by the use of transverse wave curves. This has been done in figures 4.16 and 4.17.]

At the closed end N, a node will be produced because there the air particles have no freedom of movement. At the open end A, however, the air particles have the maximum freedom of movement and hence antinode will be produced there. Under this condition, the pipe will produce the simplest stationary wave and will emit the *fundamental note*. This is shown in the first pipe of fig. 4.16.

[In the diagram, the full line represents the incident wave and broken line the reflected wave.]

If λ_0 be the wave length of the stationary wave, then the distance between a node and an antinode is $\frac{\lambda_0}{4}$ which, in the present case, is equal to the length l of the pipe. So, $l=NA=\frac{\lambda_0}{4}$ $\therefore$ $\lambda_0=4l$.

If the fundamental frequency be n_0 and the velocity of sound be V, then we know, $V=n_0\lambda_0=n_0\times 4l$. $\therefore$ $n_0=\frac{V}{4l}$.

This frequency is the lowest of all other frequencies of sound emitted by the pipe. Consequently its wave length is the longest. It is also known as the *first harmonic.**

Overtones in a closed pipe : Just as a stretched string vibrating in different loops can produce overtones (art. 4.18), a pipe can also produce overtones. A hard blowing at the open end will change the wave length of the stationary waves in air column and sounds of higher pitch will be emitted. In addition to a node at the closed end and an antinode at the open end, if there be one more node and antinode in the air column (Fig. 4.16, second pipe), *second harmonic* will be produced. If λ_1, be the wave length of the stationary wave so produced, $l=\frac{3\lambda_1}{4}$ or, $\lambda_1=\frac{4l}{3}$.

If n_1 be the frequency of the second harmonic, $V=n_1\lambda_1=n_1\cdot\frac{4l}{3}$ $\therefore$ $n_1=\frac{3V}{4l}=3n_0$.

Hence, the frequency of the second harmonic is three times the frequency of the fundamental.

In this way, the pipe may be made to emit the third, fourth etc. harmonics by overblowing and the frequencies of these overtones will be found to be five times, seven times etc. respectively of the fundamental frequency. Hence, *in a*

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* Some are of opinion that fundamental should not be called the first harmonic. The tone emitted by the pipe vibrating in its second mode [Fig. 4.13 (b)] should be called the 1st harmonic and the subsequent tones as the 2nd, 3rd. etc. harmonics. In this book, however, this opinion has not been accepted.]

closed pipe it is possible to get only those harmonics whose frequencies are odd multiples of the fundamental.

From the above equations, it may be generally said that the wave lengths of all the notes produced by a closed pipe are given by $\lambda_x=\frac{4l}{(2x+1)}=\frac{\lambda_0}{(2x+1)}$.

Hence, the frequencies $n_x=\frac{V}{\lambda_x}=(2x+1).\frac{V}{4l}=(2x+1).n_0$ where $x=0, 1, 2, 3$..etc. (*i.e.* positive integers)

4.24. Vibrations of air column in an open tube :

If any of the open ends of an open tube is blown, a longitudinal wave consisting of compressions and rarefactions travel down the pipe and proceed towards the other end wherefrom they are reflected back. A wave of compression comes back as a wave of rarefaction and *vice versa* as the open end behaves like an yielding wall. The incoming and reflected waves superimpose over each other giving rise to stationary vibrations.

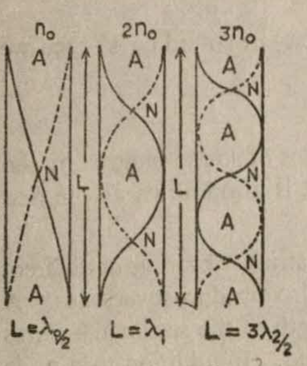

Fig. 4.17

Both the ends being free, they become antinodes and a node is formed in the middle. This mode of vibration produces the simplest wave (Fig. 4.17, first pipe) and the pipe emits the *fundamental or the first harmonic.*

If λ_0 be the wave length of the stationary wave the distance between two consecutive antinodes is $\frac{\lambda_0}{2}$, which in the present case, is equal to the length l of the pipe. So, $l=AA=\frac{\lambda_0}{2}$ $\therefore \lambda_0=2l$.

If the fundamental frequency be n_0 and the velocity of sound V, then, $V=n_0\lambda_0=n_0\times 2l$. $\therefore$ $n_0=\frac{V}{2l}$. Of all the notes the open pipe can emit, the fundamental has the longest wave length and hence the lowest frequency.

Overtones in an open tube : Like a closed tube, overtones of higher pitch may be produced in an open tube by hard blowing. For next higher mode of vibration producing the second harmonic, the pipe should be blown in such a way that in addition to two antinodes at the two open ends, one more antinode is produced at the middle of the air column [Fig. 4.17, second pipe]. If λ_1 be the wave length of the stationary wave so formed, then from the figure it is clear that $\lambda_1=l$. If the frequency of the second harmonic be n_1 then $V=n_1\lambda_1=n_1.l$.

$\therefore n_1=\frac{V}{l}=\frac{2V}{2l}=2n_0$. Hence, the frequency of the second harmonic is double that of the fundamental.

If the pipe be blown harder so that two more antinodes are produced in the air column besides those at the open ends [Fig. 4.17, third pipe] *third harmonic* will be emitted. If λ_2 be the wave length of the stationary wave so formed, $l=\frac{3\lambda_2}{2}$ or $\lambda_2=\frac{2l}{3}$. If the frequency of the third harmonic be n_2, then $V=n_2\lambda_2$ $=n_2.\frac{2l}{3}$. $\therefore$ $n_2=\frac{3V}{2l}=3.n_0$.

Hence, the frequency of the third harmonic is three times the frequency of the fundamental.

In this way, if other harmonics are produced by overblowing the pipe, their frequencies will be found to be four, five, six etc. times the frequency of the fundamental. It may, therefore, be said that *in an open pipe both odd and even harmonics are produced.* Since harmonics play an important part in musical sound, the study of open pipes is worthwhile.

In general, the wavelengths of the notes produced by an open pipe are given by $\lambda_x=\frac{2l}{(x+1)}=\frac{\lambda_0}{(x+1)}$. Hence frequencies $n_x=\frac{V}{\lambda_x}=(x+1).\frac{V}{2l}=(x+1).n_0$ where $x=0, 1, 2, 3,..$ etc. (*i.e.* a positive integer)

4.25. Frequencies of fundamental in closed and open tubes of same length :

Consider two tubes of equal length one being a closed tube and the other an open tube. If n_0 be the fundamental frequency of the closed tube, then from art. 4.22, we can write $n_0=\frac{V}{4l}$, where l is the length of the tube and V the velocity of sound.

Again, if n_0' be the fundamental frequency of the closed tube, then from art. 4.24, we have, $n_0'=\frac{V}{2l}=2\cdot\frac{V}{4l}=2.n_0$.

Hence, *the fundamental frequency of an open pipe is double that of a closed pipe of equal length or the fundamental of an open pipe is an octave higher than that of the closed one of equal length.*

For this reason, if one of the open ends of an open tube which is emitting its fundamental, be suddenly closed, the pitch of the sound will fall. On the other hand, if the lid of the closed end of a closed tube which is emitting its fundamental, be suddenly removed, the pitch of the sound will rise.

4.26. Comparison between vibrations of air column in pipes closed at one end and open at both ends :

Closed pipe	*Open pipe*
1. A pipe whose one end is closed and the other open is called a closed pipe.	1. A pipe whose both ends are open is called an open pipe.
2. In emitting notes, the closed pipe has always antinode at the open end and a node at the closed end.	2. In emitting notes, the open pipe has always two antinodes at the two open ends.
3. In a closed pipe, the wave length of fundamental tone $\lambda_0=4l$ and frequency $n_0=\frac{V}{4l}$.	3. In an open pipe, the wave length and frequency of the fundamental tone are respectively $\lambda_0=2l$ and $n_0=\frac{V}{2l}$.
4. General expression of wavelengths and frequencies of all tones emitted by a closed pipe are respectively $\lambda_x=\frac{4l}{2x+1}=\frac{\lambda_0}{2x+1}$ and $n_x=(2x+1)n_0$ [$x=0, 1, 2, 3$..etc.]	4. General expression of wavelengths and frequencies of all tones emitted by an open pipe are respectively $\lambda_x=\frac{2l}{x+1}=\frac{\lambda_0}{x+1}$ and $n_x=(x+1)n_0$ [$x=0, 1, 2, 3$..etc.]
5. The ratio of the frequencies of the fundamental and overtones is 1 : 3 : 5 : 7.	5. The ratio of the frequencies of the fundamental and overtones is 1 : 2 : 3 : 4 : 5 etc.
6. The sound is not very rich so far as musical qualities are concerned.	6. The sound is very rich so far as musical qualities are concerned.
7. Crests and troughs are reflected at the closed end without change of phase. Similarly, condensations and rarefactions are reflected without any change of phase.	7. Crests and troughs of a transverse wave and compressions and rarefactions of a longitudinal wave are reflected at the open end with a phase change of π.

4.27. Effect of temperature and humidity on the frequency of air column :

The fundamental frequencies of a closed and open pipe are respectively $n_0=\frac{V}{4l}$ and $n_0'=\frac{V}{2l}$. Now the velocity of sound in air (V) depends on the temperature of air. With the increase of temperature, velocity increases and with the decrease of temperature, velocity decreases [ref. art. 5.6]. So, from the above relations we conclude that the fundamental frequencies in an open and a closed pipe become higher and the sound shriller when the temperature of the air is increased. Similarly, the fundamental frequencies become lower and the sound flatter with the decrease of temperature of the air.

The velocity of sound in air also depends on the humidity of air in the pipe. **When the humidity of air increases, the velocity of sound in air increases and** hence the fundamental frequencies of both the pipes become shriller. On the other hand, a decrease in the humidity of air causes a lowering of the frequencies, making the sound flatter.

4.28. End-correction of open and closed pipes :

The air-particles at the open end of a pipe, closed or open, have freedom of movement and hence the vibrations at this end of the pipe extend a little more into the air outside the pipe. The antinode of the stationary wave produced in the pipe is thus situated at a distance x, say into the air outside. This distance is known as the *end-correction*. In this case, the wave-length λ of the stationary wave is such that $\lambda/4=l+x$, where l is the pipe length. In fig. 4.18(a) the end-correction in a closed pipe has been shown. In the case of an open pipe, sounding its fundamental note, the wave-length λ is given by $\lambda/2=l+x+x$, since *two* end-corrections are required [Fig. 4.18(b)]. Hence $\lambda=2(l+2x)$.

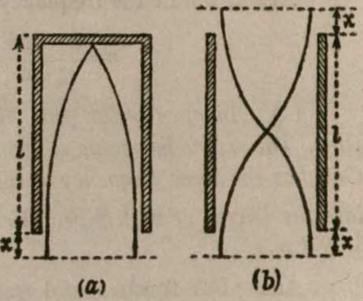

Fig. 4.18

Helmholtz and Rayleigh independently developed the theory of end-correction. The theory states that if r be the radius of the pipe, then $x=0{\cdot}6r$. So, the wider the pipe, greater is the end-correction. The theory also states that the end-correction depends on the wave length λ of the note, tending to vanish for very short wave lengths.

Now if x be the end-correction of a closed pipe of length l, then the fundamental frequency of the pipes is $n_0=\frac{V}{\lambda}=\frac{V}{4(l+x)}$.

Again $V=V_0(1+{\cdot}00183\times t)$ where t°C is the temperature of air.

$$\therefore \; n_0=\frac{V_0(1+{\cdot}00183\times t)}{4(l+x)}$$

This shows that the fundamental frequency of a closed pipe (i) increases with the increase of temperature and (ii) decreases with the increase of x.

Now, we have seen that for a wider tube, the value of x increases. Hence it may be said that of the two tubes of *same length*, the wider one emits flatter note, the temperature remaining constant.

The same results follow in the case of an open tube.

Examples : (1) *What is the minimum length of an open pipe which will be in resonance with a tuning fork of frequency 300, the velocity of sound in air being 330 metres/sec.*

Ans. We know that fundamental note is produced when the length of the pipe is minimum. For an open pipe, $n_0=\frac{V}{2l}$.

Here, $n_0=300$ and $V=330$ metres/sec. $\therefore\ l=\frac{V}{2n_0}=\frac{330\times 100}{2\times 300}=55$ cm.

(2) *The length of an open tube is 4 metres. What will be the frequency of the 8th harmonic if the air column of the pipe is vibrated ? Velocity of sound=356 metres/ sec.*

Ans. The frequency of the fundamental in an open pipe $n_0=\frac{V}{2l}=\frac{356}{2\times 4}$.

Again, if n be the frequency of the 8th harmonic, then $n=8n_0$.

$$\therefore\quad n=\frac{8\times 356}{2\times 4}=356.$$

(3) *An open organ pipe filled with air has a fundamental frequency of 500 vib/s. The second harmonic of another organ pipe closed at one end and filled with CO_2 has the same frequency as that of the second harmonic of the open pipe. Calculate the length of each pipe. Velocity of sound in air=330 m/s and that in CO_2 =264 m/s.* [*I.I.T. 1965*]

Ans. For fundamental frequency of open organ pipe, we have,

$$n_0=\frac{V}{2l}\quad \therefore\quad l=\frac{V}{2n_0}=\frac{330}{2\times 500}=0{\cdot}33 \text{ metre}=33 \text{ cm.}$$

For the second harmonic of the open pipe $n_1=2.n_0=2\times 500$

Now, for the second harmonic of closed pipe $n_1'=\frac{3V}{4l}$; but $n_1'=2\times 500$ and

$$V=264 \text{ m/s}\quad \therefore\ 2\times 500=\frac{3\times 264}{4\times l}\qquad \therefore\ l=\frac{3\times 264}{4\times 2\times 500}=0{\cdot}198 \text{ m.}=19{\cdot}8 \text{ cm.}$$

Exercises

Essay type :

1. How is a stationary wave produced ? What are their characteristics ? Explain graphically how a stationary wave is generated ? [cf. *H. S. Exam. 1978*]

2. How are stationary waves produced ? Give one example each for longitudinal stationary wave and transverse stationary wave. Explain, with diagram, what you mean by nodes and antinodes. [*H. S. 1982*]

3. Prove analytically that the distances between two consecutive nodes and antinodes in a stationary wave are equal and each is equal to half the wavelength.

4. Explain the laws of transverse vibrations of a stretched string. Write down an expression for the frequency of vibration of such a string.

5. What is a sonometer ? How would you verify experimentally the laws of transverse vibration of strings with the help of a sonometer ?

6. How would you compare the frequencies of two tuning forks by a sonometer ?

7. What are overtones and harmonics ? How are they produced in the case of transverse vibration of a stretched string ?

8. Discuss the mode of vibration of air column in an open and a closed pipe and determine the frequencies of fundamental and overtones emitted by them.

9. Explain with the help of neat diagrams how stationary waves are produced in an open and closed tubes.

10. What do you mean by interference of sound? What are the conditions of interference? How will you demonstrate it experimentally ? [*H. S. Exam. 1980*]

11. Discuss the superposition of a progressive wave and a reflected wave in an open organ pipe. [*H. S. Exam. 1980*]

12. What are beats ? How can you find out the frequency of an unknown fork with the help of beats ?

13. Explain how beats are produced when two sound waves of nearly equal frequencies are superposed on each other.

14. Given a set of standard forks of frequencies 256, 264, 272, 280 and 288, and a tuning fork whose frequency is known to be between 256 and 288, how would you determine its frequency accurately ?

Short answer type :

15. What are nodes and antinodes ? What will be the wave length in the following cases :— (i) the distance between two successive antinodes (ii) the distance between a node and the next antinode (iii) the distance between two successive nodes (iv) the distance between three successive nodes.

16. The equation of a transverse wave is given by $y=6\cos\pi/2(\cdot0050x-8t-0\cdot57)$ in which x and y are expressed in cm. and t in sec. Write down the equation of a wave when added to the given one, would produce stationary wave. [Ans. $y=6\cos\pi/2(\cdot0050x+8t-0\cdot57)$]

17. If two waves differ only in amplitude and are propagated in opposite directions through a medium, will they produce stationary waves ? Are there any nodes ? Will they transfer energy?

[**Hints :** Yes but there will be no nodes. There will be points of maximum and minimum displacements ; no transference of energy.]

18. Draw the wave patterns set up in a resonance tube open at both ends when the length of the tube is equal to the wave length and also when the length is 3/2 times the wave length. [*Jt. Entrance 1982*]

19. Mention the differences between a progressive wave and a stationary wave. [*H. S. Exam. 1982*]

20. How does the frequency of vibration of a stretched string depend on (i) the length of the string (ii) the tension of the string ?

21. How will the frequency of a note emitted by a musical instrument change if (i) the tension is quadrupled and (ii) the length is halved ?

22. Which overtones will be present in the note emitted by a wire when it is plucked at a point (i) one third of the length of the wire from one end and (ii) one fourth of the length of the wire from one end ?

23. Answer the following questions : (i) Why does an open pipe emit a note an octave higher than the fundamental when it is blown vigorously ? (ii) Why do we find differences in the quality of notes emitted by an open pipe and a closed pipe ? (iii) Will there be any change in the note emitted by an open tube, if the temperature of the air column in the tube is changed or one of the ends of the tube is suddenly closed ?

24. "Sound emitted by an open organ pipe is more musical than that emitted by an organ pipe closed at one end"—Explain. [*H. S. Exam. 1980*]

25. How will the frequency of fundamental emitted by an open organ pipe change if (i) one open end is partially closed (ii) one open end is completely closed (iii) the length of the tube is increased (iv) the diameter of the tube is increased.

[**Hints :** (i) The frequency of fundamental will be lowered (ii) it will be a closed pipe and the frequency is halved (iii) fundamental frequency $n_0=\frac{V}{2l}$. So if the length l is increased, n_0 will decrease (iv) when diameter is increased, end-correction is also increased ; consequently, the frequency is decreased.]

26. You are given two tubes of equal length—one open and the other closed. What relation will you find in the fundamental frequencies of the tubes ?

27. Explain the terms (i) fundamental (ii) overtones (iii) harmonics and (iv) octave. "All harmonics are overtones but all overtones are not harmonics."—Explain it.

28. How are beats utilised in detecting poisonous gas in a mine ?

29. Two tuning forks produce 4 beats per sec. If the prongs of a tuning fork are made a little heavy, the number of beats becomes 2 per sec ; if the prongs are made still heavier, the number of beats again becomes 4 per sec. How is this possible ?

Objective type :

30. Select the proper answer in the following questions :—

(i) Two waves of nearly equal frequencies and amplitude, travelling in the same direction, when superposed produce stationary wave/echo/beats.

(ii) When a stretched string is plucked at the middle, the harmonics which have node/antinode at that point are suppressed.

(iii) Through a gas transverse/longitudinal/stationary waves can travel.

(iv) The velocity of transverse wave in a stretched string is given by

$$V=\sqrt{\frac{E}{\rho}}\Big/V=\sqrt{\frac{T}{m}}\Big/V=\sqrt{\frac{\gamma P}{\rho}}.$$

(v) In an open tube only odd/even/all harmonics can be produced.

(vi) Two tuning forks produce 5 beats per second. The beat frequency increases when one of them is filed. If the frequency of the other fork is 512, what is the frequency of the first ? 507/512/517.

(vii) A cylindrical tube, open at both ends, has a fundamental frequency f in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of air column is now (i) $f/2$ (ii) $3f/4$ (iii) f (iv) $2f$.

(viii) Sound waves of frequency 660 *Hz*, falls normally on a perfectly reflecting wall. The shortest distance from the wall at which the air particles have maximum amplitude of vibration is (i) 0·250 m (ii) 0·125 m (iii) 0·062 m. [*I.I.T. 1984*]

Numerical problems :

31. Sound waves of frequency 220 produce stationary waves in air. If the velocity of sound wave in air be 330 metres/s, find the distance between (i) two consecutive antinodes (ii) a node and the next antinode (iii) two consecutive nodes. [Ans. (i) 75 cm (ii) 37·5 cm (iii) 75 cm.]

32. A sound wave of frequency 100 is incident normally on a rigid wall and gets reflected therefrom. How will the nodes and antinodes be situated from the wall ? Velocity of sound =340 metres/s.

[Ans. Nodes=1·7m ; 3·4m ; 5·1m. Antinodes=0·85m ; 2·55m ; 4·25m]

33. A stretched string under a tension of 1 kg. is in unison with a tuning fork of frequency 320. What alteration in the tension would make it vibrate in unison with a fork of frequency 256 ? [Ans. Tension to be decreased by 0·36 kg.]

34. The frequency of a string 36 cm. long is in unison with a fork of frequency 256. The frequency of 40 cm. long of the same wire is in unison with another fork. What is the frequency of the second fork ? [Ans. 230·4]

35. The frequency of fundamental of a stretched string of a sonometer is 250/sec. Its length is doubled and the tension is altered such that the fundamental frequency remains the same. Obtain the relation between the new tension and the original tension.

[*H. S. Exam. 1980*] [Ans. 4 : 1]

36. Two wires of the same sample (having same mass per unit length) are stretched on a sonometer. One is twice as long as the other. But they are resonating with each other. Compare their tensions. [*H.S. Exam. 1982*] [Ans. 4 : 1]

37. The frequency of transverse vibration of a stretched string is 250. When its length is 30 cm., frequency is 400. What was its initial length ? [Ans. 48 cm.]

38. A wire of length 140 cm. and of mass 52 gm. is stretched by a load of 16 kg. What is the frequency of transverse vibration of the string ? [Ans. 23 (nearly)]

39. If the fundamental be emitted by a length of 24 cm. of a wire of violin, what length of the same wire will emit the next octave ? [*H. S. Exam. 1960*] [Ans. 12 cm.]

40. A sonometer wire emits a note of frequency 150. What will be the frequency of the note emitted by the same string, if the tension be increased in the ratio 9 : 16 and the length in the ratio 1 : 2 ? [*H. S. Exam. 1962*] [Ans. 100]

41. The lengths of two wires, made of same material, are in the ratio 2 : 3. Their diameters are also equal. If the fundamental of the shorter wire is an octave higher than the longer, what is the ratio of their tensions ? [*H. S.* (*comp.*) *1966*] [Ans. 16 : 9]

42. Instead of a sonometer wire of diameter 0·9 mm, another wire of same material but of diameter 0·93 mm is used. If the tension be the same, what will be the percentage change in the frequency of the fundamental ? To keep the fundamental frequency unchanged, what should be the percentage change in the tension ? [Ans. −3·2% ; +6·8%]

[**Hints :** $n=\frac{K}{d}$ where K=constant and d=diameter. In the first case, $n_1=\frac{10K}{9}$ and in the second case $n_2=\frac{10K}{9\cdot3}$ ∴ Percentage change$=\frac{n_1-n_2}{n_1}\times 100$]

43. Two identical piano wires have a fundamental frequency of 600 vib/s, when kept under the same tension. What fractional increase in the tension of one wire will lead to the occurrence of 6 beats per sec when both wires vibrate simultaneously ? [Ans. 1/50]

44. A wire of mass 0·01 gm/cm is stretched over two sonometer bridges, 1 metre apart, by a weight of 4 kg. If the mid-point of the wire is plucked, what will be (i) the wavelength of the wave generated (ii) the frequency of the fundamental ? [Ans. (i) 200 cm. (ii) 99]

45. Two ends of a wire are rigidly fixed to two clamps x apart. The cross-section of the wire is a, tension T and the Young's modulus Y and the coefficient of linear expansion α per °C. If the temperature of the wire decreases by θ°C, how many times will the frequency increase ?

$$\left[\text{Ans. } 1+\frac{\alpha.\theta.Y.a}{2T}\right]$$

46. Two pipes—one closed and the other open—are each 25 cm long. What will be the fundamental frequencies of the pipes? Velocity of sound=336 metres/s. [Ans. 336 ; 672]

47. The pitch of the fundamental note of an open organ pipe 100 cm long is the same as that of a sonometer wire 200 cm. long with mass 1 gm per cm. Find the tension of the wire ? Velocity of sound=330 metres per sec. [Ans. 4356×10^6 dynes]

48. The length of a closed organ pipe is 90 cm. What is the frequency of the harmonic next to fundamental ? Velocity of sound=300 metres/sec. [Ans. 250]

49. Calculate the velocity of sound in a gas in which two waves of lengths 80 cm and 81 cm produce 5 beats per sec. [Ans. 324 m/s]

50. 24 tuning forks are arranged in order of increasing frequency. Each two consecutive forks produce 4 beats per second and the last fork is an octave higher than the first. Find the frequencies of the first and the last fork. [Ans. 92, 184]

51. A knife edge divides a sonometer wire into two parts which differ in length by 2 mm. ; the two parts, vibrating together, produce 1 beat per second. If the total length of the wire be 1 metre, find the frequencies of the two parts of the wire. [Ans. 250·6 ; 249·6]

52. Two tuning forks when sounded together produce 6 beats per sec. One of them is in unison with 90 cm. of a sonometer wire while the other is in unison with 91 cm. of the same wire under same tension. What are the frequencies of the forks ? [Ans. 546 ; 540]

[**Hints :** $\frac{n_1}{n_2}=\frac{91}{90}$ and $n_1-n_2=6$]

53. Two tuning forks A and B produce 5 beats when sounded together. A is in unison with 40 cm. length of a sonometer wire under a constant tension and *B* is in unison with the same wire of length 40·5 cm. under the same tension. Calculate the frequency of the forks.

[*Jt. Entrance 1982*] [Ans. 405 ; 400]

54. A sonometer wire is in unison with a tuning fork. Keeping its tension constant, if the length is increased by 1%, 3 beats are heard in a second. What is the frequency of the fork ?

[Ans. 303]

Harder problems :

55. A copper wire is held at the two ends by rigid supports. At 30°C, the wire is just taut, with negligible tension. Find the speed of transverse waves in this wire at 10°C. Y for copper$=1{\cdot}3\times10^{11}$ N/m^2 ; $\alpha=1{\cdot}7\times10^{-5}$/°C and density$=9\times10^3$ kg/m^3.

[*I.I.T. 1979*] [Ans. 70 m/s (nearly)]

56. A wire with 0·05 gm/cm linear density is stretched between two rigid supports with a force $4{\cdot}5\times10^7$ dynes. The wire is in unison with a frequency of 420. The next frequency with which the wire is in resonance is 490. What is the length of the wire ? [Ans. 214·3 cm.]

57. The fundamental frequency of longitudinal vibration of a rod clamped at its centre is 1500 Hz. If the mass of the rod is 96 gm, find the increase in its total length produced by a tension due to a load of mass 10^4 gm. ($g=980$ cm/s^2) [Ans. $1{\cdot}1\times10^{-1}$ cm]

58. A horizontal sonometer wire of length 50 cm. and mass 4·5 gm is under a fixed tension of $1{\cdot}2\times10^7$ dynes. The poles of a horse-shoe magnet are arranged to produce a horizontal transverse magnetic field at the mid-point of the wire and an alternating current passes through the wire. The wire is vibrating in resonance with the frequency of alternating current. Find the frequency of resonance. [Ans. 163]

59. The ends of a string are rigidly fixed. The temperature of the string falls by 10°C. The change in tension causes the fundamental frequency of transverse vibration of the string to increase to double its previous value. What was the original tension ? Area of cross-section of the string$=0{\cdot}01$ cm^2 ; α for the string$=16\times10^{-6}$/°C and $Y=20\times10^{11}$ dynes/cm^2.

[Ans. $10{\cdot}67\times10^5$ dynes].

60. A metal wire of diameter 1 mm. is held on two knife edges separated by a distance of 50 cm. The tension in the wire is 100 *N*. The wire, vibrating with its fundamental frequency and a vibrating tuning fork together produce 5 beats/sec. The tension in the wire is then reduced to 81 *N*. When the two are excited, beats are heard again at the same rate. Calculate the frequency of the fork and the density of the material of the wire.

[*I.I.T. 1980*] [Ans. 95 ; 12·7 gm/c.c.]

61. A uniform rope of length 12 metre and mass 6 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0·06 m is produced at the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope. [*I.I.T. 1984*] [Ans. 0·12 metre]

62. A wire of density 9 gm/cm^3 is stretched between two clamps 100 cm apart while subjected to an extension of 0·05 cm. What is the lowest frequency of transverse vibrations in the wire, assuming Young's modulus of the material to be 9×10^{11} dynes/cm^2.

[*I.I.T. 1975*] [Ans. 35·4 (nearly)]

63. The length of a sonometer wire between two fixed supports is 110 cm. Where should two bridges be placed below the wire so as to divide the wire into three segments whose fundamental frequencies are in the ratio 1 : 2 : 3 ? [Ans. 60 cm, 30 cm, 20cm]

64. A length of wire on a sonometer vibrates in unison with a fork of frequency 320 when the tension is 3 kg-wt. If the wire is replaced by one of the same material but twice the diameter, what tension would be required so that the same length is in unison with a fork of frequency 256 ? [Ans. 7·7 kg-wt (nearly)]

65. Two wires are fixed on a sonometer. The tensions are in theratio 8 : 1, thelengths in the ratio 36 : 35, the diameters in the ratio 4 : 1 and the densities are in the ratio 1 : 2. Find the frequency of beats produced if the note of the higher pitch has a frequency of 360 per sec. [*I.I.T. 1968*] [Ans. 10 per sec.]

66. Two notes of a piano have a difference of 4 octaves. The lower frequency note is emitted by 135 cm length and the higher frequency by 40 cm. length of the wire. If their tensions are equal, find the ratio of their linear densities. [Ans. 22·5 : 1]

[**Hints :** Here, $\frac{n_2}{n_1}=(2)^4$]

67. A wire is stretched elastically by 0·04 per cent. Find the ratio of the velocities of longitudinal and transverse waves on this wire. [Ans. 50]

68. An open organ-pipe is emitting its fundamental tone. It is in unison with a wire of length 35 stretched by a force of 2 kg-wt. Calculate the length of the pipe. Velocity of sound in air=330 metres/s and mass per unit length of the wire=0·04 gm/cm. [Ans. 165 cm.]

69. A swimmer sends a sound signal from the bottom of a lake to the surface. 5 beats will be heard if the signal is compared with the fundamental tone of a closed pipe of length 20 cm. What is the wavelength of sound in water ? Velocity of sound in air=360 metres/s and in water =1500 metres/s. [*I.I.T. 1974*] [Ans. 3·3 metres or 3·37 metres]

70. How many beats will be heard if the following sound waves are superposed :

$$y_1=11\cos(400\pi t-\pi x)$$
$$y_2=9\cos(410\pi t-1{\cdot}02\pi x)$$

[Ans. 5]

71. Two tuning forks *A* and *B* give 6 beats in 2 seconds. The frequency of *A* is 432. When *B* is filed, again 6 beats are produced in 2 seconds. Calculate the frequency of *B* (i) before and (ii) after filing. [Ans. (i) 429 (ii) 435]

72. The third overtone of a closed organ pipe is found to be in unison with the first overtone of an open pipe. Find the ratio of the lengths of the pipe. [Ans. 7 : 4]

73. *AB* is a cylinder of length 1 metre, fitted with a thin flexible diaphragm *C* at the middle and two other thin flexible diaphragms *A* and *B* at the ends [Fig. 4.19]. The portions *AC* and *BC* contain hydrogen and oxygen gas respectively. The diaphragms *A* and *B* are set into vibrations of same frequency. What is the minimum frequency of these vibrations for which the diaphragm *C* is a node ? Assume the velocity of sound in hydrogen is 1100 metres/s and in oxygen 300 metres/s. [*I.I.T. 1978*] [Ans. 1650]

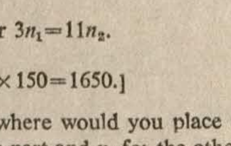

Fig. 4.19

[**Hints :** If n_1 and n_2 are the frequencies in hydrogen and oxygen, then

$$n_1=\frac{V_H}{4l}=\frac{1100}{4\times\frac{1}{2}}=550 \text{ and } n_2=\frac{V_0}{4l}=\frac{300}{4\times\frac{1}{2}}=150 \text{ So, } \frac{n_1}{n_2}=\frac{550}{150}=\frac{11}{3} \text{ or } 3n_1=11n_2.$$

So, the minimum frequency$=3\times n_1=550\times 3=1650$ or $11\times n_2=11\times 150=1650$.]

74. If the fundamental frequency of a string 60 cm long is *n*, where would you place a wooden bridge under the string to produce notes of frequency n_1 for one part and n_2 for the other so that intervals from *n* to n_1 and from n_1 to n_2 shall be the same ?

[N.B : Interval means the ratio of frequencies] [Ans. 37·1 cm from one end]

75. A tube is closed at one end and closed at the other by a vibrating diaphragm which may be assumed to be a displacement node. It is found that when the frequency of the diaphragm is 2000 Hz a stationary wave pattern is set up in the tube and the distance between adjacent nodes is then 8 cm. When the frequency is gradually reduced the stationary wave pattern disappears but another stationary wave pattern reappears at a frequency of 1600 Hz. Calculate (i) the speed of sound in air (ii) the distance between adjacent nodes at a frequency of 1600 Hz (iii) the length of the tube between the diaphragm and the closed end (iv) the next lower frequency at which a stationary wave pattern will be obtained. [Ans. (i) 320 m/s (ii) 10 cm (iii) 40 cm (iv) 1200 Hz]

76. A tuning fork of frequency 256 when sounded with an open organ pipe, produces 8 beats per sec. The frequency of the fork is lower than that of the pipe. What change should be made in the length of the pipe so that the two sounds may be in unison with each other ? Velocity of sound in air=320 metres/s. [Ans. To be increased by 1·9 cm.]

77. A string 25 cm long and having a mass of 2·5 gm is under tension. A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats per second are heard. It is observed that decreasing the tension in the string decreases the beat frequency. If the speed of sound in air is 320 m/s, find the tension in the string. [*I.I.T. 1982*] [Ans. 27×10^5 dynes]

78. The vibrations of a string of length 60 cm fixed at both end. are represented by the equation $y=4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$ where x and y are in cm. and t in second.

(i) What is the maximum displacement at $x=5$ cm? (ii) Where are the nodes located along the string ? (iii) What is the velocity of the particle at $x=7\cdot5$ cm at $=0\cdot25$ sec. ? (iv) Write the equations of the component waves whose superposition gives the above wave. [*I.I.T. 1985*]

[Ans. (i) $2\sqrt{3}$ cm. (ii) 0, 15 cm, 30 cm, 45 cm and 60 cm. (iii) 0

(iv) $y_1=2\sin\pi\left(\frac{x}{15}-96t\right)$ and $y_2=2\sin\pi\left(\frac{x}{15}+96t\right)$]

5

NATURE AND CHARACTERISTICS OF SOUND WAVES

5.1. Sound waves are elastic waves :

For the following reasons, it may be said that sound is propagated in a material medium by wave-motion.

(i) Vibrations of particles of a medium are necessary for production of waves. Similarly vibrations of particles of a medium are necessary for production of sound.

(ii) Sound moves through a medium with a definite velocity. Waves behave similarly.

(iii) When sound passes through a medium, the medium is not displaced. So is the case with a wave.

(iv) Like all waves, sound exhibits reflection, refraction, interference, and diffraction.

(v) Sound waves have actually been photographed.

We, therefore, conclude that sound is transmitted through a material medium by wave motion.

Mechanism of propagation of sound wave : Fig. 5.1 shows how a vibrating tuning fork sends out a sound wave. Any of the prongs, say the prong O, moves periodically to and fro along the path ab. The column of air in front of the prong is imagined to be divided into equi-spaced layers [Fig. 5.1(i)]. When the prong moves from a to b, it compresses the layer of air in front of it. This disturbance is then transmitted from layer to layer through air till the prong arrives at b. As a result, a pulse of compression extending over the length AB moves outward. A careful observation shows that all the layers over the region AB are not equally compressed. The reason is that the velocity of the prong is maximum at the centre of the path ab. So, the layers at the middle are more compressed than others. [Fig. 5.1(ii)]

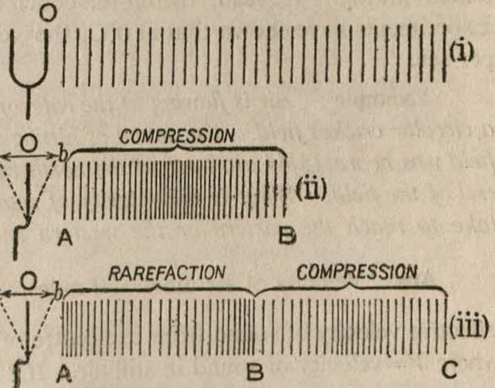

Fig. 5.1

During the reverse movement of the prong O from b to a partial vacuum will be produced behind it. Air being elastic, the layer in contact with the prong rushes behind to fill up the vacuum and a rarefaction is thus produced among the layers in contact with the prong till the prong reaches the point a. So, a pulse of rarefaction is created during this half of vibration of the prong which extends over the region *AB*, the pulse of compression produced by the prong during the previous half of vibration having moved outward to *BC* during this time [Fig. 5.1(iii)]. For the same reason as before, all the layers of the region *AB* are not equally rarefied. The central layers are more rarefied than others.

So, during a complete vibration of the prong, a compression and a rarefaction are created, whose lengths will be equal because each is created in half the vibration. As long as the fork vibrates, pulses of compression and rarefaction move outward through the air, keeping their respective positions unchanged. When such longitudinal elastic waves, consisting of compressions and rarefactions reach the ear of the listener, he hears the sound.

5.2. Sound has definite velocity :

Like all other waves, sound wave has also a definite velocity. You may have noticed that during a thunderstorm, you see the light of a distant lightning flash and hear the thunder later although both are produced at the same time. This shows that sound takes some time to cover the distance between the cloud and the earth. The velocity of light being extremely high (nearly 1,86,000 miles/sec), light reaches the earth almost immediately. Similarly, when a distant train starts and we watch for the first puff of smoke as it starts out, the arrival of the accompanying sound is not heard until an appreciable time afterwards. Watching a cricket match from a distance, you may have noticed that the sound of a bat hitting the ball is heard sometime after the actual hitting. We can, therefore, conclude that sound has a finite velocity. Experiments have shown that at 0°C, the velocity of sound in air is about 1120 ft per sec.

Example : *Air is flowing at the rate of 30 miles/hr from north to south across a circular cricket field. The sound of hitting the ball by the bat at the centre of the field was heard 13/44 sec later at the northern end and 3/11 sec later at the southern end of the field. What is the velocity of sound in still air? What time will sound take to reach the eastern or the western end of the field?*

Ans. Velocity of air flow$=30$ miles/hr$=\dfrac{30\times1760\times3}{60\times60}=44$ ft/s. Now, the effective velocity of sound from the centre of the field towards north$=(V-44)$ ft/s where V=velocity of sound in still air. If r ft be the radius of the field, the time taken by sound to reach the northern end$=\dfrac{r}{V-44}$ sec. $\therefore\ \dfrac{r}{V-44}=\dfrac{13}{44}$ .. (i)

Again, the effective velocity of sound from the centre of the field towards south $=(V+44)$ ft/s. So, $\dfrac{r}{V+44}=\dfrac{3}{11}$ .. .. (ii)

From eqn. (i) and (ii) we get, $\frac{13}{44}(V-44)=\frac{3}{11}(V+44)$

or $13V-572=12V+528$ $\therefore\ V=1100$ ft/s.

Putting this value of V in eqn. (ii), $r=\frac{3}{11}\times1144=312$ ft.

Now, the velocity of sound from the centre of the field towards east or west will not be affected by air. It will remain 1100 ft/s.

Hence time taken by sound to reach eastern or western end of the field

$=\frac{r}{1100}$ sec$=\frac{312}{1100}$ sec$=0{\cdot}3$ sec. (nearly)

5.3. Newton's formula for the velocity of sound in gases :

From a theoretical discussion of propagation of sound wave in a gas, Sir Isaac Newton established a formula for the velocity of sound. The formula is :

$V=\sqrt{\frac{E}{D}}$ where V=velocity of sound, E=modulus of elasticity of the gas and D=density of the gas.

Now, sound waves through gaseous medium are longitudinal waves and gases have only bulk elasticity. So, in the above expression, V denotes the velocity of longitudinal sound waves and E the modulus of bulk elasticity of the gas.

In the theoretical discussion, Newton assumed that during the propagation of sound waves through air or any gaseous medium, rarefactions and compressions of the layers take place slowly with consequent slow change of pressure-density condition of the medium. The heat generated in a given layer due to compression is transferred to the adjoining layers before the next rarefaction starts in the layer. Similarly, the cooling produced in a given layer due to rarefaction is not confined in the layer; it gets sufficient time to pass on to the adjoining layers. Consequently the temperature of the layers remains unaltered. According to Newton, therefore, propagation of sound through gaseous medium takes place under *isothermal condition*. In isothermal condition the bulk elasticity of a gas equals its pressure. Hence, the above formula can be written as $V=\sqrt{\frac{P}{D}}$, where P is the pressure of the gas.

In the C. G. S. system the pressure P should be expressed in dynes/sq. cm. and density D in gm/c.c.

[**N.B.** It may be proved in the following way that the bulk modulus of elasticity of a gas at constant temperature (*i.e.* isothermal condition) is numerically equal to its pressure. Consider a given mass of gas at initial pressure P and volume V. Let the pressure of this mass of gas be increased by a very small amount p and let the consequent diminution in volume be v. According to Boyle's law, $P.V.=(P+p)\,(V-v)$

or, $P.V=P.V-P.v+pV-p.v.$ or, $P.v=p.V$ [$p.v.$ being very small may be neglected.]

$$\text{or, } P=\frac{p.V}{v}=\frac{p}{v/V}.$$

Here, p is the volume stress and v/V is the volume strain. Hence,

$$P=\frac{\text{volume stress}}{\text{,, strain}}=\text{Bulk modulus}].$$

5.4. Velocity of sound in air at N. T. P. :

Normal air pressure =pressure of 76 cm. of Hg.
$=76\times 13{\cdot}59\times 980$ dynes/sq. cm.

At 0°C and at normal atmospheric pressure, the density of air=0·001293 gm/c.c. So, from Newton's formula, $V=\sqrt{\dfrac{76\times 13{\cdot}59\times 980}{0{\cdot}001293}}$

$=280$ metres/sec (nearly).

This calculated value of velocity of sound in air differs widely from the experimental value which is nearly 332 metres/sec.

5.5. Laplace's correction of Newton's formula :

This difference between the experimentally observed value of velocity of sound in air and the calculated value was first explained by the French mathematician Laplace. He pointed out that during propagation of sound waves, each layer of the medium is alternately compressed and rarefied. During compression the layer is heated and during rarefaction the layer is cooled. On account of the fact that air is a bad conductor of heat as well as a bad radiator, heat cannot flow quickly from the heated to the cooled layers. Consequently, propagation of sound waves takes place under *adiabatic conditions.** Under adiabatic condition, it may be proved in the following way that $P.V^{\gamma}=$constant, where $\gamma=\dfrac{\text{sp. heat of the gas at constant pressure}}{\text{,, ,, ,, ,, ,, ,, constant volume}}$

If the pressure P increases by a small amount p, the volume V of a certain mass of gas diminishes. Let the volume diminish by a small amount v. In this case, $PV^{\gamma}=(P+p)\,(V-v)^{\gamma}=V^{\gamma}(P+p)\,\left(1-\dfrac{v}{V}\right)^{\gamma}$.

Expanding binomially, $PV^{\gamma}=V^{\gamma}(P+p)\left(1-\dfrac{\gamma v}{V}\right)$ [the higher powers of $\dfrac{v}{V}$ may be neglected]

or, $P=(P+p)\left(1-\dfrac{\gamma v}{V}\right)=P-\dfrac{\gamma.P.v}{V}+p-\dfrac{\gamma.p.v}{V}=P-\dfrac{\gamma.P.v}{V}+p$[$p.v$ is very small]

or, $p=\dfrac{\gamma.P.v}{V}$ or $\gamma P=p\Big/\dfrac{v}{V}$. So, the bulk elasticity of the gas$=\gamma.P$.

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*At audio-frequencies, the relatively long wavelength sound waves travel through air or gas under adiabatic conditions. But at ultrasonic frequencies, short wavelength sound waves travel under conditions which are more isothermal than adiabatic and the velocity is then $V=\sqrt{P/D}$.

[**Alternative calculus proof :** The above result can be easily obtained by the application of calculus. We have $P.V^{\gamma}=$constant. Differentiating, $dP.V^{\gamma}+P.\gamma.V^{\gamma-1}.\,dV=0$

or, $dP=-P.\gamma.\dfrac{dV}{V}$ or, $P.\gamma=-\dfrac{dP}{dV/V}=\dfrac{\text{volume stress}}{\text{,, strain}}=$Bulk modulus]

Using the above relation, Laplace showed that $V=\sqrt{\frac{\gamma P}{D}}$

For air, $\gamma=1{\cdot}41$; So, $V=\sqrt{\frac{1{\cdot}41\times P}{D}}$

Applying this corrected formula of Laplace, the velocity of sound in air at N.T.P. is $V=280\sqrt{1{\cdot}41}=332{\cdot}5$ metres/sec, a result which agrees closely with the observed value and therefore, justifies Laplace's assumption.

5.6. Dependence of velocity of sound on density, pressure, temperature and humidity of the gas :

(i) **Density :** Consider two gases having densities D_1 and D_2. Pressure and temperature of the gases are the same and they are P and T respectively. If the velocities of sound in the gases be V_1 and V_2 respectively then,

$$V_1=\sqrt{\frac{\gamma P}{D_1}} \text{ and } V_2=\sqrt{\frac{\gamma P}{D_2}} \quad \therefore \quad \frac{V_1}{V_2}=\sqrt{\frac{D_2}{D_1}}$$

*So, the velocity of sound in a gas is inversely proportional to the square root of the density of the gas.** Thus, the velocity of sound in oxygen is ¼*th* the velocity in hydrogen, because the density of oxygen is 16 times that of hydrogen.

(ii) **Pressure :** If P and D be the pressure and density of a gas respectively, then from Boyle's law, it can be proved that $P\propto D$, if the temperature of the gas remains unchanged, *i.e.* $\frac{P}{D}$=constant.

So, with the change of pressure, density will also change in such a way that the ratio P/D is always constant. Consequently *changes of pressure have no effect on the velocity of sound, provided the temperature remains unchanged.*

For example, if the velocity of sound in air at 0°C and 76 cm. pressure be 332 metres/sec., then whatever may be the pressure, the velocity of sound will remain 332 metres/sec, if the temperature is not changed.

(iii) **Temperature :** When the temperature of a gas changes, its density also changes. Consequently, the velocity of sound in the gas will change. Suppose, the densities of air at 0°C and t°C are D_0 and D_t respectively. Let the velocities of sound at those temperatures be V_0 and V_t respectively.

$$\text{Then, } V_0=\sqrt{\frac{\gamma P}{D_0}} \text{ and } V_t=\sqrt{\frac{\gamma P}{D_t}} \quad \therefore \quad \frac{V_t}{V_0}=\sqrt{\frac{D_0}{D_t}}$$

Now, from Charles' law, we know, $\frac{D_0}{D_t}=\frac{T}{T_0}$, where T and T_0 are the temperatures in the absolute scale corresponding to t°C and 0°C.

$$\therefore \quad \frac{V_t}{V_0}=\sqrt{\frac{T}{T_0}} \text{ i.e. } V\propto\sqrt{T}$$

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*The above result is valid when the gases have same value of γ. But γ depends on the number of atoms in the molecule of the gas. So in comparing the velocities of sound in different gases, we should take into account the value of γ.

Again, $T=(t+273)^\circ$ K and $T_0=273^\circ$K

$$\therefore \quad \frac{V_t}{V_0}=\sqrt{1+\frac{t}{273}}=\left(1+\frac{t}{273}\right)^{\frac{1}{2}}=1+\tfrac{1}{2}\times\frac{t}{273}=1+{\cdot}00183\times t.$$

Hence, $V_t=V_0(1+{\cdot}00183\times t)$; if $V_0=33200$ cm/sec.,

$V_t=33200(1+{\cdot}00183\times t)=33200+61.t.$

It means that the velocity of sound in air or in any other gas at 0°C increases or decreases by 61 cm. due to 1°C increase or decrease of temperature. In the F.P.S. system, $V_0=1120$ ft/sec., $V=1120$ $(1+{\cdot}00183\times t)=1120+2.t.$ This means that the velocity of sound in air or in any other gas at 0°C increases of decreases by 2 ft due to 1°C increase or decrease of temperature.

[**Alternative method :**

Suppose that one gm-molecule of a gas has mass M and volume v. Then its density $D=\frac{M}{v}$. So, the velocity of sound is $V=\sqrt{\frac{\gamma.P}{D}}=\sqrt{\frac{\gamma P.v}{M}}$.

But for a gm-molecule of a gas $P.v=R_0T$ where R_0=universal gas constant and T the temperature of the gas in absolute scale. $\therefore$ $V=\sqrt{\frac{\gamma.R_0T}{M}}$

(a) As γ and R_0 are constants, $V\propto\frac{1}{\sqrt{M}}\propto\frac{1}{\sqrt{D}}$ when temperature is kept constant.

This shows that the velocity of sound in a gas is inversely proportional to the square root of the density of the gas.

(b) If T, the temperature be kept constant, the above expression shows that the velocity of sound in a gas does not depend on the pressure of the gas because, γ, M and R_0 are constants for a given gas.

(c) As v, R_0 and M are constants for a given gas, the above expression shows that $V\propto\sqrt{T}$ *i.e.* the velocity of sound in a gas is proportional to the square root of its absolute temperature.]

(iv) **Humidity :** With the increase of humidity of air, the velocity of sound in it increases. The reason is that at the same temperature and pressure the density of water vapour is about $\frac{5}{8}$ times the density of dry air. So, due to less density of moist air, the velocity of sound in it increases.

If V_d=velocity of sound in dry air at t°C and P cm. of mercury pressure, V_m=velocity of sound in moist air at the above temp. and pressure, f=aqueous tension at t°C, then, $V_d=V_m\left(1-0{\cdot}189\frac{f}{P}\right)$.

Examples : (1) *If the velocity of sound in air at 0°C be 1090 ft/sec., what will be the velocity at 20°C ?*

Ans. We know, $V_t=V_0(1+{\cdot}00183\times t)$; here $V_0=1090$ ft/sec ; $t=20$°C

Hence $V_t=1090(1+{\cdot}00183\times 20)=1090(1+{\cdot}0366)$

$=1090\times 1{\cdot}0366=1129{\cdot}89$ ft/sec.

(2) *A person sets his watch by hearing the gun fire at a distant fort. He afterwards found that his watch had gone slow by 2 sec. If the temperature at that time was 15°C and the velocity of sound at 0°C be 332 metres/sec., what was the distance of the fort from the man ?*

Ans. The fact that the watch has gone slow by 2 seconds signifies that sound has taken 2 seconds to travel the distance between the fort and the person.

Now, we know, $V_t=V_0(1+\cdot 00183\times t)$ Here, $V_0=332$ metres/sec : $t=15°\text{C}$.

$\therefore\quad V_t=332(1+\cdot 00183\times 15)=332+332\times 15\times\cdot 00183=341\cdot 11$ metres/sec.

So, the distance travelled by sound in 2 sec$=2\times 341\cdot 11=682\cdot 22$ metres.

$\therefore$ the distance of the fort from the man$=682\cdot 22$ metres.

(3) *If the velocity of sound in air at 0°C and 76 cm. pressure is 330 metres/sec., find the velocity at 50°C and 70 cm. pressure.*

Ans. We know that change of pressure has no effect on the velocity of sound. Velocity will be affected only by the change of temperature. We also know, $V_t=V_0(1+\cdot 00183\times t)$ Here, $V_0=330$ metres/sec ; $t=50°\text{C}$;

So, $V_t=330(1+\cdot 00183\times 50)=330(1+\cdot 0915)=330\times 1\cdot 0915=360\cdot 195$ metres/sec.

(4) *A stone is dropped into a well 78·4 metres deep and the sound of splash was heard 4·23 second later. Find the velocity of sound in air. $g=980$ cm/sec^2.* [*H. S. Exam. 1966*]

Ans. Suppose the stone takes 't' sec. to reach the bottom of the well. We know, $S=\frac{1}{2}g.t^2$. Here, $S=78\cdot 4\times 100$ cm. ; $g=980$ cm/sec^2.

$$\text{Hence, } 78\cdot 4\times 100=\tfrac{1}{2}\times 980\times t^2 \text{ or } t^2=\frac{78\cdot 4\times 100\times 2}{980}=16 \quad \therefore \quad t=4 \text{ sec.}$$

So, sound takes $(4\cdot 23-4)=0\cdot 23$ sec to come from the bottom to the top *i.e.* to cover a distance of 78·4 metres.

$$\text{Hence, velocity of sound}=\frac{78\cdot 4}{0\cdot 23}=340\cdot 87 \text{ metres/sec.}$$

(5) *The atmosphere surrounding the planet Jupiter is a mixture of ammonia and methane and its temperature is −130°C. If γ for the mixture is 1·3 and the molecular weight of the mixture is 16·5, find the velocity of sound on Jupiter. Universal gas constant $R_0=8\cdot 3$ joules/°K/gm-wt.*

Ans. We know, $V=\sqrt{\dfrac{\gamma.R_0T}{M}}$ Here, $\gamma=1\cdot 3$; $R_0=8\cdot 3$ joules$=8\cdot 3\times 10^7$ ergs;

$T=-130+273=143°\text{K}$; $M=16\cdot 5$

$$\therefore \quad V=\sqrt{\frac{1\cdot 3\times 8\cdot 3\times 10^7\times 143}{16\cdot 5}}=30580 \text{ cm/s}=305\cdot 8 \text{ m/s}.$$

(6) *Velocity of sound in hydrogen at 0°C is 1200 metres/s. When some amount of oxygen is mixed with hydrogen, the velocity decreases to 500 metres/s. Determine the ratio of H_2 and O_2 by volume in this mixture, given density of O_2 is 16 times that of H_2.* [*Jt. Entrance 1982*]

Ans. Let the density of hydrogen be ρ_h and that of the mixture ρ_m. If the mixture is made by taking v_1 volume of hydrogen and v_2 volume of oxygen, then, $\rho_m = \dfrac{\rho_h v_1 + 16\rho_h . v_2}{v_1 + v_2} = \dfrac{\rho_h(v_1 + 16.v_2)}{v_1 + v_2}$

Now, velocity of sound in hydrogen, $V_h = \sqrt{\dfrac{\gamma . P}{\rho_h}}$ and the velocity in the mixture, $V_m = \sqrt{\dfrac{\gamma . P}{\rho_m}} = \sqrt{\dfrac{\gamma . P(v_1 + v_2)}{\rho_h(v_1 + 16v_2)}}$

$$\therefore \quad \frac{V_m}{V_h} = \sqrt{\frac{\gamma P(v_1 + v_2)}{\rho_h(v + 16.v_2)} \times \frac{\rho_h}{\gamma . P}} = \sqrt{\frac{v_1 + v_2}{v_1 + 16.v_2}}$$

According to the problem, $\dfrac{V_m}{V_h} = \dfrac{500}{1200}$

$$\therefore \quad \frac{500}{1200} = \sqrt{\frac{v_1 + v_2}{v_1 + 16.v_2}} \quad \text{or, } \frac{25}{144} = \frac{v_1 + v_2}{v_1 + 16v_2} \quad \text{or, } 256.v_2 = 119.v_1$$

$$\therefore \quad \frac{v_1}{v_2} = \frac{256}{119}.$$

(7) *At what temperature sound travels in carbon dioxide gas with same velocity as in oxygen at 7°C, given that the molecular weights of the gases are 44 and 32 respectively.* γ *for* CO_2 *and* O_2 *are 1·33 and 1·4 respectively.*

Ans. We know $V = \sqrt{\dfrac{\gamma . R_0 T}{M}}$; For CO_2 we have, $V_{CO_2} = \sqrt{\dfrac{1{\cdot}33 . R_0 T}{44}}$

For O_2 we have $V_{O_2} = \sqrt{\dfrac{1{\cdot}4 . R_0(273 + 7)}{32}} = \sqrt{\dfrac{1{\cdot}4 \; R_0 \times 280}{32}}$

Since $V_{CO_2} = V_{O_2} \quad \therefore \quad \dfrac{1{\cdot}33T}{44} = \dfrac{1{\cdot}4 \times 280}{32} \quad \therefore \quad T = \dfrac{1{\cdot}4}{1{\cdot}33} \times \dfrac{44}{32} \times 280 = 405°\text{K}$

$\therefore$ Reqd. temperature in Celsius scale $= 405 - 273 = 132°\text{C}$.

5.7. Determination of velocity of sound by resonant air column :

Resonant air column is conveniently used for determining the velocity of sound in air at room temperature. It consists of a brass tube *A* about one and half a metre long having a diameter of about 4 cm. It is about ⅔*rd.* full of water in which is inserted a narrow glass tube *B* whose both ends are open. The tube *B* can be moved up and down by opening the key *K* (Fig. 5.2). The water surface acts as an unyielding surface and the whole arrangement becomes a closed tube of varying length. If a vibrating fork be held at the open mouth of the tube *B* and the length of the air column of the tube *B* be changed by moving the tube up and down then at a certain minimum

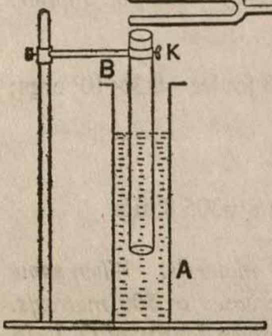

Fig. 5.2

length of the air column, resonance will occur and a loud sound will be heard. In the circumstances, the frequency of the tuning fork becomes equal to the frequency of vibration of the air column. If the resonance occurs at the minimum length of the air column, fundamental tone is produced for which we know $V=4l.n.$ where V is the velocity of sound in air, l is the length of the resonant air column and n is its frequency.

So, knowing the frequency of the tuning fork and measuring the length of the resonant air column, the velocity of sound may be found out.

End correction : When resonance takes place in the air column of the tube *B*, we assume that an antinode is formed at the top of the tube (art. 4.28). But Lord Rayleigh showed that the antinode at the top does not coincide exactly with the top of the tube, but projects slightly above it. Suppose, the antinode is formed at a distance x from the open end of the tube. The distance x is known as the *end correction.* If r be the radius of the tube, then $x=0{\cdot}6r$. So, if l be length of the tube, the distance between the node and the antinode, in this case, is $(l+x)=l+0{\cdot}6r$. Hence, $V=4(l+0{\cdot}6r)n$.

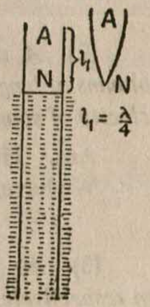

Fig. 5.3

Velocity of sound in air can, however, be determined by the resonant air column, avoiding the end correction. We have seen that fundamental tone produces resonance when the length of the air column is minimum. If the tube *B* is slowly raised and the length of the air column made three times than before, a second resonance will be produced. This shows that the air column is now emitting the second harmonic.

Suppose the wave length of the sound emitted by the tuning fork is λ ; If l_1 is the length of the air column in the first resonance, $\frac{4}{\lambda}=l_1+x$ [Fig. 5.3]. Similarly, for the second resonance, if the length be l_2 [Fig. 5.4], $\frac{3\lambda}{4}=l_2+x$.

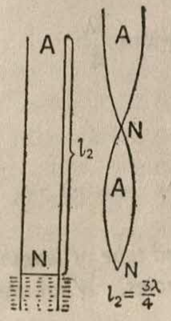

Fig. 5.4

Subtracting the first equation from the second, we get,

$\frac{\lambda}{2}=l_2-l_1$ or $\lambda=2(l_2-l_1)$ $\therefore$ $V=n\lambda=2n(l_2-l_1)$. This equation is without the end-correction.

Determination of end-correction :

For the first resonance, we have seen $\frac{\lambda}{4}=l_1+x$ and for the second resonance

$$3\lambda/4=l_2+x \quad \therefore \; 4(l_1+x)=\frac{4}{3}(l_2+x) \quad \text{or} \quad x=\frac{l_2-3l_1}{2}$$

So, knowing l_1 and l_2, the end-correction can be found out without measuring the diameter of the tube.

Examples : (1) *When a tuning fork of frequency 512 is held over a closed pipe, it is found that resonance occurs with the minimum length of 6·65 inches of air column. Calculate the velocity of sound in air.*

Ans. Here $l=6{\cdot}65$ inches ; $n=512$. Fundamental is being produced because resonance occurs with the minimum length.

So, $V=4n.l.=4\times512\times6{\cdot}65$ inches/sec. $=\dfrac{4\times512\times6{\cdot}65}{12}$ ft/sec $=1135$ ft/sec.

(2) *A tuning fork when held over a closed pipe gave resonance with air columns of lengths 27 cm and 82 cm. If the frequency of the fork be 300, find the velocity of sound in air.*

Ans. Here, $l_1=27$ cm ; $l_2=82$ cm. and $n=300$
We have, $V=2n(l_2-l_1)=2\times300\times(82-27)$ cm./sec.
$=2\times3\times55$ metres/sec $=330$ metres/sec.

(3) *An air resonance tube resounds with a tuning fork at 16·2 cm and 50·4 cm respectively. Calculate the end correction involved.*

Ans. Let the correction be x cm. So, in the first case, $16{\cdot}2+x=\dfrac{\lambda}{4}$ [λ=wavelength of the wave produced]. In the second case, $50{\cdot}4+x=\dfrac{3\lambda}{4}$

or, $\dfrac{50{\cdot}4+x}{3}=\dfrac{\lambda}{4}$

$\therefore\ 16{\cdot}2+x=\dfrac{50{\cdot}4+x}{3}$ or, $48{\cdot}6+3x=50{\cdot}4+x$ or, $2x=1{\cdot}8$ $\therefore$ $x=0{\cdot}9$ cm.

(4) ***The diameters of an open tube of length 30 cm. and a closed tube of length 25 cm are equal. They are producing first harmonics of equal frequencies. What are the end-corrections of the tubes ?***

Ans. Let the frequency of the first harmonic be n. For an open tube, $n=\dfrac{V}{(l_1+2x)}$ and for a closed tube $n=\dfrac{3V}{4(l_2+x)}$. [It is to be noted that end-correction appears at both the ends of an open tube.]

$$\therefore\ \frac{V}{l_1+2x}=\frac{3V}{4(l_2+x)} \quad \text{or,} \quad \frac{1}{30+2x}=\frac{3}{4(25+x)}$$

or, $90+6x=100+4x$ or $x=5$ cm.

(5) *A tube of certain diameter and of length 48 cm is open at both ends. Its fundamental frequency of resonance is found to be 320 Hz. The velocity of sound is 320 m/s. Estimate the diameter of the tube. One end of the tube is now closed. Calculate the lowest frequency of resonance for the tube.* [*I.I.T. 1980*]

Ans. If x be the end-correction, then for the fundamental of open tube $n_0=\frac{V}{2(l+2x)}$ $\therefore$ $320=\frac{320}{2(0{\cdot}48+2x)}$ or $0{\cdot}48+2x=0{\cdot}5$ $\therefore$ $x={\cdot}01\text{m}=1$ cm.

But $x=0{\cdot}3.d$ where d is the diameter. $\therefore$ $d=3\frac{1}{3}$ cm.

For a closed tube, $n_0=\frac{V}{4(l+x)}=\frac{320}{4(0{\cdot}48+{\cdot}01)}=163{\cdot}3$ (nearly)

(6) *Two organ pipes, one open at both ends and the other at one end only, give their fundamental notes whose frequencies differ by 25 vibrations. The length of the open pipe is 166 cm. Calculate the length of the closed pipe if the velocity of sound in air is 332 metres per sec.*

Ans. Let the frequency and the length of the open pipe be n_1 and l_1 and the corresponding values for the closed pipe be n_2 and l_2.

According to the question, $n_1-n_2=25$ or $n_2-n_1=25$.

Now, for an open pipe, we know, $n_1=\frac{V}{2l_1}=\frac{33200}{2\times166}=100$ and for a closed pipe, $l_2=\frac{V}{4n_2}=\frac{33200}{4(n_1\pm25)}=\frac{33200}{4(100\pm25)}=66{\cdot}4$ or $110{\cdot}7$ cm.

5.8. Velocity of sound in solid and liquid :

Velocity of sound in different media is different. As a general rule, sound travels faster in solids and liquids than it does in gases.

Velocity of sound in solid : The speed of sound in solids was first determined by Biot. He took several hollow iron pipes and joined them so as to obtain one hollow tube about 1000 metres long. He placed his ear against one end of the tube while a person gave a single tap with a stone at the other end. He heard two sounds, one coming through the iron, followed by another through the air of the hollow tube. From the time interval, Biot found that sound travels about 15 times faster through iron than through air.

Let the length of the pipe be l. If t be the interval of time between two sounds heard at the other end of the pipe, then, $t=\frac{l}{V}-\frac{l}{V_1}$ where V=velocity of sound in air and V_1=that in the solid pipe. $\therefore$ $\frac{l}{V_1}=\frac{l}{V}-t=\frac{l-Vt}{V}$ $\therefore$ $V_1=\frac{l.V}{l-V.t}$.

Knowing the velocity of sound in air, V_1 can be found out.

Velocity of sound in liquid : In 1827, Colladon and Strum measured the velocity of sound in water. The experiment was carried out on Lake Geneva in Switzerland, using two boats which were situated about 9 miles apart. A bell was immersed in water from a boat and an arrangement was made in such a way that as soon as a lever struck the bell, some gun-powder was set into fire producing a flash. The second boat recorded the time as soon as the flash was produced and also the time when the sound travelling through water reached an eartrumpet in the second boat. So knowing the distance and the time, the velocity of sound in water was found out. They found that the velocity of sound in water at about 8·1°C is nearly 1435 metres/sec.

Examples : (1) *A ship sent a signal to another ship at a distance. The signal was transmitted through water as well as through air. The second ship received two signals at an interval of 5 seconds. Find the distance between the ships. The velocities of sound in air and sea-water are 340 metres/sec and 1435 metres/sec respectively.*

Ans. Let the distance between the ships be x metres. If sound takes t_1 sec to travel the distance x metres in air, then, $t_1=\frac{x}{340}$ sec. Again if it takes t_3 sec to travel the distance in sea-water, then $t_2=\frac{x}{1435}$ sec. According to the question,

$$t_1-t_2=5 \text{ sec.} \quad \therefore \quad \frac{x}{340}-\frac{x}{1435}=5$$

$$\text{or, } x=\frac{5\times 340\times 1435}{1095} \text{ metres}=2{\cdot}2 \text{ kilometres (nearly).}$$

(2) *One end of an iron pipe, 1600 ft long is struck a blow. A listener at the other end hears two sounds at an interval of 1·4 sec. If the velocity of sound in air is 1087 ft/sec, find the velocity of sound in iron.*

Ans. We know, the velocity of sound in iron is given by $V_1=\frac{l.V}{l-V.t.}$

Here, $l=1600$ ft ; $V=1087$ ft/s and $t=1{\cdot}4$ sec.

$$\therefore \quad V_1=\frac{1600\times 1087}{1600-1087\times 1{\cdot}4}=\frac{1600\times 1087}{78{\cdot}2}=22{,}240 \text{ ft/s (nearly)}$$

5.9. Doppler Effect :

Nearly everyone has at some time, perhaps without realising it, observed the *Doppler effect.* The sounding horn of a car passing at high speed on the streets exhibits the phenomenon. The pitch of the horn, as the car goes by drops appreciably. A similar observation can be made by listening to the whistle of a fast moving train approaching a station. The pitch of the whistle sounds higher to a person standing on the platform as the train approaches him than it does after the train has passed by. This change in pitch is due to relative motions of the source of sound and the observer and is known as the **Doppler effect.** To see how this effect is produced, consider the following example. When the whistle on a train at rest is blown, it sends out waves travelling with the same velocity in all directions. To all stationary observers, no matter in what direction they are situated, the true pitch of the whistle is heard, since just as many waves arrive at the ear per second as there are waves leaving the whistle.

If on the other hand, the train is moving as shown in fig. 5.5, the whistle is moving away from the waves travelling to the rear and towards the waves travelling forward. The result is that the waves behind are considerably drawn out while those in front are crowded together. With each new wave sent out by the whistle, the train is farther from the preceding wave sent out to the rear and nearer to the one sent out ahead. Since the velocity of sound is the same in all directions an observer at O_1 therefore hears more waves per second and an observer at O_2 hears fewer.

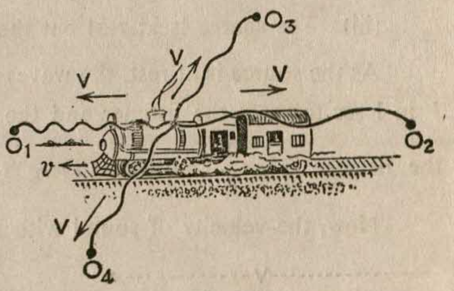

Fig. 5.5

To an observer at O_3 or at O_4 at right angles and at some distance from the moving source, the pitch remains unchanged. For these side positions, the source is neither approaching nor receding from the observer ; so nearly the same number of waves are received per second as there are waves leaving the source.

Calculation of apparent frequency :

Suppose, V=velocity of sound ; V_s=velocity of source of sound ; V_0=velocity of listener and n=actual frequency of sound.

(i) **Listener at rest but the source is moving towards the listener :**

Had the source been at rest, then n number of waves produced per sec. by the source would have travelled a distance V in 1 sec. towards the listener and their wavelength $\lambda=\frac{V}{n}$. But as the source is moving with a velocity V_s, the waves will occupy a distance $(V-V_s)$ because the source itself will move towards the listener a distance V_s in 1 second. [Fig. 5.6(*a*)]. Hence, the wavelength of the waves reaching the listener is $\lambda'=\frac{V-V_s}{n}$. But the velocity of sound$=V$.

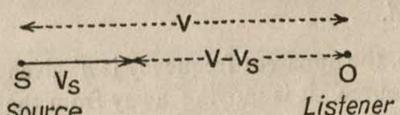

Fig. 5.6(*a*)

Hence, the apparent frequency $n'=\frac{V}{\lambda'}=\frac{V}{(V-V_s)/n}=\frac{V.n}{V-V_s}$ .. (i)

Since $(V-V_s)<V$, $n'>n$. So, *when the source moves towards the listener, the apparent frequency of the source is higher than the actual frequency.*

(ii) **Listener at rest but the source is moving away from the listener :**

In this case, the waves coming out of source will occupy a distance $(V+V_s)$ in 1 sec. Hence, the wavelength of the waves reaching the listener $\lambda'=\frac{V+V_s}{n}$ and the apparent frequency $n''=\frac{V}{\lambda'}=\frac{V.n}{V+V_s}$ .. (ii)

As $(V+V_s)>V$, $n''<n$ *i.e., the apparent frequency is lower than the actual frequency of the sound when the source moves away from the listener.*

(iii) **The source is at rest but the listener is moving towards the source :**

As the source is at rest, the waves sent out by the source will occupy a length V in 1 sec towards the listener and the wavelength of the sound waves reaching the listener is $\lambda=\frac{V}{n}$; note that *there is no change in the wavelength of the waves.*

Now, the velocity of sound with respect to the listener in this case, will not be V because the listener itself is in motion with respect to the source. The relative velocity of sound, here is $(V+V_0)$. Hence, the apparent frequency is given by $n'=\frac{V+V_0}{\lambda}=\frac{V+V_0}{V/n}=\frac{(V+V_0).n}{V}$ .. (iii)

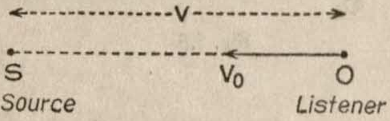

Fig. 5.6(*b*)

As $(V+V_0)>V$, $n'>n$ *i.e. when the listener moves towards the source, the apparent pitch of the sound becomes higher.*

(iv) **The source is at rest but the listener is moving away from the source :**

Since the source is at rest, here also, the wave length of the sound reaching the listener is $\lambda=\frac{V}{n}$. But as the relative velocity of sound with respect to the listener is $(V-V_0)$, the apparent frequency is $n''=\frac{V-V_0}{\lambda}=\frac{(V-V_0).n}{V}$ .. (iv)

As $(V-V_0)<V$, $n''<n$ *i.e. the apparent pitch of the sound is lowered when the listener moves away from the source of sound.*

If a moving observer approaches a source, the apparent frequency is n' given by eqn. (iii). When the observer crosses the source, it is moving away from the source and therefore the apparent frequency becomes n'' given by eqn. (iv). The change in frequency as an observer crosses a stationary source is $\Delta n=$

$$n'-n''=\frac{(V+V_0)n}{V}-\frac{(V-V_0)n}{V}. \quad =\frac{2V_0}{V}.n \quad .. \quad \text{(v)}$$

Similarly when a source approaches a stationary listener and crosses him, the change of frequency is given by [eqns. (i) and (ii)]

$$\Delta n=n'-n''=\frac{V.n}{V-V_s}-\frac{V.n}{V+V_s}=\frac{2V.V_s}{V^2-V_s^2}.n. \quad .. \quad \text{(vi)}$$

[Note : It is worthwhile to note that when the source is in motion ; the wavelength of sound waves is altered but the velocity of sound remains unaltered. Again when the listener is in motion, the wavelength remains unaltered but the velocity of sound is altered.]

5.10. General expression for Doppler effect :

The general expression for the apparent frequency of a source, caused by the various relative motions between the source and the listener—the source moving towards or away from the listener or the listener moving towards or away from the source as well as by the wind blowing along or against the direction of the sound, may be established in the following way :

Suppose V=the velocity of sound ; V_s=the velocity of source ; V_0=the velocity of the listener; ω=velocity of the wind and n=the real frequency of the source [Fig. 5.7].

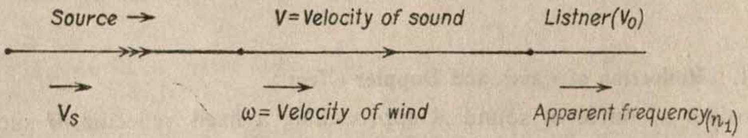

Fig. 5.7

Had there been no wind and the listener as well as the source were at rest, the frequency heard would have been the same as the real frequency and in that case $n=\frac{V}{\lambda}$.

Now, considering the motion of the wind along the direction of the sound, the effective velocity of sound=$V+\omega$ and in that case, the wavelength $\lambda=\frac{V+\omega}{n}$.

Again, if the source of sound moves with a velocity V_s, then the number of waves (n) that will be generated in 1 sec, will cover a distance $(V+\omega)-V_s$ because in 1 sec, the source itself will advance a distance V_s. As a result, the wavelength will change. If λ' be the changed wavelength, then, $\lambda'=\frac{(V+\omega)-V_s}{n}$. (i)

Now, consider the motion of the listener. As the listener moves away with a velocity V_0, the relative velocity of sound through the medium with respect to the listener=$(V+\omega)-V_0$. If n_1 be the apparent frequency of the sound heard by the listener, $n_1\lambda'=(V+\omega)-V_0$.

$$\therefore \quad n_1=\frac{(V+\omega)-V_0}{\lambda'}=\frac{(V+\omega)-V_0}{(V+\omega)-V_s}.n \quad \text{[From eqn. (i)]}$$

Similarly when the listener moves towards the source, the relative velocity of sound wave with respect to the listener=$(V+\omega)+V_0$.

$$\therefore \quad n_2=\frac{(V+\omega)+V_0}{\lambda'}=\frac{(V+\omega)+V_0}{(V+\omega)-V_s}.n \quad \text{[from eqn. (i)]}$$

Special cases :

(i) If there be no wind, $\omega=0$, so $n_1=\frac{(V-V_0)}{V-V_s}.n$ and $n_2=\frac{(V+V_0)}{V-V_s}.n$

(ii) If the listener is at rest and there is no wind, $\omega=0$, $V_0=0$; in that case $n_1=n_2=\dfrac{V}{V-V_s}.n.$

(iii) If the source is at rest and there is no wind, $\omega=0$, $V_s=0$; in that case, $n_1=\dfrac{(V-V_0)}{V}.n$; $n_1<n$ and $n_2=\dfrac{(V+V_0)}{V}.n$; $n_2>n$.

(iv) If the listener and the source both are at rest and there is no wind, $\omega=0$, $V_s=V_0=0$; in that case, $n_1=n_2=\dfrac{V}{V}.n=n$ *i.e.* the frequency remains unchanged.

5.11. Reflection of waves and Doppler effect :

Consider a source of sound A approaching a fixed reflector M such as a wall or a hill, for example. The reflected waves appear to travel towards A from M as if they came from the 'mirror image' A_1 of A in M, like light [Fig. 5.8].

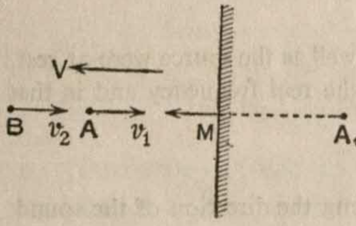

Fig. 5.8

Suppose A is a car approaching M with a velocity v_1 when sounding a note of frequency n from its horn and that another car B behind A is travelling towards reflector with a speed v_2.

The driver in car B, hears a note from M, which has an apparent pitch $n'=\dfrac{V'}{\lambda'}$ where V' is the velocity of sound relative to B and λ' is the wave length of the waves reaching B. If V be the velocity of sound then $V'=V+v_2$ and $\lambda'=\dfrac{(V-v_1)}{n}$ because the waves occupy a length $(V-v_1)$ in 1 sec due to the motion of the source A. $\therefore$ $n'=\dfrac{V'}{\lambda'}=\dfrac{(V+v_2)}{(V-v_1)}.n.$ .. (i)

The driver in the car B also hears a note directly from A. In this case, $V'=V+v_2$ and $\lambda'=\dfrac{(V+v_1)}{n}$ because the waves now occupy a length $(V+v_1)$ in 1 sec. due to the receding motion of A. $\therefore$ $n''=\dfrac{V'}{\lambda'}=\dfrac{(V+v_2)}{(V+v_1)}.n$ .. (ii)

If these two frequencies are near to each other, the driver in the car B will hear beats.

Examples : (1) *A train whistling a sound of frequency 1000 per second is approaching you with a speed 118·8 km/hour. You are also approaching the engine with the same speed in a car. What will be the apparent frequency of the whistle heard by you ? Velocity of sound in air=333 metres/sec.*

Ans. Velocity of the source or the listener$=\frac{118{\cdot}8\times1000}{60\times60}=33$ metres/sec.

We know, $n_1=\frac{V-V_0}{V-V_s}.n$. Here, $V=333$ metres/sec ; $V_0=-33$ metres/sec ; $V_s=+33$ metres/sec ; $n=1000$. So, $n_1=\frac{333+33}{333-33}\times1000=\frac{366}{300}\times1000=1220$.

(2) *Calculate the velocity at which an observer should approach a source of frequency 1000 per sec. at rest, in order to produce a Doppler shift of 40 per second. Velocity of sound in air=1120 ft/sec.*

Ans. We know, $n_1=\frac{V-V_0}{V-V_s}.n$. Here, $n_1=1000+40=1040$; $V=1120$ ft/sec; $V_s=0$; $V_0=-V_0$ (approaching). So, $1040=\frac{1120+V_0}{1120}\times1000$

$$\text{or, } \frac{1040}{1000}=1+\frac{V_0}{1120} \text{ or, } \frac{40}{1000}=\frac{V_0}{1120} \quad \therefore \quad V_0=\frac{1120\times40}{1000}=44{\cdot}8 \text{ ft/sec.}$$

(3) *A railway engine while whistling moves past an observer with a speed of 48 km/hour. Find the ratio and difference of pitch when (i) the engine is approaching and (ii) then receding from the stationary observer. Velocity of sound=300 m/s. Frequency of the whistle=1000 Hz.*

Ans. When the source is approaching the stationary observer, the apparent pitch $n'=\frac{V.n}{V-V_s}$ [eqn. (i) of art. 5.9] when the source is receding from the stationary observer the apparent pitch $n''=\frac{V.n}{V+V_s}$ [eqn. (ii)]

$$\therefore \text{ Ratio}=\frac{n'}{n''}=\frac{V+V_s}{V-V_s}=\frac{330+13{\cdot}33}{330-13{\cdot}33}=\frac{343{\cdot}33}{316{\cdot}67}=1{\cdot}08 \text{ [48 km/hr=13·33 m/s]}$$

Again, difference of pitch $\Delta n=\frac{2V.V_s}{V^2-V_s^2}\times n$ [eqn. (vi)]

$$=\frac{2\times330\times13{\cdot}33}{(330)^2-(13{\cdot}33)^2}\times1000=81 \text{ (nearly)}$$

(4) *The two horns of a motor car are not of equal frequency ; the frequency difference is 269 per second. The car is approaching a stationary observer at a speed of 30 miles/hr constantly blowing its horns. How will the listener appear to hear the frequency difference ? Velocity of sound=1120 ft/s.*

Ans. When the listener is stationary and the source of sound approaches the listener, the apparent frequency $n'=\frac{V.n}{V-V_s}$ [eqn. (i) of art. 5.9]. $\therefore$

Let the frequency of one horn be n. The pitch of other horn is $n+269$. Velocity of the car $V_s=30$ miles/hr$=44$ ft/s ; Velocity of sound $V=1120$ ft./s.

If n_1 and n_2 be the apparent frequencies, $n_1=\frac{1120\times n}{1120-44}=\frac{1120\times n}{.1076}$

and $n_2=\frac{1120(n+269)}{1120-44}=\frac{1120(n+269)}{1076}$

$\therefore$ The difference of frequencies$=n_2-n_1=\frac{1120}{1076}(n+269-n)=\frac{1120\times 269}{1076}=280.$

(5) *A car is moving towards a wall at a speed of 20 metres/s blowing its horn of frequency 500. Another car is following it along the same line with a speed of 30 metres/s. What frequencies of sounds will the driver of the second car hear ? Velocity of sound in air=330 metres/s.*

Ans. The driver of the second car will hear two sounds—one due to reflection of sound by the wall and the other straight from the first car.

As regards the reflected sound, the apparent pitch is given by $n'=\frac{(V+v_2)n}{V-v_1}$

[eqn. (i) of art. 5.11] where v_1 and v_2 are the velocities of first and second car respectively. $\therefore\ n'=\frac{(330+30)}{330-20}\times 500=\frac{360\times 500}{310}=580{\cdot}6$

As regards the direct sound, the apparent pitch is given by $n''=\frac{(V+v_2)}{(V+v_1)}.n$

[eqn. (ii) of art. 5.11] $\therefore\ n''=\frac{(330+30)}{330+20}\times 500=\frac{360\times 500}{350}=514{\cdot}3.$

(6) *A whistle, producing a note of frequency 300 is moving towards a hill with a speed of 150 cm/s away from a stationary listener. How many beats per second will the listener hear ? Velocity of sound=335 metres/s.*

Ans. In this case, the listener will hear two sounds—one reflected by the hill and the other direct from the whistle. If the frequencies of the sounds are near to each other, beats will be heard. For the reflected sound, the apparent pitch $n'=\frac{(V+v_2)}{V-v_1}\times n$ where v_1 and v_2 are the velocities of the whistle and the listener.

In the present case, $v_1=150$ cm/s$=1{\cdot}5$ m/s and $v_2=0$.

$$\therefore\ n'=\frac{(335+0)\times 300}{335-1{\cdot}5}=\frac{335\times 300}{333{\cdot}5}=301{\cdot}3$$

For the direct sound, the apparent pitch, $n''=\frac{(V+v_2)}{V+v_1}\times n$

$$=\frac{335\times 300}{335+1{\cdot}5}=\frac{335\times 300}{336{\cdot}5}=298{\cdot}7$$

$\therefore$ No. of beats heard per sec$=n'-n''=301{\cdot}3-298{\cdot}7=2{\cdot}6\simeq 3$ (nearly)

(7) *A vibrating tuning fork, tied to the end of a string 6 ft. long is whirled round in a circle. It makes 120 revolutions per minute. Calculate the difference*

of the frequencies between the highest and the lowest note heard by an observer situated in the plane of rotation. Velocity of sound=1100 ft/s.

Ans. Angular velocity of the fork $\omega=2\pi\times\frac{120}{60}=4\pi$ rad/s ; radius of the circle r=length of the string=6 ft. So, linear velocity of the fork $v=4\pi\times6=24\pi$ ft/s. The highest frequency as noted by the person at rest, is given by, $n_1=\frac{V}{V-v}\times n$ and the lowest frequency is given by $n_2=\frac{V}{V+v}.\times n$

[n=actual frequency of the fork]

$$\therefore \quad n_1-n_2=\left[\frac{1}{V-v}-\frac{1}{V+v}\right]\times V.n=\frac{2v.V.n}{(V+v)(V-v)}=\frac{2\times24\pi\times1100\times n}{(1100+24\pi)(1100-24\pi)}$$

$$=0{\cdot}14\times n \text{ [nearly]}$$

5.12. When the listener is at rest and the source is approaching him, the frequency of note heard is greater than when the source is at rest and the listener is approaching it with the same velocity :

When the source approaches the stationary listener with a velocity V_s, the frequency of the note heard $n_1=\frac{V}{V-V_s}$ n. [ref. art. 5.9(ii)]. Again when the listener approaches the stationary source with a velocity V_0, the frequency of the note heard, $n_2=\frac{V-(-V_0)}{V}n=\frac{V+V_0}{V}.n$. [ref. art 5.9(iii)]

If, now, $V_0=V_s$, then $n_2=\frac{V+V_s}{V}.n$

$$\therefore \quad n_1-n_2=\left[\frac{V}{V-V_s}-\frac{V+V_s}{V}\right].\ n=\left[\frac{V_s{}^2}{V(V-V_s)}\right].\ n=\text{a}+ve \text{ quantity.}$$

Hence, $n_1>n_2$ *i.e.* when the listener is at rest and the source is approaching him, the frequency n_1 of the note heard is greater than when the source is at rest and the listener is approaching it with same velocity.

5.13. Application of Doppler's effect in light waves :

Doppler effect is applicable to all types of waves. Light, being a wavy motion like sound, will exhibit this phenomenon. But the velocity of light being enormous, the change of frequency or wavelength of light waves due to the motion of the source or of the listener is, in general, very difficult to detect. The change becomes perceptible only when the velocity of the source is very great.

Light emitted by a source produces spectral lines of definite wavelength and hence of definite colour. When light coming from a distant star is examined spectroscopically along with the same spectral lines produced by arc light, it is found that the spectral lines of the star are moving slowly either towards the red end or towards the violet end of the spectrum. A slow motion of the spectral line towards the red-end indicates that the apparent wavelength of the light emitted by the star is increasing *i.e.* the star is receding away from the earth. On the other hand, a slow motion of the spectral lines towards the violet end indicates that the apparent wavelength is decreasing *i.e.*, the star is approaching the earth. Measuring the change of wavelength and applying Doppler's formula, the velocity with which the star approaches or recedes may be found out.

Scientists have so far collected valuable information about the binaries, the rotation of the sun about its own axis, the ring of the planet saturn etc., by the application of Doppler's effect. Recently Doppler effect in light has been applied in determining the temperature of very hot gas (plasma) to the order of millions of degree celsius.

Change of wavelength of light due to Doppler effect :

Suppose a star or a planet is moving with a velocity v *away* from the earth and emits light of wavelength λ. If the frequency of vibration of light be n, then n waves are emitted in one second where $n=\frac{C}{\lambda}$ [C=velocity of light in vacuo].

Owing to the velocity v of the source (*i.e.* the star) n waves occupy a distance $(C+v)$. Thus the apparent wavelength λ' to an observer on the earth in line with the star's motion is, $\lambda'=\frac{C+v}{n}=\frac{(C+v)}{C}.\ \lambda=\left(1+\frac{v}{C}\right).\ \lambda$

$$\therefore\ \lambda'-\lambda=\text{change in wavelength}=\frac{v.\lambda}{C}\quad ..\quad \text{(i)}$$

From the above relation, it follows that $\lambda'>\lambda$ *i.e.* there is a shift or displacement of the spectral line *towards the red* when the star or the planet is moving away from the earth. The position of a particular wavelength in the spectrum of the star is compared with that obtained in the laboratory and the difference in the wavelengths $(\lambda'-\lambda)$ is measured. From the above equation, knowing λ and C, the velocity v of the star can be determined.

If the star is moving *towards* the earth with a velocity u, the apparent wavelength λ'' is given by, $\lambda''=\frac{C-u}{n}=\frac{(C-u)}{C}\lambda=\left(1-\frac{u}{C}\right).\lambda\quad \therefore\ \lambda''-\lambda=-\frac{u}{C}.\lambda$

Since $\lambda''<\lambda$, there is a displacement of the spectral line *towards the violet* in this case.

Examples : (1) *A spectroscopic examination of light from a certain star shows that the apparent wavelength of a certain spectral line is 5001 A°, whereas the observed wavelength of the same line produced by a terrestrial source is 5000 A°. In what direction and at what speed do these figures suggest that the star is moving relative to the earth* ?

Ans. As the apparent wavelength is greater than the real wavelength, it suggests the star is moving away from the earth.

As the receiver *i.e.* the earth is at rest, the apparent frequency n_1 of the light is given by $n_1=\frac{C}{C-V_s}.n$, where C is the velocity of light wave [ref. art. 5.9(ii)].

We have, therefore, $\frac{C}{\lambda_1}=\frac{C}{C-V_s}\cdot\frac{C}{\lambda}$; where λ_1 is the apparent wavelength.

Simplifying we get, $\lambda-\lambda_1=\frac{\lambda}{C}.V_s$ or $\Delta\lambda=\frac{\lambda}{C}.V_s$

In the problem, $\Delta\lambda=-1\text{A}^\circ=-10^{-8}$ cm. $\lambda=5000\times10^{-8}$ cm. ; $C=3\times10^{10}$ cm/sec.

Hence $V_s=-\frac{3\times10^{10}\times10^{-8}}{5000\times10^{-8}}=-6\times10^6$ cm/ sec. The minus sign suggests that the star is moving away.

(2) *A policeman on duty at a road-crossing challenged a motor driver for crossing the road inspite of red signal. The driver said that due to Doppler effect he saw green signal* ($\lambda=5{\cdot}4\times10^{-7}$ *metre*) *and not red signal* ($\lambda=6{\cdot}2\times10^{-7}$ *metre*). *If the driver was correct, what was the speed with which he was driving the car* ?

Ans. When the source of light is stationary and the observer is moving towards it, the apparent frequency n_1 of the light seen by the observer is given by

$n_1=\frac{C+V_0}{C}n$ [art. 5.9(iii)] where C=velocity of light ; V_0=velocity of the observer ; n=real frequency.

If the apparent wavelength of light be λ_1 and the actual wavelength λ, then

$$\frac{C}{\lambda_1}=\frac{C+V_0}{C}\cdot\frac{C}{\lambda} \quad \therefore \quad \frac{C(\lambda-\lambda_1)}{\lambda_1.\lambda}=\frac{V_0}{\lambda}$$

Now, $\lambda_1.\lambda=\lambda^2$ (nearly) ; hence $\lambda-\lambda_1=\lambda.\frac{V_0}{C}$

Here, λ=wavelength of red light$=6{\cdot}2\times10^{-7}$ metre ; λ_1=wavelength of green light$=5{\cdot}4\times10^{-7}$ metre ; $C=3\times10^8$ metre/s.

$$\therefore\quad (6{\cdot}2-5{\cdot}4)\times10^{-7}=\frac{V_0\times6{\cdot}2\times10^{-7}}{3\times10^8} \quad \text{or,}\quad 0{\cdot}8\times3\times10^8=6{\cdot}2\times V_0$$

$$\text{or}\quad V_0=\frac{0{\cdot}8\times3\times10^8}{6{\cdot}2}\text{ m/s}=3\times10^4\text{ km/s (nearly).}$$

[N.B. Such high speed for a motor car is not practically possible]

5.14. Musical sound and noise :

We hear different kinds of sound everyday like the sound of traffic, sound of musical instruments, sound of rain and thunder, sound of bell and so on. Of these, some are pleasant and some are harsh and unpleasant. From the point of view of perception, sounds which give a pleasing sensation to the ear are called *musical sound* while those producing confused and unpleasant sensation are called *noise*.

Sound is produced by the vibration of a body and hence the source is responsible for making a sound musical or noisy. It has been found that a *regular periodic* *motion of the source produces musical sound.* When a violin string is bowed or a sitar string plucked, the strings vibrate periodically and they emit musical sound.

Noise is not easy to define but a fairly general agreement may be expected with the definition that it is a sound or combination of sounds of *constantly varying* pitch or irregular frequency. The jumbled sound of a crowd talking or shouting together has constantly varying pitch and therefore is an unpleasant sound or a noise. Similarly sound of a firing or an explosion being of irregular frequency, is also a noise.

But the same sound may appear pleasant as well as unpleasant under different circumstances. For example, the sound of a bell is generally pleasant but it may be disturbing and very noisy if it is sounded continuously. Sound of falling raindrops are not, in general, pleasing but sometimes the continuous and monotonous clattering of raindrops on tin-shed may induce a pleasant sensation. So, it is very difficult to draw a sharp line of demarcation between musical sound and noise.

5.15. Note and tone :

Note is the sound produced by the periodic vibration of a source of sound.

Tone, on the other hand, is the sound produced by the simple harmonic motion of the source of sound. Thus, the sound emitted by a tuning fork may be called a tone because the vibrations of tuning fork are simple harmonic.

Distinction between note and tone should be clearly understood. You know that white light is a mixture of seven different colours. So is a note which is a mixture of several tones. A tone has a single frequency but a note has several tones of different frequencies in it. Of these frequencies, the lowest is called the *fundamental tone*. For example, if the fundamental of a note emitted by an organ has frequency 256, the note may contain other tones of frequencies $3 \times 256 = 768$, $5 \times 256 = 1280$ etc. These tones by their presence, enhance the musical value of the note.

Besides the fundamental, other tones present in a note are known as *overtones*. Of the overtones, those which have their frequencies simple multiple of fundamental frequency, are known as *harmonics*. For example, in the note of the organ mentioned earlier the following frequencies are present : 256, 468, 502, 768, 1020, 1280 etc. Here, 256 is the fundamental frequency and others are overtones. Of the overtones, again 768, 1280 are simple multiples of 256, the fundamental frequency. Hence, they are harmonics. So *all harmonics are overtones but all overtones are not harmonics.*

Further, if the frequency of a tone is double that of another, the first is an *octave* of the second.

5.16. Characteristics of a musical note :

The following are the characteristics of a musical note :

(i) **Loudness or intensity** (ii) **Pitch** and (iii) **Quality.**

5.17. Loudness or intensity :

The intensity of a sound wave is defined as the rate of flow of energy per unit area perpendicular to the direction of the wave. It depends upon the following factors :—

(*a*) *The size of source* : The greater the size of the source, the greater is the energy imparted to the wave and hence greater is the intensity of the sound.

(*b*) *The distance between the source and the listener* : With the increase of the distance between the source and the listener, the intensity of the sound falls. If I be the intensity at a distance r, then $I \propto \frac{1}{r^2}$.

(*c*) *The amplitude of vibration* : The intensity of sound is proportional to the square of the amplitude of vibration of the source. If I be the intensity and a the amplitude, then $I \propto a^2$.

(*d*) *The density of the medium* : If the density of the medium increases, the intensity of the sound through it also increases. If I be the intensity of a sound through a medium of density ρ, then $I \propto \rho$.

(*e*) *Presence of a resonant body* : The intensity of a sound increases if there is a resonant body present in the neighbourhood of the source.

(*f*) *Motion of the medium* : If sound travels in the direction of the motion of the medium, its intensity increases.

Difference between loudness and intensity :

Loudness and intensity of sound are not the same thing. Intensity is a measurable physical quantity while loudness is a sensation which is not measurable. Intensity does not depend on the listener but loudness of a sound depends on the listener. Observations show that sounds which appear equally loud to a listener have different intensities depending on the frequency of the sound. The curves shown in fig. 5.9 represent the relation between frequency and intensity of three sounds, each of *equal loudness*. The graph shows that the intensity I_1 at P when the frequency is 1000 Hz is less than the intensity I_2 at Q when the frequency is 500 Hz, although the loudness is the same. Still, it is found that the factors which are responsible for an increase in intensity of sound are also generally responsible for an increase in its loudness.

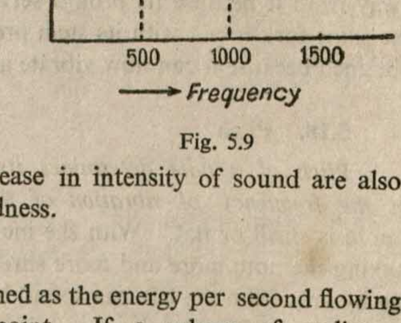

Fig. 5.9

Measure of intensity of sound :

The *intensity* of a sound at a point is defined as the energy per second flowing normally through unit area round the point. If a sphere of radius r be imagined round a small source of sound, which is emitting sound energy E per second, then the energy flowing normally through unit area of the sphere$=\frac{E}{4\pi r^2}$ where $4\pi r^2$=surface area of the sphere. Hence, the intensity of sound at a distance r from the source is $I=\frac{E}{4\pi r^2}$. This shows that $I\propto\frac{1}{r^2}$.

Suppose the displacement at any instant t of a vibrating layer of air is given by $y=a \sin \omega t$ where a=amplitude of vibration of the layer and $\omega=\frac{2\pi}{T}$.

Now, the velocity of the layer $v=\frac{dy}{dt}=\omega.a \cos \omega t$. Hence the *K.E.* of the layer$=\frac{1}{2}mv^2=\frac{1}{2}m.\omega^2.a^2 \cos^2\omega t$ .. (i) [m=mass of the air layer]

The vibrating layer of air also possesses potential energy. But we know that the total energy of the layer is always conserved and it is equal to the maximum kinetic energy because the potential energy at that moment is zero. If W be the total energy of the layer, then from eqn. (i) above we can write $W=(K.E.)_{max}$ $=\frac{1}{2}m\omega^2.a^2$. .. .. (ii)

If V be the velocity of sound in air, then the air is disturbed over distance V in 1 second. Taking 1 sq. unit of area the volume of the disturbed air$=V\times 1=V$ and its mass $m=V.\rho$. where ρ=density of air.

So, the total energy passing per unit area per sec is given by

$$W=\frac{1}{2}V.\rho.\omega^2.a^2 \quad .. \quad ..\text{(iii)}$$

According to the definition, the intensity of the sound wave is $I=\frac{1}{2}.V.\rho.\omega^2.a^2$. This shows that the intensity I of a sound wave of given frequency depends on (i) the square of the amplitude (a^2) and (ii) the density (ρ) of the medium.

Further, eqn. (ii) shows that the greater the mass m of the vibrating air, the greater is the intensity of the sound produced. This is why the sound produced by the vibrating diaphragm of a telephone ear-piece can not be heard unless the ear is placed very close to the ear-piece. On the other hand, the cone of a loudspeaker with its large surface area can set into vibration a large mass of air, giving rise to a sound of much larger intensity than the vibrating telephone ear-piece. As a result, the sound produced by a loudspeaker is audible over a long distance. It is known that the sound of a vibrating tuning fork is not heard at a small distance away from it because its prongs set a small mass of air in vibration. But when the same fork is put with its stem pressed against a table, a much louder sound is obtained because it can now vibrate a large mass of air by contact with the table.

5.18. Pitch :

Pitch of a note determines its position in the musical scale and depends on the frequency of vibration of the source. It generally signifies how far a note is shrill or flat. With the increase of the frequency, pitch becomes higher making the note more and more shrill. If the reeds of a harmonium are pressed, one by one, from the left, the sound becomes shriller because the frequency gradually increases. On the other hand, if one of the prongs of a tuning fork, be slightly loaded, its frequency will decrease and it will then emit a tone of lower pitch.

Since, the pitch of a note is proportional to the frequency of vibration, it is generally expressed by the frequency. Again, we know that the wavelength of a wave is inversely proportional to its frequency. Hence a sound of higher pitch has smaller wavelength and vice versa. The international unit of pitch is *Hertz* (Hz).

Difference between pitch and frequency :

As loudness of a sound is not identical with the intensity of the sound, so pitch of a sound is not identical with its frequency. Pitch is a sensation and is not measurable. Frequency, on the other hand, is a measurable physical quantity. As pitch of a note is proportional to its frequency, it has been customary to express pitch by frequency. We know, the frequency of a wave is inversely proportional to the wavelength of the wave. So, a sound wave of high pitch *i.e.* high frequency has a smaller wavelength. In other words, it may be said that the sound waves of smaller wavelength have high pitch and vice-versa. Further pitch of a note depends on the relative motion between the source, the listener and the wind (see Doppler effect) but frequency depends on the vibration of the source.

5.19. Quality :

The characteristic by which different sounds having same frequency and loudness but coming from different musical instruments are differentiated is called the quality of the sound.

If, for example, a particular note on the musical scale is played on a piano, a violin and a flute, it is easy to distinguish, simply by hearing the sounds, the tone of one instrument from that of the other. It is because the tones of the instruments have different quality.

Generally speaking, musical instruments do not give tones which are pure in the sense that they consist of single frequency only. In the majority of cases a musical note consists of fundamental blended with overtones. A tuning fork may be regarded as a source of sound producing a pure tone of single frequency. The wave form of its note is like simple harmonic wave as shown in fig. 5.10(i). If the same note is played on a violin and a piano respectively, their waveforms may be represented by fig. 5.10(ii) and 5.10(iii). From the diagram it is clear that all the three notes of the instruments have same amplitude (a) and same wavelength (λ). Hence their loudness and pitch are same. Now curves of the shape shown in fig. 5.10(ii) and (iii), on mathematical analysis, are found to be the sum of a number of simple harmonic wave forms—whose frequencies are multiples of n_0, the frequency of the original wave form ; the amplitudes and hence intensities of the notes, however, diminish as the frequency increases. Fig. 5.10(iv), for

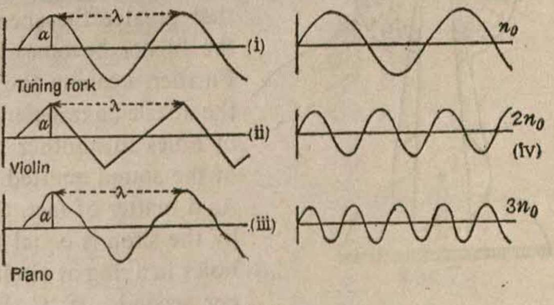

Fig. 5.10

example, might be an analysis of a curve similar to 5.10(iii) corresponding to a note produced by a piano. As human ear is sensitive to simple harmonic waves, it records the presence of notes of frequencies $2n_0$ and $3n_0$ in addition to n_0 when the note is sounded on a piano or a violin. As the intensity diminishes with the increase of frequency (the amplitude of wave forms shown in fig. 5.10(iv) is gradually diminishing), the note of frequency n_0 is predominant, intensity being proportional to the square of amplitude. In the background, however, the notes of frequencies $2n_0$, $3n_0$ etc. will be present. The frequency n_0 is called the *fundamental* while the others are known as *overtones*.

As the same tone coming from different musical instruments, has different wave forms, the mathematical analysis of each will differ. In other words, the number of overtones present in them will be different. For example, the wave form of a note of frequency n_0 from a violin may contain overtones of frequencies $2n_0$, $4n_0$, $6n_0$. The 'musical background' to the fundamental note is thus different when it is produced by different instruments. Hence *the overtones present in a note determine its quality*. The quality of a note also depends, to some extent, on the relations between the frequency of the fundamental and the frequencies of the overtones as well as the intensity of the fundamental and the intensities of the overtones. For this reason, much experience and craftsmanship are required for the design and construction of musical instruments in order to suppress unwanted overtones and enhance the desirable ones.

5.20. Determination of pitch by Seebeck's siren :

Fig. 5.11 shows a section of Seebeck's siren. It consists of a metal plate with holes drilled in concentric circles. The plate can be rotated about a horizontal spindle by means of a handle. When a jet of air is directed against the plate with the help of a pointed nozzle and a foot bellow, puffs of air escape through the holes and produce a sound. It will be found that greater the speed of rotation of the siren disc, the higher becomes the pitch of the sound emitted. Further, keeping the speed of rotation unchanged, if the nozzle be transferred from a row of smaller number of holes to another row of larger number, the pitch of the sound emitted will be found to have increased. As a matter of fact, the frequency of the note emitted by the siren is equal to the product of the number of holes in a ring or circle and the number of revolutions per second. If 'f' the frequency of the note, m the number of holes in a circle and n the number of revolution per sec., then $f = n \times m$.

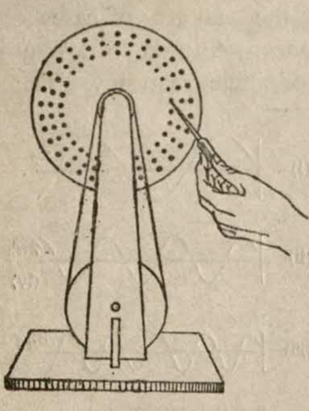

Fig. 5.11

In order to determine the pitch of a sound, the revolutions of the disc are so adjusted that the note emitted by the siren is in unison with the sound under test. Then, by applying the above relation, the unknown frequency of the sound can be found out.

Examples : (1) *The disc of a siren revolves 10 times in a second. How many holes should the disc contain in order to emit a note of frequency 480 ?*

Ans. We know, the frequency, $f=m \times n$.

Here $m=480$; $n=10$; So, $480=m \times 10$ or $m=48$

Hence, the disc should contain 48 holes.

(2) *A siren disc has 32 holes. How many revolutions should it make per minute in order to emit a sound an octave higher than another of frequency 256 ?*

Ans. The frequency of sound which is an octave higher than the frequency $256=2 \times 256$. If n=number of revolutions per second, then $2 \times 256 =32 \times n$ $\therefore$ $n=16$. $\therefore$ number of revolutions per minute$=16 \times 60=960$.

*5.21. Recording and reproduction of sound :

From time immemorial, the human mind has always been thinking of recording his own voice, music etc. and then reproducing them according to convenience. It is said that sometimes in 400 B. C. a Chinese king recorded his voice in a teak-wood box and it was reproduced by his friend just by turning the handle of the box. The story was ignored and forgotten till 1859 when Leon Scott made the first attempt to record sound. Scott's method was improved by Koenig sometimes in 1864. But in 1878 Edison set rolling the scientific methods of recording and reproduction of sound by inventing phonograph. The technique has vastly improved now-a-days after the discovery of gramophone, record-player, cinematography, tape-recorder etc.

(i) *Phonograph* : It is the first instrument to record music, speeches or recitation and then to reproduce it. It was invented by the famous inventor Thomas Alva Edison in 1878.

In this instrument, there is a conical horn [fig. 5.12], at the narrow end of which there is a thin metallic disc D. At the middle of the disc and at right angles to it, there is a thin stylus T whose pointed end is just in touch with a rotating drum W coated with a thin layer of wax. As the drum rotates, the stylus depicts a groove on the wax. The drum rotates about an axis passing through the point A and advances forward. There is a handle by means of which the drum can be rotated.

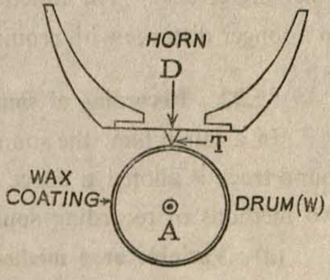

Fig. 5.12

During recording of sound, the speaker or the singer sings before the conical horn which collects the sound waves and sends them to the disc D. The disc vibrates according to the intensity of the sound and the stylus also vibrates at right angles to the disc. At the same time, the drum is rotated by the handle at a uniform rate and the stylus makes a groove on the wax. The depth of the groove is not same everywhere ; it changes with the intensity of the sound. In this way, the sound is recorded.

During reproduction, the stylus is placed at the beginning of the groove and the drum is rotated at the same uniform rate as before. When the stylus is made to move in this way a second time over the groove which it first made on the wax, it receives again and imparts to the disc *D* the same type of vibration which first fell upon it. The disc imparts its vibration to the air and thus the original sound is reproduced.

(ii) *Gramophone* : It is an improvement of the original form of phonograph. In principle both the instruments are same except certain mechanical modifications in the case of a gramophone. In the original phonograph the drum was driven by hand or an electric motor. But in a gramophone it is driven by a clock work. Moreover, instead of a drum, sound track is recorded on a flat disc so that the trace is a spiral running from near the centre to the circumference of the disc. The track is a wavy track *varying not in depth but in width.* When the gramophone needle or pin moves over the wavy track as the disc rotates horizontally, lateral motions of the pin are produced due to the varying width of the track. These lateral motions, after being amplified by lever arrangement (Fig. 5.13) are transmitted to the diaphragm of the sound-box which produces identical vibrations in air. The original sound is thus reproduced.

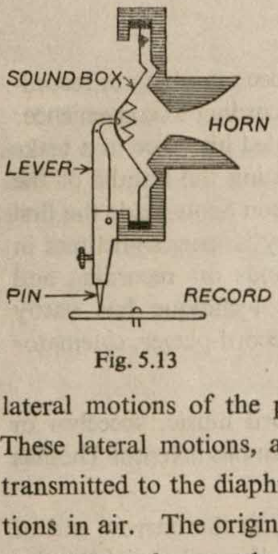

Fig. 5.13

Discs of gramophone are generally made of a suitable mixture of shellac and other ingredients. The function of the horn is to collect sound and to throw it to a longer distance with requisite intensity.

*5.22. Recording of sound on films and its reproduction :

In a talkie film, the sound is recorded by the side of the picture film. The sound track is alloted a space of $\frac{1}{10}$th of an inch wide. At present there are two methods of recording sound on a film.

(*a*) **Variable area method** : In this method of recording sound, different lengths of a slit are illuminated and its image is received on the film. This image serves as the sound track.

The chief part of the arrangement is a Duddell oscillograph *O* through which the amplified speech current coming from the microphone is fed. The oscillograph consists of a single loop of thin phosphor-bronze strip. It is situated in the field of power magnets having specially shaped pole-pieces to obtain an intense field between them (Fig. 5.14). The loop is formed by passing the phosphor-

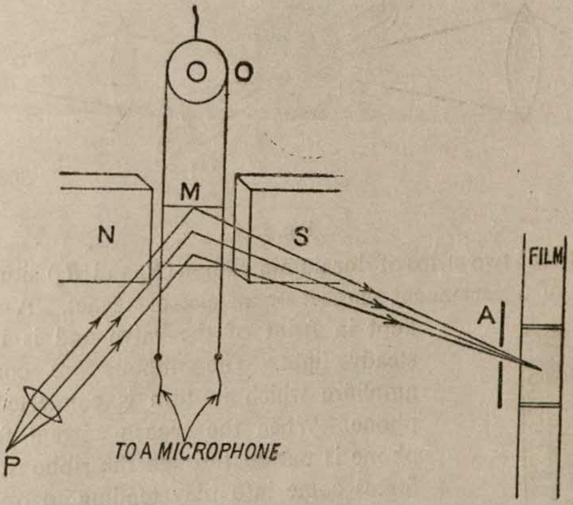

Fig. 5.14

bronze strip round a small ivory pulley. A small light mirror *M* is fixed to the loop. A beam of light is focussed on the mirror by a lens and is reflected on to the slit *A*. The beam is then collected on the film and it produces an image of the slit on the film. When the speaker speaks before the microphone, audio frequency electrical oscillations are produced in the microphone. When the amplified current passes through the loop, it is acted on by a force due to the magnetic field. Since it is an alternating current the loop vibrates and hence the mirror. As a result, the portion of the slit which is illuminated, changes with the change in strength of the speech current. At the same time, the film is drawn at a uniform speed of 1·5 ft per second and a sound track is produced on the film. When the film is developed, a sound track in the form of *variable area* but of *uniform opacity* is obtained. This is the negative record. When positive film is prepared, the sound track is converted into one of *variable area* but of *uniform transparency*. Its appearance is shown in fig. 5.15.

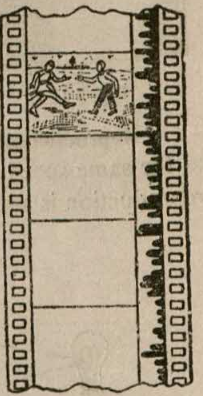

Fig. 5.15

(*b*) **Variable density method** : The main principle of this method is to feed a light of variable intensity caused by speech sound of varying quality and loudness on to the photographic film ; this would give a sound track of variable density on the film. The arrangement has been shown in fig. 5.16.

It consists of a tungsten filament lamp *L* which is imaged by a convex lens slightly out of focus at the light valve aperture. The light valve consists of a narrow slit between two strips of duralumin ribbon (R_1 and R_2) situated in a strong magnetic field of a permanent magnet or an electromagnet. A vertical slit *S* is kept in front of the valve and is illuminated with steady light. The ribbons are connected to the amplifire which in turn is connected to the microphone. When the speech current from the microphone is passed through the ribbons, electrodynamic forces come into play tending to open or close the slit. The varying slit-width of the valve modulates the light beam, which is focussed on the film with the help of another convex lens. As the film is drawn at a certain fixed speed, continuous series of parallel lines of varying opacity will be obtained on the film [Fig. 5.17]. The opacity denotes the intensity of sound and the number of lines per unit length denotes its frequency.

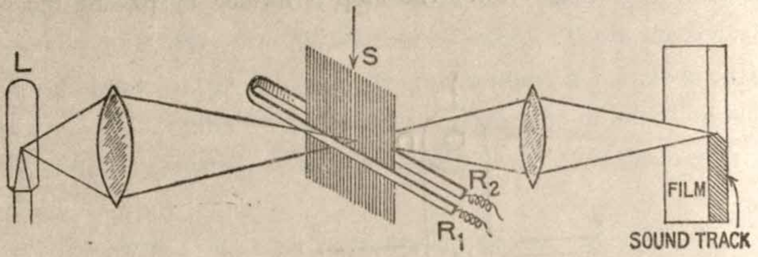

Fig. 5.16

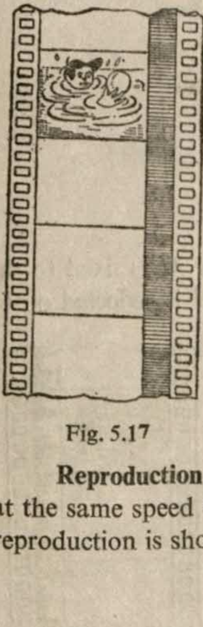

Fig. 5.17

Reproduction of Sound : During reproduction of sound, the film is drawn at the same speed at which it was drawn during recording. The arrangement for reproduction is shown in fig. 5.18.

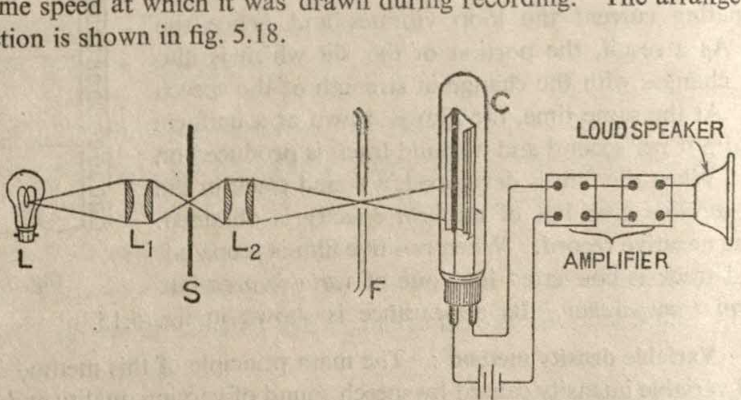

Fig. 5.18

Light from a powerful source *L* is focussed by a condensing lens *L* d$_1$an is allowed to fall on a slit S. The image of the slit is then formed on the sound track region of the film F by another similar set of lenses L_2. The intensity of the light that emerges from the film depends upon the area or density of the sound track and is made to fall on a photo-electric cell C. The variations of light intensity cause corresponding variations in the electric current through the cell. This current, having frequency and magnitude modulations identical with those of the sound record, is then amplified in the usual manner and led on the loud-speaker.

Exercises

Essay type :

1. How is the velocity of sound in gaseous medium affected by pressure and temperature of the gas ? [*H. S. Exam. 1981*]

2. What is resonant air column ? How would you determine the velocity of sound with its help ? What is end correction ? How can it be avoided ?

3. A vibrating tuning fork (say, of frequency 256) is held at the open end of a jar (100 cm. long) and water is slowly poured into the jar. What will happen ? How will you determine the velocity of sound with its help ?

4. What is Doppler effect ? Obtain an expression for the apparent pitch of the sound heard by a listener when the listener is at rest and the source of sound is moving towards the listener.

5. Prove that the Doppler effect is greater when the source approaches the observer than when the observer approaches the source with the same speed ?

6. Show that if the source moves away with the velocity of sound from an observer, who is at rest, the frequency of vibration is halved.

7. Is Doppler effect seen in the case of light waves ? Explain your answer with suitable illustrations.

8. Distinguish between noise and musical sound. What are the characteristics of a musical sound ? What enables a trained ear to distinguish between the musical sound emitted by a sitar, violin and piano at the same time ? [*H. S. Exam. 1980*]

9. Two musical sounds differ from each other in respect of loudness, pitch and quality—what does this statement mean ? What are the factors upon which these characteristics depend ?

10. Describe a siren. How can you determine the pitch of a sound by a siren ?

11. Write a short note on recording and reproduction of sound.

Short answer type :

12. Write Newton's formula for the velocity of sound in a gaseous medium. In what respect is this formula defective ? What correction did Laplace make in the formula ? [*H. S. Exam. 1978, '81*]

13. The velocity of sound is generally greater in solids than in gases at *N.T.P.* Why? [*I.I.T. 1977*]

14. In a sports meet the timing of a 200 metres straight dash is recorded at the finish by starting an accurate stop-watch on hearing the sound of the gun fired at the starting point. Will the time recorded be more accurate in summer or in winter ? [*I.I.T. 1973*]

[**Hints** : Record of time will be more accurate in summer. The velocity of sound being proportional to the square root of the absolute temperature, the sound of the gun will reach the finishing point sooner in summer than in winter.]

15. During a thunderstorm, sound of thunder is heard much after the lightning flash is seen. Why ?

16. How can the end-correction be avoided by noting two resonant heights in a resonant air column with the help of a tuning fork ? On what does the end-correction depend ?

17. If a sound is made at one end of a long hollow iron tube, two sounds are heard at the other end. Why ?

18. When a train approaches a station whistling all the time, the pitch of the whistle sound appears to increase but when the train leaves the station and goes away, the pitch appears to decrease. Why ?

19. Why does difference occur in tunes played on a violin and a piano although the tunes have the same pitch and loudness ? [cf. *H. S. Exam. 1980*]

20. What are the factors that make your voice different from that of your friend ? How can you explain the roar of a lion and buzzing of a mosquito from the point of view of the characteristics of sound ? [*Jt. Entrance 1972*]

21. What are fundamental, overtones, harmonics and octave ? [*H. S. Exam. 1981*]

22. What are fundamental and overtones ? The fundamental frequency of an open tube is an octave higher than that of a closed tube of equal length. Prove it.

23. All harmonics are overtones but all overtones are not harmonics—Explain.

24. 'A sound of higher pitch has smaller wave length and vice versa'.—Explain the statement. What is the international unit of pitch ?

25. Explain what will happen to the pitch of the fundamental note emitted by an open organ pipe when (i) its mouth is suddenly closed by hand (ii) air in it is replaced by carbon dioxide (iii) hot air is blown in the pipe and (iv) greater amount of moisture is introduced in the tube. [*Jt. Entrance 1973*]

26. Answer the following questions :—(a) When a saw starts cutting a log of wood a high-pitched sound is produced, but the pitch falls as the saw cuts into the wood. Why ? (b) The sound coming from a harmonium and violin can be easily identified even in a dark room. How is it done ? (c) The loudness of a sound diminishes as the listener moves away from the source. Why ?

27. Why the loudness of the sound emitted by a tuning fork depends on the following factors ?—(i) the force with which the fork is struck. (ii) the distance between the fork and the ear of the listener, (iii) the sound-box on which the fork is mounted ?

28. Are loudness and intensity the same thing ? Do frequency and pitch mean the same thing ? To hear the sound in a telephone we bring our ears in contact with the earpiece of the phone but the sound of a loudspeaker can be heard from a great distance. Why is this difference ?

Objective type :

29. [A]. Write *Y* where the answer is 'yes' and *N* where the answer is 'no' in the following cases :—

(i) Is the velocity of sound in a gas inversely proportional to the square root of its density ? —

(ii) A man struck one end of a long hollow iron pipe by a hammer. Another man tried to hear sound at the other end. Will he hear one sound ? —

(iii) Are all overtones of a note harmonies ? —

(iv) Will the velocity of sound be same in day and in night ? —

(v) Is Doppler effect applicable to all types of wave ? —

[B]. Each of the properties of sound listed in the column I primarily depends on one of the quantities in column II. Write down the matching pairs from the two columns. [*I.I.T. 1980*]

Column I	Column II
pitch	wave form
quantity	frequency
loudness	intensity

Numerical problems :

30. A gun is fired from a fort at a fixed hour. An observer, from a distant sets his watch by the report of the gun but finds later that it is slow by half a minute. Calculate the distance (in miles) of the fort from the observer, assuming the velocity of sound to be 1100 ft/sec. [Ans. 6¼]

31. The sound of a gun-fire reached 6 sec. after the flash was seen. If sound travels with a velocity of 1120 ft/sec., what is the distance between the gun and the recipient of the sound ? [Ans. 6720 ft.]

32. The sound of a thunderclap was heard 5·5 sec. after the flash was seen. If the temperature at the time was 20°C, find the distance of the cloud. The velocity of sound at 0°C is 1100 ft/sec. [Ans. 6271·43 ft.]

33. What time would a terrestrial sound take to reach the moon had the atmosphere been extended up to the moon ? The distance of the moon from the earth is 240,000 miles and velocity of sound 1120 ft/sec. [Ans. 314 hr. 17 mnt.]

34. On a day when temperature is 29°C, the flash of a gun-fire at 1750 metres away was seen 5 sec. before the sound is heard. Find the velocity of sound and the density of air. Atmospheric pressure at that time=76 cm. of mercury and the ratio of two specific heats=1·41. Velocity of sound at 0°C=332 m/s. [Ans. 350 metres/s (nearly) ; 1166×10^{-6} gm/c.c.]

35. If the velocity of sound at 0°C and 75 cm. pressure be 330 metres/sec., what will be the velocity at 50°C and 70 cm. pressure ? [Ans. 360·19 metres/sec.]

36. A stone is dropped into a mine 256 ft. deep and the sound is heard 4·22 sec. later. Find the velocity of sound. (g=32 ft/sec^2) [Ans. 1163·6 ft/sec.]

37. A stone is dropped from the top of a tower and the sound is heard 4·4 sec. later. Find the height of the tower. Velocity of sound in air=1000 ft/s ; g=32 ft/s. [*H. S. Exam. 1979*] [Ans. 272·2 ft.]

38. A tuning fork of frequency 500 when held over a closed pipe gives resonance with air columns of lengths 15 cm. and 49 cm. What is the velocity of sound in air ? [Ans. 340 metres/sec.]

39. A tuning fork of frequency 512 is held at the open mouth of a tube of diameter 10 cm. immersed in water. How far the tube is to be raised from water in order that resonance may occur ? Velocity of sound=330 m/s. [Ans. 13·12 cm]

40. A wind pipe 50 cm. long and open at both ends is vibrating in resonance with a tuning fork. If the velocity of sound in air is 350 metres/s, what is the frequency of the tuning fork ? [*H. S. Exam. 1982*] [Ans. 350]

41. A tuning fork of frequency 320 is held over a closed pipe, the difference between the lengths of first two resonant air columns was found to be 1·75 ft. What is the velocity of sound in air ? [Ans. 1120 ft/s]

42. When a tuning fork of frequency 320 is held over a closed pipe, the difference between the lengths of first two resonant air-columns is found to be 51·60 cm. What is the velocity of sound in air ? [Ans. 330·2 metres/sec.]

43. A fork of frequency 256 is held over a tube and a maximum sound is obtained when the column of air is 31 cm. and 97 cm. Determine the end-correction and velocity of sound in air. [Ans. 2 cm. ; 337·92 metres/sec.]

44. A tuning fork having a frequency of 340 vib/sec is vibrated just above a cylindical tube. The height of the tube is 120 cm. . Water is slowly poured into it. What is the minimum height of water required for the resonance ? [*Jt. Entrance 1985*] [Ans. 46·76 cm.]

45. A train approaches a station with a velocity of 118·8 km/hr whistling all the time with a frequency of 300. What is the frequency apparent to a person waiting on the platform of the station ? (velocity of sound in air at that time=330 metres/sec.) [Ans. 333]

46. A motor car is approaching a bend with velocity of 72 km/hr., sounding its horn all the time. A person, standing at the bend, appears to hear a frequency of 350. What is the real frequency of the horn ? Velocity of sound=330 metres/sec. [Ans. 328·8]

47. A train, whistling a sound of frequency 1000 is approaching a railway platform with a speed of 30 km/hr. What will appear to be the frequency of the sound to a man standing on the platform ? Velocity of sound=330 metres/s. [Ans. 1026]

48. At what speed should a source of sound of frequency 10,000 cycles/s advance towards a listener so as to produce a Doppler shif of 200 cycles/s ? Velocity of sound in air=1120 ft/s. [Ans. 21·96 ft/s.]

49. Two ships are approaching each other with speeds of 10 miles/hr. and 15 miles/hr. respectively. The siren of the first ship appears to be of frequency 1000 to a passenger of the second ship. What is the actual frequency of the siren ? Velocity of sound=750 miles/hr. [Ans. 966]

50. A siren disc rotates 9 times in a second. How many holes should the disc contain so that it may produce 5 beats with a tuning fork of frequency 320 ? [Ans. 35 or 36]

51. A siren disc has 200 holes and revolves 132 times in a minute. The sound it emits is an octave lower than that of a tuning fork. What is the frequency of the tuning fork ? [Ans. 880]

Harder problems :

52. Prove that in a gas for which $\gamma=1{\cdot}41$, the velocity of sound is $0{\cdot}68C$, where C is the r.m.s. velocity of the gas molecules.

53. The wavelength of the note emitted by a tuning fork of frequency 512 Hz in air at 17°C is 66·5 cm. If the density of air at S.T.P. is $1{\cdot}293\times10^{-3}$ gm/c.c., calculate the ratio of the molar sp. heats of air. Density of mercury=13·6 gm/c.c. and $g=980$ cm/s². [Ans. 1·39]

54. A man standing at one end of a closed corridor 5700 cm. long blew a short blast on a whistle. He found that the time from the blast to the sixth echo was 2 seconds. If the temperature was 17°C, what was the velocity of sound at 0°C ? [Ans. 332 metres/s]

55. An observer looking due north sees the flash of a gun 4 seconds before he records the arrival of the sound . If the temperature is 20°C and the wind is blowing from east to west with a velocity of 48 km/hr, calculate the distance between the observer and the gun. The velocity of sound in air at 0°*C* is 330 m/s. [Ans. 1366 metres]

56. A long cylindrical tube is being filled up by water at a uniform rate from a tap. An observer found that a tuning fork of frequency 300 is producing resonance with the air column in the tube after every 100 second. Calculate the volume in c.c. of the water supplied per second. The radius of the tube=10 cm and the velocity of sound=330 m/s.
[*Jt. Entrance 1976*] [Ans. 172·7 c.c./s]

57. An open organ pipe is emitting its first overtone. One end of the pipe is suddenly closed. It still emits its first overtone but the frequency difference is found to be 128. What was the frequency of the note emitted by pipe when both of its ends were open ? [Ans. 512]

58. Two organ pipes when sounded together at 24°C produce 10 beats per sec. How many beats per sec. will be heard at 4°C ? Velocity of sound in air at 0°C=332 metres/s. [Ans. 9·6]

59. Compare the velocity of sound in argon and carbon dioxide at 27°C and under a pressure of 76 cm. of mercury. The molecular weights of argon and carbon dioxide are 40 and 44. γ for argon=$\frac{5}{3}$ and γ for carbon dioxide=$\frac{4}{3}$. [Ans. $\sqrt{\frac{11}{8}}$]

60. The shortest length of a resonance tube closed at one end which resounds to a fork of frequency 256 is 30 cm. The corresponding length for a fork of frequency 480 is 12 cm. Calculate the end-correction for the tube and the velocity of sound in air.
[Ans. 8·57 cm. ; 394·98 m/s]

61. An engine, whistling a sound of frequency 133 per sec is approaching a hill with a speed of 60 metres/s. The hill produces an echo. Find the frequency of the note heard by the driver of the engine. Velocity of sound=340 metres/s. [*Jt. Entrance 1981*] [Ans. 190]

62. A car travelling at a speed of 36 km/hr sounds its horns which have a frequency 500 Hz and this is heard in another car which is travelling behind the first car in the same direction with a velocity of 20 m/s. The sound can also be heard in the second car by reflection from a bridge head. What frequencies will the driver of the second car hear ? Sound travels in air with a speed of 340 m/s. [*Joint Entrance 1984*] [Ans. 545·4 ; 514·3]

63. Two observers *A* and *B* have each a whistle of frequency 500. *B* is stationary but *A* is moving away from *B* with a speed of 180 cm/s. How many beats per sec. will *A* and *B* hear ? Velocity of sound in air=330 metres/s. [Ans. *A*=2·71 ; *B*=2·73]

64. A star is receding away from the earth with a speed of $1 \cdot 5 \times 10^7$ metre/s. The star emits a light of frequency 8×10^{14} Hz. What will be the frequency of the light received on the earth ? Velocity of light$=3 \times 10^8$ metres/s. [Ans. $7 \cdot 6 \times 10^{14}$ Hz.]

[**Hints** : $n_1 = \dfrac{c.n}{c+v_s}$ where c=velocity of light.]

65. A train is approaching a stationary observer with a speed equal to $\frac{1}{20}$th of the speed of sound and producing intense sound with its whistle in every alternate second. Find the interval in which the listener will hear the sound. [Ans. 0·95 sec.]

66. A whistle of frequency 540 vib/sec rotates in a circle of radius 2 ft. at an angular speed of 15 radians/sec. What is the lowest and the highest frequency heard by a listener a long distance away at rest with respect to the centre of the circle ? [Ans. 555.1 ; 525.6] Velocity of sound=1100 ft/sec.

67. A whistle is whirled in a circle of 100 cm radius and traverses the circular path twice per second. An observer is situated outside the circle but in its plane. What is the interval between the highest and the lowest pitch heard if the velocity of sound is 332 m/s. [Ans. 1·08]

[**Hints** : Interval means ratio of highest and lowest pitch. Highest pitch will occur when the whistle approaching the observer along the circle comes nearest to him and lowest pitch will occur when the whistle receding away along the circle goes furthest from the observer.]

6

NATURE OF LIGHT WAVES

6.1. Wave nature of light :

Ever since ancient times, people have speculated about the true nature of light. The philosopher Plato believed that light was something given off by the eye and capable of making things visible when struck by it. Others thought that light consisted of small particles shot out from luminous objects at high speed. Still others, among them the celebrated thinker Aristotle, preferred to think of light as something non-material that goes on in the space between the eye and the object seen.

In the seventeenth century the Dutch scientist Christian Huygens and the English scientist Robert Hooke suggested, independently of each other, that light might be a wave motion. Newton considered both the idea of light as a stream of minute particles darting out from luminous bodies and as a train of waves, but he definitely favoured the particle idea or *Corpuscular theory*, as it was called. According to this theory, a stream of minute particles shoot out from a luminous body and striking the eye of an observer, cause the sensation of sight. His reputation kept this theory active for nearly a century after his time, in spite of the fact that certain observations are almost impossible to explain by Corpuscular theory but are easily understood in terms of wave.

If we say that light is a wave motion, question will at once be asked 'Waves in what ?' knowing that all kinds of waves with which we are familiar, require some medium for its propagation. When this question first arose, physicists had no other way but to invent a carrier for the waves. This had to be something that filled all space, for light comes to us from the sun and stars through space that is virtually devoid of all matter. Scientists called this hypothetical carrier the *aether* but no experimental evidence for its existence has ever been obtained. But according to modern science, such idea is no longer necessary because British physicist Maxwell had shown about a century ago that light waves consist of rapidly changing electric and magnetic forces that originate in the movement of electricity in the atoms and the molecules. This is known as the '*Electro-magnetic theory of light*'.

When we look at a steady light, we have no impression of the fact that it may consist of waves, any more than hearing a steady tone gives any impression of wave-like nature of sound. For that matter, if sound or light were carried by streams of minute particles, the individual ones would fail to reveal themselves as long as they were very small and very numerous. The same thing, we know, is true for the molecules of matter. So, we must face the fact that we cannot hope to 'see' individual light waves ; our acceptance of the wave theory will have to depend on indirect evidence.

In art. 9.18 of Heat, it has been pointed out that the family of waves which light belongs to, is known as the *electromagnetic wave* family. Gamma rays, visible light, radiant heat, wireless waves—all are included in this family. All of them travel through aether with a speed of 1,86,000 miles per second. Each one of them possesses all the optical properties like reflection, refraction, interference, diffraction etc. Only difference that they exhibit is the difference in wavelengths. If the wavelength lies in the range of 25 metres to 5000 metres, the waves are wireless waves which are used in broadcasting radio signals. These waves do not produce the sensation of sight in human eyes. If the wavelength becomes smaller and falls in the region between 4×10^{-5} cm to 8×10^{-5} cm, they produce the sensation of light and we call them visible light waves.

6.2. Difference between light waves and sound waves :

Although light and sound are both waves, yet there are some basic differences between them. The differences are as follows :

(i) Sound waves are longitudinal in nature but light waves are transverse. Sound waves do not exhibit polarisation but light waves may be polarised.

(ii) Sound waves are mechanical elastic waves while light waves are electromagnetic waves.

(iii) Velocity of sound in air is very small but the velocity of light in air is tremendous. In the same way, the wavelength of sound wave in air is very big compared to the wavelength of light in air.

(iv) For the propagation of sound, a material medium solid, liquid or gas—is necessary. Sound waves can not travel through vacuum. But light waves do not require any medium for its propagation. It can travel through vacuum.

(v) Velocity of sound in a gaseous medium is dependent on the temperature of the medium. With the increase of temperature, the velocity of sound increases and vice versa. But temperature of a gaseous medium has no effect on the velocity of light through it.

6.3. Velocity of light :

The velocity of light is so enormous that all early attempts to determine it failed and it was generally believed, till the later part of the seventeenth century that light had infinite velocity. In 1675, Olaf Romer, a Professor of Astronomy in the university of Copenhegen, first determined the velocity of light from some astronomical observations. Later, Fizeau, Michelson, Anderson and others devised various methods from which it is now known that light travels with the tremendous speed of about 1,86,000 miles per second. So, the speed of light, though enormous, is not infinite. It is perhaps the limit to all speeds in the universe and according to the inexorable laws of nature, nothing can perhaps exceed the speed of light.

The sun is about 93,000,000 miles away from the earth and light, travelling with the above speed takes about 8·3 minutes to cover the distance. But there are stars and planets in the heaven which are very much further away from the earth than the sun and light from these heavenly bodies takes much longer time to reach the earth. If any incident observed at any moment from the earth in those heavenly bodies, then it should be borne in mind that the incident happened long ago. For example the light that reaches the earth at this moment from the nearest fixed star '*Alpha centauri*', started its journey about 4·4 years ago. If the brightest star in the heaven '*Sirius*' suddenly dies out to-day, it will continue to send light to the earth for about 8·8 years more. It may, therefore, be said that many of the stars that we see in the heaven to-day, might have died out long ago !

6.4. Light year :

The countless number of stars that adorn the sky, are so far apart from one another that an attempt to express their distances in miles would yield staggering figures. The astrophysicists use '*light year*' as the unit of length to express such astronomical distances.

Definition : *Light year is the distance that light travels in one year with a velocity of 186,000 miles/sec or 300,000 Kilometres/sec.*

So $1 \text{ light year} = 186{,}000 \times 365 \times 24 \times 60 \times 60$ miles

$= 5{\cdot}86 \times 10^{12}$ miles (nearly).

or, $1 \text{ light year} = 300{,}000 \times 365 \times 24 \times 60 \times 60$ kilometres

$= 9{\cdot}46 \times 10^{12}$ kilometres (nearly).

6.5. Romer's method :

Romer first determined the velocity of light by observing the eclipses of one of the satellites of the planet Jupiter.

The planet Jupiter has four bright and seven small and dim satellites. They move round the planet in their own orbits. During their orbital motion, whenever they enter into the shadow-cone of the planet, they become invisible from the earth *i.e.*, they are eclipsed. Romer kept the innermost satellite under observation because its plane of orbit coincides with that of the planet Jupiter. Now, if light travels with infinite velocity, it will take no time to travel from one place to another. It means that as soon as the satellite enters the shadow-cone of the planet, the observer on the surface of the earth will record it at once and there will be no time-lag between the commencement of the eclipse and its recording by the observer on the earth. Consequently, if the eclipses of the satellite are continuously watched the time-interval between each two successive eclipses will always be equal because the motion of the heavenly bodies, including the earth, is uniform. But actual observations made by Romer revealed that it was not so. He found that time-intervals between two successive eclipses of the satellite gradually shortened as the earth and the Jupiter approached each other and gradually lengthened when they receded from each other in their orbital motions. Romer was of opinion that such variation of time-interval was due to the fact that light travels with a finite velocity.

Jupiter makes one complete revolution round the sun in 11·86 terrestrial years. So, in their orbital motion, the earth and the Jupiter, at some time, will come nearest to each other such as E_1 and J_1 respectively (Fig. 6.1). This is known as the *position of conjunction.* If light has finite velocity, then the light emitted by the satellite of Jupiter, just at the commencement of its eclipse, has to travel a distance J_1E_1 to reach the earth. If R and r be the radii of orbits of the earth and Jupiter respectively then, $J_1E_1=R-r$ and therefore, the eclipse will be seen from the earth $\frac{R-r}{C}$ seconds after the commencement, where C is the velocity of light.

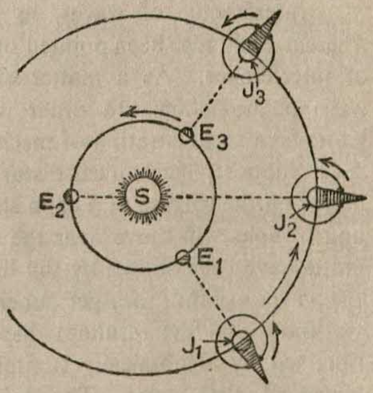

Fig. 6.1

After a period about seven months, the earth and the Jupiter will move farthest from each other such as E_2 and J_2 respectively. This position is called the *position of opposition.* Suppose at this moment, the satellite suffers *nth* eclipse. It will be seen on the earth $\frac{R+r}{C}$ second after the commencement because $J_2E_2=R+r$.

It t be the periodic time of the satellite, then the actual time-interval between the first and the *nth* eclipse$=(n-1)t$; but the time-interval as observed from the earth is,

$$T_1=\left[(n-1)t+\frac{R+r}{C}-\frac{R-r}{C}\right]=(n-1)t+\frac{2r}{C} \quad .. \quad \text{(i)}$$

After about seven months more, the earth and the Jupiter will again come in the position of conjunction as E_3 and J_3. During this time, there will be again n number of eclipses of the satellite and the time-interval between the first and the last eclipse as observed from the earth is given by,

$$T_2=\left[(n-1)t-\frac{R+r}{C}+\frac{R-r}{C}\right]=(n-1)t-\frac{2r}{C} \quad .. \quad \text{(ii)} \quad \therefore \quad T_1-T_2=\frac{4r}{C}.$$

The value of (T_1-T_2) as observed by Romer was 33 minutes. Taking the value of r as 9×10^7 miles, the velocity of light comes out as $C=186{,}000$ miles/sec. (nearly).

The result obtained from this method cannot be very correct because (i) the orbits of the earth, the satellite and the Jupiter were taken circular but they are actually elliptical (ii) the radius of the earth's orbit (r) was not known correctly at that time.

The most recent and correct value of the velocity of light in vacuum is as follows :

$$\begin{aligned} C&=299{,}774\pm6 \text{ kilometres/sec.}\\ &=2{\cdot}99774\times10^{10} \text{ cm/sec.}\\ &\fallingdotseq 186{,}285 \text{ miles/sec.}\end{aligned}$$

6.6. Interference of light :

Interference of waves, in general, had been mentioned earlier in art. 4.8 wherein it had also been pointed out that light waves also exhibit the phenomenon of interference. As a matter of fact, interference of light proves that light is a wave phenomenon. In order to understand the interference of light clearly, let us take an illustration of mechanical wave.

Suppose, mechanical disturbances of identical nature are being made at *A* and *B* on the surface of a calm sheet of water. Waves consisting of circular crests and troughs will move over the surface of water from *A* and *B*. In the fig. 6.2 crests have been shown by full line circles and troughs by dotted line circles. As the waves expand, they get superimposed. From the principle of superposition, we know that the resultant displacement at a point on the surface of water is obtained by algebraically summing up the displacements produced by the two waves at that point. The points where a crest is falling on a trough (the intersections of continuous lines and broken lines) are marked by putting 0 there. At these points waves interfere destructively with each other and the resultant displacements at those points are zero. Consequently, the intensity there is also zero. From the figure it is seen that those points lie on straight lines passing through points *G*, *F*, etc. After half the time-period of the wave, crest will be converted into a trough and vice versa, but those points will remain as points of destructive interference. There will be no flow of energy through those regions.

Again the points where a crest falls on a crest or a trough on a trough (intersections of two full lines or two broken lines respectively) are marked by putting × there. It is clear that the displacements at those points will be doubled or the intensity quadrupled. Those points lie on straight lines passing through *C*, *D*, *E*, etc. The waves are said to interfere constructively at those points. After half the time period of the wave, a crest will change into a trough and a trough into a crest, but all those points will always produce constructive interference. It is needless to mention that maximum flow of energy takes place through those regions.

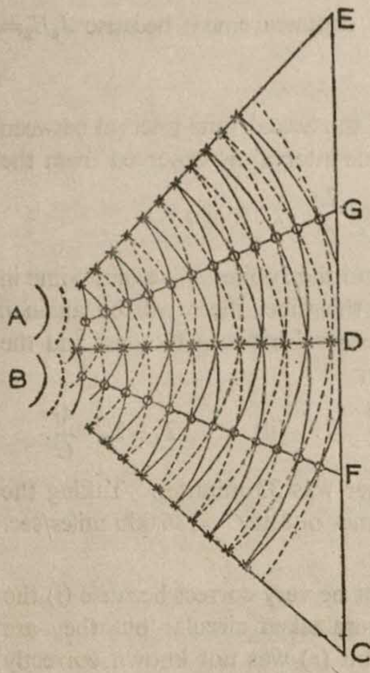

Fig. 6.2

The same thing happens in the case of light. Instead of the surface of water, if it is supposed that *A* and *B* are two optical sources continuously sending out identical waves in the surrounding optical medium and *CDE* is a screen, then we will find very bright points at *C*, *D*, *E* etc. and dark points at *F*, *G* etc. In other words, the screen will be intersected by alternate bright and dark points. This was first experimentally demonstrated by Young in 1801.

6.7. Young's experiment :

Sun rays were allowed to pass through a narrow slit (S) placed normal to the plane of the paper and were incident on two very fine slits A and B parallel to S and separated by a distance of about half a millimeter [Fig. 6.3]. In passing through the slits A and B, the waves were split up into two parts, which while travelling through the medium got superposed on each other and caused interference. Interference bands C, D, E etc. were observed on the screen in the region where light fell from both the sources A and B. The bands looked like as shown in fig. 6.4. In the figure F, C, D, etc. denote bright coloured bands. The bands were coloured because the sun light is white. With monochromatic light, alternate bright and dark bands were noticed. The bands disappeared when one of the slits A or B was covered. In this way, Dr. Thomas Young demonstrated experimentally the phenomenon of interference and hence established the wave nature of light.

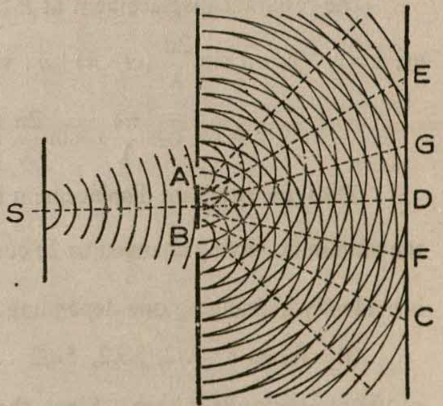

Fig. 6.3

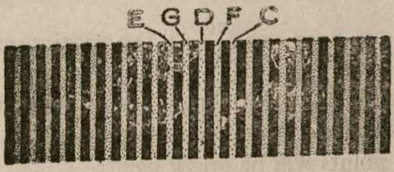

Fig. 6.4

In this connection, it may be mentioned that from the interference phenomenon, the wavelength of light can be determined.

6.8. Mathematical analysis of interference :

Let S_1 and S_2 be two sources of monochromatic light, each emitting waves of wavelength λ. Let the waves start from the sources in phase and get superphased at a point P [Fig. 6.5]. From the principle of superposition, we know that the resultant displacement at P at any instant is the algebraic sum of the displacements at P due to the individual waves. If the waves, coming from the source S_1 produce displacement y_1 at P at an instant t, then $y_1 = a \sin \frac{2\pi}{\lambda}(ct-x)$ where $a =$ amplitude of the wave; $c =$ velocity of light waves; $x =$ distance of the point P from the source S_1.

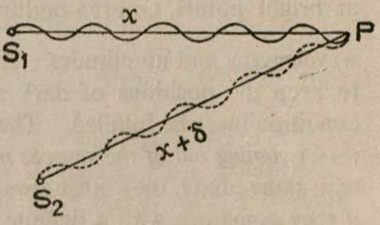

Fig. 6.5

Similarly, if the waves coming from the source S_2 produce displacement y_2

at P at the same instant, then $y_2 = a \sin \frac{2\pi}{\lambda} [ct-(x+\delta)]$ where δ=path-difference of the point P from the sources S_1 and S_2.

The resultant displacement at P from the principle of superposition, is given by, $y=y_1+y_2=a \sin \frac{2\pi}{\lambda}(ct-x)+a \,.\, \sin \frac{2\pi}{\lambda}[ct-(x+\delta)]$

$$=2a.\cos\frac{\pi\delta}{\lambda}\cdot\sin\frac{2\pi}{\lambda}\left[ct-\left(x+\frac{\delta}{2}\right)\right] \quad .. \quad \text{(i)}$$

Thus, the resultant displacement has the same wave-form as the original ones, the amplitude having changed to $2a \cos \frac{\pi\delta}{\lambda}$ in place of a. The resultant amplitude is evidently a changing one depending on the path-difference δ.

(i) When $\delta=\lambda/2$, $3/\lambda2$, $5\lambda/2$ .... $(2n+1)\lambda/2$, we have $\cos\frac{\pi\delta}{\lambda}=0$ *i.e.* the resultant amplitude is zero. Since the intensity of light at a point is proportional to the square of the amplitude there, we can say that all points where the path-difference δ is an *odd multiple* of $\lambda/2$, the intensity will be zero and these points will be *dark points*.

(ii) When $\delta=0$, $2\lambda/2$, $4\lambda/2$ .. $2n.\lambda/2$ we have $\cos\frac{\pi\delta}{\lambda}=1$ the maximum value and the resultant amplitude=$2a$=double the amplitude of the individual wave. Hence at all points where the path-difference δ is an even multiple of $\lambda/2$, the intensity of light will be maximum and the points will be *bright points*.

6.9. Conditions of permanent interference :

Mathematical analysis shows that at dark points the interfering waves should have a path-difference equal to an odd multiple of $\frac{\lambda}{2}$ (λ being the wavelength) and at bright points, an even multiple of $\frac{\lambda}{2}$. Further, the waves should have equal wavelengths and amplitudes. But to produce permanent interference pattern *i.e.* to keep the positions of dark and bright points permanent, another important condition must be fulfilled. The condition is that the *phase-relation between the waves coming out of the sources must always be maintained.* If the waves come out with same phase, they must always come out with same phase ; on the other hand, if they come out with a definite phase-difference, they must maintain that phase-difference always. If, for some reason, there be any change of phase in one source, there must, at once, be the corresponding change of phase in the other. If this does not happen, the positions of dark and bright points will rapidly change. Eye will not be able to follow such quick change. Eye will see a general illumination instead of dark and bright fringes. The following are, therefore, the conditions of *permanent interference pattern.*

(i) The sources must continuously emit waves of equal wavelength and equal amplitude.

(ii) The sources must maintain a constant phase-relation between them. If there be any change of phase in any of them, there must be the corresponding change in the other at the same time. Such sources are called **coherent sources.**

(iii) The sources must be close together otherwise the dark and bright points may come very near to each other and may get mixed up.

(iv) At the dark points, the path-difference between the waves should be equal to an odd multiple of half the wavelength and at bright points, the path difference should be equal to an even multiple of half the wavelength.

Why two identical candles cannot produce permanent interference fringe ?

From the conditions stated above, we can easily understand why waves coming from two separate but identical sources, like two candles or two sodium vapour lamps, cannot produce permanent interference fringe.. Light is emitted not by bulk matter as a whole but by individual atoms. Sound, on the other, is emitted by bulk matter like the whole prong of a tuning fork. When light is given out by a candle, millions of atoms take part in the process. Hence, a constant phase relation between two independent sources is meaningless because we really have two separate sets of millions of atoms. Phase relation between two such sources continually changes causing a continual change in the positions of dark and bright points and thus no interference can be seen.

Example : *Two sources A and B are sending out waves of wavelength* $\lambda=4$ *cm. and of equal intensity and the separation AB is 2 cm. Find the intensity of the resultant waves for points along* $\overrightarrow{AB}$, *along* $\overrightarrow{BA}$ *and along the perpendicular bisector of AB when* (*i*) *A and B are in same phase* (*ii*) *A and B are in the opposite phase.*

Ans. (i) When the sources are in the same phase, the only phase difference is due to the path-difference ($BE-AE$) where E is a point on AB produced. Along the perpendicular bisector *i.e.* when E coincides with D which is on the perpendicular bisector, the path-difference is zero. Hence, the maximum amplitude is $2a$ and the maximum intensity $4I$, where a and I are amplitude and intensity respectively, due to a single source.

Along $\overrightarrow{AB}$, the path-difference$=BE-AE=2$ cm.$=-\lambda/2$. Along $\overrightarrow{BA}$, the path-difference$=+\lambda/2$. Hence the minimum intensity is zero along $\overrightarrow{AB}$ or $\overrightarrow{BA}$.

(ii) When the sources are in the opposite phase or have a phase difference of π, the source A remaining ahead of B, the point D on the perpendicular bisector will have a phase difference of π due to the sources only because the path-difference of D from A and B *i.e.* $AD-BD=0$. Hence in this direction the intensity will be zero.

Along $\overrightarrow{AB}$, we have $BE-AE=-\lambda/2$, but as A is ahead of B by phase π, this introduces a path difference of $+\lambda/2$ between them. Hence, the net path-difference being zero, the waves are in same phase and maximum intensity of $4I$ is produced.

Along $\overrightarrow{BA}$, we have path-diff. $= +\lambda/2$ but initial phase-diff. between A and B being $+\lambda/2$, the net path-diff. $= \lambda$ which again produces maximum intensity $4I$.

6.10. Diffraction of light ; approximate rectilinear motion of light :

From geometrical optics we know that light travels in straight line. Production of shadow of an opaque obstacle with sharp edge held in the path of the light, is usually taken as a proof of rectilinear propagation of light. But a close examination of the shadow shows that it is not so sharp as exact rectilinear propagation of light demands. The shadow is much broader than expected. This deviation from a very sharp shadow leads us to believe that light can slightly bend round corners and propagation of light is approximately rectilinear. This phenomenon is known as **diffraction of light.**

Diffraction is a characteristic of wave motion. With the increase of wavelength, the phenomenon of diffraction also increases. Sound, for example, has longer wavelength and its diffraction is also appreciable. When we speak in one room, the sound can easily be heard in the adjoining room. In this case, sound waves turn easily round the corner and enter the adjoining room. But light possesses very limited diffraction power because its wavelength is extremely small. For this reason, light on one side of a wall cannot be seen from the other side.

Experiment : S is a very small source of light and PQ an obstacle with sharp edge in front of it (Fig. 6.6). MN is a screen behind the obstacle. The laws of geometrical optics give the shadow AB on the screen MN. The edges of the shadow AB ought to be very sharp as light travels in straight lines. The portions of the screen above the point A and below the point B will receive light from the source and will be illuminated. But a careful examination of the shadow reveals that darkness does not abruptly commence from the points A or B *i.e.*, the edges of the shadow are not very sharp. Light is found to encroach a little in the geometrical shadow below A and above B.

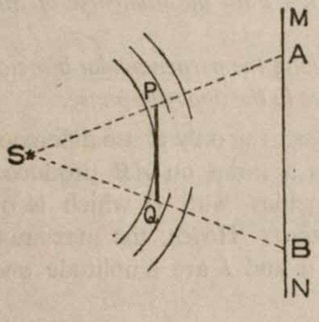

Fig. 6.6

The encroachment of light in the geometrical shadow was first observed by Grimaldi (1618—1663). Sir Isaac Newton tried to explain the phenomenon by his corpuscular theory but failed. Later, in 1815, Fresnel gave a satisfactory explanation of it on the basis of wave theory and established conclusively the wave nature of light. Fresnel not only explained the phenomenon of diffraction but also proved that light travels approximately in straight line.

It is worthwhile to mention, that like interference, diffraction can also be used to determine the wavelength of light.

6.11. Polarisation of light :

The large group of experiments, illustrating the diffraction and interference of light, is usually regarded as a proof that light, like sound, is a wave motion.

These experiments do not, however, reveal the kind of waves we are dealing with *i.e.* whether the waves are longitudinal or transverse. This information is obtained by a new phenomenon known as *polarisation of light*.

Huygens first observed polarisation of light in 1690. We will see later that only transverse waves exhibit polarisation. Since light exhibits polarisation, we can regard light as a transverse wave motion. It may be mentioned here that sound waves in air, being longitudinal, have no polarisation effect.

Experiment with tourmaline crystal : Tourmaline crystal is a chemical composition of oxides of several metals. It is a hexagonal crystal, almost transparent with a tinge of greenish colour. The largest diagonal of its cross-section (*ab* in the fig. 6.7) is called its optic axis.

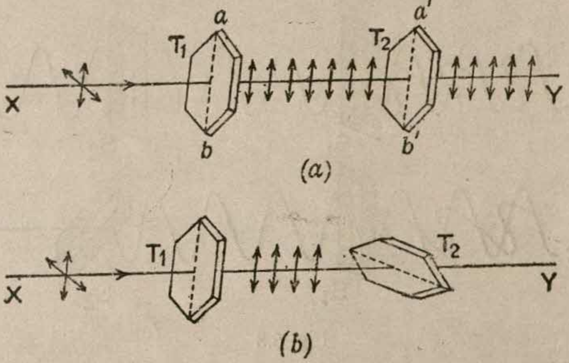

Fig. 6.7

T_1 is a thin slice of tourmaline crystal, upon whose flat surface a beam of light, travelling along the line XY, is incident normally. Some of the light emerges on the other side of the crystal, with almost unchanged intensity but probably with some colouration. Now, if the crystal be rotated about the line XY as axis, the emergent ray will not undergo any change of intensity or characteristics. The emergent beam is, then, allowed to fall on another similar crystal T_2. If the optic axis $a'b'$ of the second crystal be parallel to the optic axis *ab* of the first, the light will come out of the second crystal T_2 in its full strength [Fig. 6.7(*a*)]. But, keeping the crystal T_1 fixed if the crystal-T_2 be slowly rotated about the line XY as axis, the intensity of the emergent beam will gradually diminish. Light will be completely cut off when the two axes of the crystals will be mutually perpendicular to each other [Fig. 6.7(*b*)]. In this condition, the two crystals are said to be in **crossed position.**

If the crystal T_2 be further rotated in the same direction, the axes will again approach the parallel position, and the light will also begin to emerge slowly with increasing intensity from the crystal T_2. When the second crystal T_2 is rotated through 180°, the axes will be again parallel, and light will come out of the second crystal with its initial intensity.

From the above experiment of the tourmaline crystals, we know that the beam of light, passing through the first crystal, acquires a one-sided property due to which it can pass through the second crystal when the axes of the crystals are parallel to each other and can be totally stopped when the axes at right angles to each other. This characteristic property of light is called *polarisation*.

The phenomenon can be best understood by the following mechanical illustration. A rubber cord CD passes through two narrow slits S_1 and S_2 in cardboard screens [Fig. 6.8]. One end D is fixed ; a longitudinal wave is set up in CD by moving C forward and backward along the string. The wave passes through the first and second slits without being affected at all, in whatever position the slits may be arranged. A longitudinal wave motion has the same properties with respect to all planes throughout its line of advance.

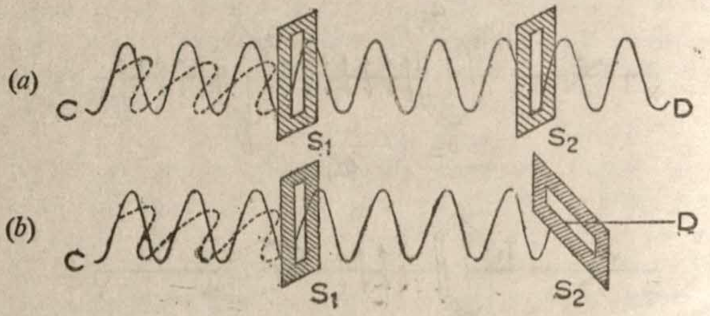

Fig. 6.8

Suppose C is now moved in a circular manner or is vibrated haphazardly in directions perpendicular to CD, the portion of the cord from C to S_1 shows circular motion, beyond S_1 it shows linear motion parallel to the length of the slit S_1 the other components of vibration being cut off by the slit S_1. This linear motion parallel to S_1 passes through S_2 if the latter is parallel to the former [Fig. 6.8(a)]. If S_2 is perpendicular to S_1 [Fig. 6.8(b)] these vibrations are also stopped and the portion S_2D of the cord remains undisturbed.

Let us now come back to the experiment of tourmaline described a little while ago. The second crystal T_2, in its crossed position, can totally stop the light—this shows that light cannot be longitudinal wave motion because we have seen that the longitudinal wave motion is not affected whatever may be the positions of the slits S_1 and S_2. Hence, we may regard ordinary light as transverse wave motion in which the particles of the medium may vibrate in all possible directions in a plane perpendicular to the direction of propagation of the wave (Fig. 6.9). This ordinary light is known as **unpolarised light.** When a beam of unpolarised light passes through a tourmaline crystal T_1, the crystal absorbs all other vibrations except those which are parallel to its optic axis ab [Fig. 6.7(a)]. As a result, the emergent beam contains vibrations which are confined in a particular direction in a plane perpendicular to the direction of the

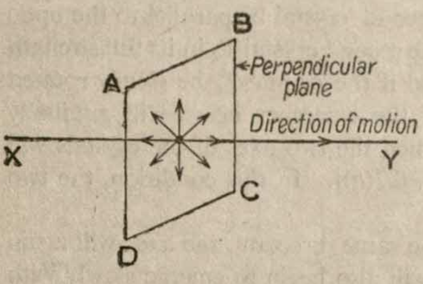

Fig. 6.9

beam. This *one sidedness* is known as polarisation and the beam is called a polarised beam. The polarised beam can pass through the second crystal T_2 when the axis of the crystal T_2 is parallel to the vibration of the polarised beam and is completely cut off when the said axis is at right angles to the vibration of the polarised beam.

From the above experiment of tourmaline crystals we come to know the following :—

(i) Light is a transverse wave motion ; the vibrations of the particles of the medium in a plane perpendicular to the propagation of light create such waves and the light is known as ordinary or unpolarised light.

(ii) When a beam of unpolarised light falls on a tourmaline crystal, the crystal absorbs all vibrations except one in a particular direction, giving the beam a property of one sidedness. The beam, in that condition, is known as a polarised beam. In fig. 6.7(*a*), the crystal T_1 which produces polarisation, is known as **polariser.**

(iii) Whether the beam is polarised or not, may be tested by another similar crystal T_2. If the beam is polarised, the crystal T_2 will stop it totally in a particular position. For this reason, the crystal T_2 in the fig. 6.7(*a*) is called an **analyser.**

(iv) When the polarised beam, coming out of the first crystal is totally cut off by the second, the polariser and the analyser are said to be in **crossed** position.

Exercises

Essay type :

1. Mention and describe, in brief, two theories advanced for the purpose of explaining the nature of light,

2. What is the nature of light waves according to the modern ideas ?

3. Explain Romer's method of determining the velocity of light.

4. What is interference of light ? Describe Young's experiment to demonstrate interference of light. [*H. S. Exam. 1980*]

5. Explain what you know about polarisation. How does it help to determine the character of a wave ? Can you obtain polarisation of sound waves ? [*H. S. Exam. 1980*]

Short answer type :

6. What is the value of the velocity of light ? 'Many of the stars that we see in the sky, might have been extinct long ago.'—Explain the statement.

7. What is that quantity whose unit is 'light-year' ? What is its magnitude ?

8. What do you know of the nature of light from interference ? Enumerate the conditions of permanent interference pattern.

9. Two identical candles placed side by side will not produce interference fringe. Why ?

10. The two slits in a Young's double slit experiment are illuminated by two different sodium lamps emitting light of the same wavelength. No interference pattern will be observed, on the screen. Is it correct ? [*I. I. T. 1984*]

11. What is the difference between light waves and sound waves ? [*H. S. Exam. 1979*]

12. How do you know that light does not travel exactly in straight line ? What is that phenomenon called ?

13. How is light wave polarised ? If plane-polarised light is seen through an analyser which is gradually rotated ,what will be observed ? [*H. S. Exam. 1980*]

Objective type :

14. Select the correct answer :—

(i) What is the nature of light waves ? *Ans.* (a) longitudinal, (b) transverse, (c) stationary, (d) not at all waves but particle.

(ii) What type of medium does light wave require for propagation ? *Ans.* (a) solid, (b) liquid, (c) ether (d) vacuum (e) no material medium.

(iii) If two coherent sources of white light are used to form interference pattern what will be the nature of pattern ? *Ans.* (a) dark and bright fringes (b) no fringes at all (c) coloured fringes.

(v) What type of wave does not show polarisation ? *Ans.* (a) stationary (b) transverse, (c) longitudinal.

APPENDIX

*** Moment of inertia of bodies having simple shapes.**

(i) *Moment of inertia of a thin uniform rod rotating about an axis passing through its centre and perpendicular to its length.*

In fig. 1, AB represents a thin uniform rod of mass M and length l. The axis of rotation CD passes through its middle point O and is perpendicular to its length. Since the rod is uniform, its mass per unit length$=\frac{M}{l}$. Consider an element of length dx at a distance x from its axis of rotation. The mass of this element$=\frac{M}{l}\times dx$. Since the element is very thin, all of its particles may be supposed to be equidistant from the axis and the distance$=x$. Hence, the moment of inertia of the element about the given axis$=\frac{M}{l}\times dx\times x^2$.

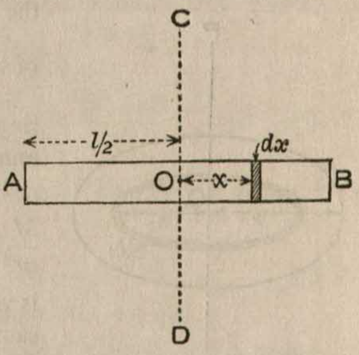

Fig. 1

For finding the moment of inertia I of the whole rod, the above expression is to be integrated between the limits $x=0$ and $x=\frac{l}{2}$ and then multiplied by 2 to include the other half AO.

$$\therefore\ I=2\int_0^{\frac{l}{2}}\frac{M}{l}.x.^2.dx=\frac{2M}{l}\int_0^{\frac{l}{2}}x^2.dx=\frac{2M}{l}\left[\frac{x^3}{3}\right]_0^{\frac{l}{2}}=\frac{2M}{l}\times\frac{l^3}{24}=\frac{Ml^2}{12}$$

(ii) *Moment of inertia of the above rod when the axis of rotation passes through one end and is perpendicular to its length.*

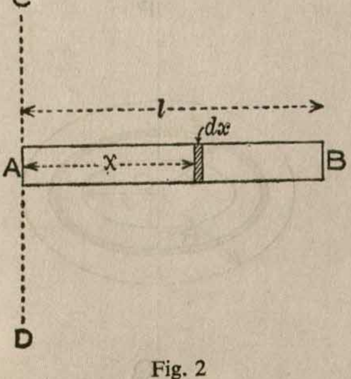

Fig. 2

The axis of rotation CD now passes through the end, say A, and is perpendicular to the length of the rod [Fig. 2]. The mass of the rod$=M$ and its length$=l$. Since the rod is uniform, the mass per unit length of the rod$=\frac{M}{l}$. As before, consider a very thin element of length dx at a distance x from the axis. The moment of inertia of the element about the axis $CD=\frac{M}{l}\times dx\times x^2$. To get the moment of inertia I of the whole rod, the above expression is to be integrated between limits $x=0$ and $x=l$.

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* Included in the syllabi of I. S. C., S. S. C., J. E. E. and I. I. T examinations.

$$\therefore\ I=\int_0^l \frac{M}{l}.\ x^2.dx=\frac{M}{l}\int_0^l x^2.dx=\frac{M}{l}\left[\frac{x^3}{3}\right]_0^l=\frac{Ml^2}{3}.$$

(iii) *Moment of inertia of a thin circular disc about an axis passing through its centre and perpendicular to its plane.*

Fig. 3 shows a thin circular disc of mass M and radius r. The axis of rotation PQ passes through the centre O and is perpendicular to its plane. Now, the area of the disc$=\pi r^2$ and the mass per unit area of the disc$=\frac{M}{\pi r^2}$. Consider a thin concentric annular ring of radius x and width dx. The area of this annular ring=length of the circumference×width $=2\pi x dx$. Hence, the mass of the annular ring$=\frac{M}{\pi r^2}\times 2\pi x.dx=\frac{2M}{r^2}.x.dx$. Since, the width of the ring is very small, all of the particles of the ring are at the same distance from the axis PQ and this distance is x. So, the moment of inertia of the annular ring about the given axis$=\frac{2M}{r^2}x.dx\times x^2=\frac{2M}{r^2}.x^3.dx$.

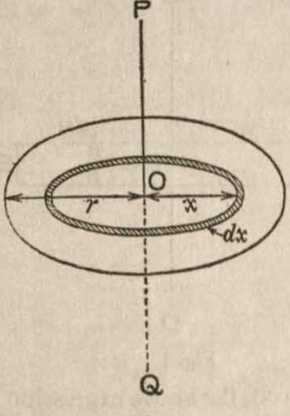

Fig. 3

Since the whole disc may be supposed to be divided into numerous such concentric rings of radii ranging from zero to r, we can get the moment of inertia I of the entire disc by integrating the above expression between limits $x=0$ to $x=r$. Thus, the moment of inertia of the disc

$$I=\int_0^r \frac{2M}{r^2}.x^3.dx=\frac{2M}{r^2}\int_0^r x^3.dx=\frac{2M}{r^2}\left[\frac{x^4}{4}\right]_0^r=\tfrac{1}{2}M.r^2$$

(iv) *Moment of inertia of a flat ring about an axis passing through its centre and perpendicular to its plane.*

If a concentric circular portion is cut out from a thin circular disc, we get a flat ring. Suppose the external and internal radii of such a flat ring are R and r respectively, and its mass$=M$. The axis of rotation PQ passes through its centre O and is perpendicular to its plane [Fig. 4]. The surface area of the flat ring$=\pi R^2-\pi r^2=\pi(R^2-r^2)$. So mass per unit area of the ring$=\frac{M}{\pi(R^2-r^2)}$.

The flat ring may be supposed to be composed of numerous thin concentric rings, one of which has a radius x and the width dx as shown in the figure. The

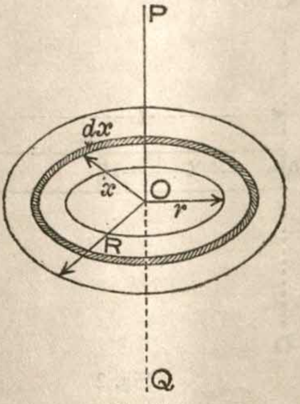

Fig. 4

surface area of the thin ring$=2\pi x.dx.$ and its mass$=\dfrac{M}{\pi(R^2-r^2)}\times 2\pi x.dx$

$$=\frac{2Mx}{(R^2-r^2)}.\ dx.$$

As the ring is very thin, its moment of inertia about the axis PQ $=\dfrac{2Mx}{(R^2-r^2)}\ .\ dx\times x^2=\dfrac{2Mx^3}{(R^2-r^2)}.dx.$ Now, integrating this expression between limits $x=R$ and $x=r$, the moment of inertia of the flat ring,

$$I=\int_r^R \frac{2Mx^3}{(R^2-r^2)}dx=\frac{2M}{(R^2-r^2)}\left[\frac{x^4}{4}\right]_r^R=\frac{2M}{(R^2-r^2)}\times\frac{(R^4-r^4)}{4}=\frac{M}{2}(R^2+r^2).$$

Note : If $R=r$ *i.e.*, if the body in question is a thin circular ring of radius R, $I=\dfrac{M}{2}(R^2+R^2)=M.R^2.$

(v) *Moment of inertia of a hollow cylinder about its own axis.*

Consider a hollow cylinder of external and internal radii R and r respectively and mass M rotating about its own axis PQ [Fig. 5]. Cross-sectional area of the hollow cylinder$=\pi(R^2-r^2)$. If l be the length of the cylinder, its volume $=\pi(R^2-r^2).l.$ Hence, the density of the material of the cylinder$=\dfrac{M}{\pi(R^2-r^2)l}.$ Suppose, the hollow cylinder is made up of a large mumber of thin co-axial cylinders, one of which has a radius x (say) and width dx. The cross-sectional -area of the thin annular cylinder$=2\pi x.dx.$ and its volume$=2\pi x.dx.l.$ So, the mass of the annular cylinder$=\dfrac{M}{\pi(R^2-r^2)l}\times 2\pi x.dx.l=\dfrac{2Mx}{(R^2-r^2)}.dx.$

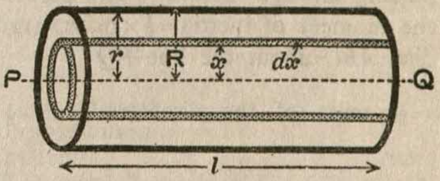

Fig. 5

As the annular cylinder is very thin, all of its particles may be supposed to be equidistant from the axis of rotation PQ and this distance is x. Hence, the moment of inertia of the annular cylinder$=\dfrac{2Mx}{(R^2-r^2)}.dx\times x^2=\dfrac{2M}{(R^2-r^2)}\ x^3.dx.$

Now integrating the above expression between $x=R$ and $x=r$, we get the moment of inertia of the whole cylinder. So, the required moment of inertia,

$$I=\int_r^R \frac{2M}{(R^2-r^2)}\ x^3.dx=\frac{2M}{(R^2-r^2)}\left[\frac{x^4}{4}\right]_r^R=\frac{2M}{(R^2-r^2)}\ \frac{(R^4-r^4)}{4}=\frac{M}{2}(R^2+r^2).$$

[**Note** : If $r=0$, the cylinder becomes solid and in that case, $I=\dfrac{M.R^2}{2}$]

(vi) *Moment of inertia of a solid sphere rotating about any of its diameters.*

Consider a solid sphere of radius r and mass M. O is the centre of the sphere and the diameter PQ is the axis of rotation. The volume of the sphere$=\frac{4}{3}\pi r^3$ and its density $\rho=\frac{M}{\frac{4}{3}\pi r^3}=\frac{3M}{4\pi r^3}$.

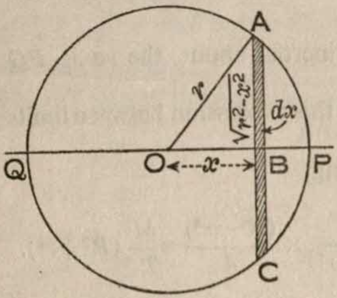

Fig. 6

Suppose the solid sphere is divided into a large number of thin circular discs, with their planes perpendicular to the line PQ. Consider such a thin disc at a distance x from the centre O and of thickness dx [Fig. 6]. It is easy to see that the line PQ passes through the centre of this disc. So, the radius of the disc$=AB=\sqrt{r^2-x^2}$. Hence the surface area of the disc$=\pi(\sqrt{r^2-x^2})^2=\pi(r^2-x^2)$ and its volume$=\pi(r^2-x^2).dx$.

$\therefore$ Mass of the disc$=$volume$\times$density $(\rho)=\pi(r^2-x^2)dx\times\frac{3M}{4\pi r^3}=\frac{3M}{4r^3}(r^2-x^2)dx$.

Now, as the sphere rotates about the diameter PQ, the disc also rotates about the same line. In the case of a circular disc rotating about a line passing through the centre and perpendicular to the plane of the disc, we know the moment of inertia$=\frac{1}{2}\times$mass$\times$(radius)^2. So, the moment of inertia of the disc ABC about the line PQ

$$=\tfrac{1}{2}\times\text{mass of the disc}\times(\text{radius})^2=\tfrac{1}{2}\times\frac{3M}{4r^3}(r^2-x^2)dx\times(\sqrt{r^2-x^2})^2$$

$$=\frac{3M}{8r^3}(r^2-x^2)^2.dx.$$

Now integrating the above expression between the limits $x=0$ and $x=r$, and then multiplying by 2, we get the moment of inertia of the whole sphere. So, the required moment of inertia,

$$I=2\int_0^r\frac{3M}{8r^3}(r^2-x^2)^2.dx=\frac{3M}{4r^3}\int_0^r(r^4-2r^2x^2+x^4)dx.$$

$$=\frac{3M}{4r^3}\left[r^4\int_0^r 1.dx-2r^2\int_0^r x^2dx+\int_0^r x^4dx\right]$$

$$=\frac{3M}{4r^3}\left[r^5-2r^2\times\frac{r^3}{3}+\frac{r^5}{5}\right]=\frac{3M}{4r^3}\times\frac{8r^5}{15}=\frac{2}{5}.Mr^2$$

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WEST BENGAL HIGHER SECONDARY EXAMINATION, 1984

PHYSICS—First Paper

Group 'A'

1. (*a*) Establish graphically the equation $s=ut+\frac{1}{2}ft^2$ where u, f, s and t have usual meaning.

(*b*) Uniform retardation is produced by applying brake when a train is moving with uniform speed of 45 miles per hour and the train comes to a stop after one minute. Calculate the rate of retardation and find also the distance traversed by the train after the application of the brake. [Ans. 1·1 ft/sec^2 ; 1980 ft]

2. (*a*) Write down the laws of static friction. What do you mean by the angle of repose of an inclined plane ? Obtain the relation between the coefficient of friction and the angle of repose.

(*b*) A body, moving on the ground with a speed of 15 miles per hour, comes to rest. The coefficient of friction between the body and the ground is 0·22. Calculate the distance traversed by the body on the ground before it comes to a stop. [Ans. 1·1 ft (nearly)]

Or, Give an example of the disadvantage of friction. How is it minimised ?

3. (*a*) Define work and power. Write down their units in the M. K. S. system.

(*b*) Obtain the relation between Horse Power and Watt.

(*c*) A train moves with the speed of 50 miles/hour against a force of 1,200 lb. wt. Calculate the power of the engine in H.P. [Ans. 80]

4. (*a*) What do you mean by fictitious force ? Why is the centrifugal force known as a fictitious force ?

(*b*) How is the weight of a terrestrial body affected by the earth's diurnal rotation ?

(*c*) A stone of mass 500 gms is tied at one end of a thread and is rotated along a circular path of diameter 200 cm in a vertical plane with the speed of 4 metre/sec. Calculate the tension of the thread when the stone is at its highest and lowest positions. $g=980$ cm/sec^2. [Ans. $3·1\times10^5$ dynes ; $12·9\times10^5$ dynes]

Group 'B'

5. (*a*) Discuss how the acceleration due to gravity changes as one goes about the earth's surface and also within the earth. What would be the weight of a body, placed at the centre of the earth ? A straight tunnel is dug passing from the north-pole to the south-pole of the earth. What would be the motion of a body if it is dropped into the tunnel ?

(*b*) A body is dropped from a balloon when it is at a height of 390 metres from the earth's surface during its ascent. It is found that the body reaches the ground after 10 seconds. If the acceleration due to gravity be 980 cm/sec^2, calculate the speed of ascent of the balloon at the time when the body is dropped from it. [Ans. 10 metre/sec]

6. (*a*) State Archimedes' Principle and verify the same experimentally.

(*b*) The weights of a body in air and water are respectively 50 gms and 30 gms. What would be its weight in a liquid of sp. gr. 0·8 ? Calculate the sp. gr. of the body in relation to the liquid. [34 gm ; 3·12]

7. (*a*) What do you mean by the standard atmospheric pressure ? What will be your conclusion if there is sudden drop of height of mercury in a barometer ? Calculate the value of the standard atmospheric pressure in the C.G.S. unit. g=980 cm/sec^2 ; Density of mercury=13·6 gm/c.c. [Ans. 1·013×10^6 dynes/cm^2]

(*b*) Explain the principle of action of a siphon. Explain under what conditions a siphon does not work.

8. (*a*) Define the coefficient of linear expansion of a soild. Establish the relation between the coefficient of linear expansion and the coefficient of superficial expansion of a soild.

(*b*) The density of brass at 0°C is 7·8 gm/c.c. Calculate its density at 100°C. Coefficient of linear expansion of brass=18·0×10^{-6}/°C. [Ans. 7·76 gm/c.c.]

(*c*) Two plates of copper and iron of identical shape and size are rivetted together. What will happen to the combined system if the temperature be raised ? Explain your answer with a diagram. Give an example of the practical application of such a bimetallic strip.

9. (*a*) Define the pressure and volume coefficients of a gas. Prove that in the case of an ideal gas these two coefficients are equal.

(*b*) Define the sp. heat and the latent heat of fusion. A calorimeter of water equivalent 30 gms contains 270 gms of water at 30°C. A piece of ice of mass 10 gms at −10°C is dropped into the calorimeter. Calculate the final temperature of the mixture. Sp. heat of ice=0·5. Latent heat of fusion of ice=80 cal/gm. [Ans. 26·3°C (nearly)]

10. (*a*) Define dew point and relative humidity. How are they affected by the temperature of a room ?

The temperature and the dew point in a room are respectively 26·5°C and 14°C on a certain day. The aqueous tensions at 14°C, 26°C and 27°C are respectively 12·00 mm, 25·00 mm and 26·60 mm of mercury. Calculate the relative humidity of the room. [Ans. 46·5%]

(*b*) What do you mean by mechanical equivalent of heat ? The temperature of water at the top of a waterfall is less than that at its bottom by 0·47°C. If the work done by water in falling through the height of waterfall is fully converted into heat, calculate the height of the waterfall. g=980 cm/sec^2 ; J=4·2×10^7 ergs/cal. [Ans. 201·4 metre]

Group 'C'

11. (*a*) What do you understand by simple harmonic motion ? What is meant by time period ?

(*b*) What do you mean by forced vibration and resonance ? Explain their difference by giving examples.

12. (*a*) Show that the sum of the potential energy and kinetic energy of a particle executing S.H.M. is a constant.

(*b*) The time period and amplitude of a particle, executing S. H. M., are respectively 12 secs and 13 cm. Calculate the velocity of the particle when it is 5 cm away from the mean position of rest. [Ans. 6·28 cm/sec]

13. (*a*) How is a stationary wave formed ? What are its characteristics ?

(*b*) Show that only the odd harmonics are produced in the case of vibratio of an air column in a tube, closed at one end.

WEST BENGAL HIGHER SECONDARY EXAMINATION, 1985

PHYSICS—First Paper

Group 'A'

1. (*a*) Establish graphically the equation $v^2=u^2+2fs$ where u, v, f and s have usual meaning.

(*b*) The initial velocity of a train is 30 miles per hour and the velocity of the train becomes 15 miles per hour after moving through 363 ft with uniform retardation. How far will it move further before stopping if it moves with the same uniform retardation? [Ans. 60·5 ft]

2. (*a*) Define power and write down its units in C.G.S. and M.K.S. systems.

(*b*) What do you understand by the law of conservation of energy? Give a few examples.

(*c*) If a body of mass 100 gms be falling freely from a height in a downward direction, calculate its kinetic energy 5 seconds after the beginning of downward falling. [$g=980$ cm/sec^2] [Ans. $1200·5\times10^5$ erg.]

3. (*a*) What do you mean by relative velocity? Two particles are moving with velocities u and v respectively making an angle θ between them. Obtain the relative velocity of one particle with respect to the other.

(*b*) One ship is moving due east with a velocity of 9 kms per hour and another is moving due south with a velocity of 12 kms per hour. What would be the velocity and direction of motion of the first ship with reference to the second? [Ans. 15 km/hour, $\tan^{-1}\frac{4}{3}$ to the east]

Group 'B'

4. (*a*) State the laws of gravitational attraction and explain what you mean by universality of the law of gravitation.

(*b*) Write down and explain the laws of a freely falling body.

(*c*) A body is thrown vertically upward with a velocity of 80 ft per sec. If the acceleration due to gravity be 32 ft/sec^2 find the time of ascent. How far will it ascend and what will be its velocity? [Ans. 100 ft, $40\sqrt{2}$ ft/sec]

5. (*a*) Write down Hooke's law of elasticity. Explain how you can verify the law experimentally. What do you understand by elastic limit? Define Young's modulus of elasticity and Poisson's ratio.

(*b*) An 8 kg weight is suspended at the end of an iron wire 2 metres long and 1 mm in diameter. Calculate the elongation of the wire if Young's modulus of iron be 2×10^{12} dynes/cm^2. [$g=980$ cm/sec^2] [Ans. 0·1 cm. (nearly)]

6. (*a*) State Pascal's law of transmission of pressure within a fluid. Show how multiplication of thrust can be explained with the help of this law. Explain the statement—'The principle of conservation of energy is not violated inspite of multiplication of thrust'.

(*b*) The diameters of two pistons of a hydraulic press are 4 inches and 40 inches respectively. The arms of the lever of the press are respectively 6 inches and 3 ft. A force of 50 lb is applied at the end of the longer arm of the lever. Calculate the thrust on the larger piston. [Ans. 30000 lb.]

7. (*a*) What is the distinction between heat and temperature? Establish the relation between the Celsius and Fahrenheit scales.

(*b*) Obtain the temperature at which the readings of Celsius and Fahrenheit thermometers will be the same.

(*c*) The lower and upper fixed points of a thermometer are 0·5° and 101° respectively. What will be the reading of this thermometer at 60°C? [Ans. 6·08°]

8. (*a*) Define real and apparent coefficients of expansion of a liquid and establish the relation between them.

(*b*) The internal volume of a glass flask is v c.c. Find the volume of mercury that must be kept within the flask so that the volume above the mercury remains the same at all temperatures. Coeff. of vol. expansion of mercury=0·00018/°C. Coeff. of linear expansion of glass=0·000009/°C. [Ans. 3/20]

(*c*) An alloy contains 60% copper and 40% nickel. A piece of this alloy of mass 50 gms is heated to 50°C and then dropped into 140 gms of water at 20°C kept in a calorimeter of water-equivalent of 10 gm. Calculate the final temperature of the mixture. Sp. ht. of copper=0·09 ; Sp. ht. of nickel=0·11.
[Ans. 20·95°C (nearly)]

9. (*a*) Write down the different methods for transmission of heat and explain the difference between them. Explain why ventilators are placed near the ceiling of a room.

(*b*) There are five glass windows in a room, the area of the glass of each window and the thickness of the glass being 2 sq.m. and 2 mm respectively. The temperatures of the inner and outer surfaces of the glass are 20°C and —5°C respectively. Calculate the amount of heat flowing out from the room per min. through the windows by conduction. Conductivity of glass=0·002 C.G.S. Unit.
[Ans. 15×10^5 cal.]

Or, (*a*) Write down the basic assumptions of the kinetic theory of an ideal gas. Explain the concept of pressure and temperature of an ideal gas with the help of the kinetic theory.

(*b*) The R. M. S. velocity of hydrogen molecule at N.T.P. is 1·85 km/sec. Calculate the density of hydrogen gas. [Ans. 8·88 gm/c.c.]

Group 'C'

10. (*a*) What do you understand by beat? Explain the formation of beats.

(*b*) Two tuning forks produce 6 beats per sec. The frequency of one of th
forks is 312/sec. If the mass of a prong of the fork of unknown frequen
increased gradually by fixing small amounts of wax on it, at a time the
frequency gradually decreases to 3/sec. Calculate the frequency ow do they
Explain whether it is possible to produce 6 beats per sec. ag rnace and then
the mass of the prong further. equivalent 50 gm.

11. (*a*) Write down and explain the laws of transver (*b*) A sonometer wire of length 75 cm is in unison with a tuning fork. If the length of the wire be decreased by 3 cm then it produces 6 beats per sec. with the fork. Calculate the frequency of the fork. [Ans. 144]

12. (*a*) A person sets his watch at 27°C by hearing the sound of a gun-fire from the fort at a fixed hour and finds that the watch runs behind correct time by 10 seconds. Explain the reason. If the velocity of sound at 0°C be 330 metres/sec. calculate the distance between the person and the fort.

[Ans. 3460·8 metres]

(*b*) An engine is advancing towards a hill with a velocity of 30 miles/hour by whistling sound of frequency 224/sec. Calculate the frequency of sound which is heard by the driver of the engine after the reflection of the sound from the hill. Assume the velocity of sound in air to be 1080 ft/sec. [Ans. 305·9]

WEST BENGAL HIGHER SECONDARY EXAMINATION, 1986

PHYSICS—First Paper

Group 'A'

1. (*a*) Establish graphically the equation $S=ut+\frac{1}{2}ft^2$, where u, f, S and t have their usual meaning.

(*b*) A particle moving along a straight line with uniform acceleration describes 145 cm and 185 cm in the sixth and the tenth second respectively. What distance will it cover in the sixteenth second? [Ans. 245 cm.]

(*c*) A particle moves '*a*' metres along a straight line and then it moves '*b*' metres along a direction making angle θ with the original direction. Obtain the resultant displacement of the particle. [Ans. $a^2+b^2-2ab\cos\theta$]

2. (*a*) Define centripetal force and establish an expression for the same. Why is centrifugal force called a pseudo force?

(*b*) Explain the effect of diurnal motion of the earth on the weight of a body on the surface of the earth.

(*c*) Establish the relation between linear velocity and angular velocity.

3. (*a*) What is work? Define joule, erg and watt.

(*b*) What is meant by potential energy and kinetic energy?

(*c*) Deduce an expression for the kinetic energy of a body moving with linear velocity u.

Group 'B'

4. (*a*) Write down the laws of simple pendulum.

Explain how the height of a hill can be determined by determining the a pendulum at the foot and top of the hill.

(*b*) The th of a simple pendulum is four times the length of another the time period of the pendulum of longer length is 6 secs, od of the other pendulum.

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40 inches respectively.

5. (*a*) Define Young's modulus of elasticity. Same weight is suspended at the lower ends of two wires of same length but of different material. The diameters of the wires are 0·4 mm and 0·6 mm respectively. If the elongation of the first wire is three times the elongation of the second wire, compare the Young's modulii of the materials of the two wires. [Ans. 3 : 4]

(*b*) What should be the velocity of an artificial satellite of the earth so that it may revolve round the earth along a circular orbit 300 km above the earth's surface? Explain weightlessness of a body on an artificial satellite. What is meant by escape-velocity?

g=980 cm/sec^2 ; Radius of the earth=6400 km. [Ans. 7·62 km/sec]

6. (*a*) State Archimedes' Principle. How would you verify the same experimentally?

(*b*) A body is floating on water with a portion of the body immersed. Will the body float up or drown if the entire air above the body be removed? Give reason for your answer.

(*c*) The internal and external diameter of a hollow sphere is 6 cm and 8 cm respectively. If it floats just fully immersed in a liquid of sp. gr. 1·2, find out the density of the material of the sphere. [Ans. 2·08 gm/c.c.]

7. (*a*) What do you mean by the statement that normal atmospheric pressure is 76 cm of mercury column?

(*b*) What is a siphon? Explain its principle of action. Mention the conditions for the working of a siphon.

(*c*) Calculate the limiting height of an obstacle for siphoning kerosene oil of sp. gr. 0·80 from one vessel to another when the atmospheric pressure is 76 cm of mercury. Density of Hg=13·6 gm/c.c. [Ans. 12·92 metre]

Group 'C'

8. (*a*) Define coefficients of linear expansion, surface expansion and volume expansion of a solid. Establish the relation between them.

(*b*) A metre scale, made of steel, is correct at 10°C. What would be the correct distance between two consecutive cm-markings on this scale at 30°C and 60° F?

A brass rod measured with the help of the above scale at 30°C is found to be 5 metre long. What would be the correct length of the rod at 10°C? Coeff. of linear expansion of steel=12×10^{-6}/°C. Coeff. of linear expansion of brass=18×10^{-6}/°C. [Ans. 1·00024 cm ; 1·00072 cm ; 5·1 cm]

9. (*a*) State Boyle's law and Charles' law. Establish the equation of state for a perfect gas. How can you obtain the absolute scale of temperature from Charles' law ?

(*b*) Deduce the relation between the volume and pressure coefficient of an ideal gas.

10. (*a*) Define water-equivalent and thermal capacity. How do they differ? A block of platinum of mass 200 gms is heated in a furnace and then dropped into 650 gms of water at 10°C, kept in a vessel of water equivalent 50 gm.

If the final temperature of the mixture be 25°C, calculate the temperature of the furnace. Sp. heat of platinum=0·030. [Ans. 1·775°C]

(*b*) Distinguish between vapour and gas. Define dew point and relative humidity.

The dew point and the temperature on a certain day were 10°C and 16·5°C respectively. The saturation vapour pressures at 10°C, 16°C and 17°C are respectively 9·10 mm, 13·50 mm and 14·40 mm of mercury column. Calculate the relative humidity of the day. [Ans. 65·2%]

11. Write notes on : (*a*) Effect of pressure on melting point and boiling point. (*b*) First law of thermodynamics.

Group 'D'

12. (*a*) What is simple harmonic motion? Mention its characteristics. Show that the principle of conservation of mechanical energy holds good in such motion.

(*b*) The amplitude of a particle of mass 4 gm executing simple harmonic motion is 5 cm. The acceleration of the particle is 12 cm/sec^2 when it is 3 cm away from the mean position. Calculate the velocity of the particle and the force acting on the particle when it is 4 cm away from the mean position.

[Ans. 6 cm/sec. ; 64 dyne]

13. (*a*) What are forced vibration and resonance?

(*b*) A vibrating tuning fork of frequency 256 vibrations per sec is held near the upper open end of a vertical tube 200 cm long filled with water and water is allowed to run out slowly from the lower end. Neglecting end correction, calculate the positions of water surface in the tube for first and second resonances. Velocity of sound in air=320 m/sec. [Ans. 168·75 cm ; 106·75 cm]

14. (*a*) Write down Newton's formula for the velocity of sound in air. What is its defect? How was the formula modified by Laplace?

(*b*) Write down the difference between longitudinal and transverse wave. How would you establish that light is a transverse wave ?

WEST BENGAL HIGHER SECONDARY EXAMINATION, 1987

PHYSICS—First Paper

Group A

1. (*a*) Draw and explain the velocity—time graph of a particle moving with uniform acceleration. Explain the nature of the graph when the acceleration decreases with time.

(*b*) A train begins to move with an acceleration of 2 metre/sec^2 when a man is 9 metres away from a door of the train. The man begins to run and catches it in 3 sec. What is his acceleration ? [Ans. 4 metre/sec^2]

(*c*) Two particles are dropped from the top of a tower at an interval of 2 seconds. Find their relative velocity and relative acceleration at a moment when both of them are falling. [Ans. 2g, 0]

2. (*a*) State Newton's first and second laws of motion, and define force from the first law.

(*b*) Mass is the measure of inertia—explain.

(*c*) What do you mean by static friction and kinetic friction ?

3. (*a*) Determine the value of potential energy of a body of mass m, situated at a height h.

(*b*) Show that the law of conservation of energy is obeyed by a body falling freely under gravity.

(*c*) Show that the momentum of a body having mass m and kinetic energy E is $\sqrt{2mE}$.

Group B

4. (*a*) State Newton's law of gravitation. What is universal gravitational constant ? Find the relation between the said constant and the acceleration due to gravity.

(*b*) The mass and the radius of the earth are respectively 81 times and 4 times the values for the moon. Compare the accelerations on their surfaces. [Ans. 5.06]

(*c*) Why does a body feel weightless in an artificial satellite ?

5. (*a*) State and explain Hooke's law.

(*b*) Draw and explain the load-elongation characteristic of an elastic solid material.

(*c*) If the strain be 1% of 0.1, find the change in length of a wire 5 metre long. If the cross-section is 1 sq.mm. and load 10 kg.wt., what is the ratio of stress and strain ? [Ans. $\frac{1}{200}$; 1.96×10^{11}]

(*d*) Define Poisson's ratio and bulk modulus.

6. (*a*) Compare specific gravity and density.

(*b*) A body of density d_1 is placed slowly on the surface of a liquid of density d_2 and depth h. After time t, the body reaches the bottom of the liquid. Determine the value of t. $\left[\text{Ans. } \left\{\frac{2hd_1}{g(d_1-d_2)}\right\}^{\frac{1}{2}}\right]$

(*c*) State Pascal's law and with its help, explain the principle of operation of a hydraulic press.

7. (*a*) Explain with the help of diagram the working principle of a common pump used for lifting a water. Discuss if there is any upper limit of the height to which water can be raised by such a pump.

(*b*) If water is used in place of mercury in a barometer, what will be the height of the water column at normal atmospheric pressure ? Density of mercury 13.6 gm/c.c. [Ans. 10.34 metre]

Group C

8. (*a*) Define the real and apparent coefficients of expansion of a liquid and establish the relation between them.

(*b*) What do you understand by anomalous expansion of water ? Explain with the help of a diagram the variation of volume of a certain amount of water, if temperature changes from 0°C to 10°C.

(*c*) Give three practical examples where expansion of solids has been made use of.

9. (*a*) Define latent heat of ice, dew point and relative humidity.

(*b*) What is the difference between evaporation and boiling ?

(*c*) When two pieces of ice are pressed together, they form a single piece. Explain why.

(*d*) If 64,800 calorie of heat is extracted from 100 gm of steam at 100°C, what will be the result ? Latent heat of steam=540 cal/gm ; latent heat of ice=80 cal/gm [Ans. 10 gm ice and 90 gm water.]

10. (*a*) Write down the postulates of the kinetic theory.

(*b*) What is mechanical equivalent of heat ?

(*c*) Explain isothermal and adiabatic processes.

(*d*) A meteorite of mass 42 kg has its velocity reduced from 15 km/sec to 5 km/sec while crossing the atmosphere of the earth. What will be the amount of heat produced for this change of velocity ? $J=4.2\times10^7$ ergs/cal. [Ans. 10^9 Cal.]

11. (*a*) Define thermal conductivity.

(*b*) Write the differences between conduction and convection of heat.

(*c*) Wind blows from sea to land during daytime but from land to sea at night.—Explain.

(*d*) A metallic vessel having surface area of 1 sq. metre and thickness 0.5 cm is filled with ice and then placed inside water at 100°C. How much ice will melt in 36 seconds ? Thermal conductivity of metal=0.02 c.g.s. units, latent heat of ice=80 cal/gm. [Ans. 18×10^5 gm]

Group D

12. (*a*) What do you mean by time period ?

(*b*) If the angular amplitude and the length of a simple pendulum are respectively 0 and l, find the velocity of the bob at the lowest position.

(*c*) Show graphically the resultant motion of a particle on which two simple harmonic motions of the same frequency are superposed (i) in phase and (ii) out of phase.

13. (*a*) Explain the difference between progressive and stationary waves.

(*b*) An aeroplane is moving parallel to the ground with a velocity v. A sound from the plane reaches it again by reflection from the ground after t sec. Find the height of the plane, if the velocity of sound be V. [Ans. $\frac{1}{2}\,t\sqrt{V^2-v^2}$]

(*c*) What are beats ?

14. (*a*) State the laws of transverse vibration of a string.

(*b*) Determine the relation between the length of a closed pipe and the wavelength of the vibrating air column inside the pipe.

(*c*) What are the characteristics of musical sound ?

WEST BENGAL HIGHER SECONDARY EXAMINATION, 1988

PHYSICS—First Paper

Group—A

1. (*a*) Show the subtraction of two vectors by a figure. What do you mean by the resolution of a vector ? How do you find out the rectangular components of a vector ?

(b) A man can reach just the opposite point of the bank by swimming in time t_1 and can swim the same distance down the current in time t_2. If the speed of the man in still water be u and that of current be v, then find the ratio of t_1 and t_2. [Ans. $\sqrt{\{(u+v)/(u-v)\}}$]

(*c*) A man is walking at the rate of 2 km per hour. The rain appears to him to fall vertically with a speed of 2 km/hour. Find out the real value and direction of velocity of rainfall. [Ans. $2\sqrt{2}$ km/h ; 45°]

2. (*a*) State Newton's second law of motion. How can you establish the first law from his second law ?

(*b*) Show that if a weight is hung from the mid-point of a wire, it cannot be kept horizontal.

(*c*) Action and reaction are equal and opposite—describe an easy experiment to show this.

(*d*) A mass of 2.0 kg is hanging from the hook of a spring balance fixed to the roof of a lift. What will be the reading of spring balance when the lift is (i) ascending with an acceleration of 0.2 m/s^2; (ii) descending with an acceleration of 0.1 m/s^2, (iii) moving up uniformly with a velocity of 0.15 m/s ?
[Ans. (i) 2.04 kg (ii) 1.98 kg (iii) 2 kg]

3. (*a*) Find the relation between angular velocity and linear velocity. Obtain the expression for centripetal accleration.

(b) The weight of a body on the surface is less due to diurnal motion of the earth—explain.

(*c*) Define Joule and Horse-power.

(*d*) A cycle rider is moving with a speed of 18 km/hour along a curved path of radius 20 meter. Find the angle of his inclination with the vertical. (g=980 cm/sec^2) [Ans. 7°16′]

Group—B

4. (*a*) Find out how acceleration due to gravity at a place varies with altitude.

(*b*) What do you mean by a geo-stationary satellite ?

(*e*) Derive an expression for the velocity of an artificial satellite moving in a circular orbit.

(*d*) A satellite is moving round the earth in a circular path of 300 km from the surface. Find out its velocity. Radius of the earth=6400 km. g=980 cm/sec^2
[Ans. 7.7 km/s (nearly)]

5. (*a*) Define Young's modulus and Poisson's ratio.

(*b*) Find out the work done per unit volume in the case of extension of a metal wire.

(*c*) What is a spring balance ?

(*d*) Two ends of a steel wire of diameter 4 mm are fixed by supports at 300°C. What will be the force exerted on the supports if temperature falls down to 30°C ? Coefficient of linear expansion of steel$=10\times10^{-4}/°C$; Young's modulus of steel$=1.1\times10^{12}$ dynes/cm^2. [Ans. 37.31×10^9 : dynes]

6. (*a*) Explain the principle of action of a siphon mentioning clearly the conditions under which it will be in action.

(*b*) Explain wheather the rate of flow of liquid through a siphon will change if the atmospheric pressure changes.

(*c*) How can you determine the height of a mountain with the help of a barometer, if the average density of air is known ?

(0) Kerosine oil has to be brought over an obstacle by the action of siphon. What should be the maximum height of the obstacle for this purpose ? Sp. gr. of kerosine$=0.8$; atmospheric pressure$=76$ cm of Hg. [Ans. 12.9 metre]

7. (*a*) State and explain Archimedes' principle.

(*b*) What is buoyancy ? Show how and on what factors its value depends.

(*c*) A floating body loses its weight—explain.

(*d*) The weight of a body in air is 0.4 gm. Its weight along with an immerser in water is 3.37 gm. The weight of immerser in air is 4.0 gm. Find the specific gravity of the body. Sp. gr. of immerser$=8.0$. [Ans. 0.75]

Group—C

8. (*a*) Derive the relation between the coefficient of cubical expansion and coefficient of linear expansion of a solid.

(*b*) A metal scale does not give correct value at all temperatures—explain.

(*c*) The density of glass at 10°C and 60°C is respectively 2.6 gm/cc and 2.596 gm/cc. Find out the average value of the coefficient of linear expansion of glass in this temperature range. [Ans. 1.027×10^{-5}]

(*d*) Explain wheather the base angle of an isoscales triangle made by copper rods will change by heating.

9. (*a*) State Boyle's law and Charles' law. Derive the ideal gas equation from the same. What is the universal gas constant ?

(*b*) How can you get Boyle's law and Avogadro's law from the kinetic theory of gases ?

(*c*) How do you get the idea of the absolute zero of temperature from Charles' law ?

10. (*a*) What are the factors on which the rate of evaporation depends ?

(*b*) State and explain the fundamental principle of calorimetry ; mention the conditions under which it is applicable.

(*c*) State the influence of pressure on the melting point of a substance.

(*d*) A piece of metal of mass 10 gm was heated to 100°C and then dropped in a mixture of water and ice. The volume of mixture decreased by 0.1 cc though the temperature remained the same. Find the specific heat of the metal. The latent heat of ice $=80$ cal/gm. Densities of ice and water at 0°C are 0.916 and 1.0 gm/cc respectively. [Ans. 0.087]

11. (*a*) State and explain the first law of thermodynamics.

(*b*) Describe a method for determining the mechanical equivalent of heat.

(*c*) At which temperature a piece of wood and a piece of metal will appear equally hot on touching and why ?

(*d*) The velocity of a meteorite of mass 42 kg changed from 15 km/sec to 5 km/sec while passing through the atmosphere. Calculate the amount of heat produced during the process. $J=4.2\times10^7$ ergs/cal. [Ans. 10^9 Cal]

Group—D

12. (*a*) What is resonance ? Explain with an example.

(*b*) Show that the equation $x=a \sin wt$ represents a simple harmonic motion.

(*c*) Explain the terms amplitude, phase and time period of a simple harmonic motion.

13. (*a*) Discuss the correction made by Laplace to Newton's formula for the velocity of sound in gas.

(*b*) Explain what information is obtained about its nature from polarisation of light. Why there is no polarisation of sound waves ?

(*c*) Find out the velocity of sound in hydrogen gas at N.T.P. The density of hydrogen at N.T.P. $=0.090$ gm/litre ; $\gamma=1.4$. [Ans. 1255 m/s (nearly)]

14. (*a*) What is interference of waves ? Describe an experiment to show the interference of sound wave.

(*b*) Derive the relation between the fundamentals produced in two tubes of same length—one open, the other is closed.

(*c*) 5 beats are produced when two tuning forks *A* and *B* are sounded together. Find out the frequencies of the forks, if they produce resonance with 36 cm and 37 cm respectively of air column of a tube closed at one end. [185, 180]

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